Hide and Seek Search:
Why Angels Hide and Entrepreneurs Seek

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ABSTRACT

Angel capital investment poses a puzzle for search theory. In the investment literature, angel investors are often described as hiding from entrepreneurs seeking angel capital investment. Hiding behaviour by "angels" forces entrepreneurs to engage in costly search for angels. In our model, high productivity entrepreneurs find it profitable to seek out hiding angels, whereas low productivity entrepreneurs don't search. The hiding strategy by angels screens out low productivity entrepreneurs who would otherwise inundate angels. This "hide and seek search" increases aggregate surplus relative to the situation where angels cannot hide. However, it is not always social surplus maximizing because angels tend to hide too often. Hide and seek search stands in contrast to the traditional search theory, where the search friction represents inherent physical and informational impediments to trade, as well as directed search, where inherent coordination problems generate impediments to matches.

Keywords: *Angel Capital Market, Angel Investors, Entrepreneurs, Search Friction.*

1. INTRODUCTION

> "...the search is also extremely inconvenient for the seller, entrepreneur, because angel investors prize their privacy. For good reason, they make themselves extremely difficult to find. The entrepreneur has a difficult time indeed locating investors with discretionary net worth, the inclination to subject themselves to the high levels of risk associated with this type of investment and the skills necessary to evaluate and add value to these ventures."
>
> Benjamin and Margulis (2001 p. 15)

The angel capital market is usually described as highly inefficient because entrepreneurs arduously search for angel investors. The difficulty for entrepreneurs in finding angels is usually attributed to a lack of information about angels and their preference for anonymity.[1] Benjamin and Margulis (2001) reinforce this view by claiming that information about angels is simply not readily available. Furthermore, they state (in the opening quote): "For good reason they (angels) make themselves extremely difficult to find". According to Van Osnabrugge and Robinson (2000), angels would be swamped with hundreds of project proposals were information about them widely available. The implication we explore in this paper is that angels deliberately hide to avoid being swamped by entrepreneurs with inferior project proposals. When angels hide, only entrepreneurs with superior proposals find it profitable to seek them out.

Entrepreneurs seeking angels in the venture capital market appears to be a classic search story. There are scattered agents on both sides of the market who do not know much about each other. However, as the literature indicates, the search problem in the angel capital market does not spring from the standard spatial, information and coordination impediments. Rather angels face a market that is too "thick" and would rather face fewer and higher productivity entrepreneurs. Angels erect additional barriers to matching giving the appearance of a classic search environment. The additional barriers effectively induce greater search by entrepreneurs.

[1] See, for example, Wetzel (1987), Harrison and Mason (1992), and Freear, Sohl and Wetzel (1994).

Is forcing greater search by entrepreneurs the best screening mechanism? In contrast to angel investors, venture capitalists are typically characterized as easy to find. Venture capitalists are usually distinguished from angels as having greater funds as well as a greater flexibility to take on and support different types of projects. They usually take on several projects whereas angels usually only take on one project at a time. Sometimes an entrepreneur first secures an angel investor and then, if the project is successful, subsequently both the angel and entrepreneur approach a venture capitalist to grow the business. There have been efforts to form angel consortiums and networks to screen and direct entrepreneurs. In fact a formal group of angels sometimes form a venture capital firm. However, these alternative structures are not the norm, even though they are often supported by governments. The fact that the angel capital market is as large as the venture capital market and contains mostly lone angels suggests that this structure fills an important role.[2] Also, the fact that these lone angels choose to hide and force greater search on entrepreneurs suggest that hiding is an optimal strategy.

In this paper, we develop a theoretical model of hide and seek search. Unlike the traditional search models, there are no natural impediments to angels seeking an entrepreneur. If angels do not hide, they can encounter entrepreneurs with certainty. The problem is that there are low and high productivity entrepreneurs and angels prefer to match with high productivity entrepreneurs but can't identify them prior to matching and forming a firm. Hiding provides a way to screen entrepreneurs. By hiding sufficiently hard (being appropriately elusive) angels can discourage low productivity entrepreneurs from searching while not discouraging high productivity entrepreneurs. Only high productivity entrepreneurs find it worthwhile to incur the costs of search.

In our model, the hiding strategy is profit maximizing for angels only when they suffer a loss from financing a low productivity entrepreneur. Hiding may generate more social surplus than

[2] Angel investing is an important source of funds for entrepreneurs. Riding (2008) finds that there are about 15,800 angels investing at least $1.9 billion annually in the entrepreneurial firms in Canada. In contrast, venture capital firms invest less than half that amount, at about $870 million annually. Madill, Haines, and Riding (2005) note that business angel investors not only constitute an important source of financing, they also provide significant non-financial inputs to the growth and viability of the firms through, among other things, mentoring their industry experience and contacts. In this paper, we abstract from the other roles angel investors might play in startup firms.

not hiding. However, sometimes angels hide when it is socially optimal for them not to hide. The hide and seek strategy used by angels may or may not coincide with maximizing social surplus.

The paper proceeds as follows. The next section describes the angel capital market and provides details on entrepreneurs and the angels. We then solve the angel's optimization problem. Finally, the efficiency of the angel capital market is examined. The conclusion summarizes the model and discusses direction for future research.

2. THE FRAMEWORK

2.1 OVERVIEW

There are two groups of risk neutral agents in the angel capital market: angels and entrepreneurs. Angels have capital but no projects and the entrepreneurs have projects but no capital. Angels and the entrepreneurs engage in a matching process. Once matched, the angel and the entrepreneur found a firm and the angel finances the entrepreneur's project. For simplicity, we assume that angels can finance only one project and each entrepreneur has only one project. Thus the number of firms is the same as the number of successful matches.

A project yields a return that depends on the productivity level of the entrepreneur. We assume the productivity level of each entrepreneur is private information and is only revealed after angel investment takes place. The angel and the entrepreneur share the return of the project according to an exogenous sharing rule determined at the time of a match.

A Pissarides-like function matches searching angels with searching entrepreneurs. The key feature in our model is that we allow the angels to choose how hard to hide (degree of elusiveness) which affects the rate of matching. No hiding results in complete matching for angels. The probability of matching decreases with the intensity of hiding. We assume all angels hide equally hard, i.e. the chosen level is the "industry standard" in the angel capital market. Consequently, the number of matches depends on three components: the mass of searching entrepreneurs, the mass of searching angels, and the level of elusiveness chosen by the angels.

The timing of actions and events is as follows:

- Stage 1: Angels choose a how hard to hide.
- Stage 2: Entrepreneurs decide whether to search.
- Stage 3: The matching technology matches searching angels with searching entrepreneurs.
- Stage 4: Once matched, each angel-entrepreneur pair decides whether to form a firm.
- Stage 5: The return is realized and shared between the parties according to an exogenous sharing rule known to both parties.

2.2 ENTREPRENEURS

The number of entrepreneurs is denoted by E. Entrepreneurs are identical except for their productivity level. The number of high productivity entrepreneurs is denoted by E_H, where $E_H = \lambda E$ and $0 < \lambda < 1$ is the fraction of entrepreneurs that are high productivity. The remaining entrepreneurs, $E_L = (1-\lambda)E$, are low productivity entrepreneurs. We assume the mass of entrepreneurs is $E \in \left[1, \overline{E}\right]$ where $\overline{E} \geq 1$ is specified at the end of this subsection.

