THE RELATIONSHIP BETWEEN EMISSIONS AND INCOME GROWTH FOR A TRANSBOUNDARY POLLUTANT

Peter Kennedy

Emma Hutchinson

Department of Economics

University of Victoria

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ABSTRACT

We identify a *pollution-spillover effect* in the relationship between emissions and income growth for a transboundary pollutant. This effect causes countries that are otherwise identical to follow very different emissions paths as they grow, due only to differences in the positions they occupy along the growth path. Emissions from a country of any given income depend on *when* it reaches that income level relative to other countries moving along the same income growth path. This variation across countries arises whenever the damage function is strictly convex in the level of global emissions. In such a setting, the emissions level for a country at any point in time depends on the level of emissions from all other countries at that time. This means that a country of given income will behave differently depending on whether other countries have high emissions (at the peak of their emissions paths) or low emissions (at the early or late stages of their emissions paths). The behavior of any individual country, at any point in time, therefore depends on its relative position in the global income distribution.

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1. INTRODUCTION

The relationship between pollution and income growth is a complex and controversial one. Much of the debate has focused on the environmental Kuznets curve (EKC) hypothesis which predicts an inverted U-shaped relationship between income and emissions [Grossman and Krueger (1991, 1995)] . In the early stages of growth this relationship is dominated by output growth (a *scale effect*), and emissions rise as income rises. At the same time, income growth stimulates the adoption of cleaner technologies (a *technique effect*), and this effect can eventually offset the scale effect to the extent that emissions eventually start to decline as growth continues.

A number of different mechanisms have been proposed as the source of the technique effect. The most commonly cited are a preference-based linkage between income and the demand for environmental quality [Grossman and Krueger (1991) and Copeland and Taylor (1994)], and a technological channel based on increasing returns to abatement [Andreoni and Levinson (2001)]. Dinda (2004) provides a useful survey of the literature.

The EKC hypothesis underlies an oft-made argument that rapidly rising pollution levels in many emerging economies is only a short-term problem. As they continue to grow, these economies are forecast to follow the same emissions path traveled historically by the advanced economies of today, and their emissions will eventually fall in the same way that advanced-economy emissions have declined from earlier peaks, at least in the case of some pollutants. This extrapolative interpretation of the EKC hypothesis is especially controversial.

Critics of the EKC hypothesis have questioned both the underlying theory and the empirical methodologies employed to test it [Stern (2004)]. For example, Harbaugh *et. al*. (2002) reexamine the original Grossman and Krueger (1995) analysis with an updated data set and do not find a turning point in emissions. Millimet, List and Stengos (2003) argue that many of the early parametric models employed to estimate EKCs are subject to serious misspecification problems. Similarly, Müller-Fürstenberger and Wagner (2007) identify a number of problems associated with commonly-used variable transformations in the estimation of reduced-form EKC models and in the calibration of integrated-assessment (structural) models.

The theory behind the EKC has also been questioned. Arrow *et. al.* (1995) argue that the hypothesis ignores the deleterious impact of current pollution on future growth via irreversible damage to natural capital. Prieur (2009) formalizes this argument in a model of a stock pollutant and finds that an EKC is unlikely to emerge in that model. Arrow *et. al.* (1995) also argue that the observed decline in emissions for some pollutants in developed economies over time may not reflect a technique effect as much as a trade-induced change in the *composition* of production that has seen highly polluting manufacturing migrate to lower-income countries. (See Brock and Taylor (2009) for a formal treatment of the composition effect). Emerging economies may find it more difficult to follow that same path precisely because they are followers.

In this paper we provide an additional argument as to why the historical path followed by the advanced economies of today may not provide a good guide to the future behavior of emerging economies, specifically with respect to transboundary pollutants. We identify a *pollution-spillover effect* that introduces an important asymmetry across countries as they grow, even when standard scale and technique effects would otherwise produce a shared and well-behaved EKC for those countries. In particular, we show that countries that are otherwise identical can follow very different emissions paths as they grow because emissions from a country of any given income depend on *when* it reaches that income level relative to other countries moving along the same income growth path. This asymmetry arises even in the absence of stock-pollutant effects or trade-induced composition effects.

The variation across countries in our model arises whenever the damage function is strictly convex in the level of global emissions. In such a setting, the optimal emissions level for a country at any point in time depends on the level of emissions from all other countries at that time; emissions are strategic substitutes. This means that a country of given income will behave differently depending on whether other countries have high emissions (at the peak of their emissions paths) or low emissions (at the early or late stages of their emissions paths). The behavior of any individual country, at any point in time, therefore depends on its relative position in the global income distribution.

When calibrated roughly to actual GDP data, our model indicates that the emissions paths followed by large advanced economies to date may not provide a good guide at all to the behavior of the emerging economies that follow them on the income growth path. These emerging economies have higher emissions than more advanced economies did at the same stage of growth purely because these emerging economies have developed later.

The rest of our paper is organized as follows. In section 2 we present our model. In section 3 we characterize the non-cooperative equilibrium among countries. In section 4 we examine the income-emissions relationship in that equilibrium as incomes rise, and show that the timing of emissions growth-and-decline for an individual country *relative to* the evolution of aggregate global emissions, is central to individual behavior. In section 5 we compare the non-cooperative equilibrium with the solution to a planning problem in which emissions are chosen to minimize total global cost. We show that emissions fall more quickly in the minimum-cost solution than in the non-cooperative equilibrium but the cross-country dynamics are similar in both formulations. In section 6 we conclude with some summary remarks and highlight some of the main limitations of our analysis. Most of the technical details are placed in an Appendix.

2. THE MODEL

Let $y_i(t) > 0$ denote aggregate income in country *i* at date *t*. To minimize notational clutter we denote income for country *i* at date $t = 0$ as $y_i \equiv y_i(0)$. Income grows exogenously at rate $r > 0$ for all countries. Thus, country *i* has income $y_i(t) = y_i e^{rt}$ at date *t*. This means that all countries are on the same income growth path, and differ only in terms of their relative positions on that growth path at any point in time.

We make this strong assumption on growth rates for two reasons. First, it makes the mathematical analysis tractable. Second, and more importantly, we wish to highlight the complexity introduced into emissions paths by the pollution-spillover effect even in an otherwise simple setting. Differential rates of growth or a negative feedback effect between emissions and growth would further add to that complexity and to the possibility that emissions paths differ across countries. We revisit these additional sources of asymmetry in Section 6.

Emissions from country *i* at date t are denoted $z_i(t)$, and depend on its economic output, as measured by $y_i(t)$, and the abatement technology in use, denoted $a_i(t) \in [0,1]$:

(1) $$z_i(t) = y_i(t)[1 - a_i(t)]$$

The associated damage to country *i* at date *t* is

(2) $$d_i(t) = \delta_i [z_i(t) + \phi Z_{-i}(t)]^2$$

where $\delta_i > 0$ is the damage parameter for country *i*, $\phi \in [0,1]$ measures the degree to which emissions are transboundary in their impact, and $Z_{-i}(t) = \sum_{j \neq i} z_j(t)$ is aggregate emissions at date *t* from all countries other than country *i*. If $\phi = 0$ then the pollutant causes no damage beyond national boundaries; if $\phi = 1$ then the pollutant is fully global in its impact.

The quadratic specification of the damage function in (2) implies that damage is strictly convex in global emissions when $\phi > 0$. This strict convexity is central to our results.[1] It means that emissions are strategic substitutes: the marginal domestic damage from domestic emissions is increasing in the emissions from all other countries.

Though not all pollutants exhibit a strictly convex damage function, many certainly do. Strict convexity can arise from two main sources. The first relates to physical threshold effects in natural systems, especially those that exhibit chaotic dynamics. Such systems may exhibit only mild effects from pollution up to a certain exposure but then "flip" to a new path once a critical exposure level is passed. A sequence of such threshold effects can often be approximated by a smooth but strictly convex function like the one in our model. The second source of strict convexity is preference-based. The damage function in our model relates to the *value* of physical damage, and this will typically exhibit strict convexity when preferences over goods and services – including environmental services – are strictly convex, even if the underlying physical impact of pollution is linear in emissions.

It is important to stress that damage in our model stems for the flow of emissions and not from a stock of the pollutant that accumulates over time. This limits the applicability of the model – it cannot adequately capture the carbon-dioxide problem, for example – but our purpose here is to highlight the asymmetry that can arise among emissions paths even when there are no stock effects at work. In section 6 we speculate on how the introduction of stock effects might alter our results.

We assume that there are increasing returns to abatement, and this drives the technique effect in our model (as in Andreoni and Levinson (2001)). In particular, abatement cost for the economy as a whole is $a_i(t)^2$. This is clearly an extreme form of increasing returns since the cost of abatement does not depend at all on the level of production to which it is applied. Our central results do not hinge on this extreme assumption. On the contrary, our purpose is to work with as simple a model as possible in which there are transparent scale and techniques effects, so as to highlight the complicating role of the pollution-spillover effect. The analysis can be extended to less extreme specifications of the abatement cost function.

[1] The quadratic specification *per se* is less important but it does allow us to derive analytical solutions.

Countries are ordered from largest to smallest according to their incomes at $t = 0$, such that $y_1 \geq y_2 \geq ... \geq y_n$. The mean of those incomes at $t = 0$ is

(3) $$\mu = \frac{1}{n}\sum_{j=1}^{n} y_j$$

where *n* is the number of countries, and the variance at $t = 0$ is

(4) $$\sigma^2 = \frac{1}{n}\sum_{j=1}^{n}(y_j - \mu)^2$$

We make no assumptions about the distribution of income other than that μ and σ^2 are finite, and that $y_i > 0 \;\; \forall i$. These properties of the distribution nonetheless dictate certain relationships between key parameters. The most important of these for the interpretation of our results is that there exists a lower bound on y_1, given by

(5) $$y_{1L} = \mu + \frac{\sigma^2}{\mu}$$

That is, the income of the largest country cannot be less than y_{1L}. Note that this is not an assumption; *any* empirical distribution with strictly positive support must satisfy this condition (see Lemma 1 in the Appendix).

