COOPERATIVE ACTION ON GREENHOUSE GAS EMISSIONS AND THE DISTRIBUTION OF GLOBAL OUTPUT AND DAMAGE

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ABSTRACT

This paper compares the performance of two allowance allocation rules in an international climate change treaty. I construct a model in which countries differ according to both GDP and an idiosyncratic damage parameter that links global emissions to damage for an individual country. Allowances are allocated to treaty members according to an allocation rule based on a single allocation parameter. The model can be solved analytically to determine upper and lower bounds on this allocation parameter that ensure that a treaty of any given composition is internally and externally stable. I focus on grand coalition treaties. The first treaty examined uses a simple proportional rule in which allocations are set as some fraction of emissions in the non-cooperative equilibrium. The second treaty adds a coverage-contingent element to the allocation rule such that the emissions reduction required of each treaty member is weighted by the fraction of global emissions that treaty members as a whole emit in the non-cooperative equilibrium. I show that the coverage-contingent treaty outperforms the simpler treaty except when all countries have the same emissions intensity in the non-cooperative equilibrium (in which case neither treaty can reduce emissions).

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1. INTRODUCTION

Negotiations to date towards a global treaty on climate change have been largely unsuccessful. The Kyoto Protocol of 1997 required no reductions from China, and the U.S. did not ratify the treaty. It thereby left out the two biggest emitters of greenhouse gases; together these two countries currently account for around 45% of global emissions.[1] Subsequent rounds of negotiations have not yet produced a viable successor to the Kyoto treaty.

This failure to strike a meaningful treaty is depressingly consistent with predictions from theory. The earliest work on international environmental treaties identified a fundamental free-rider problem with treaty formation: each country has an incentive to remain outside the treaty and instead free-ride on the abatement undertaken by treaty members [Hoel (1992), Carraro and Siniscalco (1993) and Barrett (1994)].

Subsequent work has shown that a system of transfers among treaty members can dramatically improve the prospects for a treaty when countries differ in terms of their net benefits from the collective action. [Barrett (1992), Hoel and Schneider (1997) and Barrett (2001)]. The simplest transfer system involves initial allowance allocations, coupled with allowance trading. Countries who purchase additional allowances transfer wealth to the countries who sell those allowances. These transfers effectively share the cooperative gains among treaty members, and thereby create stronger incentives for countries to join the treaty. Nonetheless, the free-rider problem cannot be eliminated entirely.

In this paper I propose an allowance allocation rule that further reduces the incentive to free-ride by including a *coverage-contingent* element to allowance allocations. In particular, the emissions reduction required of each treaty member is weighted by the fraction of global emissions that treaty members as a whole emit in the non-cooperative equilibrium. This coverage-contingent rule reduces the incentive for countries to remain outside the treaty and free-ride on reductions undertaken by treaty members. The incentive effect is especially strong for large existing emitters.

I examine this coverage-contingent rule in the context of a model in which countries differ according to both GDP and an idiosyncratic damage parameter that links global emissions to damage for an individual country on the basis of its vulnerability to climate change (as

[1] Based on emissions data from *World Resources Institute, Climate Analysis Indicators Tool (Version 9), 2012*.

determined for example by its average elevation or its dependence on agriculture). The distribution of these country-characteristics is unrestricted.

Treaty allowances are determined by an exogenous rule with a single endogenous parameter that allocates allowances to a member country on the basis of some verifiable characteristic of that country (such as current emissions). The same rule applies to all treaty members, and a country either accepts the allocation parameter or remains outside the treaty. Member countries are free to trade allowances among themselves once a treaty is struck. The model can be solved analytically to determine upper and lower bounds on this allocation parameter that ensure that a treaty of any given composition is internally and externally stable.

I compare the coverage-contingent allocation rule with a simple proportional rule in which allowances for each country are set as a fraction of its emissions in the non-cooperative equilibrium. The coverage-contingent rule makes that fraction contingent on the coverage of the treaty. I focus on grand-coalition treaties so that the two rules are directly comparable, and show that the coverage-contingent rule outperforms the simpler rule in terms of the reduction in global emissions achieved, except when all countries have the same emissions intensity in the non-cooperative equilibrium (in which case there is no emissions trading, and neither treaty can reduce global emissions). Results from a calibrated Monte Carlo experiment suggest that the performance advantage of the coverage-contingent treaty could be large.

The rest of the paper is organized as follows. Section 2 situates my work in the existing literature on international environmental treaties. Section 3 describes the analytical model, and Section 4 characterizes the non-cooperative equilibrium. Section 5 derives analytical results on treaty composition for a general class of single-parameter allowance-allocation rules (SPAARs). Section 6 then compares treaty outcomes under two specific rules: the simple proportional rule, and the coverage-contingent rule. Section 7 provides some concluding remarks. An Appendix contains proofs and derivations not presented in the main text.

2. RELATED LITERATURE

There are two broad branches to the literature on international environmental treaties. One is rooted in cooperative game theory, while the second is rooted in non-cooperative games. Most recent work in the area has focused on the non-cooperative game-theoretic approach, and my paper does the same.

The standard non-cooperative model involves an open-membership, single coalition game. This means that the game between countries involves a single coalition of member countries, and a group of non-member countries who act independently and non-cooperatively. Membership of the coalition is open to any country willing to meet the abatement requirements specified by the treaty. The game comprises two stages. In the first stage each country decides whether or not to join the treaty. The second stage involves abatement actions by coalition members and non-members. My analysis employs this same standard approach.

The equilibrium requirement is subgame perfection: each country correctly anticipates the equilibrium in the second stage when deciding whether or not to join the treaty in the first stage. If a coalition exists in equilibrium then it must exhibit internal and external stability, as proposed in the context of cartel formation by d'Aspremont *et. al.* (1983): no member of the coalition wishes to leave it unilaterally, and no non-member wishes to join it unilaterally.

In a setting with asymmetric countries, the stability of a coalition depends critically on how the gains from cooperation are shared among its members via transfers within the coalition. There are two main approaches to constructing these transfers: outcome-based schemes, and allocation-based schemes [Rose *et. al.* (1998)]. Under an outcome-based scheme, each coalition member receives the payoff they would realize as a non-member, plus a share of the remaining surplus, as determined by a specified sharing rule. The stability of any candidate coalition is sensitive to the choice of sharing rule, and some rules perform better than others in simulations and laboratory experiments [Carraro *et. al.* (2006), McGinty (2007), Nagashima *et. al.* (2009), McGinty *et. al.* (2012)].

Allocation-based schemes implement transfers via the allocation of tradable emissions allowances [Barrett (1992)]. As noted in the introduction, these schemes transfer wealth from allowance-buyers to allowance-sellers within the coalition. Allocation-based schemes have attracted most attention in actual treaty negotiations because they are easily understood and simple to define. It is this connection to real-world negotiations that motivates my focus on these schemes in this paper.[2]

Allocation-based transfer schemes can use a variety of rules for determining the initial allocation of allowances, and different rules typically yield different outcomes with respect to the

[2] It should be noted that more sophisticated outcome-based schemes can support treaties that may Pareto-dominate treaties supported by allocation-based schemes [Nagashima *et. al.* (2009)].

set of stable coalitions when studied in simulation models [Weikard *et. al.* (2006), Altamirano-Cabrera and Finus (2006) and Nagashima *et. al.* (2009)]. Altamirano-Cabrera and Finus (2006) distinguish between "pragmatic schemes" based on some baseline level of historical emissions, and "equitable schemes" that are based on alternative benchmarks such as population or *per-capita* GDP. Treaty negotiations in practice have been framed in terms of "pragmatic schemes", and with good reason. Allocations based on historical emissions might be inequitable by some argument, but schemes that depart too drastically from *status quo* emissions-shares implicitly embody transfers that are simply too large for big historical emitters to tolerate in a stable coalition.[3] The two allocation rules I examine are pragmatic ones: they are based on non-cooperative equilibrium emissions levels.

