Competition Within a Cartel: Correction

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Abstract

The model of league conduct formulated by Ferguson, Jones, and Stewart (2000) contains an algebraic error. This note provides the relevant correction and shows that the empirical results given in their article are robust to it.

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Ferguson, Jones, and Stewart (2000) formulate and estimate a model of Major League Baseball that distinguishes between the behavior of teams and that of the league. The league is modeled as establishing rules within which teams compete for players. Unfortunately their derivation made a simple algebraic error that has implications—minor, it turns out—for the econometric model. This note corrects the error and shows that their empirical findings are robust to it.

The problem arises in connect with their equation (2), which specifies wins w_t as depending on unobservable team quality Q_t according to

$$w_t = \frac{T}{2} \frac{Q_t}{\sum_{\tau=1}^{T} Q_\tau}.$$

The derivative $\partial w_t / \partial Q_t$ plays a role in the subsequent revenue-maximization problem. The correct derivation of this derivative is

$$\begin{aligned}
\frac{\partial w_t}{\partial Q_t} &= \frac{T}{2}[(\textstyle\sum Q_\tau)^{-1} - Q_t(\textstyle\sum Q_\tau)^{-2}] \\
&= \frac{T}{2} \frac{1}{\sum Q_\tau} \left(1 - \frac{Q_t}{\sum Q_\tau}\right) \\
&= \frac{w_t}{Q_t} \left(1 - \frac{2}{T} w_t\right).
\end{aligned}$$

Unfortunately the factor $2/T$ was omitted and so the subsequent derivations used the incorrect expression $(w_t/Q_t)(1 - w_t)$.

Substituting the correct expression into equation (7) yields

$$(1-\mu)\alpha \frac{\partial R^*(w_t, x_t)}{\partial w_t} \frac{w_t}{Q_t} \left(1 - \frac{2}{T} w_t\right) - \lambda_t = 0,$$

where λ_t is the Lagrange multiplier having the interpretation as marginal cost $a(\rho_t)$. Multiplying through by Q_t and using the cost function $c(\rho_t, Q_t) = a(\rho_t)Q_t$ given in equation (9), equation (10) becomes

$$(1-\mu)\alpha \frac{\partial R^*(w_t, x_t)}{\partial w_t} w_t \left(1 - \frac{2}{T} w_t\right) - c(\rho_t, Q_t) = 0. \tag{1}$$

This differs from the original equation (10) only by the factor $2/T$.

Given the choice of functional forms described in the article, an intermediate result implicit in the derivation (and which is not affected by the correction) is that

$$\frac{\partial R^*(w_t, x_t)}{\partial w_t} = \frac{1}{2\gamma} \left(\theta(x)\phi(w)\right)^2 [\phi_1 w^{-1} - \phi_2 (1-w)^{-1}].$$

Rearranging (1), team costs C_t are given by the cost function

$$\begin{aligned}
C_t = c(\rho_t, Q_t) &= \alpha(1-\mu) \frac{\partial R^*(w_t, x_t)}{\partial w_t} w_t \left(1 - \frac{2}{T} w_t\right) \\
&= \alpha(1-\mu) \frac{1}{2\gamma} \left(\theta(x)\phi(w)\right)^2 [\phi_1 w^{-1} - \phi_2(1-w)^{-1}] w_t (1-w_t) \frac{1-(2w_t/T)}{1-w_t} \\
&= \alpha(1-\mu) \frac{1}{2\gamma} \left(\theta(x)\phi(w)\right)^2 [\phi_1(1-w_t) - \phi_2 w_t] \frac{1-(2w_t/T)}{1-w_t}.
\end{aligned}$$

Comparing with equation (19) of the article, it is evident that the effect of the correction is to introduce into that equation the multiplicative factor

$$\frac{1-(2w_t/T)}{1-w_t}.$$

The logarithm of this expression should therefore appear as an additive term in the third equation of the 3-equation econometric model on p. 428; the first two equations are unaffected. Reestimating the entire model with this modification yields the results of Table 1, which replace those of Table 5 of the article.[1]

Table 1: System Estimation Results

Parameter	Estimate	Asymptotic t-Statistic*
θ^*	3.816	18.70
θ_1	0.281	12.38
θ_2	0.329	5.16
θ_3	0.025	2.02
ϕ_1	1.171	7.46
ϕ_2	0.607	5.96
γ^*	0.919	35.34
β_0	−0.441	−3.73
β_1	0.139	2.27

* Computed using heteroskedasticity-consistent standard errors.

These results are quantitatively very similar—in some cases identical—to those of the article, and their qualitative implications are unchanged. First, the estimate of β_1 continues to be positive and statistically significant, supporting the hypothesis of a structural change in league conduct between 1986–88 and 1989–1991. Second, the interpretation of the parameter estimates in terms of attendance demand is unaffected. As before, previous season's attendance (θ_1), per capita income (θ_2), and the number of other professional sports in the city (θ_3) all affect demand positively. The estimates of γ^* and θ_2 and their t statistics are virtually identical to the originals, and therefore so are the implied price and income demand elasticities of −0.96 and 0.55. Finally, although the new estimates of ϕ_1 and ϕ_2 differ slightly from before, these values still yield a maximum of the $\phi(w_t)$ function at $w_t = \hat{\phi}_1/(\hat{\phi}_1 + \hat{\phi}_2) = 0.66$; the hypothesized "uncertainty of game outcome effect" is therefore manifested as described in the article.

References

Ferguson, Donald G., Jones, J.C.H., and Kenneth G. Stewart, "Competition Within a Cartel: League Conduct and Team Conduct in the Market for Baseball Player Services," *Review of Economics and Statistics* 82 (August 2000), 422–430.

[1] Replication files are available at `http://web.uvic.ca/~kstewart/baseball.html` .