3. EMISSIONS IN THE NON-COOPERATIVE EQUILIBRIUM

We focus our attention on the non-cooperative equilibrium (NCE). In the absence of a global treaty to control transboundary emissions, the NCE is the most reasonable description of behavior. The impediments to a global treaty on a transboundary pollutant are varied and partly political in nature, but from a theoretical perspective the biggest obstacle is the well-known free-rider problem: each country has an incentive to remain outside the treaty and instead free-ride on the abatement undertaken by treaty members [Hoel (1992), Carraro and Siniscalco (1993) and Barrett (1994)]. A stable treaty coalition – one which no member wishes to leave and no non-member wishes to join – is typically unable to achieve significant abatement relative to the NCE even when within-treaty transfers are available. [Barrett (2001)]. Accordingly, we do not examine treaty equilibria in this paper. Instead we focus primarily on the NCE, and in Section 5 we compare the associated dynamics with those of the globally efficient solution that a global planner (or *ideal* treaty) might implement.

In the NCE, the policy-maker in country *i* chooses at each point in time the technology that minimizes the sum of abatement cost and domestic damage:

(6) $$\min_{a_i(t)} \; a_i(t)^2 + \delta_i \left[y_i(t)[1 - a_i(t)] + \phi Z_{-i}(t) \right]^2$$

We assume that no country can commit to using a technology at date *t* if that technology is not optimal for it at that date.

As a benchmark, first consider the case where the pollutant is purely local: $\phi = 0$. In that setting, the solution to (6) yields an emissions path against income given by

(7) $$z_i(t) = \frac{y_i(t)}{1 + \delta_i y_i(t)^2}$$

This income-emissions path has an inverted U-shape, with peak emissions for country *i* occurring when its income reaches $y_i^* \equiv \delta_i^{-1/2}$. Thus, the model yields a conventional EKC for country *i*, reflecting the evolving relative strengths of scale and technique effects as income grows over time. Note that countries may not all follow the *same* path – since δ_i can differ across countries – but the path for any given country is a well-behaved EKC.

Now suppose that the pollutant is fully global in its impact: $\phi = 1$. To focus on differences that arise across countries due solely to differences in their positions along the income growth path, we henceforth assume that all countries are identical with respect to δ_i. This allows us to solve the model in terms of key parameters of the income distribution, and to focus specifically on the pollution-spillover effect as a source of heterogeneity in the behaviour of countries over time.

Solving (6) when $\phi = 1$ and $\delta_i = \delta \;\; \forall i$ yields a best-response function for country *i* with respect to $Z_{-i}(t)$:

(8) $$z_i(t) = \frac{y_i(t) - \delta Z_{-i}(t) y_i(t)^2}{1 + \delta y_i(t)^2}$$

Crucially, note that emissions for country *i* at any date *t* are declining in aggregate emissions from all other countries; that is, emissions are strategic substitutes. This reflects the fact that an increase in $Z_{-i}(t)$ raises domestic marginal damage for country *i*, because damage is strictly convex and emissions are transboundary. The optimal response for country *i* is to reduce its own emissions.[2]

Setting $Z_{-i}(t) = Z(t) - z_i(t)$ in (8), and summing across *i*, we can solve for equilibrium aggregate emissions, and the emissions for country i. (See the Appendix). These are given by

(9) $$Z^*(t) = \frac{N_t \mu_t}{1 + \delta N_t (\mu_t^2 + \sigma_t^2)}$$

and

(10) $$z_i^*(t) = y_i(t) - \delta Z^*(t) y_i(t)^2$$

respectively, where N_t is the number of countries that have positive emissions at date *t*, and where μ_t and σ_t^2 are the mean and variance of the income distribution across those countries at date t.

The time-dependence of N_t arises here because equilibrium emissions for any country $i < n$ eventually decline to zero at some finite date T_i^*. Hence, (8) is a valid description of behavior only in the period prior to date T_i^*. This means that the interaction among countries comprises a sequence of *n* periods, where the end of a period occurs when one of the countries in that period reaches zero emissions.

We can show from (10) that the exiting country in any period is always the largest country remaining in that period. In particular, setting $z_i^*(t) = 0$ and solving for *t* when $y_i = y_1$ and $N_t = n$ yields

(11) $$T_1^* = -\frac{1}{2r} \log\left(n\delta\mu(y_1 - y_{1L})\right)$$

This is positive and decreasing in y_1 if country 1 has positive emissions at $t = 0$. (See the Appendix for the details). Thus, if the largest country has positive emissions in the first period then it is the first country to reach zero emissions. The first period therefore comprises $t \in [0, T_1]$.

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[2] Strict convexity of the damage function is one mechanism through which strategic interaction can arise but there are others as well. For example, emissions can be strategic substitutes in a setting with a linear damage function via the impact of unilateral abatement measures on the global price of fossil fuels and the movement of footloose capital. Conversely, Golombek and Hoel (2004) argue that technology spillovers associated with abatement in one country could lead to more abatement in another, such that emissions effectively become strategic complements. In a similar vein, Ebert and Welsch (2011) show that emissions can be strategic complements when abatement and defensive action against damage are substitute policies.

We confine our discussion in the main text to this first period since all proceeding periods exhibit the same basic properties. Thus, we set $N_t = n$ in (9) for the analysis that follows. A description of the equilibrium in later periods is included in the Appendix.

Note that the emissions path in (10) describes a quadratic *cross-section* relationship between income and emissions at any point in time. However, the coefficient on the quadratic term in (10) is *not constant over time*. In particular, μ_t and σ_t^2 are continually evolving as incomes grow, and so too therefore is $Z^*(t)$. Thus, the cross-section relationship does not provide a prediction of how each individual country will behave as its income grows over time. On the contrary, emissions for a country at any given income level depend on *when* that income level is reached relative to other countries, and this in turn depends on where that country sits in the income distribution. We characterize this relationship between emissions paths and the income distribution in the next section.

4. THE RELATIONSHIP BETWEEN INCOME AND EMISSIONS OVER TIME

There are three distinct cases with respect to the pattern of the income-emissions relationship that can arise in this model. These cases are distinguished by the size of the largest country, which determines the length of period 1. This in turn determines which countries reach an emissions peak within that period, and whether or not their emissions follow a well-behaved inverted U-shape over time. In the main text we report full results for only one of these cases so as not to obfuscate the main message with unnecessary detail. The other two cases are discussed briefly at the end of this section, and described in more detail in the Appendix.

The case upon which we focus is where $4y_{1L}/3 \le y_1 < 2y_{1L}$, henceforth referred to as Case 1. This case accords best with actual GDP data. In particular, using IMF data on GDP for the years 2003 – 2012, we calculated μ and σ^2 in each of those years, and then constructed $y_{1L} = \mu + \sigma^2/\mu$ for each year.[3] This parameter has an average value of 7443 over the ten-year period. In comparison the largest country, the United States, had an income of 13960 on average over that period. Thus, in every year, the data tell us that $4y_{1L}/3 < y_1 < 2y_{1L}$.

[3] Data on income are drawn from International Monetary Fund, *World Economic Outlook Database*, October 2013. Values are in billions of dollars, calculated at purchasing-power-parity exchange rates.

The key relationships in this case are summarized in Figure 1. The figure partitions the income distribution at $t = 0$ (depicted on the horizontal axis) into two key regions, divided by a critical income level equal to

$$\tilde{y} = \frac{y_1^2}{3y_1 - 2y_{1L}} \tag{12}$$

This income level identifies a pivotal country whose emissions reach a local maximum at exactly $t = T_1^*$. (Recall that T_1^* is the date at which the largest country reaches zero emissions, marking the end of period 1). Countries with an income smaller than $\tilde{y}$ do not reach an emissions peak during period 1; countries with an income larger than $\tilde{y}$ do.

First consider the largest countries in the income distribution: those in region 2 of Figure 1 (whose initial incomes are greater than $\tilde{y}$). These countries follow an inverted U-shaped emissions path over time, with country *i* reaching an emissions peak at some date $t_i^* < T_1^*$, and where $\partial t_i^* / \partial y_i < 0$. That is, within this group of countries, larger ones reach peak-emissions at an earlier date than smaller ones. In this respect, the patterns so far are consistent with a standard EKC hypothesis.

However, now consider the relationship between income and peak-emissions. This is depicted in Figure 2. Ignore the dashed curve for the moment and focus on the solid curve in region 2 labeled $z_i^*(t_i^*)$. This curve tells us that peak-emissions vary across countries, and that these peak-emissions are *not* monotonic in initial income. In particular, peak-emissions are lowest for a country with initial income y_{1L}, henceforth referred to as "country *L*". Countries either side of country *L* in the income distribution have higher peak emissions.

The reason for this pattern relates directly to the interaction among countries via the pollution-spillover effect, and the *timing* of peak-emissions for each country in relation to when aggregate emissions reach a peak.

To see this, consider the behavior of aggregate emissions over time. In the Appendix we show that aggregate emissions follow an inverted U-shaped path, reaching a maximum within period 1 at

$$t^* = -\frac{1}{2r}\log(n\delta(\mu^2 + \sigma^2)) \tag{13}$$

We also show that country *L* reaches its own peak-emissions at this same date. Thus, countries either side of this country in the income distribution face lower other-country emissions as they approach their own peak-emissions, and by virtue of the fact that emissions are strategic substitutes, they therefore have higher peak-emissions than country *L*. This country has the misfortune of reaching its own emissions peak just as global emissions as a whole are at their highest.

The pollution-spillover effect also influences the income levels at which different countries reach peak-emissions. Figure 3 illustrates the relationship between initial income for country *i* and its "peak-emissions income", depicted as the curve labeled $y_i(t_i^*)$. Note that this peak-emissions income varies across countries, and is not monotonic in initial income.

The reason for this again relates to the evolution of aggregate emissions over time, both in terms of level *and gradient*. Think again of country L. We know that this country reaches peak-emissions on the same date that aggregate emissions peak. Each country *behind* country *L* along the income growth path (with initial income less than y_{1L}) therefore faces a lower level of other-country emissions as it approaches its emissions peak than country *L* did at its emissions peak. Moreover, each country behind country *L* faces *falling* other-country emissions as its income grows towards its own emissions peak. These two factors together cause those countries to decelerate their technology adoption as their incomes grow, relative to the countries just ahead of them on the income growth path. Thus, they reach peak-emissions at an income level *higher* than that for country *L*. Hence, $y_i(t_i^*)$ is declining in y_i at and below y_{1L} in Figure 3.