3. THE MODEL

Let $y_i > 0$ denote the economic output of country *i* (as measured by its GDP). Aggregate output is $Y = \sum_{i=1}^{n} y_i$, where *n* is the number of countries. Emissions from country *i* are denoted z_i . These emissions are a function of y_i and the production-technology used in country *i*, denoted $x_i \in [0,1]$:

(1) $$z_i = (1 - x_i) y_i$$

This technology is determined endogenously via domestic policy, and varies across countries in equilibrium.[4] The cost of producing output y_i using technology x_i is

(2) $$c(y_i, x_i) = x_i^2 y_i$$

Thus, production cost is increasing and strictly convex in the cleanliness of the technology used.

Global emissions are denoted

(3) $$Z = \sum_{i=1}^{n} z_i$$

and the associated damage to country *i* is

(4) $$D(\delta_i, y_i, Z) = \delta_i y_i Z^2$$

[3] For example, in a simulation model with 12 regions (and hence 4084 possible non-singleton coalitions), Altamirano-Cabrera and Finus (2006) find stable coalitions only when pragmatic schemes are used.
[4] It should be noted that in the absence of emissions-related damage, all countries would choose $x = 0$, and therefore have the same emissions intensity. This is a strong assumption; it ignores other important country-specific determinants of emissions intensity, such as natural capital endowments and historical factors.

where $\delta_i > 0$ is the "damage parameter" for country *i*.

This specification of the damage function has a number of important properties. First, damage is strictly convex in global emissions. This is a fairly standard assumption in the literature though it should be noted that climate change is ideally modeled as a stock pollutant problem. The key implication of strict convexity is that emissions are strategic substitutes in the non-cooperative game between countries. This in turn means that abatement by any coalition of countries will induce an increase in emissions from countries outside the coalition.

Second, climate-related damage to any country is linear in its GDP. This reflects the fact that damage rises with the scale of economic activity affected by adverse climate-related events. For example, crop losses from a prolonged drought of any given severity are more-or-less proportional to the size of the crop affected. Similarly, extreme weather events that disrupt power grids and transportation links have larger *absolute* impacts when they affect a larger scale of industrial activity.

Third, some countries are more vulnerable to climate-related damage than others, due to factors related to geography and economic composition (especially with respect to the relative importance of agriculture). Hence, two countries with the same GDP could suffer very different damages, as reflected by differences in their idiosyncratic δ values.

To capture these two determinants of damage jointly, it will prove useful to define a summary variable $v_i = \delta_i y_i$, henceforth referred to as the "vulnerability-weighted output" (VWO) for country *i*. It will also prove useful to define

(5) $$S = \sum_{i=1}^{n} \delta_i y_i^2$$

and

(6) $$\tilde{v} = \frac{S}{Y} = \sum_{i=1}^{n} w_i v_i$$

where $w_i = y_i / Y$ is the share of global GDP for country i. This weighted average of the VWOs will be a pivotal parameter in some results.

4. THE NON-COOPERATIVE EQUILIBRIUM

In the non-cooperative equilibrium (NCE), the policy-maker in country *i* chooses a technology that minimizes the sum of domestic production cost and domestic damage:

$$(7) \qquad \min_{x_i} \; x_i^2 y_i + \delta_i y_i [(1-x_i) y_i + Z_{-i}]^2$$

where Z_{-i} denotes aggregate emissions from all countries other than country *i*. Solving (7) yields a technology choice for country *i* as a function of global emissions:

$$(8) \qquad x_i(Z) = \delta_i y_i Z$$

This in turn yields a best-response function in terms of domestic emissions:

$$(9) \qquad z_i(Z_{-i}) = \frac{y_i - \delta_i y_i^2 Z_{-i}}{1 + \delta_i y_i^2}$$

Note that emissions for country *i* are declining in Z_{-i}; that is, emissions are strategic substitutes. As noted earlier, this property of the best-response function stems directly from the strict convexity of the damage function in aggregate emissions.

Setting $Z_{-i} = Z - z_i$ in (9), summing across *i*, and solving for *Z* yields equilibrium global emissions (see the Appendix):

$$(10) \qquad Z^* = \frac{Y}{1+S}$$

Upon substitution of (10) into (8) we then obtain the equilibrium technology for country *i*:

$$(11) \qquad x_i^* = \delta_i y_i \left(\frac{Y}{1+S} \right)$$

where the bracketed term is the same for all countries.

Expression (11) tells us that countries with higher GDPs and higher damage parameters adopt cleaner technologies in equilibrium. The reasoning behind the role of δ_i is straightforward: greater vulnerability to damage motivates a lower level of emissions for any given GDP.

The role of y_i is more nuanced. On one hand, emissions cause greater absolute damage for a high-GDP country, so y_i plays a role similar to δ_i. On the other hand, the aggregate cost of production, using any given technology, is also rising in y_i. These two forces are mutually offsetting in this model. In particular, inspection of (7) reveals that y_i can be taken outside the maximand, leaving only its role in the emissions function as a determinant of the technology choice. That remaining role of y_i arises because higher-GDP countries generate more emissions for any given technology used. Critically, this means that higher-GDP countries have *greater control over global emissions* – and hence, over the environmental damage they suffer – than do smaller-GDP countries. This in turn means that higher-GDP countries have more incentive to adopt the technologies required to reduce those emissions.

Substitution of (11) into (1) yields equilibrium emissions for country *i*:

$$(12) \qquad z_i^* = y_i - \left(\frac{Y}{1+S}\right)\delta_i y_i^2$$

In the special case where δ_i is the same for all countries, this solution for equilibrium emissions yields a type of environmental Kuznets curve (EKC): an inverted U-shaped relationship between output and emissions. In general, an EKC reflects the opposing forces of *scale* and *technique* effects as output rises. The scale effect is driven by simple arithmetic: emissions rise as output rises for any given technology. The technique effect captures the subtler notion that countries adopt increasingly cleaner technologies as output grows.[5] In this model, the technique effect derives from the fact that higher-GDP countries have greater control over global emissions than do smaller-GDP countries, and hence have more incentive to adopt cleaner technologies. On balance, these two effects produce an inverted U-shaped relationship between output and emissions, but only when δ_i is the same for all countries. If δ_i is *not* the same for all countries – as seems likely in reality – then this simple quadratic relationship between output and emissions no longer holds.

5. THE COMPOSITION OF A STABLE "SPAAR" TREATY

I examine an open-membership, single coalition game. This means that the game between countries involves a single coalition of treaty members, and a group of non-member countries who act independently and non-cooperatively. Membership of the coalition is open to any country willing to meet the abatement requirements specified by the treaty.

The game comprises two stages. In the first stage each country decides whether or not to join the treaty. The second stage involves abatement actions by treaty members and non-members. The equilibrium requirement is subgame perfection: each country correctly anticipates the equilibrium in the second stage when deciding whether or not to join the treaty in the first stage. If a coalition exists in equilibrium then it must exhibit internal and external stability: no

[5] Andreoni and Levinson (2001) derive a technique effect based on increasing returns to abatement. Copeland and Taylor (1994) derive an effect motivated by preferences over environmental quality. The latter typically yields an EKC between *per-capita* income and *per-capita* emissions, and it is this form of the EKC that has attracted most attention in empirical work. See Dinda (2004) for a useful survey of the literature.

member of the coalition wishes to leave it unilaterally, and no non-member wishes to join it unilaterally.

I focus on treaties that use a *single-parameter allowance-allocation rule (SPAAR).*[6] The rule allocates to member-country *i* an emissions allowance a_i based on some verifiable characteristic of that country, denoted q_i. The rule specifies an allocation mechanism $m(q_i, \alpha)$ and a single allocation parameter α that translates the rule into an actual allowance for member-country *i*:

(13) $a_i(\alpha) = m(q_i, \alpha) \ \ \forall i \in C$

where *C* denotes the set of treaty members. Without loss of generality, the mechanism is structured such that a higher value of α means a smaller allowance for country *i*.

