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The Market Provision of Club Goods in the Presence of Scale Economies

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A natural club monopoly exists when economies of scale are so significant relative to population size that in the efficient allocation the entire population is included in a single club. Under these circumstances an uncontested monopolist will inefficiently exclude some individuals from the club if marginal congestion costs are sufficiently high when the entire population is included. If admitting that the entire population is profit-maximizing, then the facility size chosen is also efficient. A perfectly contestable monopoly is always efficient. 'External' economies of scale exist when the cost of each facility declines as more facilities are provided. This externality leads a competitive market to provide too few clubs, but a monopolist will provide an efficient allocation because it internalizes the cost externality.

Introduction

Much is already known about the technological conditions under which club goods will be efficiently provided by the market. Berglas (1976) has shown that if the efficient number of clubs is an integer, a competitive market will provide the club good efficiently even if there are increasing returns to scale in its provision. This result arises because decreasing costs in provision are offset by increasing congestion costs as the size of the club increases. If scale economies are very significant, then clubs will be large and oligopoly theory becomes more relevant than perfect competition. Scotchmer (1985a) shows that, when there are perceived interdependencies between clubs, price will be set above marginal congestion cost and there will tend to be too many clubs (relative to the efficient number) in a free entry equilibrium.

The results of Berglas and Scotchmer relate to the conventional notion of scale economies where it is cheaper (over some range) to expand the size of a club facility than to provide a number of smaller facilities. A second type of scale economy exists when the cost of each facility declines as more club facilities are provided. We will refer to this as 'external' economies of scale because the economy is external to the individual club. The importance of distinguishing between the two types of scale economies when assessing the efficiency of market provision was first recognized by Berglas (1981) and was again emphasized by Berglas and Pines (1981, 1984). Those authors argue that external economies of scale are analogous to increasing returns to scale in the production of private goods and have nothing to do with clubs *per se*.[1] This paper takes a somewhat different view. I believe that external economies of scale in club provision are importantly different from increasing returns to scale in private good production because a competitive club equilibrium can be sustained in the presence of external economies of scale (as I later demonstrate), while a competitive private good equilibrium can not generally be sustained in the presence of increasing returns. External economies of scale in club provision are more closely analogous to 'multi-plant' economies of scale in private good production, wherein the cost of production declines as more plants are built.[2]

The present paper contributes two new sets of results on the efficiency of market provision in the presence of scale economies. The first relates to the conventional notion of scale economies (which we will sometimes refer to as 'internal' economies of scale). We consider the case of natural monopoly and show that an unregulated monopolist may (inefficiently) exclude some population members from the club. We then admit the possibility of contestability and show that a perfectly contestable monopoly will achieve an efficient outcome. The second set of results relates to 'external' economies of scale. We offer a number of reasons why external economies of scale may arise and consider the efficiency of market provision when they are present. We show that club facilities will be under-provided by a competitive market but that a monopolist will provide an efficient allocation because it internalizes the cost externality.

The paper proceeds as follows. In the next section a simple model is presented in which we explicitly distinguish between internal and external economies of scale. In Section II we set up the planner's problem and derive the efficiency conditions for club good provision when the technology can exhibit both types of scale economies. These conditions are then used as a benchmark for assessing the efficiency of market provision. Section III considers provision by a natural monopoly. In Section IV we examine competitive and monopoly provision when there are external scale economies. Some summary remarks are presented in the final section.

I. The Model

Let x denote the size of the club facility and let n denote the number of users of the facility (club members). Let y denote the quantity of a numeraire purely private commodity. Suppose there are N identical individuals in the population, each with utility function $u(x, y, n)$.[3] Assume that $u_x > 0$ and $u_n < 0$, reflecting the fact that utility is decreasing in congestion and that congestion increases as n increases or x decreases. Assume also that $u_y > 0$. Let the economy's technology be

$$N\bar{y} = Ny + kC(x, k), \tag{1}$$

where the total endowment $N\bar{y}$ of the numeraire good can be consumed (Ny) or transformed into the provision of k club facilities of size x each costing C.[4] Internal scale economies exist when it is cheaper (over some range) to expand the size of a club facility than to make a number of smaller separate facilities providing the same service. In specifying the technology in equation (1) we have also allowed each facility's cost to depend on the total number of facilities provided. This allows for the possibility of external economies of scale, which we now define.