Each entrepreneur is endowed with a project but no capital. An entrepreneur who is searching for an angel incurs a search cost $\eta > 0$ in the process. This search cost includes shoe leather cost, costs of composing a business plan, opportunity cost, etc. Entrepreneurs either choose to search or not to search (in which case they effectively leave the angel capital market). The number of searching high productivity entrepreneurs is denoted $e_H \leq E_H$, and the number of searching low productivity entrepreneurs is denoted $e_L \leq E_L$. For example, if $e_L < E_L$, then $E_L - e_L > 0$ low productivity entrepreneurs are not searching.

A firm is the outcome of an agreement to match between an entrepreneur and an angel. The firm produces a return that depends on the productivity level of the entrepreneur. In particular, a high productivity entrepreneur's project yields a return of R_H and a low productivity entrepreneur's project yields a return of R_L, where $R_L < R_H$. We assume the productivity level of each entrepreneur is private information and is only revealed after angel investment takes place. Also, we assume that in a match entrepreneurs receive an exogenously determined fixed proportion $1-\sigma$ of the project return, where $0 < \sigma < 1$.

As entrepreneurs are risk neutral their payoff is determined by their expected profits. In a match a low productivity entrepreneur's profit after search cost is $(1-\sigma)R_L - \eta$. Low productivity entrepreneurs will only be active when this profit is non-negative $(1-\sigma)R_L - \eta \geq 0$

which implies $R_L \geq \frac{\eta}{(1-\sigma)}$ or $\overline{E} \equiv \frac{(1-\sigma)R_L}{\eta} \geq 1$. This is the same condition for the upper bound on *E* which we derive later.

2.3 ANGELS

The number of angel investors is denoted A. For simplicity, we normalize this mass to $A = 1$. There is a relative scarcity of angels to entrepreneurs in the sense that $A \leq E$. Each angel has capital *K* but no project. An angel can retain his capital in which case they earn a zero net return. Alternatively, an angel may be able to enter a match and finance an entrepreneur's project; in which case, the angel receives a share σ of the project's return. We denote the measure of active angels in the market by $a \leq A = 1$. The number of non-active angels (those that choose to leave the market) is $1-a$. Later we concentrate the analysis by making assumptions where it is always profitable for an angel to be active so that $a = 1$ in equilibrium.

A key feature in our model is that angels can choose how hard to hide, denoted by $h \geq 0$. A higher h corresponds to hiding harder (i.e. being more elusive). Such behavior by all angels would result in a smaller number of matches. To simplify the analysis, we allow angels to choose the level of h at no cost. In general it might be more (or less) costly to be more elusive. Our results generalize to costly evasion under circumstances we discuss later.

2.4 MATCHING TECHNOLOGY AND PROBABILITY OF A MATCH

The matching technology has been studied at length in the labour literature. The angel capital market closely resembles the labour market in at least two ways. First, angels and entrepreneurs appear to engage in a search process in the same way as employers and unemployed workers in the labour market. Second, there may be traditional search frictions that characterize the angel capital market much like in the labour market. Consequently, we employ the Pissarides' matching technology used in the labour market as our starting point.

Equation 1 shows the matching technology in the angel capital market.

$$m(a,e,h) = \left(\frac{1}{h}a\right)^{\alpha} e^{\beta} \tag{1}$$

where α and β are parameters bounded between $(0,1)$, and $m \leq \min[a,\ e]$ is the number of resulting matches between active angels and searching entrepreneurs. Apart from the parameter h which captures intensity of hiding by angels, the matching function is completely analogous to the standard matching technology in the labour literature. All else constant, an increase in e or a increases the number of matches at a diminishing rate (assuming $e > 0$ and $a > 0$). The probability of a match for an active angel and for a searching entrepreneur denoted by p_a and p_e respectively are

$$p_a(a,e,h) = \frac{\left(\frac{1}{h}a\right)^{\alpha} e^{\beta}}{a} = \left(\frac{1}{h}\right)^{\alpha} a^{\alpha-1} e^{\beta} \tag{2}$$

$$p_e(a,e,h) = \frac{\left(\frac{1}{h}a\right)^{\alpha} e^{\beta}}{e} = \left(\frac{1}{h}\right)^{\alpha} a^{\alpha} e^{\beta-1} \tag{3}$$

The key feature in our analysis is the inclusion of the hiding parameter *h*. Observe that both probabilities (2) and (3) are decreasing in h. The harder angels hide the the smaller is the probability that both angels and entrepreneurs will find a match. Critically, we allow angels not to hide. Not hiding in some sense corresponds to Van Osnabrugge and Robinson (2000) claim that an angel would be swamped with hundreds of project proposals if their information become widely known. Not hiding implies complete matching, $m = \min\{a,\ e\}$. Assuming $a < e$, we have $m = a$ and $p_a(a,e,h) = 1$, where $h = \left(a^{\alpha-1}e^{\beta}\right)^{1/\alpha}$. Without loss of generality we restrict the analysis to consider the range $h \geq h_{\min} \equiv E^{\beta/\alpha}$; this corresponds to $p_a(1,E,h_{\min}) = 1$ in our model.[3]

[3] More generally we could have formulated the matching function $m(a,e,h) = (f(h)a)^{\alpha} e^{\beta}$ where *f*(*h*) is an inverse convex function $f'(h) < 0$ and $f''(h) > 0$. Our explicit form *f*(*h*) = 1/*h* satisfies this requirement. Observe that the

Conversely, for a sufficiently high value of h, active angels can completely avoid entrepreneurs and the probability of an entrepreneur being matched goes to zero; i.e, $\lim_{h\to\infty} p_a(a,e,h) \to 0$ and $\lim_{h\to\infty} p_e(a,e,h) \to 0$.

3. SOLVING THE MODEL

Below we examine the agents' problems. Entrepreneurs choose whether to search or not to search. Angels choose a degree of elusiveness to maximize their expected return from search.

3.1 ENTREPRENEURS' PROBLEMS

The expected return from search for a high productivity entrepreneur is given by:

$$\pi_{e_H}(a,e,h) = p_e(a,e,h)(1-\sigma)R_H - \eta \tag{4}$$

It is the difference between the expected return of the project and the search cost η. A high productivity entrepreneur chooses to search if and only if $\pi_{e_H}(a,e,h) \geq 0$. Similarly, the expected return from search for a low productivity entrepreneur is

$$\pi_{e_L}(a,e,h) = p_e(a,e,h)(1-\sigma)R_L - \eta \tag{5}$$

and a low productivity entrepreneur chooses to search if and only if $\pi_{e_L}(a,e,h) \geq 0$.

Observe that the expected returns between the types differ only because of the different project returns. As $R_H > R_L$, it follows that $\pi_{e_H} > \pi_{e_L}$. Thus, if the low productivity entrepreneurs choose to search, $\pi_{e_L} \geq 0$, then so do the high productivity entrepreneurs, $\pi_{e_H} > 0$. The converse isn't necessarily true. That is, if high productivity entrepreneurs choose to search, $\pi_{e_H} \geq 0$, it may be the case that low productivity entrepreneurs choose not to search $\pi_{e_L} < 0$. This later situtation occurs if the probability p_e lies within the following bounds

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lower bound $h = \left(a^{\alpha-1}e^{\beta}\right)^{1/\alpha}$ is decreasing in a so setting a=1 is not restrictive for h_{min}. In our model there exists a critical $h_E \geq h_{min}$ for which $e(h) = E$ for $h \leq h_E$. Finally, we could have used the more awkward specification $f(h)$ = $1/(h + E^{\beta/\alpha})$ in which case h = 0 implies $p_a(1, E, 0) = 1$.