For a country with initial income marginally higher than y_{1L}, peak-emissions are approached when other-country emissions are high *and rising*. This causes that country to accelerate its technology adoption relative to the countries behind it along the income growth path. Thus, $y_i(t_i^*)$ is still declining in y_i for $y_i > y_{1L}$. In contrast, a country with initial income *well* above y_{1L} faces other-country emissions that are rising but *low in level* as it approaches its own emissions peak. At some point high enough in the income distribution, the level effect becomes more important than the gradient effect, and the peak-emissions income begins to rise with initial income. That is, the relationship between y_i and $y_i(t_i^*)$ in Figure 3 eventually turns positive (at $y_i = 4y_{1L}/3$).

The central message from the results so far is that while all countries with initial incomes greater than $\tilde{y}$ follow inverted U-shaped emissions paths, those paths differ markedly across countries depending on when their emissions reach a peak relative to the evolution of aggregate emissions, which in turn depends on their position in the global income distribution.

Now consider countries with initial incomes less than $\tilde{y}$ (those in region 1 of Figure 1). Emissions for these countries do not reach a peak in period 1. The highest emissions levels for these countries during period 1 occur at the end of the period, at $t = T_1^*$, when their emissions are *still rising*. The relationship between emissions at $t = T_1^*$ and initial income is depicted in Figure 2, for all countries, as the dashed curve labeled $z_i^*(T_1^*)$. This curve is simply the graph of (10) at $t = T_1^*$.

The key feature of this graph is that those countries in region 1 with an initial income close to $y_1 / 2$ already have *higher* emissions at $t = T_1^*$, when their emissions are still rising, than the larger countries of region 2 had at the *turning point* of their emissions. The reason is that aggregate emissions are low and falling at $t = T_1^*$, whereas aggregate emissions were high and/or rising when the higher-income countries of region 2 reached their emissions peaks. Thus, the behavior of large advanced economies whose emissions have peaked, or will peak soon, does not provide a good guide to the behavior of the emerging economies that follow them on the income growth path. These emerging economies could have much higher emissions than more advanced economies did at the same point of development, purely because these emerging economies have developed later.

Summary of Cases 2 and 3

Case 2 is where $y_1 \geq 2y_1$. In this case the largest country is so large relative to most other countries that its emissions fall to zero before aggregate emissions peak; that is, $T_1^* < t^*$. The same basic patterns that arise in Case 1 also arise in this case except that peak-emissions are increasing in y_i for *all* countries with $y_i > \tilde{y}$. (Region 2A in Figure 2 vanishes in this case because $\tilde{y} > y_{1L}$). This reflects the fact that aggregate emissions are rising throughout period 1 in this case, so successively smaller countries behind country 1 who approach their emissions peak in period 1 do so at successively higher levels of aggregate emissions, and accelerate their technology adoption accordingly.

Case 3 is where $y_1 < 4y_{1L}/3$. In this case the gap between the largest country and the others is smaller than in Case 1. As a consequence, the highest income countries within region 1 of Figure 1 now have *three* turning points in their income-emissions path: a local maximum followed by a local minimum followed by another local maximum.[4]

The reason for this relates to the *rate* at which aggregate emissions fall. Aggregate emissions are falling when these countries approach their emissions peak but not fast enough to delay those peaks beyond T_1^*, as they do in Case 1 where a very large country pulls aggregate emissions down quickly as it descends from its own peak. Thus, the highest-income countries in region 1 reach an emissions peak before T_1^*. However, as aggregate emissions continue to fall – reducing the marginal domestic damage of own-emissions – these countries decelerate their technology adoption to the point where the scale effect of continued income growth re-dominates the technique effect, and emissions begin to rise again. Thus, a local maximum is followed by a local minimum after which emissions rise again. Emissions for these countries do eventually start to fall again (some time after T_1^*), and so the local minimum is eventually followed by another local maximum.

5. GLOBAL EFFICIENCY

In this section we compare the NCE with the minimum-global-cost solution (MGCS). We continue to focus on the case where the pollutant is purely global and all countries suffer the same damage.

The MGCS minimizes the sum of global abatement cost and global damage (as determined by global emissions):

$$\text{(14)} \qquad \min_{\{a(t)\}} \sum_{i=1}^{n} a_i(t)^2 + n\delta\left[\sum_{i=1}^{n} y_i(t)[1-a_i(t)]\right]^2$$

It is straightforward to solve this planning problem for the optimal technology for each country, and to derive the associated emissions paths. (See the Appendix). We confine consideration to

[4] Egli and Steger (2007) also derive dynamic paths with "false peaks" – an N-shaped EKC – but via a mechanism entirely different from the pollution-spillover effect we have identified here.

the period in which all countries have positive emissions. During that period, aggregate emissions are

(15) $$Z^{**}(t) = \frac{n\mu_t}{1+\delta n^2(\mu_t^2 + \sigma_t^2)}$$

and emissions for country i are

(16) $$z_i^{**}(t) = y_i(t) - \delta n Z^{**}(t) y_i(t)^2$$

The largest country reaches zero emissions before any other country, at date

(17) $$T_1^{**} = T_1^* - \frac{\log(n)}{2r}$$

Note that this date is *earlier* than the corresponding date in the NCE (for any $n > 1$). In the NCE, each country ignores the negative externality that its emissions impose on other countries. In contrast, a global planner takes account of this external cost and so sets a faster rate of technology adoption, thereby causing emissions to decline earlier than in the NCE.

This difference in timing pervades all aspects of the relationship between the NCE and MGCS, so it will prove useful to identify it specifically as the *non-cooperative lag*, henceforth denoted

(18) $$\lambda(n,r) = \frac{\log(n)}{2r}$$

Note that $\lambda(n,r)$ is increasing in *n* and decreasing in *r* (the income growth rate). The first of these properties reflects the fundamental externality at the heart of the distortion: the transboundary pollutant causes more external damage when there are more countries affected. The second property reflects the fact that a faster growth rate compresses the emissions paths in an inversely-proportional way, in both the NCE and the MGCS, and so the *absolute* size of the non-cooperative lag shrinks as *r* rises.

Now consider the timing and magnitude of peak-emissions in the MGCS. The basic pattern with respect to timing is essentially identical to that in the NCE except for the impact of the non-cooperative lag. In particular, there are still three distinct cases of interest, as determined by the size of the largest country, and the critical income thresholds that distinguish those three cases are unchanged. We will again focus on Case 1 (where $4y_{1L}/3 < y_1 < 2y_{1L}$).

Recall that in this case, there exists a pivotal country with income $\tilde{y}$ (see expression (12) above) whose emissions in the NCE reach a local maximum at $t = T_1^*$; countries with an income smaller than $\tilde{y}$ do not reach an emissions peak before that date, while countries with an income larger than $\tilde{y}$ do. The same pivotal country plays the same separating role in the MGCS. In particular, a country with income $\tilde{y}$ reaches its emissions peak in the MGCS at $t = T_1^{**}$. Countries with income greater than $\tilde{y}$ follow an inverted U-shaped emissions path over time, but country *i* reaches that emissions peak earlier than it does in the NCE, *by an amount equal to the non-cooperative lag*. (See the Appendix). Emissions for countries with income less than $\tilde{y}$ are still rising at $t = T_1^{**}$. Global emissions in the MGCS follow an inverted U-shaped path but reach a peak earlier than they do in the NCE, also by the length of the non-cooperative lag.

Since peak-emissions occur earlier in the MGCS than in the NCE, it follows that the magnitude of those peak-emissions is lower in the MGCS. In particular, the ratio of peak-emissions in the NCE to peak-emissions in the MGCS is $\sqrt{n}$, for all individual countries with initial incomes greater than $\tilde{y}$, and for aggregate emissions. Similarly, for countries with initial incomes lower than $\tilde{y}$ (whose emissions are still rising at T_1^* in the NCE and at T_1^{**} in the MGCS), emissions at T_1^{**} in the MGCS are a fraction $1/\sqrt{n}$ of their emissions at T_1^* in the NCE. (See the Appendix).

The relationship between peak-emissions and initial income follows the same pattern as in the NCE (recall Figure 2). In fact, the only difference between Figure 2 and a comparable figure for the MGCS would be the units on the vertical axis; the units on the MGCS figure would simply be scaled by a factor $1/\sqrt{n}$. Thus, in the MGCS, emerging economies can have higher emissions than more advanced economies did at the same point of development, just as they can in the NCE. This lack of symmetry along the growth path in both cases reflects the strict convexity of the damage function; it is not a manifestation of non-cooperative behavior *per se*. However, the unpriced pollution spillovers in the NCE amplify the difference in emissions from countries at comparable stages of development, by a factor of $\sqrt{n}$.

Individual-Country Costs in the NCE and the MGCS

In a setting where all countries have the same initial income, the MGCS would Pareto-dominate the NCE at every date along the growth path. The welfare comparison is more complicated when countries are heterogeneous. For the sake of brevity, we explore that comparison here only for

Case 1 (where $4y_{1L}/3 < y_1 < 2y_{1L}$). We also restrict attention to the period in which all countries have positive emissions.

As a starting point, consider the time path of domestic cost for country *i* in the NCE (calculated at each date as the sum of abatement cost and domestic damage from equilibrium global emissions). This domestic cost initially rises over time as technology cost and aggregate emissions both grow, but it eventually reaches a turning point for at least some countries after aggregate emissions begin to decline, even though technology cost continues to rise with income. In particular, in the Appendix we show that in the NCE, country *i* reaches a turning point in its domestic cost before T_1^* if and only if its initial income is less than

(19) $$\bar{y} \equiv \left(\frac{(2y_{1L} - y_1)n\mu}{2} \right)^{1/2}$$

The turning point occurs earlier for small economies than for larger ones because small economies are still facing a relatively flat technology-adoption-cost curve when aggregate emissions begin to fall; recall that abatement cost is strictly convex.

Domestic cost paths in the MGCS also follow an inverse U-shaped pattern for at least some countries. In the Appendix we show that country *i* reaches a turning point in its domestic cost before T_1^{**} if and only if its initial income is less than $\bar{y}/\sqrt{n}$. This turning point occurs earlier for small economies than for larger ones, just as it does in the NCE, and for the same reason: marginal technology-adoption-cost is lower for more polluting technologies.