Definition 1. The technology exhibits *external economies of scale* (in the sense of being external to the club) if $C(x, k)$ is decreasing in k for a given x, that is if $C_k < 0$.

External economies of scale will arise when provision of the club good requires the input of a public factor. The simplest example is the existence of a fixed cost in the production of the club facility, such as the cost of the mould in swimming pool production. Once the mould has been made, it can be used to produce many pools, and a declining proportion of its cost will be attributable to each pool produced. Public factors will also be important when provision of the club service requires more than the production of the facility itself. For example, the provision of an airline service between two cities requires the use of aircraft (club facilities) together with airport infrastructure. This infrastructure is a public (or, more precisely, a club) factor, the cost of which must be apportioned to each aircraft using it. The full cost of each flight is reduced as more aircraft make use of the available infrastructure.

II. Efficient Provision

The planner's problem is to choose x, y and n to maximize the utility of a representative club member subject to the economy's technology. (Since k and n are related through $k \equiv N/n$, choosing n implies a choice of k also.) The problem is formulated as follows:

$$\max_{x,y,n} u(x, y, n), \qquad \text{s.t. } N\bar{y} = Ny + (N/n)C(x, N/n). \tag{2}$$

The first-order conditions reduce to

$$n(u_x/u_y) = C_x \tag{3}$$

$$-n(u_n/u_y) = [C(x, N/n)/n] + (NC_k/n^2). \tag{4}$$

A sufficient condition for a maximum is the following.

Assumption 1. $u[x, \bar{y} - C(x, N/n)/n, n]$ is quasi-concave in x and n.

Condition (3) is the familiar Samuelson condition for public goods. It states that for each club the sum of the marginal utilities of x in terms of y must equal the marginal provision cost. It determines the optimal facility size for a given number of members. Condition (4) determines the optimal number of users per club, or, equivalently, the optimal number of clubs. It states that the marginal congestion cost (in terms of y) imposed on all existing users by the addition of one more user must equal the marginal (net) reduction in shared provision cost achieved by admitting that additional user. This marginal reduction in shared provision cost comprises both the wider sharing of the existing cost and the change in the unit cost of the club facility stemming from the reduced number of facilities provided as more users are allocated to fewer clubs. This second term will be negative if there are external economies of scale.

These two conditions provide a benchmark for assessing the efficiency of a market mechanism in providing the club good. However, the first-best solution will generally be feasible only if the efficient number of clubs $k^* = N/n^*$ is an integer. The reason is quite straightforward. When k^* is not an integer, there will be residual population members $r < n^*$ who are omitted from optimally sized clubs, and these individuals will generally not derive the same utility as members of optimally sized clubs. We will abstract from this issue here and henceforth assume that k^* is an integer.[5]

The following result will also prove useful for assessing the efficiency of market provision. The result generalizes a similar result by Scotchmer (1985a) to allow for external scale economies.

Lemma 1. If Assumption 1 holds, then

$$-n(u_n/u_y) \gtreqless C(x,k)/n + NC_k/n^2 \qquad \text{as } n \gtreqless n^*.$$

The proof is contained in the Appendix.

III. Provision by a Natural Monopoly

In this section we will assume that there are no external economies of scale since we wish to focus on the importance of internal scale economies. We define a natural monopoly in the following way.

Definition 2. A *natural monopoly* exists in the provision of the club facility if and only if there exists an x^* such that $[\bar{y} - C(x^*)/N] \geq 0$ and

$$u(x^*, [\bar{y} - C(x^*)/N], N) \geq u(x, [\bar{y} - C(x)/n], n) \qquad \forall x, n$$

such that $[\bar{y} - C(x)/n] \geq 0$.