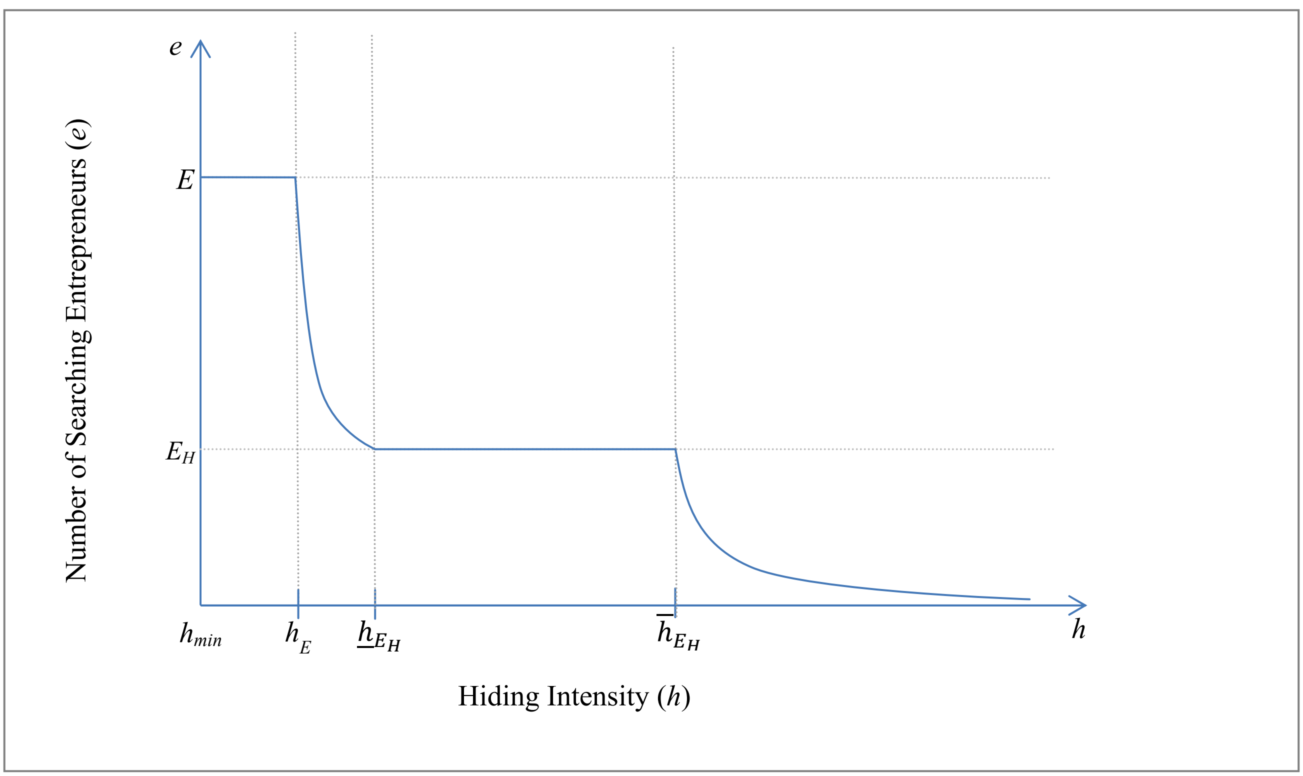
$$\frac{\eta}{(1-\sigma)R_H} \le p_e < \frac{\eta}{(1-\sigma)R_L}$$

or equivalently, $\frac{R_L}{R_H}/\overline{E} \le p_e < 1/\overline{E}$ where $\overline{E} \equiv \frac{(1-\sigma)R_L}{\eta} \ge 1$. If p_e is less than the left hand side bound, then no low productivity entrepreneurs search; if it is greater than the right hand side bound, all entrepreneurs search. As p_e is inversely related to h in equation (3) we can, holding a constant, derive the number of searching entrepreneurs as a function of h.

$$e(h) = \begin{cases} E & \text{for } h_{\min} \le h \le h_E \\ \left[\frac{a}{h}\right]^{\frac{\alpha}{1-\beta}} \overline{E}^{\frac{1}{1-\beta}} & \text{for } h_E < h < \underline{h}_{E_H} \\ E_H & \text{for } \underline{h}_{E_H} \le h \le \overline{h}_{E_H} \\ \left[\frac{a}{h}\right]^{\frac{\alpha}{1-\beta}} \left[\overline{E}\frac{R_H}{R_L}\right]^{\frac{1}{1-\beta}} & \text{for } h > \overline{h}_{E_H} \end{cases} \qquad (6)$$

Figure 1 illustrates this relationship for a = 1.

Figure 1: Number of Searching Entrepreneurs (*e*) as a function of Hiding Intensity (*h*)



Recall that $h_{\min}$ is where there is no hiding in the sense that an angel matches with certainty, $p_a\left(1,E,h_{min}\right) = 1$, and h_E is maximum level of hiding at which all entrepreneurs search, $e = E$. At h_E an angel's matching probability is $p_a\left(a,E,h_E\right) = \frac{\eta}{(1-\sigma)R_L}\frac{E}{a}$. We assume throughout that $h_{\min} \le h_E$. This implies $p_a\left(a,E,h_E\right) = \frac{\eta}{(1-\sigma)R_L}\frac{E}{a} \le 1$. Setting a to its maximum, a=1, yields our previously stated upper bound $E \le \overline{E} \equiv \frac{\left(1-\sigma\right)R_L}{\eta} \ge 1$.

As we increase h past h_E some low productivity entrepreneurs choose to be inactive, $e_L < E_L$, and for $h \ge \underline{h}_{E_H}$ all low productivity entrepreneurs are inactive $e_L = 0$. In the interval $[\underline{h}_{E_H}, \overline{h}_{E_H}]$ all high productivity entrepreneurs are active, $e = e_H = E_H$. For h greater than $\overline{h}_{E_H}$ only some high productivity entrepreneurs are active, $e = e_H < E_H$, and in the limit as $h \to \infty$, then $e = e_H \to 0$.

Choosing h in the interval $[\underline{h}_{E_H}, \overline{h}_{E_H}]$ loses low productivity entrepreneurs while retaining high productivity entrepreneurs. Observe that $h = \underline{h}_{E_H}$ is the lowest value of h consistent with losing low productivity entrepreneurs. It corresponds to the highest matching probability in the interval. In the next subsection, we show that $h > \underline{h}_{E_H}$ gives less profits for angels than $h = \underline{h}_{E_H}$.