Now consider the relationship between the cost path for a given country in the MGCS and its cost path in the NCE. This relationship depends critically on the position of the country in the income distribution. In the Appendix we show that the initial income distribution can be partitioned into three distinct regions. First consider countries with initial income below

(20) $$y_{low} = \left(\frac{2n(\mu^2 + \sigma^2)}{n+1} \right)^{1/2}$$

Domestic cost for these countries is lower in the MGCS than in the NCE at every date prior to T_1^{**}. Thus, these countries are always better off in the MGCS.

Next consider countries with initial income greater than y_{low} but less than

(21) $$y_{high} = \left(\frac{(n+1)(\mu^2 + \sigma^2)}{2} \right)^{1/2}$$

For any country *i* in this region of the income distribution, domestic cost is *lower* in the NCE than in the MGCS during a period prior to an idiosyncratic threshold date, henceforth denoted $\omega(y_i)$, and higher in the NCE during the period after that threshold date. Thus, these countries are initially worse off under the MGCS but eventually become better off as time passes.

The analytical expression for $\omega(y_i)$ is reported in the Appendix, and its key properties are illustrated in Figure 4. The horizontal axis in the figure depicts initial incomes (as in the previous figures). The vertical axis measures time. The curve labeled $\omega(y_i)$ splits time into two periods for each country: the NCE has lower cost in the period below the curve, and higher cost in the period above the curve, in comparison with the MGCS. Critically, note that $\omega'(y_i) > 0$: larger economies must wait longer before the MGCS delivers a cost advantage over the NCE; we explain why in a moment.

The third key segment of the initial income distribution covers incomes above y_{high}. Domestic cost for countries in this income segment is higher in the MGCS than in the NCE at every date prior to T_1^{**}; the MGCS never delivers a cost advantage in this period for these countries.

The key to these results is that all countries enjoy the same benefits of lower aggregate emissions (as yielded by the MGCS relative to the NCE) because all countries suffer the same damage from emissions. However, the technology gap between the MGCS and the NCE at any date is *not* the same for all countries. This technology gap is increasing in economic size because the external cost imposed by a dirty technology is higher for a big economy than for a small one (because emissions are proportional to income for any given technology in place). Moreover, the marginal cost of cleaner technology is rising. In combination, these two effects mean that the cost difference between the MGCS technology and the NCE technology is higher for larger economies than for smaller ones, at every date.

Because the largest economies may not enjoy a cost advantage from the MGCS at every date means that in present value terms, they may be worse off overall in the MGCS relative to the NCE. While it is not possible to find closed-form solutions for present-value-costs in this model, the monotonicity of $\omega(y_i)$ in y_i dictates that small economies necessarily benefit more than larger economies from a switch to the MGCS from the NCE, and that some of the largest economies may not benefit at all.

6. CONCLUSION

The central message of this paper is that a pollution-spillover effect – arising when the pollutant is transboundary and damage is strictly convex – has a significant impact on the relationship between emissions and income growth. Countries that differ only with respect to their position in the global income distribution follow quite different paths over time. In particular, the *timing* of emissions growth-and-decline for an individual country relative to the evolution of aggregate emissions, is central to individual behavior. This means that the behavior of any individual country, at any point in time, depends on its relative position in the global income distribution.

Our results demonstrate that even a simple model based on scale and technique effects that would otherwise yield a conventional EKC can produce dynamics in which no two countries follow the same emissions path over time when the pollutant is transboundary. Thus, any relationship estimated from cross-section data might tell us very little about the emissions paths that individual countries will follow as their incomes grow over time. In particular, our results suggest that emerging economies will have higher emissions than more advanced economies did at the same stage of growth purely because these emerging economies have developed later.

Our analysis of the MGCS demonstrates that these cross-country differences in emissions paths also arise in the solution to a planning problem. Relative to the NCE, emissions in the MGCS are lower and peak sooner, but the strict convexity of the damage function still produces very different paths for early and late developers. This asymmetry also has interesting welfare implications: imposition of the MGCS by a global planner would provide greater net benefit to late developers than to early ones.

Our model abstracts from a number of important issues that often arise in real transboundary pollution problems. First and foremost, our model does not adequately capture important aspects of a stock-pollutant problem (like greenhouse gases or persistent organic pollutants). In particular, some types of strategic behavior can arise in a stock-pollutant setting that do not arise in our model, and these could produce dynamics effects that run counter to those we have identified. For example, the damaging impact from emissions added to a large existing stock may be much higher than for those added to a small stock (due to threshold effects), thereby creating for late developers a higher cost of emissions than early developers incurred at the same stage of growth. All other things equal, this would tend to dampen emissions from those late developers (in contradiction to our results).[5] It seems reasonable to suppose that this stock effect could outweigh the spillover effect we have identified if the pollutant is sufficiently long-lived and threshold effects are sufficiently strong.

We have also abstracted here from the possibility that growth rates are endogenous. More generally, our damage function is artificially simple: all countries suffer the same damage regardless of economic size, and future growth is unaffected by current damage. This facilitates a simple exposition of the cross-country interaction we highlight here but it is also a limiting aspect of the model. A generalized specification might treat emissions as a damaging public factor in production whose impact reduces net output, and consequently, reduces investment in growth-inducing capital. Such an effect would undoubtedly complicate the income-emissions paths and possibly change them drastically.

Richer models than the one we have presented would undoubtedly allow for a greater range of possibilities with respect to the income-emissions relationship, and could capture aspects of real pollution problems that we have ignored here. However, it seems safe to presume that our central point would continue to hold: when the pollutant is transboundary and the damage function is strictly convex, countries that are otherwise identical can follow very different emissions paths as they grow due purely to differences in the positions they occupy along the growth path.

[5] We are grateful to an anonymous referee for this point.

APPENDIX

Lemma 1

(a) If $y_i > 0 \ \ \forall i$ then there is an upper bound on σ^2, given by

(A1) $$\sigma_{\max}^2 = (n-1)\mu^2$$

(b) For any given μ and σ^2, there are lower and upper bounds on y_1, given by y_{1L} and y_{1H} respectively, where

(A2) $$y_{1L} = \mu + \frac{\sigma^2}{\mu}, \text{ and}$$

(A3) $$y_{1H} = \frac{\mu + (\mu^2 + 4n\sigma^2)^{1/2}}{2}$$

Proof.

(a) If $y_i > 0 \ \ \forall i$ then $\left(\sum_{i=1}^{n} y_i\right)^2 \geq \sum_{i=1}^{n} y_i^2$. Since $\sum_{i=1}^{n} y_i = n\mu$, it follows that $n\mu^2 \geq \sum_{i=1}^{n} y_i^2 / n$. But $\sum_{i=1}^{n} y_i^2 / n = \sigma^2 + \mu^2$. Thus, $n\mu^2 \geq \sigma^2 + \mu^2$ or $\sigma^2 \leq (n-1)\mu^2$.

(b) Let $\{y_1\}$ denote the set of possible values of y_1. Let y_{1L} denote the infimum of $\{y_1\}$, and set $y_{1l} = \mu + q$, where $q > 0$. Since $y_i > 0 \ \ \forall i$, y_{1L} occurs where $y_i = 0 \ \ \forall i > 1$. For this distribution,

(A4) $$\mu = \frac{\mu + q}{n}, \text{ and}$$

(A5) $$\sigma^2 = \frac{(n-1)\mu^2 + q^2}{n}$$

Solving (A4) and (A5) for q yields

(A6) $$q = \frac{\sigma^2}{\mu}$$

Thus,

(A7) $$y_{1L} = \mu + \frac{\sigma^2}{\mu}$$

Let y_{1H} denote the supremum of $\{y_1\}$. Since $y_i > 0 \ \ \forall i$, the smallest possible variance, given μ and $y_1 = y_{1H}$, occurs where $y_i = \mu$ for $i = 2,...,k$ and $y_i = 0$ for $i = k+1,...,n$, such that

(A8) $$\mu = \frac{(k-1)\mu + y_{1H}}{n}, \text{ and}$$

(A9) $$\sigma^2 = \frac{(n-k-1)\mu^2 + (y_{1H} - \mu)^2}{n}$$

Solving (A8) and (A9) for y_{1H} yields

(A10) $$y_{1H} = \frac{\mu + (\mu^2 + 4n\sigma^2)^{1/2}}{2}$$

Derivation of (9) and (10)

From (8) in the text, we obtain

(A11) $$z_i(t)[1 + \delta y_i(t)^2] = y_i(t) - \delta Z_{-i}(t) y_i(t)^2$$

Setting $Z_{-i}(t) = Z(t) - z_i(t)$, and collecting terms, yields

(A12) $$z_i(t) = y_i(t) - \delta Z(t) y_i(t)^2$$

Summing across *i* and solving for $Z(t)$ then yields equilibrium aggregate emissions:

(A13) $$Z^*(t) = \frac{\sum_{i=1}^{n} y_i(t)}{1 + \delta \sum_{i=1}^{n} y_i(t)^2} = \frac{n\mu_t}{1 + n\delta(\mu_t^2 + \sigma_t^2)}$$

This is expression (9) in the text when $N_t = n$. Setting $Z_{-i}(t) = Z(t) - z_i(t)$ in (A11) and solving for $z_i(t)$ yields

(A14) $$z_i^*(t) = y_i(t) - \delta\left(\frac{n\mu_t}{1 + n\delta(\mu_t^2 + \sigma_t^2)}\right) y_i(t)^2$$

This is expression (10) in the text.

Derivation of (11) and its Properties

Setting $z_i^*(t) = 0$ from (A14), and solving for *t* when $y_i = y_1$ yields

(A15) $$T_1^* = -\frac{1}{2r}\log\left(n\delta\mu(y_1 - y_{1L})\right)$$

This is expression (11) in the text. If country 1 has positive emissions at $t = 0$ then from (A14) we know that

(A16) $$y_1 - \delta\left(\frac{n\mu}{1+n\delta(\mu^2+\sigma^2)}\right)y_1^2 > 0$$

Using (A7), we can write this as

(A17) $$\delta\left(\frac{n\mu}{1+n\delta\mu y_{1L}}\right)y_1 < 1$$

which simplifies to

(A18) $$n\delta\mu(y_1 - y_{1L}) < 1$$

We know that $y_1 > y_{1L}$ when $\sigma^2 > 0$ (by Lemma 1), so the term inside the logarithm in (A15) is positive and less than one. Thus, $T_1^* > 0$. It is clearly decreasing in y_1.