That is, the operation of a single facility of size x^* with N users is feasible and yields a utility level that is at least as high as any other feasible allocation (x, n). A natural monopoly will arise when internal economies of scale are so significant that the congestion costs incurred by admitting the entire population into a single club are more than offset by the provision cost reductions achieved by expanding the size of the facility. Efficient provision in this case entails a single club with a membership of N and a facility size given by (3) with $n = N$.[6]

Natural monopoly can be viewed as a special case of non-replicability, and it will be useful to relate the notion to that more general issue. We have seen in Section II that clubs will be non-replicable when the efficient number of clubs is not an integer.[7] An extreme instance of this is where the efficient club membership is so large relative to the population size that only one club can form. Some individuals may then be excluded. Helpman and Hillman (1977) have shown (for fixed utilization) that in this case members must be distinguished from non-members, and efficiency requires that the total net benefits to the entire population be maximized. A natural monopoly (as we have defined it) arises when internal scale economies are so extensive that the efficient club membership is at least as large as the population and no individuals should be excluded. (The need to distinguish between members and non-members, therefore, does not arise.) We next consider the efficiency of the unregulated market outcome under these circumstances.[8]

The natural monopolist's problem is to choose the size of the facility, the price of membership P (in terms of the private good) and the number of members to be admitted so as to maximize profit. The level of utility offered to members must be at least as great as their reservation utility u^0, or else they will not join the club. This reservation utility is the utility a consumer could obtain from an alternative source or from not consuming the club good at all. Initially we assume that the only alternative source is a member-owned and -operated club. The utility provided by such a club will depend on the accessibility of the technology needed to produce the facility and on the cost of coordinating a coalition of members.[9] For our purposes it can be taken as given. Under these circumstances, the monopolist's problem is

$$\max_{x,P,n} nP - C(x) \qquad \text{s.t.} \begin{cases} u(x, \bar{y} - P, n) \geq u^0 \\ N - n \geq 0. \end{cases} \tag{5}$$

It is easily shown that $u(x, \bar{y} - P, n) \geq u^0$ will hold as a strict equality, and so the Kuhn–Tucker conditions for the problem reduce to

$$n(u_x / u_y) = C_x \tag{6}$$

$$-n(u_n / u_y) = P - \upsilon \tag{7}$$

$$(N - n) \geq 0, \qquad \upsilon \geq 0, \qquad \upsilon(N - n) = 0, \tag{8}$$

where υ is the multiplier on the second constraint. A comparison of (6) with (3) reveals that the Samuelson condition is satisfied under the unregulated outcome, which means that the facility size chosen is efficient for the number of users admitted. However, the number of users admitted may not be efficient. If marginal congestion cost is sufficiently high at $n = N$, then the monopolist may find it profitable to restrict membership to $n < N$, thereby improving the utility of those admitted which in turn allows the monopolist to charge a higher admission price. High marginal congestion cost at $n = N$ is most likely when scale economies have been exhausted as n approaches N. If considerable scale economies remain at some allocation (x, n), the monopolist can profitably increase x, thereby reducing marginal congestion cost for that level of n. Hence membership will be limited to $n < N$ only if there remain no significant unexploited scale economies at that allocation.

These characteristics of the outcome are illustrated in Figure 1, where we have depicted in (n, P) space an indifference curve for a representative individual and isoprofit contours for the firm. The curves are drawn so as to embody the optimal choice of x for any (n, P) pair as given by condition (6) above. Application of the envelope theorem for maximum value functions then allows us to ignore changes in x along the curves. Along the indifference curve we have $du = -u_y dP + u_n dn = 0$, while along the isoprofit contours we have $d\Pi = ndP + Pdn = 0$. Notice that the slope of the indifference curve is the marginal congestion cost imposed on a representative member (u_n / u_y). If