3.2 THE REPRESENTATIVE ANGEL'S PROBLEM

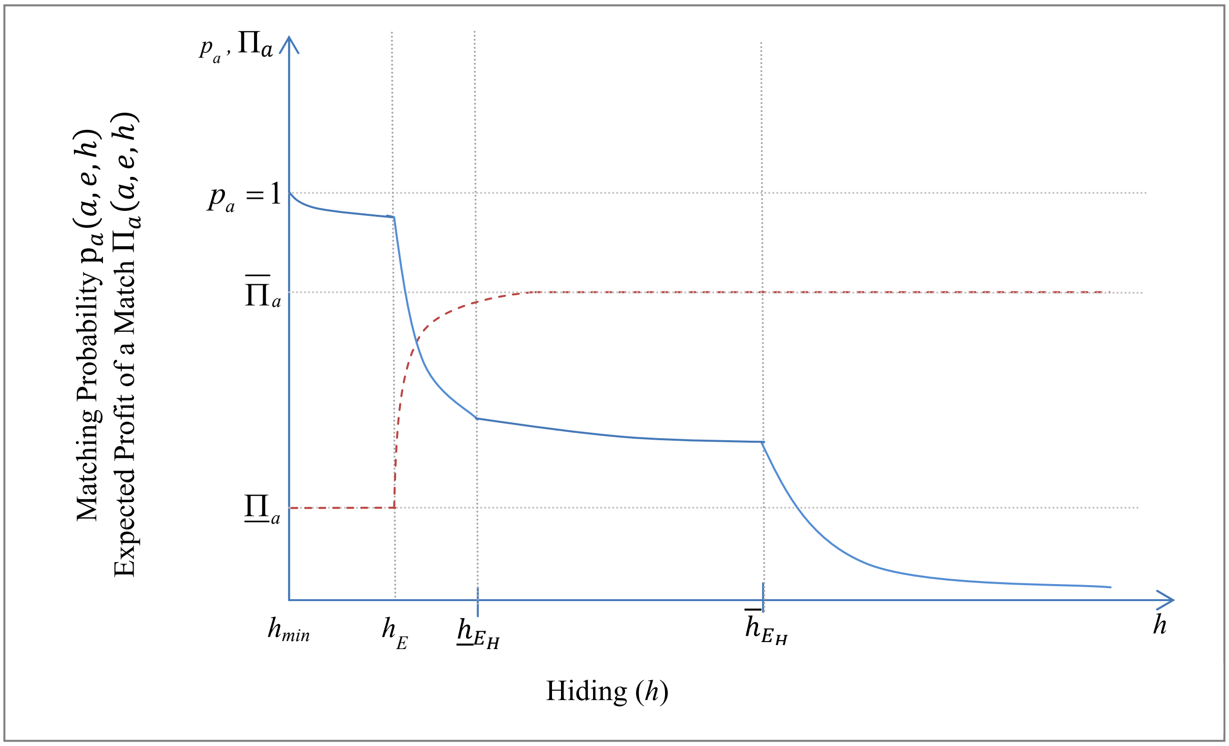
The representative angel's expected profit function is

$$\pi_a(a,e,h) = p_a(a,e,h)\Pi_a(a,e,h) \tag{7}$$

where $\Pi_a\left(a,e,h\right) = \frac{e_H(a,e,h)}{e(a,e,h)}\sigma R_H + \frac{e_L(a,e,h)}{e(a,e,h)}\sigma R_L - K$ is the expected profit of a match for an angel. It is bounded: $\Pi_a\left(a,e,h\right) \in \left[\underline{\Pi}_a, \overline{\Pi}_a\right]$, where $\underline{\Pi}_a \equiv \lambda\sigma R_H + (1-\lambda)\sigma R_L - K$ and $\overline{\Pi}_a = \sigma R_H - K$. At the lower bound all entrepreneurs search, $e = E$, and at the upper bound only high productivity entrepreneurs search, $e = e_H$. In Figure 2, the expected profit of a match

$\Pi_a(a,e,h)$ is drawn versus h, holding $a=1$ but with $e(h)$ endogenous as describe in (6). The lower bound $\underline{\Pi}_a$ obtains for $h_{\min} \le h \le h_E$, and the upper bound $\overline{\Pi}_a$ obtains for $h \ge \underline{h}_{E_H}$.

Figure 2: Angel's Matching Probability (p_a) and Expected Profit of a Match (Π_a) as a function of Hiding (h)



Though unnecessary to our results, it is convenient to assume that the lower bound on profits is non-negative, $\underline{\Pi}_a \ge 0$ (or equivalently $\frac{(1-\lambda)(\sigma R_L - K)}{\lambda(\sigma R_H - K)} \ge -1$, an expression we use later). Then $\Pi_a(a,e,h) \ge 0$ and $\pi_a(a,e,h) \ge 0$ as $p_a(a,e,h) > 0$. Since profits are always non-negative for angels, all angels will be active so that $a = A = 1$. This simplifies our analysis and also shows that our results go through even when the presence of low productivity entrepreneurs is profitable.

The other component of expected profits is the matching function $p_a(a,e,h)$. Figure 2 illustrates that $p_a(1,e,h)$ is decreasing in h. Taking the derivative of (2) with respect to h subject to (6) yields:

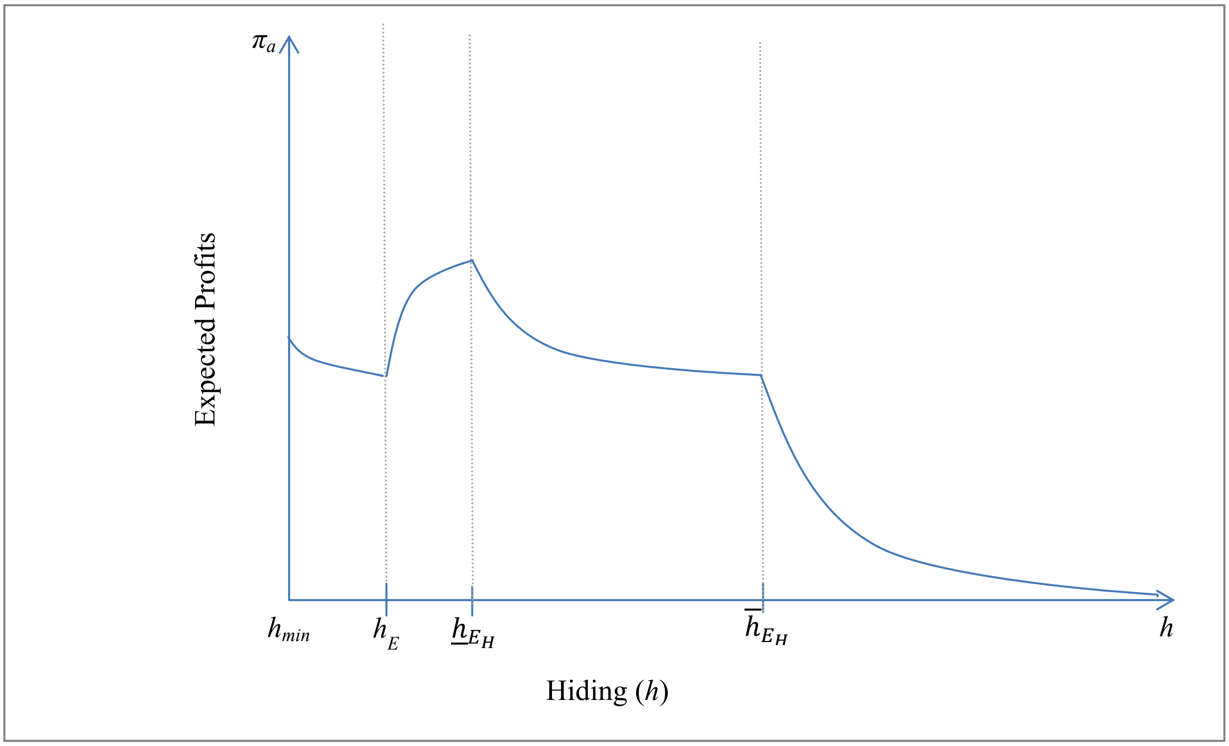
$$\frac{\partial p_a(1,e(h),h)}{\partial h} = -\alpha\left[\frac{1}{h}\right]^{\alpha+1} e(h)^{\beta} + \beta\left[\frac{1}{h}\right]^{\alpha} e'(h)^{\beta-1} < 0 \qquad (8)$$

The first term on the right hand side describes the direct effect of an increase in h. An increase in h makes it more difficult for existing entrepreneurs to find angels and hence reduces the probability of an angel meeting an existing entrepreneur. This effect is negative throughout the range of h. The second term describes the indirect effect through $e(h)$ as described by (6). It is non-negative as $e'(h) \le 0$ is shown in Figure 1. An increase in h makes it more costly for the entrepreneurs to search. Over parts of the range, $h_E < h < \underline{h}_{E_H}$ and $h > \overline{h}_{E_H}$, some existing entrepreneurs will cease searching. The kinks in Figure 2 correspond to this indirect effect turning on and off. In particular, the indirect effect turns off at $h = \underline{h}_{E_H}$, thus the slope of p_a is less steep at this point.