Derivation of (13) and the Characterization of Individual-Country Turning Points

The turning point for aggregate emissions is found by differentiating (A13) with respect to *t*:

(A19) $$t^* = -\frac{1}{2r}\log(n\delta(\mu^2+\sigma^2))$$

This is expression (13) in the text.

Next consider individual-country turning points. Differentiating (A14) with respect to *t* yields the following potential turning points:

(A20) $$t_i^+ = \frac{1}{2r}\log\left(\frac{3y_i - 2y_{1L} + y_i^{1/2}(9y_i - 8y_{1L})^{1/2}}{2n\delta\mu(y_{1L} - y_i)y_{1L}}\right)$$

and

(A21) $$t_i^* = \frac{1}{2r}\log\left(\frac{3y_i - 2y_{1L} - y_i^{1/2}(9y_i - 8y_{1L})^{1/2}}{2n\delta\mu(y_{1L} - y_i)y_{1L}}\right)$$

The first of these roots is real and finite if and only if $y_i \geq 8y_{1L}/9$ *and* either

(A22) $$y_i < y_{1L} \text{ and } 3y_i - 2y_{1L} + y_i^{1/2}(9y_i - 8y_{1L})^{1/2} > 0$$

or

(A23) $$y_i > y_{1L} \text{ and } 3y_i - 2y_{1L} + y_i^{1/2}(9y_i - 8y_{1L})^{1/2} < 0$$

The conditions in (A22) are mutually implied but (A23) is a contradiction. Thus, t_i^+ is real and finite if and only if

(A24) $$8y_{1L}/9 \leq y_i < y_{1L}$$

The second root is real and finite for any $y_i \geq 8y_{1L}/9$, and at $y_i = y_{1L}$ it reduces to

(A25) $$t^* = -\frac{1}{2r}\log(n\delta(\mu^2 + \sigma^2))$$

Moreover, if t_i^* is real and finite then $\partial^2 z_i(t_i^*)/\partial t_i^2 < 0$ (and hence a local maximum), and if t_i^+ is real and finite then $\partial^2 z_i(t_i^+)/\partial t_i^2 > 0$ (and hence a local minimum). If both roots are real and finite then $t_i^* < t_i^+$.

Now compare these turning point dates with T_1^* from (A15). Setting $t_i^* = T_1$ and solving for y_i yields

(A26) $$\tilde{y}^* = \frac{y_1^2}{3y_1 - 2y_{1L}}$$

where $t_i^* > T_1^*$ for $y_i < \tilde{y}^*$, and $t_i^* < T_1^*$ for $y_i > \tilde{y}^*$. This critical income level is plotted against y_1 for $y_1 \geq 4y_{1L}/3$ as the upper curve in Figure A1, where y_1 is plotted on the *vertical* axis. The horizontal axis partitions the income distribution into key regions (as in Figures 1 – 3 from the text).

Setting $t_i^+ = T_1$ and solving for y_i yields

(A27) $$\tilde{y}^+ = \frac{y_1^2}{3y_1 - 2y_{1L}}$$

where $t_i^+ > T_1^*$ for $y_i > \tilde{y}^*$, and $t_i^+ < T_1^*$ for $y_i < \tilde{y}^*$. This critical income level is plotted against y_1 for $y_1 \leq 4y_{1L}/3$ as the lower curve in Figure A1.

Note from (A26) and (A27) that the two curves plotted in Figure A1 are simply two parts of the same function, identified as equation (12) in the text, but its interpretation depends on whether it is evaluated at values of y_1 above or below $4y_{1L}/3$. This distinguishes Case 1 from Case 3.

Case 1 applies to the region at and above $y_1 = 4y_{1L}/3$ but below $y_1 = 2y_{1L}$. Countries with incomes higher than $\tilde{y}$ in this case are those to the right of the upper curve, and these countries reach an emissions peak at $t_i^* < T_1^*$. For these countries, either $t_i^+ \geq T_1^*$ (if $y_i \leq y_{1L}$) or t_i^+ is not real (if $y_i > y_{1L}$). Countries with incomes below $\tilde{y}$ are those to the left of the upper curve, and these countries do not reach a turning point before T_1^*.

Case 3 applies to the region above $y_1 = y_{1L}$ but below $y_1 = 4y_{1L}/3$. (Recall that y_{1L} is the lower bound on y_1). Countries with incomes higher than $\tilde{y}$ in this case are those to the right of the lower curve, and these countries reach an emissions peak at $t_i^* < T_1^*$. Countries with incomes below $\tilde{y}$ are those to the left of the lower curve. For those countries with incomes below $\tilde{y}$ but above $8y_{1L}/9$, $t_i^* < t_i^+ < T_1$; these countries reach both turning points during period 1. Countries with incomes below $8y_{1L}/9$ do not reach a turning point before T_1^*.

Case 2 applies to the region at and above $y_1 = 2y_{1L}$. This case is essentially identical to Case 1 except that $\tilde{y} \geq y_{1L}$, and $t_i^* > T_1^*$; that is, aggregate emissions do not reach a turning point before T_1^*.

Emissions and Income at the Turning Points in Case 1 – 3

Case 1: $4y_{1L}/3 \leq y_1 < 2y_{1L}$

Setting $t = t_i^*$ in (A14) yields

(A28) $$z_i^*(t_i^*) = \frac{y_i(3y_i - \theta_i)}{2y_{1L}(\theta_i - y_i)}\left(\frac{2(3y_i - 2y_{1L} + \theta_i)(y_i - y_{1L})}{n\delta\mu y_{1L}}\right)^{1/2}$$

where

(A29) $$\theta_i = y_i^{1/2}(9y_i - 8y_{1L})^{1/2}$$

It is straightforward to show that $z_i^*(t_i^*)$ reaches a minimum at $y_i = y_{1L}$, as depicted in Figure 2.

Emissions for country *i* at $t = T_1^*$ are given by

(A30) $$z_i^*(T_1^*) = \frac{y_i(y_1 - y_i)}{y_1\left(n\delta\mu(y_1 - y_{1L})\right)^{1/2}}$$

This is the dashed curve in Figure 2. Differentiation of (A30) with respect to y_i shows that $z_i^*(T_1^*)$ reaches a maximum at $y_i = y_1/2$. Evaluating $z_i^*(T_1^*)$ at this income level yields

(A31) $$\hat{z}(T_1^*) = \frac{y_1}{4\left(n\delta\mu(y_1 - y_{1L})\right)^{1/2}}$$

It is then straightforward to show that $\hat{z}(T_1^*) \geq z_1^*(t_1^*)$ if and only if $y_1 \leq 2y_{1L}$. Thus, in Case 1, $\hat{z}(T_1^*)$ exceeds peak emissions for all countries in region 2, as depicted in Figure 2.

Income at peak emissions is found by evaluating $y_i(t)$ at t_i^*:

(A32) $$y_i(t_i^*) = \frac{y_i}{2}\left(\frac{2(2y_{1L} - 3y_i + \theta_i)}{n\delta\mu y_{1L}(y_i - y_{1L})}\right)^{1/2}$$

It is straightforward to show that $y_i(t_i^*)$ reaches a minimum at $y_i = 4y_{1L}/3$, as depicted in Figure 3.

Case 2: $y_1 \geq 2y_1$

In this case $T_1^* < t^*$. The dynamics are summarized in Figure A3. There are two noteworthy differences between these dynamics and those of Case 1. First, region 2A in Figures 1 and 2 vanishes in this case because $\tilde{y} > y_{1L}$ when $y_1 > 2y_{1L}$. This means that peak emissions are increasing in y_i for all countries in region 2. Second, if y_1 is large enough then region 2B-1 in Figure 1 also vanishes, in which case peak-emissions income is increasing in y_i for all countries in region 2; that is, region 2B-2 in Figure 3 is the only remaining component of region 2.

Case 3: $y_1 < 4y_{1L}/3$

The dynamics of this case are summarized in Figure A3. There are two key differences between this case and Case 1. First, the turning point in $y_i(t_i^*)$ depicted in Figure 3 no longer occurs; the peak-emissions income is declining in y_i for *all* countries in region 2 (and so region 2B-2 vanishes). Second, a new sub-region arises in region 1, labeled region 1B in Figure A2. Countries in this region – who in Case 1 did not reach an emissions peak in period 1 – now have *two* turning points in their emissions paths during period 1: a local maximum a date t_i^*, and a local minimum at a later date $t_i^+ > t_i^*$.

Equilibrium in Later Periods

The second period begins at $t = T_1^*$. The remaining $n-1$ countries with positive emissions start this second stage with incomes equal to $y_2 e^{rT_1^*} \geq ... \geq y_n e^{rT_1^*}$. The mean and variance of these incomes is $\mu_2 e^{rT_1^*}$ and $\sigma_2^2 e^{2rT_1^*}$ respectively, where

(A33) $$\mu_2 = \frac{n\mu - y_1}{n-1}$$, and

(A34) $$\sigma_2^2 = \frac{n(n-1)\sigma^2 - n\mu^2 + 2n\mu y_1 - ny_1^2}{(n-1)^2},$$

and the lower bound on the income of country 2 – the largest country among the remaining $n-1$ emitting countries – is $y_{2L}e^{rT_1^*}$, where

(A35) $$y_{2L} = \mu_2 + \frac{\sigma_2^2}{\mu_2}$$

The equilibrium in this period is as described for the first period but with $\{n, \mu, \sigma^2, y_{1L}\}$ replaced by $\{n-1, \mu_2, \sigma_2^2, y_{2L}\}$. The emissions paths for the remaining $n-1$ emitting countries are continuous at $t = T_1^*$, but they are not differentiable at that point.