Neither $m(.)$ nor α are country-specific – the same rule applies to *all* treaty members – and a country either accepts the allocation parameter or remains outside the treaty. Allowances may be traded among treaty members once a treaty is struck.

5.1 Equilibrium Emissions

If all countries are members of the treaty then global emissions are simply equal to $Z(\alpha) = \sum_{i=1}^{n} m(q_i, \alpha)$. However, if a treaty does not have universal membership then global emissions are the equilibrium outcome of the cooperative behavior of treaty members, and the non-cooperative behavior of the non-members.

Let Y_C denote the aggregate GDP of treaty members, and let $Z_C(\alpha)$ denote the collective emissions of treaty members under a treaty with allocation parameter α. Let *N* denote the set of non-member countries, let $Y_N = Y - Y_C$ denote their aggregate GDP, and let $Z_N(\alpha)$ denote their non-cooperative collective emissions. Let $Z(\alpha)$ denote equilibrium global emissions.

Each non-member country plays the non-cooperative strategy described by (8) in section 4. Thus, the *collective* non-cooperative best response to global emissions by non-members is

(14) $Z_N(\alpha) = Y_N - Z(\alpha) S_N$

where $S_N = \sum_{j \in N} \delta_j y_j^2$. Setting $Z_N(\alpha) = Z(\alpha) - Z_C(\alpha)$, and solving for $Z(\alpha)$ yields equilibrium global emissions under the treaty:

[6] Hoel (1992), Eyckmans (1999), and Endres and Finus (2002) also study single parameter allocation rules.

(15) $$Z(\alpha) = \frac{Z_C(\alpha) + Y_N}{1 + S_N}$$

Substituting (15) back into (14) yields equilibrium emissions from non-members:

(16) $$Z_N(\alpha) = \frac{Y_N - Z_C(\alpha) S_N}{1 + S_N}$$

5.2 Membership Payoffs and Treaty Composition

Recall that treaty members can trade allowances once a treaty is struck. Let p denote the market price of those allowances. Facing this price, member-country *i* solves the following cost-minimization problem:

(17) $$\min_{x_i} \; x_i^2 y_i + \delta_i y_i Z(\alpha)^2 - p[a_i(\alpha) - (1 - x_i) y_i]$$

where these three additive terms measure abatement cost, domestic damage, and proceeds from the sale of allowances respectively. The implied demand for emissions by country *i* is

(18) $$z_i(p) = \left(1 - \frac{p}{2}\right) y_i \quad \forall i \in C$$

Summing across $i \in C$ in (18) then yields aggregate demand for emissions by treaty members. By equating this aggregate demand to total emissions allowed for treaty members, and solving for *p*, we can obtain the equilibrium price of allowances as a function of α :

(19) $$p = P(\alpha) \equiv 2\left(1 - \frac{Z_C(\alpha)}{Y_C}\right)$$

At this price, demand for allowances (and hence, emissions) by member-country *i* is

(20) $$z_i^C(\alpha) = \left(\frac{y_i}{Y_C}\right) Z_C(\alpha) \quad \forall i \in C$$

where the "*C*" superscript denotes membership of the treaty. Equation (20) tells us that the emissions share for member-country *i* among all treaty members is equal to its share of aggregate GDP among treaty members. The technology choice associated with this level of emissions is

(21) $$x_i^C(\alpha) = 1 - \frac{Z_C(\alpha)}{Y_C} \quad \forall i \in C$$

Note from (21) that all treaty members choose the *same* technology. This solution minimizes aggregate abatement cost for treaty members as a group, subject to meeting their collective commitment under the treaty.

Total cost for member-country *i* under the treaty – damage plus abatement cost, less proceeds from the sale of allowances – reduces to

$$(22)\qquad c_i^C(\alpha) = y_i\left(\delta_i Z(\alpha)^2 + \left(1-\frac{Z_C(\alpha)}{Y_C}\right)\left(1+\frac{Z_C(\alpha)}{Y_C}-\frac{2a_i(\alpha)}{y_i}\right)\right)$$

Now consider the conditions for internal and external stability of the coalition. Internal stability requires that no coalition member can achieve a lower cost by leaving the treaty to act non-cooperatively. If member-country *i* does leave the treaty then it reverts to its non-cooperative strategy, and chooses technology

$$(23)\qquad x_i^L(\alpha) = \delta_i y_i Z(\alpha,-i)$$

where the "*L*" superscript indicates that country *i* has left the treaty, and $Z(\alpha,-i)$ denotes *equilibrium* global emissions if it leaves. Note that member-country *i* anticipates its own behavioral change and the equilibrium behavioral changes of all other countries – both treaty members and non-treaty members – if it leaves the treaty. Total cost for country *i* as a non-member is the sum of its damage and abatement cost, which reduces to

$$(24)\qquad c_i^L(\alpha) = \delta_i y_i (1+\delta_i y_i^2) Z(\alpha,-i)^2$$

Internal stability requires that $c_i^L(\alpha) \ge c_i^C(\alpha)\ \ \forall i \in C$. This condition can be stated as

$$(25)\qquad \pi_i^C(\alpha) \equiv c_i^L(\alpha) - c_i^C(\alpha) \ge 0\ \ \forall i \in C$$

where $\pi_i^C(\alpha)$ is the payoff to member-country *i* from remaining in the treaty.

Next consider the requirements for external stability. External stability requires that no non-member can achieve a lower cost by joining the treaty than by acting non-cooperatively. If non-member-country *j* remains outside the treaty then its total cost is the sum of its damage and abatement cost under its non-cooperative strategy. This reduces to

$$(26)\qquad c_j^N(\alpha) = \delta_j y_j (1+\delta_j y_j^2) Z(\alpha)^2$$

where the "*N*" superscript indicates non-membership. If instead country *j* joins the treaty and receives an allowance $a_j(\alpha)$ then its total cost (damage plus abatement cost, less proceeds from the sale of allowances) reduces to

$$(27) \qquad c_j^J(\alpha) = y_j\left(\delta_j Z(\alpha,+j)^2 + \left(1-\frac{Z_C(\alpha,+j)}{Y_C+y_j}\right)\left(1+\frac{Z_C(\alpha,+j)}{Y_C+y_j}-\frac{2a_j(\alpha)}{y_j}\right)\right)$$

where the "*J*" superscript indicates that country *j* has joined the treaty, $Z_C(\alpha,+j)$ denotes emissions for the expanded coalition when country *j* joins, and $Z(\alpha,+j)$ denotes equilibrium global emissions given the expanded treaty. Note that country *j* anticipates its own behavioral change and the equilibrium behavioral changes of all other countries – both treaty members and non-treaty members – if it joins the treaty.[7]

External stability requires that $c_j^J(\alpha) > c_j^N(\alpha) \ \ \forall j \in N$. This condition can be stated as

$$(28) \qquad \pi_j^N(\alpha) \equiv c_j^J(\alpha) - c_j^N(\alpha) > 0 \ \ \forall j \in N$$

where $\pi_j^N(\alpha)$ is the payoff to non-member-country *j* from remaining outside the treaty. Conditions (25) and (28) together describe a stable coalition.[8]

It is important to stress that for any given distribution of y_i and δ_i there may not exist a stable coalition that can reduce emissions below their NCE levels. Conditions (25) and (28) are *requirements* for a stable coalition but there may not be a solution to these conditions for any value of α other than that which implements the NCE. In the next section I show that the existence of a stable coalition hinges on whether or not there are gains from trade in allowance trading, and how those gains are distributed across the members of a candidate coalition. This in turn depends on the distribution of y_i and δ_i across countries.