3.3. WHEN HIDING MAXIMIZES ANGEL PROFITS

Comparing the two schedules in Figure 2 reveals three logical candidates for the profit maximizing h. First $h = \underline{h}_{E_H}$ generates greater profits than any $h > \underline{h}_{E_H}$. Second, there is the possibility of a profit maximizing h internal to the range $h_E \le h \le \underline{h}_{E_H}$ because payoff Π_a is increasing in h but p_a is decreasing in h. Third, $h = h_{\min}$ generates greater profits than $h_{\min} < h \le h_E$. The first two possibilities involve angels hiding sufficiently to discourage some entrepreneurs from searching. The third possibility is when angels do not hide. Figure 3 draws the profit function for the case where $\sigma R_L < K$ and angels are best off hiding at $\underline{h}_{E_H}$.

Figure 3: When Hiding Maximizes Angels' Expected Profits



In the Appendix we show that over the intermediate interval $h_E \leq h \leq \underline{h}_{E_H}$ the maximum is h_E when financing the low productivity project is profitable, $\sigma R_L - K > 0$, and the maximum is $\underline{h}_{E_H}$ when it is unprofitable, $\sigma R_L - K \leq 0$. As $h_{\min} \leq h_E$, we can identify the maximum by comparing the profits at $h_{\min} = E^{\beta/\alpha}$ to $\underline{h}_{E_H} = \left[\frac{\overline{E}}{(\lambda E)^{1-\beta}}\right]^{\frac{1}{\alpha}}$, where $E \leq \overline{E} \equiv \frac{(1-\sigma)R_L}{\eta} \geq 1$. The specific condition for whether angels are best off hiding at $\underline{h}_{E_H}$ or not hiding at h_{min} depends on whether E lies between the upper bound and the following term:

$$\underline{E} \equiv \max\left\{\overline{E} + \left[\frac{(1-\lambda)(\sigma R_L - K)}{\lambda(\sigma R_H - K)}\right]\overline{E}, 1\right\}$$

Hiding occurs over the whole range when $\underline{E}=1$. It can be shown that $\underline{E}=1$ if and only if $\underline{\Pi}_a \leq \frac{\lambda\eta}{(1-\sigma)R_L}\overline{\Pi}_a$. There always exists combinations of parameters for which this condition is satisfied as we have assumed $0 \leq \underline{\Pi}_a < \overline{\Pi}_a$, $R_L > 0$, $0 < \lambda < 1$, $0 < \sigma < 1$ and $\eta > 0$.

Proposition 1. Angels are best off by either hiding at $h = \underline{h}_{E_H}$, so as to completely discourage low productivity entrepreneurs from searching but not discourage high productivity entrepreneurs from searching, or not hiding at $h = h_{\min}$ so that all entrepreneurs search.

(i) Low productivity project is unprofitable for angels, $\sigma R_L - K \leq 0$. There exists an interval $E \in \left[\underline{E}, \overline{E}\right]$ over which angels are best off hiding at $h = \underline{h}_{E_H}$, and when the lower bound profits are sufficiently small, $\underline{\Pi}_a \leq \frac{\lambda\eta}{(1-\sigma)R_L}\overline{\Pi}_a$, angels are best off hiding over the entire range $E \in \left[1, \overline{E}\right]$. Otherwise, $E \in \left[1, \underline{E}\right)$ angels are best off not hiding, $h = h_{\min}$.

(ii) Low productivity project is profitable for angels, $\sigma R_L - K > 0$. Angels are always best off not hiding, $h = h_{\min}$.

The proof to all the propositions are found in the Appendix. First observe that if financing a low productivity project yields a profit, $\sigma R_L - K > 0$, then $\underline{E} > \overline{E}$; the condition in the proposition is never satisfied and angels are best off not hiding. Angels only hide when financing a low productivity project does yield a profit, $\sigma R_L - K \leq 0$. Only then is $E \in \left[\underline{E}, \overline{E}\right]$ non-empty. When angels suffer a loss, $\sigma R_L - K < 0$, then $\underline{E} < \overline{E}$ and $\underline{E}$ is increasing in R_L and decreasing in R_H , K, σ, λ and η. The lower bound $\underline{E}=1$ occurs when $\underline{\Pi}_a \le \dfrac{\lambda\eta}{(1-\sigma)R_L}\overline{\Pi}_a$; this condition is satisfied for sufficiently small values of R_L or large values of R_H , K, σ, λ and η.[4]

An alternative way to understand when hiding is preferable is revealed by rewriting the lower bound condition

$$\underline{E} \equiv \max\left\{\frac{1}{\lambda}\left(\underline{\Pi}_a / \overline{\Pi}_a\right)\overline{E},1\right\}$$

where $\overline{\Pi}_a \equiv \sigma R_H - K$. Then, $\lambda \ge \underline{\Pi}_a / \overline{\Pi}_a$ is required for $\underline{E} \le \overline{E}$. Hiding is only desirable when the proportion of high productivity entrepreneurs is sufficiently great that it is worthwhile increasing h which has the negative consequence of reducing the chance of meeting a high quality entrepreneur.

4. SOCIAL WELFARE

When does the representative angel's hiding choice maximize social surplus? To answer this question, we look at the constrained welfare optimum where the planner is constrained by the profit participation constraints of agents as well as the sharing rule σ. As before all angels receive non-negative profits if active and as before all participate, a =1. The participation decision for entrepreneurs is also the same as before, $e(h)$ as described in (6). Thus, for our comparison, the planner only chooses h.

4.1 MAXIMIZING SOCIAL SURPLUS

[4] Alternatively, $\underline{\Pi}_a \equiv \lambda\sigma R_H + (1-\lambda)\sigma R_L - K \ge 0$ implies $\left[\dfrac{(1-\lambda)(\sigma R_L - K)}{\lambda(\sigma R_H - K)}\right] \ge -1$. If $\underline{\Pi}_a\left(a,e,h\right) \to 0$ then $\left[\dfrac{(1-\lambda)(\sigma R_L - K)}{\lambda(\sigma R_H - K)}\right] \to -1$ and $\underline{E} = 1$.

Expected social surplus is simply the population weighted sum of the expected profits of angels and entrepreneurs. Substituting a = 1 and $e(h)$, the planner's problem is to choose h to maximize the following welfare function:

$$W(1,e(h),h) = m(1,e(h),h)\Pi_W(1,e(h),h) - \eta e(h) \qquad (9)$$

where m is the number of matches given by equation (1) , and the expected social surplus in a match is given by $\Pi_W(1,e(h),h) = \frac{e_H(1,e(h),h)}{e(1,e(h),h)}R_H + \frac{e_L(1,e(h),h)}{e(1,e(h),h)}R_L - K$. $\Pi_W(a,e,h)$ is bounded $\Pi_W\left(a,e,h\right) \in \left[\underline{\Pi}_W, \overline{\Pi}_W\right]$, where $\underline{\Pi}_W \equiv \lambda R_H + (1-\lambda)R_L - K$ and $\overline{\Pi}_W = R_H - K$.