The results from the first and second periods generalize in a straightforward way to subsequent periods. In particular, period $s+1$ begins at $t = T_S^*$, where

(A36) $$T_S^* = \frac{1}{2r}\log\left(\frac{1}{n\delta\mu_S(y_S - y_{SL})}\right),$$

(A37) $$y_{SL} = \mu_S + \frac{\sigma_S^2}{\mu_S},$$

(A38) $$\mu_S = \frac{n\mu - \sum_{j=1}^{S-1} y_j}{n-s+1}$$, and

(A39) $$\sigma_S^2 = \frac{n^2\sigma^2 - n(s-1)(\mu^2 + \sigma^2) + 2n\mu\sum_{j=1}^{S-1} y_j - (n-s+1)\sum_{j=1}^{S-1} y_j^2 - \left[\sum_{j=1}^{S-1} y_j\right]^2}{(n-s+1)^2}$$

It is important to note that the behavior of the economy in any period depends on the entire *initial* income distribution since this determines the evolution of the mean and variance among incomes of the remaining emitters in each period, as well as the size of the largest remaining emitter in each period. However, for any given list of initial incomes, the results for period 1, and their generalizations using equations (A36) – (A39), fully describe the equilibrium.

The Minimum Global Cost Solution (MGCS)

Setting the derivative of (14) with respect to $a_i(t)$ equal to zero and solving for $a_i(t)$ yields

(A40) $$a_i(t) = n\delta Z(t) y_i(t)$$

The corresponding level of emissions is

(A41) $$z_i(t) = y_i(t) - n\delta Z(t) y_i(t)^2$$

Note that $a_i(t)$ is increasing in $y_i(t)$, and could be greater than one for $y_i(t)$ sufficiently large. The MGCS requires zero emissions for these large countries. We confine consideration to the period in which all countries have positive emissions. Thus, summing across *i* in (A41) and solving for $Z(t)$ yields

(A42) $$Z^{**}(t) = \frac{n\mu_t}{1 + \delta n^2 (\mu_t^2 + \sigma_t^2)}$$

This is expression (15) in the text. Substituting (A42) into (A41) yields expression (16) in the text.

Turning Points in the MGCS Paths

Setting the derivative of (A41) with respect to t equal to zero and solving for t yields

(A43) $$t_i^{++} = \frac{1}{2r}\log\left(\frac{3y_i - 2y_{1L} + y_i^{1/2}(9y_i - 8y_{1L})^{1/2}}{2n^2\delta\mu(y_{1L} - y_i)y_{1L}}\right) = t_i^{+} - \lambda(n,r)$$

and

(A44) $$t_i^{**} = \frac{1}{2r}\log\left(\frac{3y_i - 2y_{1L} - y_i^{1/2}(9y_i - 8y_{1L})^{1/2}}{2n^2\delta\mu(y_{1L} - y_i)y_{1L}}\right) = t_i^{*} - \lambda(n,r)$$

Note that these turning points for the MGCS are earlier than those for the NCE (see expressions (A20) and (A21) above) by an amount exactly equal to the non-cooperative lag.

The turning point for aggregate emissions is found by setting the derivative of (A42) with respect to *t* equal to zero and solving for *t*:

(A45) $$t^{**} = -\frac{1}{2r}\log(\delta n^2(\mu^2 + \sigma^2))$$

From (A45) and (A19) it is clear that $t^{**} = t^{*} - \lambda(n,r)$.

Emissions at the Turning Points in the MGCS Paths (Case 1)

In Case 1 ($4y_{1L}/3 < y_1 < 2y_{1L}$) there is only one turning point in the emissions path for county *i*, at t_i^{**} (the second root, at t_i^{++}, is not real in this case). Setting $t = t_i^{**}$ in (16) yields

(A46) $$z_i^{**}(t_i^{**}) = \frac{y_i(3y_i - \theta_i)}{2y_{1L}(\theta_i - y_i)}\left(\frac{2(3y_i - 2y_{1L} + \theta_i)(y_i - y_{1L})}{n^2\delta\mu y_{1L}}\right)^{1/2}$$

where θ_i is given by (A29). Compare (A46) with (A28) to see that $z_i^{**}(t_i^{**}) = z_i^*(t_i^*)/\sqrt{n}$.

Similarly, emissions for country *i* at $t = T_1^{**}$ are given by

(A47) $$z_i^{**}(T_1^{**}) = \frac{y_i(y_1 - y_i)}{y_1\left(n^2\delta\mu(y_1 - y_{1L})\right)^{1/2}}$$

Compare (A47) with (A30) to see that $z_i^{**}(T_1^{**}) = z_i^*(T_1^*)/\sqrt{n}$. Substituting (A19) into (A13) yields peak-aggregate-emissions in the NCE:

(A47) $$Z^*(t^*) = \frac{\mu}{2}\left(\frac{n}{\delta(\mu^2 + \sigma^2)}\right)^{1/2}$$

In comparison, substituting (A45) into (A42) yields peak-aggregate-emissions in the MGCS:

(A48) $$Z^{**}(t^{**}) = \frac{\mu}{2}\left(\frac{1}{\delta(\mu^2 + \sigma^2)}\right)^{1/2}$$

Clearly, $Z^{**}(t^{**}) = Z^*(t^*)/\sqrt{n}$.

Thus, for individual-country emissions at an interior turning point and at T_1^{**}, and for aggregate emissions at its turning point, emissions in the MGCS are simply a fraction $1/\sqrt{n}$ of the corresponding emissions levels in the NCE. The relationship between these emissions levels and income therefore follows the same pattern as depicted in Figure 2 but with an appropriate scaling of the vertical axis.

Individual-Country Costs in the NCE and the MGCS

Domestic cost for country *i* at date *t* is

(A49) $$c_i(t) = a_i(t)^2 + \delta Z(t)^2$$

We compare this cost under the NCE and MGCS scenarios.

Evaluating (A49) at the NCE values yields

(A50) $$c_i^*(t) = \frac{\delta n^2 \mu_t^2 (1+\delta y_i(t)^2)}{(1+\delta n(\mu_t^2 + \sigma_t^2))^2}$$

Setting the derivative of (A50) with respect to *t* equal to zero and solving for the turning point date in terms of initial values yields

(A51) $$\tau_i^* = -\frac{1}{2r}\log(\delta(n(\mu^2+\sigma^2) - 2y_i^2))$$

This turning point is real for any $y_i < 2y_{1L}$ and so it is real for all y_i in Case 1 (where $y_1 < 2y_{1L}$). The turning point occurs prior to T_1^* if and only if $y_i < \bar{y}$ as reported in expression (19) in the text. It is clear from (A51) that τ_i^* is increasing in y_i; smaller economies reach a turning point earlier than larger ones in the NCE.

Evaluating (A49) at the MGCS values yields

(A52) $$c_i^{**}(t) = \frac{\delta n^2 \mu_t^2 (1+n^2\delta y_i(t)^2)}{(1+\delta n^2(\mu_t^2 + \sigma_t^2))^2}$$

Setting the derivative of (A52) with respect to *t* equal to zero and solving for the turning point date in terms of initial values yields

(A53) $$\tau_i^{**} = -\frac{1}{2r}\log(n^2\delta((\mu^2+\sigma^2) - 2y_i^2))$$

This turning point is real for all y_i in Case 1. The turning point occurs prior to T_1^{**} if and only if $y_i < \bar{y}/\sqrt{n}$. It is clear from (A53) that τ_i^{**} is increasing in y_i; smaller economies reach a turning point earlier than larger ones in the MGCS.

Now compare $c_i^*(t)$ and $c_i^{**}(t)$. Setting $c_i^*(t) = c_i^{**}(t)$ and solving for *t*, it is straightforward to show that $c_i^*(t) > c_i^{**}(t)$ if and only if *t* is less than

(A54) $$\omega(y_i) = -\frac{1}{2r}\log\left(\frac{(n+1)y_i^2 - 2n(\mu^2+\sigma^2)}{n^2\delta(\mu^2+\sigma^2)((n+1)(\mu^2+\sigma^2) - 2y_i^2)}\right)$$

This threshold date is negative for all $y_i < y_{low}$, where y_{low} is reported in expression (20) in the text. For $y_i < y_{low}$, $c_i^*(t) > c_i^{**}(t)$ for all $t > 0$. Conversely, $\omega(y_i) \to \infty$ as $y_i \to y_{high}$, where y_{high} is reported in expression (21) in the text. For $y_i \geq y_{high}$, $c_i^*(t) < c_i^{**}(t)$ for all finite $t > 0$. For intermediate values ($y_{low} < y_i < y_{high}$), $\omega(y_i)$ is finite and positive, and increasing in y_i, as illustrated in Figure 4.

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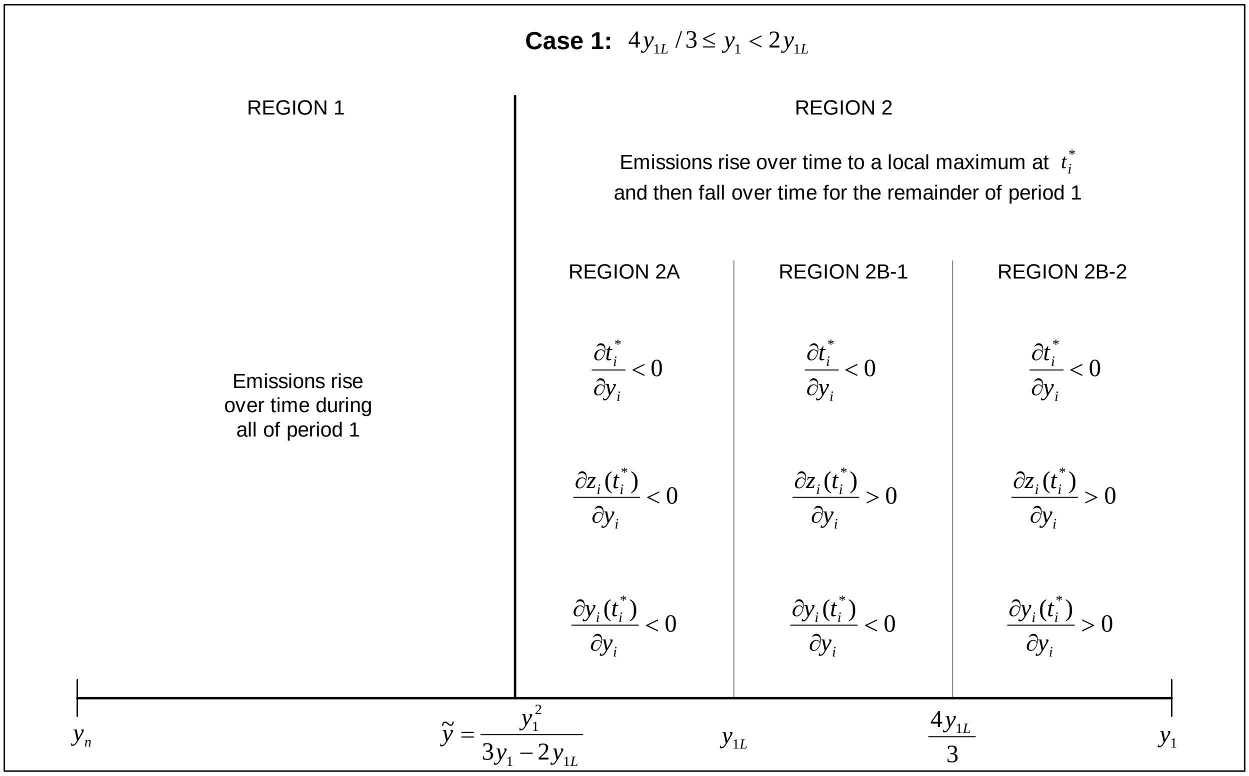