6. A COMPARISON OF ALLOCATION RULES

My primary focus here is the relative performance of two "pragmatic" allocation rules based on NCE emissions: a simple proportional rule, and a rule that adds a coverage-contingent element. However, it is useful to begin with a rule based on GDP. This serves to demonstrate the critical

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[7] Note in particular that country *j* correctly anticipates how total emissions from the expanded coalition will change if it joins. This is important to country *j* because this total determines the price at which its allocation can be traded if it joins. Note too that emissions from the expanded coalition do not necessarily increase by the allocation awarded to country *j*; we need to allow for the possibility that allocations are coverage-contingent, which means that all allocations may change if country *j* joins the treaty.

[8] Note that (28) is stated as a strict inequality while (25) is stated as a weak inequality. This implies that an indifferent member-country always chooses to remain in the treaty while an indifferent non-member country always chooses to join the treaty. Thus, all non-member countries strictly prefer to remain outside the treaty. I have not explored the possibility that an indifferent country instead plays a mixed strategy.

importance of emissions trading in supporting a treaty, and illuminates some key properties of the model in a transparent way.

6.1 Allowances are Proportional to GDP

Recall from (21) above that least-cost implementation of *any* emissions target under a treaty requires that emissions intensities are equalized across treaty members; this is precisely what emissions trading among members achieves. Suppose a treaty attempts to achieve this outcome directly. In particular, consider an allocation rule that requires all treaty members to adopt an intensity standard equal to a fixed value, $1-\rho$. Then the emissions allowance for member-country i is

(29) $\quad a_i = (1-\rho)y_i$

That is, the allowance is proportional to GDP.

Since this allocation rule implements the least-cost solution directly, no emissions trading occurs under this treaty even though trading is permitted. This means that there is effectively no mechanism for sharing the gains from cooperation among treaty members. A key message from the existing literature is that a stable treaty can achieve very little under these conditions, and this received wisdom can be demonstrated analytically in the context of this model. In particular, the following result holds.

PROPOSITION 1. If allowances are proportional to GDP (meaning that no allowance trading occurs among treaty members) then no stable treaty can achieve a reduction in global emissions.
Proof. If allowances are allocated according to (29) then collective emissions from treaty members are

(30) $\quad Z_C(\rho) = (1-\rho)Y_C$

By setting $Z_C(\alpha) = Z_C(\rho)$ in (15) above, we can find equilibrium global emissions under the treaty:

(31) $\quad Z(\rho) = \dfrac{Y - \rho Y_C}{1 + S_N}$

Setting $\alpha = \rho$ in (25), and making the substitutions for $Z_C(\rho)$ and $Z(\rho)$ from (30) and (31) yields the internal stability condition for this treaty:

(32) $$\pi_i^C(\rho) \equiv \delta_i y_i (1+\delta_i y_i^2) Z(\rho,-i)^2 - \rho^2 y_i - \frac{\delta_i y_i (Y-\rho Y_C)^2}{(1+S_N)^2} \geq 0 \quad \forall i \in C$$

where

(33) $$Z(\rho,-i) = \frac{Y-\rho(Y_C - y_i)}{1+S_N+\delta_i y_i^2}$$

is global emissions if member-country *i* leaves the treaty to act non-cooperatively.

Next consider the external stability condition for this treaty. If non-member-country *j* joins the treaty then collective emissions from the expanded coalition become

(34) $$Z_C(\rho,+j) = (1-\rho)(Y_C + y_j)$$

Setting $\alpha = \rho$ in (28) and making the substitutions for $Z_C(\rho,+j)$ and $Z(\rho)$ from (34) and (31) yields the external stability condition:

(35) $$\pi_J^N(\rho) \equiv \delta_j y_j Z(\rho,+j)^2 + \rho^2 y_j - \frac{\delta_j y_j (1+\delta_j y_j^2)(Y-\rho Y_C)^2}{(1+S_N)^2} > 0 \quad \forall j \in N$$

where

(36) $$Z(\rho,+j) = \frac{Y-\rho(Y_C + y_j)}{1+S_N-\delta_j y_j^2}$$

is global emissions if member-country *j* joins the treaty.

The internal and external stability conditions can now be used to derive the highest value of ρ that will just keep country *i* in the treaty, and the lowest value of ρ that will just keep country *j* out of the treaty. In particular, setting $\pi_i^C(\rho) = 0$ and solving for ρ yields:

(37) $$\overline{\rho}_i^C = \frac{v_i Y}{1+v_i Y_C + S_N}$$

It is straightforward to show that at any $\rho > \overline{\rho}_i^C$, member-country *i* prefers to leave the treaty (because a higher value of ρ means a smaller allowance within the treaty). Similarly, setting $\pi_j^N(\rho) = 0$ and solving for ρ yields:

(38) $$\overline{\rho}_j^N = \frac{v_j Y}{1+v_j Y_C + S_N}$$

At any $\rho \leq \overline{\rho}_j^N$, non-member-country *j* prefers to join the treaty.

Note that $\overline{\rho}_i^C$ is increasing in v_i, $\overline{\rho}_j^N$ is increasing in v_j, and $\overline{\rho}_i^C = \overline{\rho}_j^N$ when $v_i = v_j$. Thus, under this allocation rule, the set of countries can be partitioned into treaty members and non-members based on their VWOs. Moreover, we can identify the *limiting member-country* whose VWO is just high enough to make its membership in the treaty worthwhile. In particular, the member-country with the smallest VWO limits the strictness of the treaty in terms of the intensity standard the treaty can adopt and still be internally stable. Let v_L denote the VWO of this limiting member-country.

Now suppose that a treaty is struck with $\rho = \overline{\rho}_L$ given by (37) evaluated at $v_i = v_L$. Global emissions under this treaty are given by (31) with $\rho = \overline{\rho}_L$. Comparing this with global emissions under the NCE from (10), it is straightforward to show that the treaty achieves a reduction in global emissions if and only if

(39) $$v_L > \frac{S - S_N}{Y_C}$$

The RHS of this condition can be written as

(40) $$\frac{S - S_N}{Y_C} = \frac{S_C}{Y_C} = \sum_{i \in C} w_{Ci} v_i$$

where $S_C = \sum_{i \in C} \delta_i y_i^2$, $w_{Ci} = y_i / Y_C$ is the GDP share of country *i* among *member* countries, and $\sum_{i \in C} w_{Ci} = 1$. Thus, condition (39) can be written as

(41) $$v_L > \sum_{i \in C} w_{Ci} v_i$$

This condition can *never* hold since v_L is the smallest member of $\{v_{i \in C}\}$. (See the Appendix).□

The failure of this treaty stems from the fact that it offers no incentive for relatively low-VWO countries to join it. It requires the lowest-VWO countries in a treaty – those with the highest emissions intensities in the NCE – to make significant cuts, but asks relatively little of the highest-VWO countries, whose emissions intensities are relatively low in the NCE. Critically, there is no mechanism through which the gains from cooperation can be shared because there is no emissions trading (even though trading is permitted). Thus, the least-cost solution that emissions trading would otherwise achieve cannot be induced *directly* with allowance allocations. The result illustrates the fact that emissions trading plays two crucial roles in a treaty:

it minimizes the collective cost of achieving an emissions target; and it transfers wealth among treaty members in a way that can motivate some countries to join a treaty that they would otherwise refuse to join.

6.2 Allowances are Proportional to Existing Emissions

Now suppose that each treaty member receives an allowance equal to a fraction of its NCE emissions. In particular,

(42) $$a_i = (1-\beta) z_i^*$$

Thus, the treaty requires that all members make the same percentage reduction in emissions, where the required percentage reduction is equal to β. I will henceforth refer to this rule as an equal-percentage-reduction (EPR) rule.[9]

Substituting z_i^* from (12) into (42), and summing across $i \in C$, yields collective emissions for treaty members under the EPR rule:

(43) $$Z_C(\beta) = (1-\beta)\left(Y_C - \frac{YS_C}{1+S} \right)$$

Equilibrium demand for allowances by country *i* is then given by (20) with $Z_C(\alpha) = Z_C(\beta)$. If country *i* receives an allocation based on (42) then it is straightforward to identify the marginal buyer of allowances within the treaty. This marginal buyer is described in the following result.