The planner's optimization problem has many of the same features as that of the representative angel's problem. When a =1, then $m(1,e(h),h) = p_a(1,e(h),h)$. Observe that the expected social surplus in a match can be written

$$\Pi_W\left(1,e(h),h\right) = \Pi_a\left(1,e(h),h\right) + (1-\sigma)\left(\frac{e_H\left(1,e(h),h\right)}{e\left(1,e(h),h\right)}R_H + \frac{e_L\left(1,e(h),h\right)}{e\left(1,e(h),h\right)}R_L\right).$$

Thus, the expected social surplus in a match responds in the same way to h as the angel's profit. The product $m\Pi_W(a,e,h)$ has the same extrema and general shape as the representative angel's profit maximizing function $\pi_a(a,e,h) = p_a(a,e,h)\Pi_a(a,e,h)$. Thus, the planner's problem differs from the angel's problem by including the entrepreneurs' costs of participation, $\eta e(h)$. These costs are draw in Figure 4 along with the profiles for number of matches and social surplus.

Figure 4: Number of Matches (*m*), Expected Social Surplus of a Match (Π_W), Total Search Cost ($\eta e(h)$) as a function of Hiding (*h*)

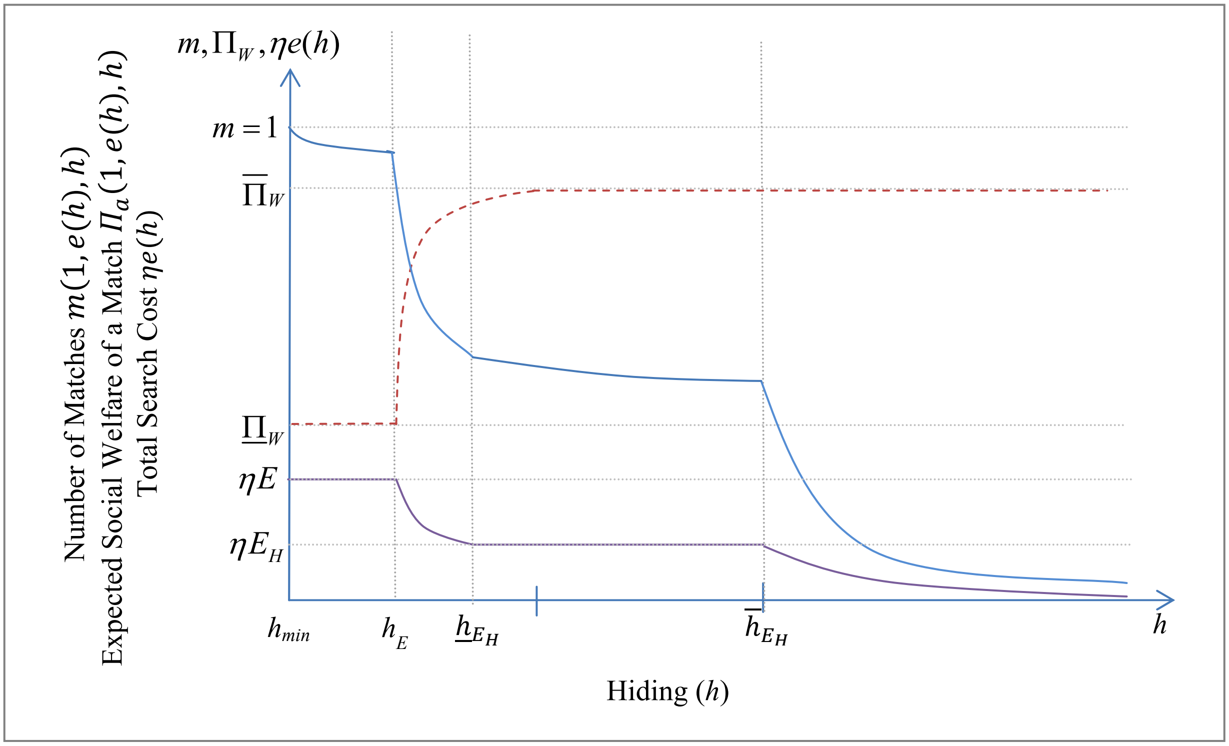


Figure 4 reveals that the social welfare optimization problem is analogous to the representative angel's optimization problem except for the inclusion of the search cost incurred by the searching entrepreneurs. Similar to the angel's problem, there is no internal optimum in the intermediate interval $h_E \leq h \leq \underline{h}_{E_H}$.[5] Two candidates emerge as possible choices for maximizing *h*. First $h = \underline{h}_{E_H}$ generates greater social surplus than any $h > \underline{h}_{E_H}$. Second, $h = h_{\min}$ generates greater social surplus than $h_{\min} < h \leq h_E$. The former implies hiding is socially optimal and the latter implies hiding is not socially optimal.

[5] In the Appendix we show that over the interval $h_E \leq h \leq \underline{h}_{E_H}$ the maximum is h_E when the low productivity project is profitable, $\sigma R_L > K$, and the maximum is $\underline{h}_{E_H}$ when it is not profitable, $\sigma R_L \leq K$.

It is socially optimal to hide when $W(1,e(\underline{h}_{E_H}),\underline{h}_{E_H}) \geq W(1,e(h_{\min}),h_{\min})$. In the Appendix we show this requires that the number of entrepreneurs is sufficiently large:

$$E \geq \underline{\underline{E}} \equiv \max\left\{\left[\frac{\underline{\Pi}_W}{\lambda\Psi\overline{\Pi}_W}\right]\overline{E},1\right\}$$

where $\Psi = 1+\frac{(1-\lambda)(1-\sigma)R_L}{\lambda(R_H-K)} > 1$. It can be shown that $\underline{\underline{E}} = 1$ if and only if the lower bound surplus is sufficiently small, $\underline{\Pi}_W \leq \frac{\lambda\eta}{(1-\sigma)R_L}\Psi\overline{\Pi}_W$. As we have assumed $0 \leq \underline{\Pi}_W < \overline{\Pi}_W$, $R_L > 0$, $0 < \lambda < 1$, $0 < \sigma < 1$ and $\eta > 0$ there always exists combinations of parameters for which $\underline{\Pi}_a \leq \frac{\lambda\eta}{(1-\sigma)R_L}\overline{\Pi}_a$ is satisfied. As before we consider the range of $E \leq \overline{E}$.

Proposition 2. To maximize social surplus angels should act as follows.

(i) Low productivity project is unprofitable for angels, $\sigma R_L - K \leq 0$. There exists an interval $E \in \left[\underline{\underline{E}},\overline{E}\right]$ over which hiding at $h = \underline{h}_{E_H}$ maximizes social surplus, and if $\underline{\Pi}_W \leq \frac{\lambda\eta}{(1-\sigma)R_L}\Psi\overline{\Pi}_W$ it is socially optimal to hide over the entire range $E \in \left[1,\overline{E}\right]$. Otherwise, $E \in \left[1,\underline{\underline{E}}\right)$ and not hiding at $h = h_{\min}$ is socially optimal.

(ii) Low productivity project is profitable for angels, $\sigma R_L - K > 0$. Not hiding at $h = h_{\min}$ maximizes social surplus.

It turns out that to maximize social surplus, angels should only hide when low productivity projects are unprofitable for angels. This is a similar feature to the angel's profit maximizing problem. However, we show in the next section that there is a range of E where angels should not hide according to the surplus maximizing criteria.