FIGURE 1: Properties of the Equilibrium (Case 1)

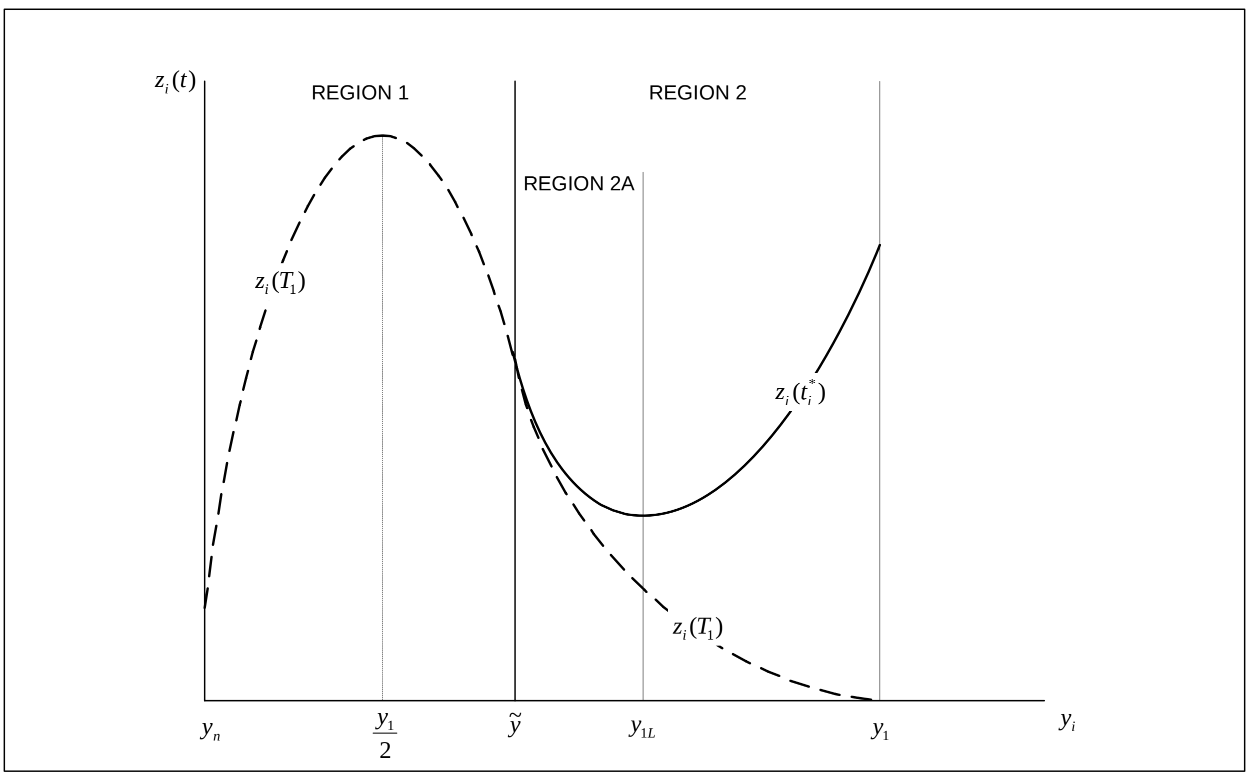

FIGURE 2: Peak-Emissions during Period 1 (Case 1)

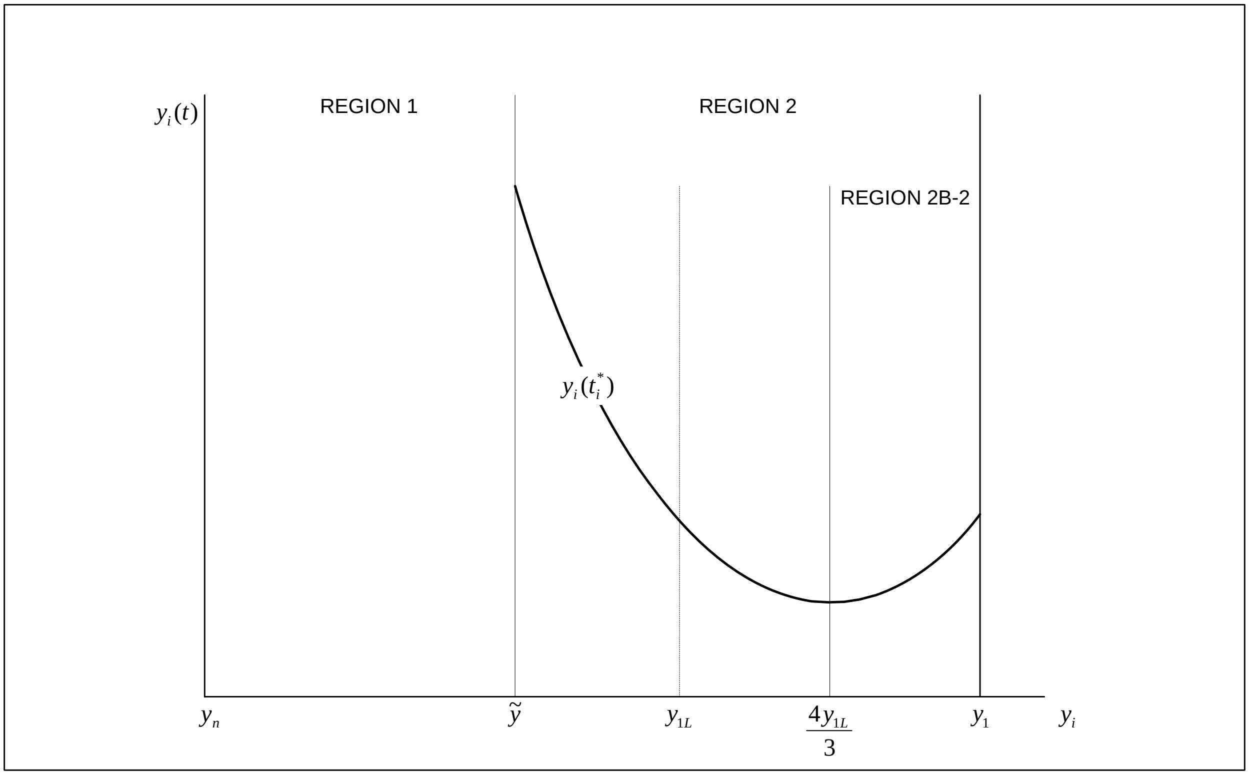

FIGURE 3: Income at Peak-Emissions during Period 1 (Case 1)

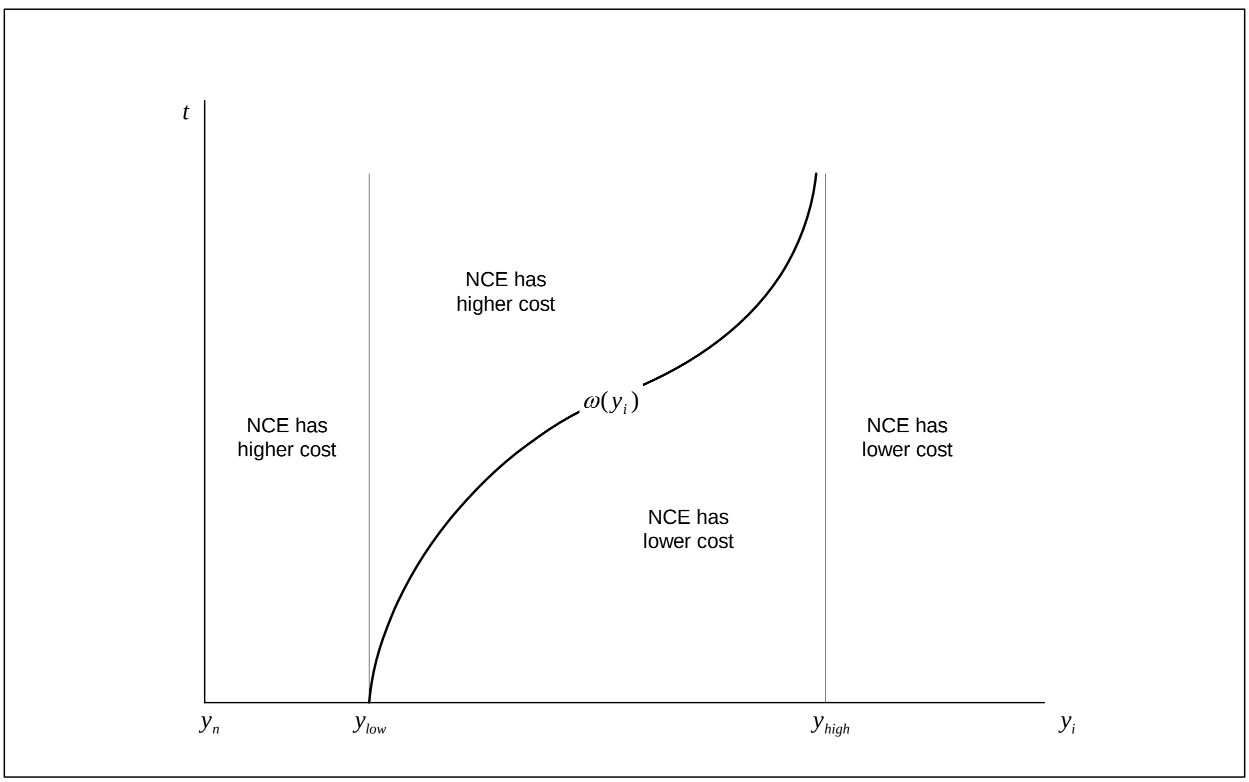

FIGURE 4: Cost Comparisons over Time (Case 1)