PROPOSITION 2. Let

(44) $$\tilde{v}_C = \sum_{i \in C} w_{Ci} v_i$$

where $w_{Ci} = y_i / Y_C$ is the GDP share of country *i* among member countries. Member-country *i* is a seller of allowances under an EPR treaty if and only if $v_i < \tilde{v}_C$, and a buyer of allowances if and only if $v_i > \tilde{v}_C$.

Proof. See the Appendix.

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[9] This type of allocation rule has been studied before in the literature. For example, see Eyckmans (1999), and Endres and Finus (2002).

This result tells us that member-countries with the highest VWOs are buyers of allowances. In the NCE, high-VWO countries have lower emissions intensities than low-VWO countries. This relative difference is preserved when allowances are based on the EPR rule. Allowance trading then brings emissions intensities into equality across member countries; thus, high-VWO countries are buyers, while low-VWO countries are sellers. The only country that does not trade allowances is one whose VWO happens to be just equal to $\tilde{v}_C$. This country will prove to be a pivotal player.

Now consider the internal and external stability conditions for the EPR treaty. Setting $Z_C(\alpha) = Z_C(\beta)$ in (15) above yields equilibrium global emissions under the EPR treaty:

$$(45) \qquad Z(\beta) = \frac{Y - \beta Y_C - (1-\beta)\left(\dfrac{YS_C}{1+S}\right)}{1+S_N}$$

If member-country *i* leaves the treaty to act non-cooperatively then global emissions become

$$(46) \qquad Z(\beta,-i) = \frac{Y - \beta(Y_C - y_i) - (1-\beta)\left(\dfrac{Y(S_C - \delta_i y_i^2)}{1+S}\right)}{1+(S_N + \delta_i y_i^2)}$$

Conversely, if non-member-country *j* joins the treaty then collective emissions from the expanded coalition become

$$(47) \qquad Z_C(\beta,+j) = (1-\beta)\left(Y_C + y_j - \frac{Y(S_C + \delta_j y_j^2)}{1+S}\right)$$

and global emissions become

$$(48) \qquad Z(\beta,+j) = \frac{Y - \beta(Y_C + y_j) - (1-\beta)\left(\dfrac{Y(S_C + \delta_j y_j^2)}{1+S}\right)}{1+(S_N - \delta_j y_j^2)}$$

Setting $\alpha = \beta$ in (25) and (28) and making the substitutions from (42), (43) and (45) – (48) yields the internal and external stability conditions respectively for the EPR treaty.

These conditions can be used to place bounds on the value of β that ensures the stability of a treaty of any given composition. In particular, setting $\pi_i^C(\beta) = 0$ and solving for β yields the highest value of β that will just keep country *i* in a treaty of any given composition, as reflected in Y_C and S_C. Let $\overline{\beta}_i^C$ denote this limiting value for member-country *i*. Similarly, setting $\pi_i^N(\beta) = 0$ and solving for β yields the lowest value of β that will just keep country *j* out of a treaty with any given Y_C and S_C. Let $\overline{\beta}_j^N$ denote this limiting value for non-member-country *j*.

Analytical solutions can be found for both $\overline{\beta}_i^C$ and $\overline{\beta}_j^N$ but they are too complicated to report here. Moreover, one must have data on individual country incomes and damage parameters in order to fully characterize the set of stable coalitions. For the purposes of presenting transparent analytical results, it is more helpful to focus on the grand-coalition (GC) treaty, for which $Y_C = Y$ and $S_C = S$. This also facilitates a direct comparison between the EPR treaty and the coverage-contingent treaty examined in the next section.

A GC treaty is externally stable by definition so we can focus exclusively on the requirements for internal stability in characterizing the EPR-GC treaty. These are described in the following proposition.

PROPOSITION 3. The EPR-GC treaty is stable if and only if $\beta \leq \overline{\beta}_i^{GC} \ \ \forall i$, where

$$(49) \qquad \overline{\beta}_i^{GC} = (v_i - \widetilde{v})\left(\frac{Y^2(1+y_i v_i)(v_i - \widetilde{v}) + \Phi Y(1+S)(1+y_i v_i)^{\frac{1}{2}}}{Y^2(v_i-\widetilde{v})^2(1+y_i v_i) - (1+S)^2}\right)$$

and where $\Phi = 1$ if $v_i < \widetilde{v}$ and $\Phi = -1$ otherwise. Thus, the largest possible β in a stable EPR-GC treaty is the smallest of these $\overline{\beta}_i^{GC}$ values among all countries.

Proof. See the Appendix.

Note from (49) that $\overline{\beta}_i^{GC} = 0$ at $v_i = \widetilde{v}$. Thus, if there exists a country for whom $v_i = \widetilde{v}$, then the GC cannot achieve *any* reduction in emissions. The limiting role of this country relates directly to the potential gains from emissions trading. Recall from Proposition 2 that a country with $v_i = \widetilde{v}$ does not buy or sell allowances in equilibrium. In contrast, countries with $v_i > \widetilde{v}$ buy allowances in equilibrium, while countries with $v_i < \widetilde{v}$ sell allowances in equilibrium. These countries therefore gain from treaty membership, via allowance trading, even if $\beta = 0$. They are therefore willing to accept a $\beta > 0$ in return for those gains from trade. A country with $v_i = \widetilde{v}$ derives no gains from trade, and is therefore unwilling to accept any $\beta > 0$.

If all countries happen to have the *same* VWO then $\tilde{v} = \sum_{i=1}^{n} w_i v = v$, where $v_i = v \ \ \forall i$. In that case, no country is willing to accept a $\beta > 0$. Note too that in this special case, equilibrium emissions for country *i* are proportional to its GDP. Thus, the EPR rule in this case is equivalent to one in which allocations are proportional to GDP, as examined in Section 6.1 above.

In contrast, if there are no countries with $v_i = \tilde{v}$ then the EPR-GC treaty can achieve some reduction in emissions. The size of that reduction is limited by the smallest $\overline{\beta}_i^{GC}$ value among countries, and the identity of this limiting country depends on the actual distribution of y_i and δ_i. In particular, it is *not* necessarily true that the country whose v_i is closest to $\tilde{v}$ has the lowest $\overline{\beta}_i^{GC}$ value; neither branch of $\overline{\beta}_i^{GC}$ in (49) is monotonic in v_i.

It is instructive to illustrate the relationship between $\overline{\beta}_i^{GC}$ and v_i in terms of iso-value contours in (y, δ) space, as depicted in Figure 1. The heavy central contour corresponds to $\overline{\beta}^{GC} = 0$ *and* to $v = \tilde{v}$. This contour partitions the space into allowance-sellers (where $v_i < \tilde{v}$ and $\overline{\beta}_i^{GC} > 0$) and allowance-buyers (where $v_i > \tilde{v}$ and $\overline{\beta}_i^{GC} > 0$). The two solid contours labeled $\overline{\beta}_k^{GC} > 0$ both correspond to some arbitrary value $\overline{\beta}_k^{GC}$; the lower contour corresponds to $\Phi = 1$ in (49) and the upper contour corresponds to $\Phi = -1$. (Thus, the topography resembles that of a river valley, with $\overline{\beta}^{GC} = 0$ at the valley floor). The dashed contours correspond to two different values of *v*, one lower than $\tilde{v}$, the other higher than $\tilde{v}$. (These contours map a topography resembling that of a hill sloping up away from the origin). From (49), it is straightforward to show that an iso-$\overline{\beta}^{GC}$ contour has slope

(50) $$\left.\frac{d\delta_i}{dy_i}\right|_{\overline{\beta}^{GC}} = -\frac{2(Y - y_i S + 2\delta_i y_i^2 Y)\delta_i}{(3\delta_i y_i^2 Y - y_i S + 2Y) y_i}$$

In comparison, an iso-*v* contour has slope $-\delta_i / y_i$. These slopes are equal at any point if and only if $v_i = \tilde{v}$. If $v_i < \tilde{v}$ then the iso-*v* contour has steeper slope at any given point, and if $v_i > \tilde{v}$ then the iso-$\overline{\beta}^{GC}$ contour has steeper slope at that point. Thus, two countries – for example, those labeled A and B in Figure 1 – can have the same the v_i value but lie on either side of a given iso-$\overline{\beta}^{GC}$ contour because they have different GDPs.