4.2 WHEN ANGELS SHOULD AND SHOULD NOT HIDE

Comparing Propositions 1 and 2 reveal that there is only one case in which hiding prescriptions differ: $\sigma R_L - K < 0$ and $\underline{E} \neq \underline{\underline{E}}$. In the Appendix we show that $\underline{E} - \underline{\underline{E}} < 0$ provided that $\underline{\underline{E}} > 1$ or equivalently $\underline{\Pi}_W > \frac{\lambda\eta}{(1-\sigma)R_L}\Psi\overline{\Pi}_W$ i.e., $\underline{\underline{E}}$ is not already at its lower bound, $\underline{\underline{E}} = 1$. We have the following proposition.

Proposition 3. Angels profit maximizing hiding behavior maximizes social surplus in all cases except one: when the low productivity project is unprofitable for angels, $\sigma R_L - K < 0$, and social surplus maximization does not always involve hiding, $\underline{\Pi}_W > \frac{\lambda\eta}{(1-\sigma)R_L}\Psi\overline{\Pi}_W$, there exists an interval $E \in \left[\underline{E}, \underline{\underline{E}}\right)$ where angels hide at $h = \underline{h}_{E_H}$ but social surplus maximization involves not hiding at $h = h_{\min}$.

Angels may hide over a greater range of *E* than is socially desirable. In the Appendix we show that the source of the divergence in outcomes boils down to the following term: $(1-\sigma)\left[\lambda R_H + (1-\lambda)R_L\right] > 0$ which describes the average return across high and low productivity entrepreneurs. The planner hides over a shorter interval because they take into account the this average return from including low quality entrepreneurs. Other factors affect the length of the interval $E \in \left[\underline{E}, \underline{\underline{E}}\right)$. For example, increasing entrepreneur's search cost, *η,* decreases the interval.

5. CONCLUSION

Search in the angel capital market is best described as hide and seek. Angels hide to avoid being inundated by the low productivity entrepreneurs. At the same time angels hope to be sought out and found by the high productivity entrepreneurs. They can do this by choosing to be elusive but not too elusive. In this way, only high productivity entrepreneurs enter the search, as they are the only ones that generate sufficient surplus to compensate for the higher search cost. Thus, in the angel capital market, search is induced and acts like a screening mechanism.

Search in the angel capital market does not fit into either the traditional matching search theory (e.g. Mortensen and Pissardes (1994)) or directed search (e.g. Julien et. al. (2000), Buddett et. al. (2001)). These theories assume inherent frictions related to physical, information or coordination impediments. In the angel capital market, the main impediment is an endogenous search friction created by the angels hiding behaviour.

In this paper, we develop a theoretical model to capture this phenomenon by utilizing the standard matching technology commonly used in the labour literature. We extend the model by allowing the angels to change the matching technology by choosing how hard to hide. Not hiding corresponds to complete matching for angels. Hiding results in incomplete matching. In our model, angels only hide to discourage low quality entrepreneurs when encountering them results in negative profits.

We examine when angels profit-maximizing hiding choice maximizes social surplus. Whenever angels are best off not hiding this also maximize social surplus. When angels are best of not hiding this may or may not maximize social surplus. We show that there is always a range of parameters where hiding maximizes both angels profits as well as social surplus. On the other hand, there may be a case where angels are best off hiding but this lowers social surplus. In this case, there is generally too few trades.

Our model can be interpreted in two ways. For generality, we have developed the model as if there a continuum of angels being active and we normalized this number to a =1. The choice of the hiding intensity then is a collective decision for angels. However, we could as well interpret the model as having only one angel, in which case the choice of hiding intensity is an individual decision. This later interpretation is consistent with the angel capital market being highly heterogeneous and the interaction between angels being minimal.

Our hide and seek model is quite simple. We have not included hiding costs. If hiding costs were positive then the parameter space over which hiding would be optimal for angels and the planner would be smaller. With prohibitive hiding costs, angels would never hide. We have set our model so that no hiding corresponds to complete matching. We did this deliberately to make our point that search maybe simply induced. However, in a more general Pissarides type model, zero hiding might well correspond to incomplete matching and we could model negative hiding similar to advertising. There could be costs to both hiding and advertising. Depending on parameters different angels might take different strategies. Much remains to do to flesh out hide and seek search.

6. APPENDIX

6.1 PROOF OF PROPOSITION 1

The representative entrepreneur's expected profit is given by (7): $\pi_a = p_a(a,e,h)\Pi_a\left(a,e,h\right)$. Recall that $p_a > 0$ and we assumed that $\Pi_a\left(a,e,h\right) > 0$. Thus, $\pi_a > 0$ and all angels choose to be active $a=A = 1$. Substituting for p_a from (2) and e from (6) into the profit function (7) gives the unconstrained representative angel's profit maximization problem over the full range $h \geq h_{min}$.

$$\max_h \pi_a = \begin{cases} \frac{1}{h^\alpha} E^\beta \left[\lambda\sigma R_H + [1-\lambda]\sigma R_L - K\right] & \text{for } h_{\min} \leq h \leq h_E \\ \frac{E_H}{\overline{E}}\sigma\left[R_H - R_L\right] + \left(\frac{1}{h}\right)^{\frac{\alpha}{1-\beta}} \left(\overline{E}\right)^{\frac{\beta}{1-\beta}} \left[\sigma R_L - K\right] & \text{for } h_E < h < \underline{h}_{E_H} \\ \frac{1}{h^\alpha} E_H{}^\beta \left[\sigma R_H - K\right] & \text{for } \underline{h}_{E_H} \leq h \leq \overline{h}_{E_H} \\ \left(\frac{1}{h}\right)^{\frac{\alpha}{1-\beta}} \left[\overline{E}\frac{R_H}{R_L}\right]^{\frac{1}{1-\beta}} \left[\sigma R_H - K\right] & \text{for } h > \overline{h}_{E_H} \end{cases}$$

For $h_{\min} \leq h \leq h_E$ profit is declining in h so that h_{min} gives the greatest profit. As h_{min} corresponds to p_a = 1, we find $h_{\min} \equiv E^{\beta/\alpha}$ and the corresponding profit is given by:

$$\pi_a(1, e(h_{\min}), h_{\min}) = \lambda\sigma R_H + \left(1-\lambda\right)\sigma R_L - K$$

Similarly, for $h \geq \underline{h}_{E_H}$ profit is declining in h so that $\underline{h}_{E_H}$ gives the greatest profit. As $\underline{h}_{E_H}$ corresponds the smallest value of h that gives $e = E_H = \lambda E$, we find $\underline{h}_{E_H} \equiv \left[\frac{\overline{E}}{(\lambda E)^{1-\beta}}\right]^{\frac{1}{\alpha}}$ and

$$\pi_a\left(1, e(\underline{h}_{E_H}), \underline{h}_{E_H}\right) = \frac{\lambda E}{\overline{E}}\left(\sigma R_H - K\right)$$

Over the intermediate interval $h_E \leq h \leq \underline{h}_{E_H}$ the change in expected profits depends on the sign of $\sigma R_L - K$: if $\sigma R_L - K < 0$, then expected profits are increasing and the maximum is at $\underline{h}_{E_H}$; if $\sigma R_L - K > 0$, then expected profits are decreasing and the maximum is at h_E; if $\sigma R_L - K = 0$, the expected profits are constant, $h = h_{min} = \underline{h}_{E_H}$. As profits at h_{min} exceeds profits at h_E finding the maximum h reduces to comparing profits at h_{min} to $\underline{h}_{E_H}$.