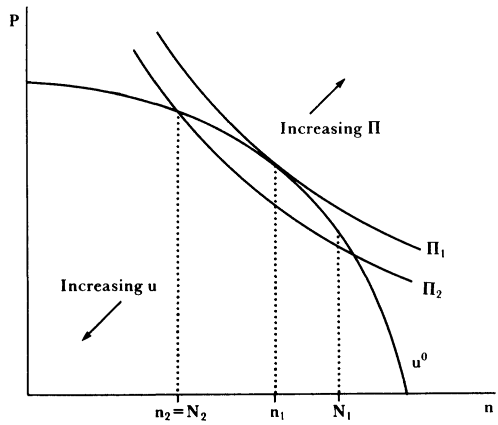

FIGURE 1

marginal congestion cost is very high (in absolute value) at $n = N$ (such as if $N = N_1$ in Figure 1), then it is optimal for the firm to choose $n_1 < N_1$. Conversely, if marginal congestion cost is still low at $n = N$ (such as if $N = N_2$), then it is optimal for the firm to choose $n_2 = N_2$. Notice that the $n_1 < N_1$ solution is an interior one in which the slopes of the curves are equated: $(u_n/u_y) = -P/n$. This is consistent with condition (7) since $\upsilon = 0$ when the membership constraint is not binding. Conversely, at $n_2 = N_2$ we have $-P/n < (u_n/u_y)$, which is consistent with $\upsilon > 0$ in condition (7) since the membership constraint is binding. Notice that price is set above marginal congestion cost in the unrestricted membership outcome but there is no resulting efficiency loss because utilization is perfectly inelastic for $u \geq u^0$.[10]

Cornes and Sandler (1986, p. 191) have suggested that the monopoly solution will correspond to the member-owned and -operated club solution. This is true in so far as member utility under the monopoly outcome corresponds to member reservation utility, but the values of n and x under the two solutions need not be the same. The facility size and membership under the member-owned and -operated club will be chosen to maximize utility subject to coordination costs and any restrictions on access to the technology. Conversely, under the monopoly outcome facility size and membership are chosen to maximize profit subject to providing the reservation utility. Only where there is perfect access to the technology and there are no coordination costs can the two outcomes be expected to coincide. In this case the member-owned and -operated club can achieve the first-best utility given by $u^* = u(x^*, [\bar{y} - C(x^*)/N], N)$, and since this is a maximum over all possible allocations, the monopolist must also choose the efficient allocation in order to meet the utility constraint.

Another circumstance in which the monopolist may choose the efficient outcome is where a potential entrant firm possessed the same technology, that is, where the market is contestable. The conventional contestability model assumes that there may be a sunk cost E associated with entry and that potential entrants expect the currently prevailing price to persist after entry.[11] Under these conditions, the monopolist's profit must not exceed the sunk entry fee; otherwise the potential entrant can enter, offer a slightly lower price, thereby capturing the entire market, and make sufficient profit to cover the sunk cost of entry. Hence contestability adds the following constraint to the monopolist's problem:

$$(9) \qquad nP - C(x) \leq E.$$

Depending on the value of E, the incumbent monopolist's choice of x, n and P may be closer to the efficient solution. In the extreme case where E is higher than the unconstrained monopoly profit, the possibility of entry has no affect on the monopolist's behaviour and the outcome is precisely the same as where there are no potential entrants. At the opposite extreme, E may be equal to zero, in which case the market is perfectly contestable. The equilibrium must then involve zero profits, and it then follows that the monopolist chooses $n = N$ in equilibrium. To see this, suppose the incumbent monopolist offers a utility level u^1 with an allocation (x^1, n) with $n < N$. Then another firm could enter, offer $u^2 > u^1$ with an allocation (x^2, N), and by Definition 2 make non-negative profits. Hence to preclude entry the monopolist must choose $n = N$. It further follows that the monopolist must offer x^*, since if it does not an entry could offer a higher utility with an allocation (x^*, N) and by Definition 2 still make non-negative profit. The following proposition follows immediately from these arguments.[12]

Proposition 1. A perfectly contestable natural monopoly is efficient.

Where E is greater than zero but less than the value of unconstrained monopoly profit, an intermediate solution will arise in which the monopolist's choice of x and n may not be efficient but the distortion will generally be lower than in the unconstrained case.

IV. Market Provision with External Economies of Scale

Competitive market provision

In the competitive case each firm operates only one club and takes as given the utility it must offer its members (much like price-taking behaviour in the conventional private good model). Let $\bar{u}$ denote this utility. Each firm's problem is to choose x, n and P to maximize profit subject to the utility constraint. For simplicity we assume that all firms are identical, so we can consider the choice problem of the representative firm:

$$(10) \qquad \max_{x,n,P} nP - C(x, k) \qquad \text{s.t. } u(x, \bar{y} - P, n) = \bar{u}.$$

The first-order conditions for this problem reduce to

$$(11) \qquad n(u_x/u_y) = C_x;$$

$$(12) \qquad -n(u_n/u_y) = P.$$

Entry is free, so profits are zero in equilibrium. Hence $nP = C(x, k)$. Upon substitution into (12), we then obtain

$$(13) \qquad -n(u_n/u_y) = C(x, k)/n.$$

Condition (11) indicates that the Samuelson condition is satisfied in the competitive equilibrium, but a comparison of (13) with (4) reveals that the outcome is efficient only if there are no external economies of scale ($C_k = 0$).[13]

Proposition 2. A competitive market provides fewer facilities (each with too many users) than is efficient when there are external economies of scale. The converse is true when there are external diseconomies.