6.3 A Coverage-Contingent EPR Treaty

Now consider a coverage-contingent EPR rule in which the reductions undertaken by treaty members depend on the fraction of existing global emissions covered by the treaty. In particular, suppose the allowance for member-country *i* is

(51) $$a_i = [1-\omega(\theta)]z_i^* \quad \forall i \in C$$

where

(52) $$\omega(\theta) = \theta\left(\frac{\sum_{i\in C} z_i^*}{Z^*}\right)$$

and $\theta \in [0,1]$ is the allocation parameter specified under the treaty. This means that the actual reduction associated with any given θ is increasing in the fraction of NCE global emissions covered by the treaty.

I focus exclusively on the GC for this coverage-contingent treaty. This means that β and θ are directly comparable in terms of the reduction in global emissions achieved by a treaty. The GC is externally stable by definition so to characterize the treaty we need to consider only the requirements for internal stability. I first construct the internal stability condition for a treaty of *any* composition and then focus on the GC.

Substituting for z_i^* and Z^* in (52), and then summing across $i \in C$ in (51), yields aggregate emissions from treaty members under the coverage-contingent allocation rule:

(53) $$Z_C(\theta) = \left(1-\theta\frac{Y_C(1+S)-YS_C}{Y}\right)\left(Y_C - \frac{YS_C}{1+S}\right)$$

From (15), equilibrium global emissions under the treaty are

(54) $$Z(\theta) = Z^* - \theta\left(\frac{Y^2S_C^2 + Y_C(1+S)[Y_C(1+S)-2YS_C]}{Y(1+S)(1+S_N)}\right)$$

If member-country *i* leaves the treaty to act non-cooperatively then global emissions become

(55) $$Z(\theta,-i) = Z^* - \theta\left(\frac{Y^2(S_C-\delta_i y_i^2)^2 + (Y_C-y_i)(1+S)[(Y_C-y_i)(1+S)-2Y(S_C-\delta_i y_i^2)]}{Y(1+S)(1+S_N+\delta_i y_i^2)}\right)$$

Setting $\alpha = \theta$ in (25) and making the substitution for $Z(\theta,-i)$ from (55) yields the internal stability condition: $\pi_i^C(\theta) \geq 0 \;\; \forall i \in C$.

Setting $\pi_i^C(\theta)=0$ and solving for θ yields the highest value of θ that will just keep country *i* in a treaty with any given composition, as reflected in Y_C and S_C. Let $\overline{\theta}_i^C$ denote this limiting value for member-country *i*. Setting $Y_C = Y$ and $S_C = S$ then yields these limiting values for each member of the GC treaty, denoted $\overline{\theta}_i^{GC}$ for country *i*.

The analytical solution for $\overline{\theta}_i^{GC}$ is too complicated to provide useful insights but its topography in (y,δ) space is illuminating. Figure 2 plots three iso-$\overline{\theta}^{GC}$ contours, where $\theta_3 > \theta_2 > \theta_1 > 0$. (These contours map a topography resembling that of a curved peninsular with a ridge along its spine). The figure also plots an iso-*v* contour, labeled $v = v_k$, drawn for some arbitrary value v_k. Along that iso-*v* contour, $\overline{\theta}_i^{GC}$ initially rises as y_i rises to some point C, and then falls as y_i continues to rise thereafter. Thus, $\overline{\theta}_i^{GC}$ is not monotonic in v_i. The limiting country in the treaty (the country with the smallest $\overline{\theta}_i^{GC}$) therefore depends on the particular distribution of y_i and δ_i among countries. This limiting country is typically *not* the same country that limits the simple EPR treaty.

Now compare the performance of the coverage-contingent treaty with that of the simple EPR treaty. In the GC treaty, $\omega(\theta)=1$ (since all NCE emissions are covered). Thus, the reduction in aggregate emissions achieved by the coverage-contingent GC treaty is $\min_{i\in GC}\{\overline{\theta}_i^{GC}\}$. In comparison, the reduction in aggregate emissions achieved by the simple EPR-GC treaty is $\min_{i\in GC}\{\overline{\beta}_i^{GC}\}$. The following result describes the relative performance of the two treaties.

PROPOSITION 4. If all countries have positive emissions in the non-cooperative equilibrium, and countries are not identical with respect to v_i, then $\min_{i\in GC}\{\overline{\theta}_i^{GC}\} > \min_{i\in GC}\{\overline{\beta}_i^{GC}\}$.

Proof. See the Appendix.

This result tells us that the coverage-contingent GC treaty typically achieves a greater reduction in global emissions than the simple EPR-GC treaty. Only in the special case where all countries have the same v_i does the coverage-contingent treaty not outperform the simple EPR treaty, in which case both treaties are completely ineffective.

The superior performance of the coverage-contingent treaty stems from the way in which it discourages free-riding, especially by the largest existing emitters. If a country exits the treaty, then the reductions undertaken by the remaining treaty members are reduced, thereby diminishing the gains to the exiting country. The difference is small for countries with low emissions in the NCE, since their withdrawal has little impact on the share of global emissions covered, but these countries have relatively high $\bar{\beta}^{GC}$ values anyway. The constraint on what the EPR treaty can achieve comes from those relatively large emitters who gain little from emissions trading. These countries still gain little from emissions trading in the coverage-contingent treaty, but the benefits they enjoy as non-members (via reduced global emissions) are much smaller when the treaty is coverage-contingent than when it is not. Thus, they have a much stronger incentive to join the coverage-contingent treaty. Of course, if all countries happen to be identical with respect to v_i then the coverage-contingent treaty performs no better than the simple EPR treaty: the GC cannot achieve an emissions reduction under either treaty.

6.4 A Calibrated Monte Carlo Experiment

The performance gap between the two allocation rules naturally depends on the particular distribution of y and δ among countries, so no measure of relative performance can be constructed analytically. However, a simple calibrated example provides some interesting results.

I used GDP and emissions data on the ten largest economies in the world (treating the EU as a single economy).[10] Together these economies account for about 77% of global emissions. (No other economy accounts for more than 1.9%). Given the absence of any reliable data on δ for these ten economies, I assigned a δ value to each economy based on random draws from a uniform distribution, repeated 10,000 times. The support for that distribution was set at $[0, \delta_{\max}]$, where $\delta_{\max}$ was chosen to ensure that aggregate NCE emissions averaged over the 10,000 repetitions of the experiment matched actual aggregate emissions for the ten countries. The purpose of this calibration is to ensure that the experiment is conducted in a relevant region of the parameter space, as predicted by the theoretical model.

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[10] GDP data is from *International Monetary Fund, World Economic Outlook Database, October 2012*. GDP is calculated at purchasing power parity. Emissions data is from *World Resources Institute, Climate Analysis Indicators Tool (Version 9), 2012*.

For each repetition of the experiment, I constructed $\overline{\beta}_i^{GC}$ and $\overline{\theta}_i^{GC}$ for each of the ten economies, and identified the minimum values of $\overline{\beta}_i^{GC}$ and $\overline{\theta}_i^{GC}$ (denoted $\overline{\beta}_{\min}^{GC}$ and $\overline{\theta}_{\min}^{GC}$ respectively) for each repetition. I then calculated the average of these minimum values over the 10,000 repetitions, and also recorded the two extremes of the performance gap within the sample.