Angels are best off hiding if and only if $\pi_a\left(1, e(\underline{h}_{E_H}), \underline{h}_{E_H}\right) > \pi_a\left(1, e(h_{\min}), h_{\min}\right)$. Substituting the appropriate values for h this condition implies

$$\frac{\lambda E}{\overline{E}}\left(\sigma R_H - K\right) \geq \lambda\sigma\left(R_H - R_L\right) + \left(\sigma R_L - K\right)$$

This equation can be rewritten as

$$\lambda\left(\sigma R_H - K\right)\left(\frac{E}{\overline{E}} - 1\right) \geq \left(1-\lambda\right)\left(\sigma R_L - K\right)$$

Rearranging this equation gives the condition used in defining the lower bound $\underline{E}$:

$$E \geq \underline{E} \equiv \max\left\{\overline{E} + \left[\frac{\left(1-\lambda\right)}{\lambda}\frac{\left(\sigma R_L - K\right)}{\left(\sigma R_H - K\right)}\right]\overline{E}, 1\right\}$$

As we have restricted $E \leq \overline{E}$, angels are best off hiding if and only if $E \in \left[\underline{E}, \overline{E}\right]$. As described in the text, this interval is non-empty if and only if $\sigma R_L - K \leq 0$. Otherwise, angels are best off not hiding at h_{min}. This completes the proof. ■

6.2 PROOF OF PROPOSITION 2

The social welfare function is given by (9). The planner's problem is as follows:

$$\max_h W = \begin{cases} \dfrac{1}{h^{\alpha}} E^{\beta}\left[\lambda R_H + \left[1-\lambda\right] R_L - K\right] - \eta E & \text{for } h_{\min} \leq h \leq h_E \\ \dfrac{E_H}{\overline{E}}\left(R_H - R_L\right) + \left(\dfrac{1}{h}\right)^{\frac{\alpha}{1-\beta}}\left(\overline{E}\right)^{\frac{\beta}{1-\beta}}\left(\sigma R_L - K\right) & \text{for } h_E < h < \underline{h}_{E_H} \\ \dfrac{1}{h^{\alpha}} {E_H}^{\beta}\left[R_H - K\right] - \eta E_H & \text{for } \underline{h}_{E_H} \leq h \leq \overline{h}_{E_H} \\ \left[\dfrac{1}{h}\right]^{\frac{\alpha}{1-\beta}}\left[\overline{E}\dfrac{R_H}{R_L}\right]^{\frac{\beta}{1-\beta}}\left\{R_H - K - \eta\left[\overline{E}\dfrac{R_H}{R_L}\right]\right\} & \text{for } h > \overline{h}_{E_H} \end{cases}$$

For $h_{\min} \le h \le h_E$ social welfare is declining in *h* so that $h_{\min} \equiv E^{\beta/\alpha}$ gives the greatest social welfare. The corresponding social welfare is given by:

$$W(1, e(h_{\min}), h_{\min}) = \lambda R_H + (1-\lambda) R_L - K - \eta E$$

Similarly, for $h \ge \underline{h}_{E_H}$ profit is declining in *h* so that $\underline{h}_{E_H} \equiv \left[\frac{\overline{E}}{(\lambda E)^{1-\beta}}\right]^{\frac{1}{\alpha}}$ gives the greatest social surplus. The corresponding social welfare is given by:

$$W\left(1, e(\underline{h}_{E_H}), \underline{h}_{E_H}\right) = \frac{\lambda E}{\overline{E}}\left(R_H - K\right) - \eta \lambda E$$

In the interval $h_E < h < \underline{h}_{E_H}$ is the social surplus is increasing in *h* if $\sigma R_L - K < 0$, constant if $\sigma R_L - K = 0$ and decreasing if $\sigma R_L - K > 0$. Thus parallel to the representative angel's problem, finding social maximization *h* reduces to comparing social welfare at h_{min} to $\underline{h}_{E_H}$.

Hiding is social optimal if and only if $W\left(1, e(\underline{h}_{E_H}), \underline{h}_{E_H}\right) \ge W\left(1, e(h_{\min}), h_{\min}\right)$. Substituting the appropriate values for *h* this condition implies

$$\frac{\lambda E}{\overline{E}}\left(R_H - K\right) - \eta \lambda E \ge \lambda R_H + (1-\lambda) R_L - K - \eta E$$

Rearranging this equation implies the condition for the lower bound $\underline{\underline{E}}$ used in Proposition 2:

$$E \ge \underline{\underline{E}} \equiv \max\left\{\frac{1}{\Psi}\left[\frac{\underline{\underline{\Pi}}_W}{\lambda \overline{\overline{\Pi}}_W}\right]\overline{E}, 1\right\}$$

where $\Psi = 1 + \frac{(1-\lambda)(1-\sigma) R_L}{\lambda(R_H - K)} > 1$. As we restrict $E \le \overline{E}$, social surplus maximization involves angels hiding when $E \in \left[\underline{\underline{E}}, \overline{E}\right]$.

Finally, we show that $\underline{\underline{E}} \le \overline{E}$ if and only if $\sigma R_L - K \le 0$. Substituting yields

$$\underline{\underline{E}} = \overline{E}\frac{\left[\frac{(1-\lambda)\left(R_L - K\right)}{\lambda\left(R_H - K\right)}\right]}{\Psi} + \frac{1}{\Psi}\overline{E} \le \overline{E}$$

The term $\overline{E}$ cancels and the inequality can be written

$$\left[\frac{(1-\lambda)\left(R_L-K\right)}{\lambda\left(R_H-K\right)}\right]\leq\Psi-1$$

Substituting for Ψ gives $\sigma R_L - K \leq 0$. This completes the proof. ■

6.3 PROOF OF PROPOSITION 3

We want to show

$$\underline{E}-\underline{\underline{E}}=\max\left\{\frac{\Pi_a}{\lambda\overline{\overline{\Pi}}_a}\overline{E},1\right\}-\max\left\{\frac{1}{\Psi}\frac{\Pi_W}{\lambda\overline{\overline{\Pi}}_W}\overline{E},1\right\}<0$$

whenever $\sigma R_L - K < 0$ and $\underline{\underline{E}} > 1$. Ignoring the lower bound values of 1 gives the unconstrained difference

$$\frac{\overline{E}}{\lambda}\frac{\Pi_a}{\overline{\overline{\Pi}}_a}-\frac{\overline{E}}{\lambda}\frac{\Pi_W}{\Psi\overline{\overline{\Pi}}_W}=\frac{\overline{E}}{\lambda}\left[\frac{\Pi_a}{\overline{\overline{\Pi}}_a}-\frac{\Pi_W}{\Psi\overline{\overline{\Pi}}_W}\right]<0$$

The common term $\frac{\overline{E}}{\lambda}$ cancels and substituting gives the following inequality

$$1+\left[\frac{\left(1-\lambda\right)}{\lambda}\frac{(\sigma R_L-K)}{\left(\sigma R_H-K\right)}\right]-\left[\frac{(1-\lambda)\left(R_L-K\right)}{\lambda\left(R_H-K\right)}\right]\frac{1}{\Psi}-\frac{1}{\Psi}<0$$

Substituting for Ψ and rearranging gives

$$\frac{\left(1-\lambda\right)\left(\sigma R_L-K\right)}{\lambda\left(\sigma R_H-K\right)}<\frac{\left(1-\lambda\right)\left(\sigma R_L-K\right)}{\lambda\left(R_H-K\right)+\left(1-\lambda\right)\left(1-\sigma\right)R_L}$$

As $\sigma R_L - K < 0$, the above expression can be simplified as $\left(1-\sigma\right)\left[\lambda R_H+\left(1-\lambda\right)R_L\right]>0$ which is true statement. This completes the proof. ■

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