Proof. Let $\bar{k}$ denote the number of facilities in the competitive outcome and let k^* denote the efficient number of clubs. In the competitive outcome $-(u_n/u_y) = C(x, k)/n^2$, and if $C_k < 0$, it follows that $-(u_n/u_y) > C(x, k)/n^2 + NC_k/n^2$. Then by Lemma 1, $\bar{n} > n^*$ and so $\bar{k} < k^*$. Q.E.D.

The intuition behind this result is straightforward. Each firm operating only one club and acting independently has no incentive to take account of the cost-saving it could bestow on the industry by attracting fewer users and allowing more clubs to operate. Indeed, it has a disincentive to do so since profits to a competitive firm under the efficient outcome will be negative. To see this, substitute (4) into (12) to obtain

$$(14) \qquad nP = C(x, k) + NC_k/n,$$

and recall that $C_k < 0$. Firms will not enter with the prospect of negative profits, and this leaves fewer incumbent firms than is efficient.[14]

Monopoly provision

Where external economies of scale are considerable, a single firm may be able to produce k club facilities more cheaply than a greater number of firms can in aggregate (an 'external' natural monopoly). Since such a firm will internalize the positive cost affects that were external to the competitive market, we might expect provision by a monopolist to be efficient. To investigate this question we assume that internal economies of scale are not so significant as to imply that efficiency dictates only one club. We also ignore the possibility of contestability in this case.[15] The monopolist's problem is to choose x, P, n and k to maximize profit subject to a utility constraint:

$$(15) \qquad \max_{x,P,k,n} knP - kC(x, k) \qquad \text{s.t.} \begin{cases} u(x, \bar{y} - P, n) \geq u^0 \\ N - nk \geq 0. \end{cases}$$

The reservation utility is that offered by a member-owned and -operated club since there are (by assumption) no competing firms. It is easily shown that the first constraint will hold as a strict equality. The second constraint states that total membership is limited by the population size. The Kuhn-Tucker conditions for this problem reduce to

$$(16) \qquad n(u_x/u_y) = C_x,$$

$$(17) \qquad -n(u_n/u_y) = P - \upsilon,$$

$$(18) \qquad nP - C(x, k) - kC_k - \upsilon n = 0,$$

$$(19) \qquad N - nk \geq 0, \qquad \upsilon \geq 0, \qquad \upsilon(N - nk) = 0,$$

where υ is the multiplier on the second constraint. To determine whether or not $N - nk \geq 0$ will hold as a strict equality, suppose it does not so that $\upsilon = 0$. From condition (18), this implies that $nP - C(X, k) = kC_k$, but this in turn implies that profits are negative (since $C_k < 0$), which cannot be an optimum. Hence it must be that $N = nk$; that is, all members of the population purchase club services in equilibrium. Substituting (18) into (17) then yields

$$(20) \qquad -n(u_n/u_y) = C(x, k)/n + NC_k/n^2.$$

The following proposition follows directly from a comparison of (16) with (3) and (20) with (4).

Proposition 3. A monopoly is efficient when there are external economies of scale.

The result arises because the monopolist internalizes the external scale economies and so chooses the efficient number of clubs. Moreover, the monopolist can not benefit from denying admission to some individuals (unlike the natural monopolist of Section III) because those individuals could always be profitably included in a new club, the provision of which would lower the cost of all existing clubs. Notice, however, that the monopolist extracts the entire surplus from club members, leaving them only with their reservation utility.