The results are as follows. The average of the $\overline{\beta}_{\min}^{GC}$ values across the 10,000 repetitions is 0.116. This means that the EPR treaty is able to achieve a reduction in emissions of around 12% on average. In contrast, the average of the $\overline{\theta}_{\min}^{GC}$ values is 0.216, implying an emissions reduction of just under 22% on average under the coverage-contingent treaty.

Among the 10,000 repetitions, the smallest performance gap is 3 percentage points, when neither treaty performs well: $\overline{\theta}_{\min}^{GC} = 0.037$ and $\overline{\beta}_{\min}^{GC} = 0.007$ in that case. The largest performance gap is 22.1 percentage points (when $\overline{\theta}_{\min}^{GC} = 0.303$ and $\overline{\beta}_{\min}^{GC} = 0.082$).

7. CONCLUSION

My results show that a coverage-contingent treaty outperforms a simple EPR treaty in terms of the emissions reduction achieved. The coverage-contingent allowance allocation rule reduces the incentive for countries to remain outside the treaty and free-ride on the reductions undertaken by treaty members.

It is important to be clear that this performance advantage does not necessarily mean that the coverage-contingent treaty Pareto-dominates the simple-EPR treaty. In particular, depending on the distribution of y and δ, some countries (but not *all* countries) may prefer to be a member of the simple EPR-GC treaty than to be a member of the coverage-contingent GC treaty. These countries nonetheless prefer to join the coverage-contingent treaty than to act non-cooperatively.

This raises the important question of how a treaty process chooses between different allocation rules. That question is complicated further by the possibility that for any given allocation rule, a strict subset of countries may prefer a smaller treaty to the GC treaty if it can achieve a greater reduction in global emissions by omitting the limiting country in the GC. While I have not investigated these issues here, the model is well-suited to pursuing them because it allows the derivation of analytical solutions for *any* treaty composition. Coupled with a more comprehensive calibration of the model using data-driven estimates for $\{\delta_i\}$, further work in this direction may produce useful insights into whether, and through what mechanism, successful cooperative action might be achieved.

APPENDIX

Derivation of (10)

Set $Z_{-i} = Z - z_i$ in (9) and cross-multiply to obtain

(A1) $$z_i(1+\delta_i y_i^2) = y_i - \delta_i y_i^2 (Z - z_i)$$

Collect terms to obtain

(A2) $$z_i = y_i - \delta_i y_i^2 Z$$

and then sum across *i* to obtain

(A3) $$Z = Y - Z\sum_{i=1}^{n}\delta_i y_i^2 = Y - ZS$$

which then yields (10). □

Analysis of (41)

Rewrite condition (41) as

(A4) $$\sum_{i\neq L} w_{Ci} v_i < v_L - w_{CL} v_L \text{ for } i \in C$$

The RHS can be written as

(A5) $$v_L(1 - w_{CL}) = v_L \sum_{i\neq L} w_{Ci}$$

since $w_{CL} + \sum_{i\neq L} w_{Ci} = 1$. Thus, the condition (41) becomes

(A6) $$\sum_{i\neq L} w_{iC} v_i < \sum_{i\neq L} w_{iC} v_L$$

This can never hold since $v_L = \min_{i\in C}\{v_i\}$ and $v_i \geq 0 \ \ \forall i$. □

Proof of Proposition 2

We know from (20) that demand for allowances by member country *i* is

(A7) $$z_i^C(\beta) = \left(\frac{y_i}{Y_C}\right) Z_C(\beta) \quad \forall i \in C$$

where

(A8) $$Z_C(\beta) = (1-\beta)\left(Y_C - \frac{YS_C}{1+S}\right)$$

The emissions allowance for country *i* under the EPR treaty is

(A9) $$a_i(\beta) = (1-\beta)y_i\left(1 - \frac{v_i Y}{1+S}\right)$$

where $v_i = \delta_i y_i$. Thus, allowance sales by country *i* are

(A10) $$s_i(\beta) \equiv a_i(\beta) - z_i^C(\alpha) = (\tilde{v}_C - v_i)(1-\beta)\frac{y_i Y}{1+S}$$

where

(A11) $$\tilde{v}_C = \frac{S_C}{Y_C} = \sum_{i \in C} w_{Ci} v_i$$

It follows that $s_i(\beta) > 0$ if and only if $v_i < \tilde{v}_C$, and $s_i(\beta) < 0$ if and only if $v_i > \tilde{v}_C$. □

Sketch proof of Proposition 3

Setting $\pi_i^C(\beta) = 0$ and solving for β yields two roots for each value of v_i as a function of Y_C and S_C. As noted in the text, these solutions are too complicated to report usefully here but they are easily generated using *Maple* (*Version 15*). The solutions are much simpler when $Y_C = Y$ and $S_C = S$, and reduce to the roots reported in the text. Note that these two roots are symmetric around a common intercept at $v_i = \tilde{v}$. □

Proof of Proposition 4

Setting $Y = Y_C$ and $S = S_C$ in $\pi_i^C(\theta) = 0$, and solving for θ using *Maple* (*Version 15*), yields the solution for $\overline{\theta}_i^{GC}$ as a function of y_i and δ_i (too complicated to report here). From Proposition 3, let $\overline{\beta}_{Li}^{GC}$ denote the solution for $\overline{\beta}_i^{GC}$ when $v_i < \tilde{v}$, and let $\overline{\beta}_{Hi}^{GC}$ denote the solution for $\overline{\beta}_i^{GC}$ when $v_i > \tilde{v}$. Setting $\overline{\theta}_i^{GC} = \overline{\beta}_{Li}^{GC}$ and solving for y_i yields a *single* real root, at $y_i = 0$, for any δ_i. Thus, $\overline{\theta}_i^{GC}$ lies either everywhere above or everywhere below $\overline{\beta}_{LI}^{GC}$ for $y_i > 0$. At $y_i = 0$, $\partial(\overline{\theta}_i^{GC} - \overline{\beta}_{Li}^{GC}) / \partial y_i = \delta_i Y(1+S)/(1+2S)^2 > 0$ for any $\delta_i > 0$. Thus, $\overline{\theta}_i^{GC}$ must lie everywhere above $\overline{\beta}_{LI}^{GC}$ for $y_i > 0$ and $\delta_i > 0$.

Setting $\overline{\theta}_i^{GC} = \overline{\beta}_{Hi}^{GC}$ and solving for y_i also yields a *single* real root, at $y_i = (1+S)/(\delta_i Y)$. This is an iso-*v* contour in (y_i, δ_i) space, along which $v_i = v^* \equiv (1+S)/Y$. Crucially, along this contour, $z_i^* = 0$; see (12) in the text. If $z_i^* > 0 \ \ \forall i$ (all countries have positive emissions in the NCE), then $v_i < v^* \ \ \forall i$. Thus, if $z_i^* > 0 \ \ \forall i$ then $\overline{\theta}_i^{GC}$ lies either everywhere above or everywhere below $\overline{\beta}_{Hi}^{GC}$. At $y_i = (1+S)/(\delta_i Y)$, $\partial(\overline{\theta}_i^{GC} - \overline{\beta}_{Hi}^{GC})/\partial y_i$ and $\partial(\overline{\theta}_i^{GC} - \overline{\beta}_{Hi}^{GC})/\partial \delta_i$ are both complicated expressions but they are *both* negative if

$$\text{(A12)} \quad \delta_i > -\frac{(1+S)[1+S+(S^2+2S+5)^{\frac{1}{2}}]}{2Y^2}$$

This holds for any $\delta_i \geq 0$. Thus, $\overline{\theta}_i^{GC}$ must lie everywhere above $\overline{\beta}_{Hi}^{GC}$ for $v_i < v^*$.