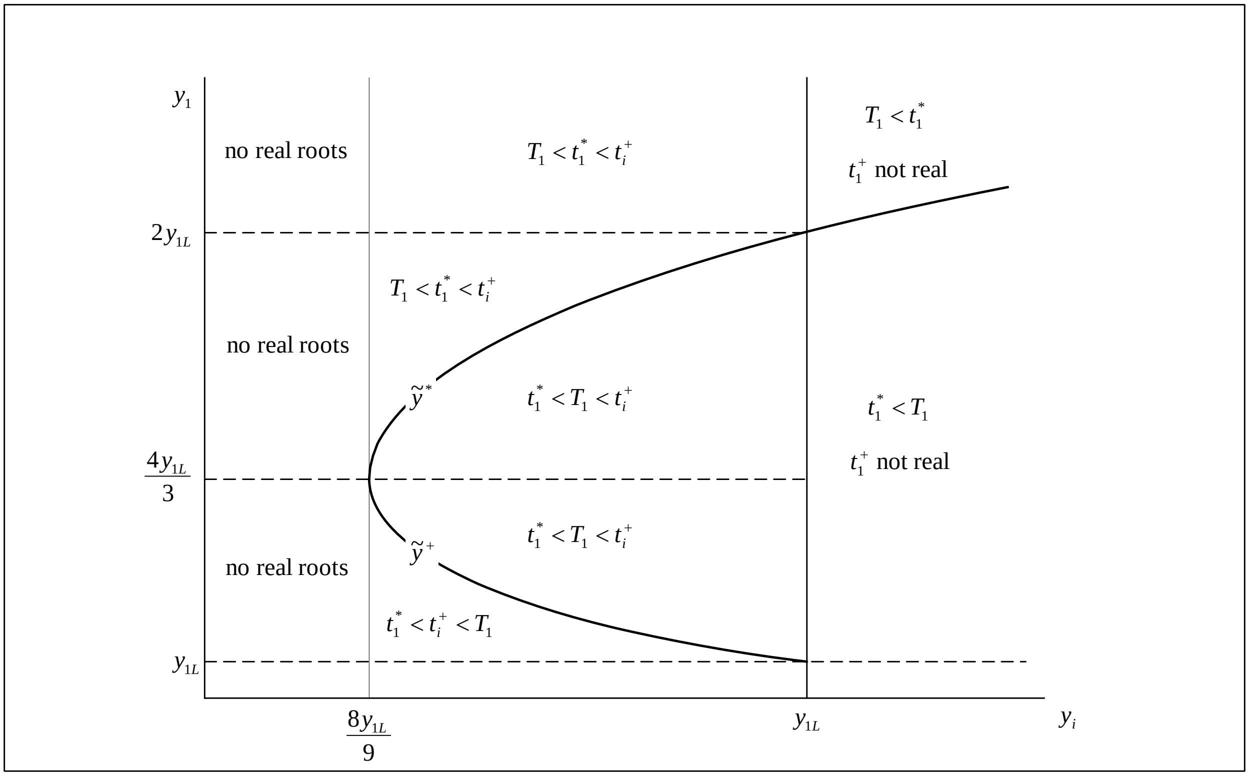

FIGURE A1: Turning Points in the Emissions Paths

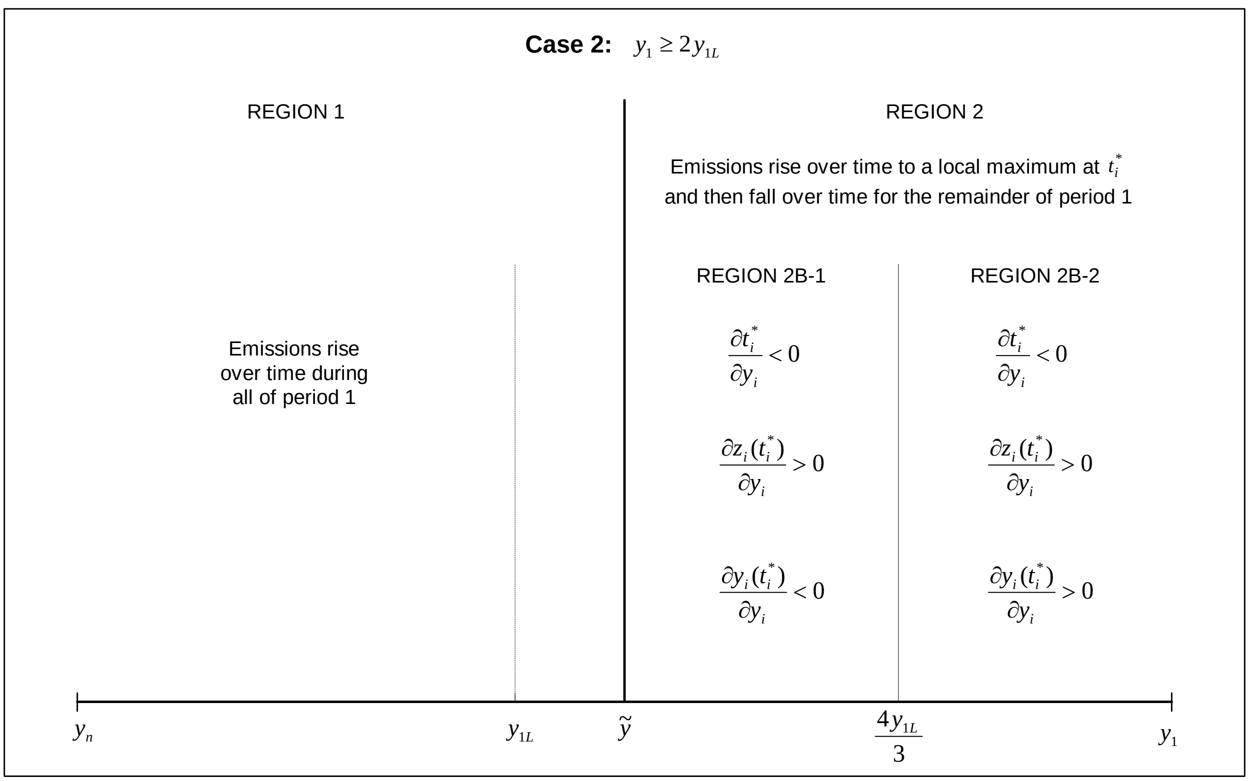

FIGURE A2: Properties of the Equilibrium in Case 2

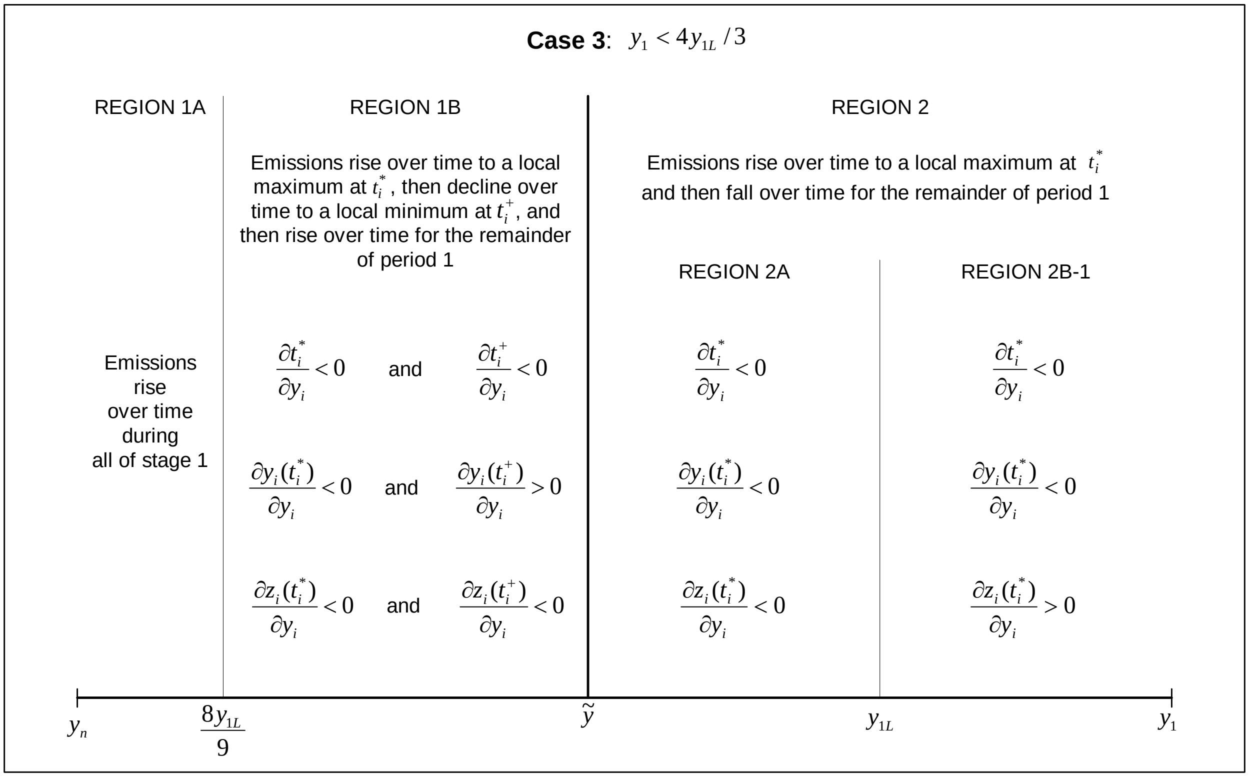

FIGURE A3: Properties of the Equilibrium in Case 3