Thus, while the allocation is efficient, it may not be satisfactory from a distributive viewpoint.[16]

V. Conclusion

When the production technology exhibits economies of scale, the market provision of club goods need not be efficient. In the presence of internal economies of scale a competitive market will be efficient, but if those economies are large an oligopolistic market structure may arise, in which club facilities will generally be over-provided. I have extended consideration to the case of natural monopoly and shown that, while a perfectly contestable monopoly is always efficient, an uncontested monopolist will inefficiently exclude some individuals from club membership if marginal congestion costs are sufficiently high when the entire population is included. If scale economies are sufficiently large, then the uncontested monopolist will choose an efficient allocation.

We also considered the case of 'external' economies of scale wherein the cost of each facility declines as more facilities are provided. This externality leads a competitive market to provide too few clubs, but a monopolist will provide an efficient allocation because it internalizes the cost externality.

APPENDIX: PROOF OF LEMMA 1

If Assumption 1 holds, then $u(x(n), \bar{y} - C[x(n), N/n]/n, n)$ is strictly quasi-concave in n (see Scotchmer 1985a, Lemma 2). It then follows that $\mathrm{d}u/\mathrm{d}n \gtreqless 0$ as $n \lesseqgtr n^*$ where

$$\mathrm{d}u/\mathrm{d}n = u_x(\partial x/\partial n) - u_y[nC_x(\partial x/\partial n) - NC_k/n - C(\cdot)]/n^2 + u_n.$$

Multiplying and dividing by (u_y/n) yields

$$\mathrm{d}u/\mathrm{d}n = (u_y/n)[n(u_x/u_y) - C_x](\partial x/\partial n) + (u_y/n)[NC_k/n^2 + C(\cdot)/n + n(u_n/u_y)].$$

The first term is zero by (3), and the result follows immediately from the second term.

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NOTES

1. Berglas and Pines (1984, p. 393. These authors use the term 'returns to scale in aggregate production' to describe what I have called external economies of scale. I prefer the latter term because it emphasizes the external nature of the non-convexity.
2. See Scherer (1980, pp. 100–19) for a discussion of a multi-plant economies.
3. Notice that I have assumed club utilization to be fixed. Variable utilization introduces an inefficiency in monopoly provision (through the distortion of the utilization choice) from which we wish to abstract so as to focus on the role of scale economies. Some of my results must be qualified when utilization is variable, and I bring this to readers' attention at the appropriate points in the paper.
4. This assumes that all facilities are the same size. This assumption is not restrictive provided the optimal number of clubs is an integer and the population is homogeneous.
5. For a more complete discussion of the 'integer problem', see Scotchmer (1985a).
6. Condition (4) must hold only as a weak inequality in this case since the solution is not an interior one. The appropriate condition is, $-n(u_n/u_y) \leq [C(x)/n]$.
7. Non-replicability may also arise when the club good in question is non-reproducable such as a wilderness area, a beach or a work of art.

8. Hillman (1978) also considers provision of a monopolist, but he does not consider the case of natural monopoly.
9. Coordination costs are likely to be especially high when production involves significant economies of scale because the optimal number of members is large.
10. Berglas *et al.* (1982, p. 347) have shown that if utilization is variable then utility is maximized when all individuals are included in the club. This implies that the natural monopolist will never restrict membership when utilization is variable. However, the outcome will not be efficient because utilization price is set above marginal congestion cost and the utilization choice is thereby distorted.
11. This latter assumption is rather dubious since it violates the subgame perfection criterion for an equilibrium.
12. This proposition will also be true when utilization is variable.
13. It is in this sense that Boadway's (1980) claim that efficient provision by a competitive market requires constant returns to scale is correct. However, Boadway made no distinction between what we have called external economies of scale and the (internal) economies of scale to which the Berglas (1976) result referred. This created an apparent inconsistency between the two results (see Berglas 1981).
14. Proposition 2 is also true when utilization is variable. The proof requires a generalization of Lemma 1 that can be found in Kennedy (1988, p. 9).
15. We will see that the uncontested monopoly outcome in this case is efficient and so contestability has distributive implications only.
16. When utilization is variable the unregulated monopoly outcome will not be efficient because price will be set above marginal congestion cost. Whether the outcome is better than the competitive equilibrium will depend on the benefits of internalizing the provision cost externality relative to the welfare cost of the distortion in the utilization choice. These will in turn depend on the relative magnitudes of the external economy and the utilization elasticity.

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