Since $\overline{\theta}_i^{GC}$ lies everywhere above $\overline{\beta}_{LI}^{GC}$ for $y_i > 0$ and $\delta_i > 0$, and $\overline{\theta}_i^{GC}$ lies everywhere above $\overline{\beta}_{Hi}^{GC}$ for $v_i < v^*$, it follows that $\min_{i \in GC}\{\overline{\theta}_i^{GC}\} > \min_{i \in GC}\{\overline{\beta}_i^{GC}\}$ if all countries have positive emissions in the NCE, *except* when v_i is the same for all countries. In that special case, $\overline{\theta}_i^{GC} = \overline{\beta}_{Li}^{GC} = \overline{\beta}_{Hi}^{GC} = 0 \ \ \forall i$. □

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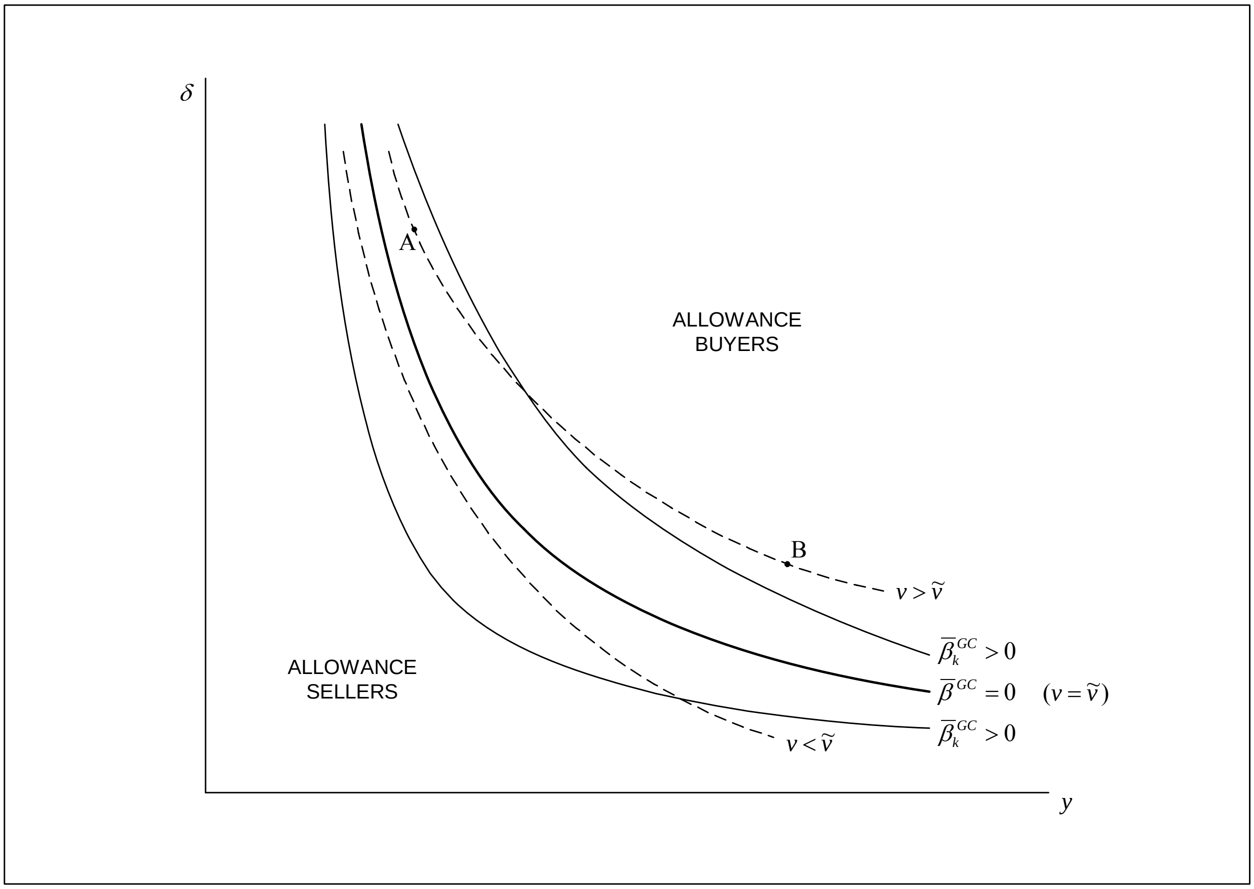

δ
A
ALLOWANCE
BUYERS
B
$v > \tilde{v}$
$\overline{\beta}_k^{GC} > 0$
ALLOWANCE
SELLERS
$\overline{\beta}^{GC} = 0 \quad (v = \tilde{v})$
$v < \tilde{v}$
$\overline{\beta}_k^{GC} > 0$
y

FIGURE 1

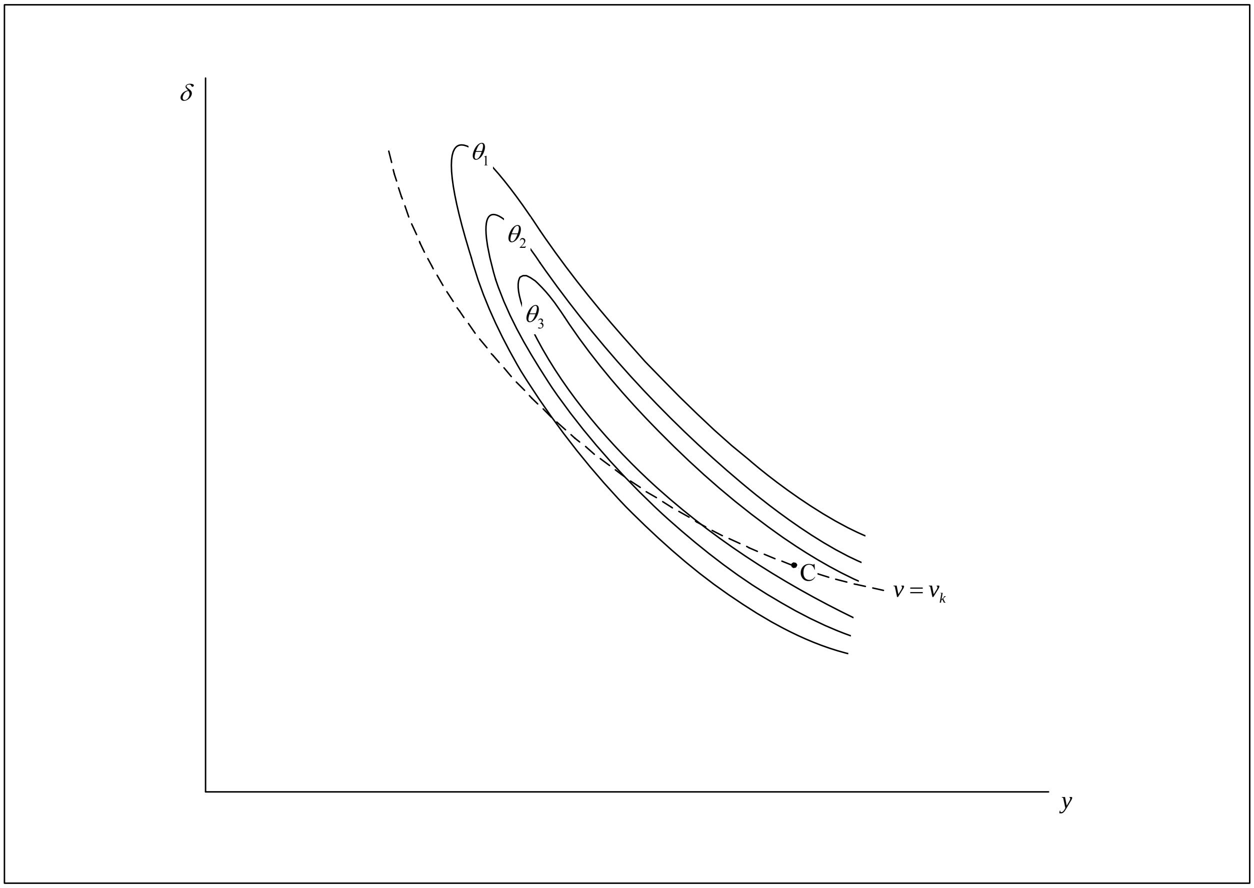

FIGURE 2