ENVIRONMENTAL ECONOMICS

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CONTENTS

3

4

1. INTRODUCTION AND OVERVIEW

OUTLINE

2

1.1 WHAT IS ENVIRONMENTAL ECONOMICS?

What is Environmental Economics?

- Environmental economics is the analysis of
  - **externalities** associated with resource allocation decisions, as they pertain to the environment
  - **policy intervention** to correct those externalities
  - the **distributional consequences** of environmental impacts and policies

4

What is Environmental Economics?

- To examine these issues we need
  - an understanding of how resource allocation decisions are made in a market context, and why externalities arise
  - some basic knowledge about the natural system and the science of environmental impacts
  - well-formulated criteria to guide corrective policy intervention

5

What is Environmental Economics?

- What distinguishes an economic approach to the study of environmental issues from other ways of thinking?

- There are two parts to the answer:
  - the methodology of economic analysis
  - the foundation of value

6

What is Environmental Economics?

- The methodology of economic analysis comprises two key concepts:
  - **optimization** (to characterize the individual actions of economic agents)
  - **equilibrium** (to characterize interaction among economic agents)
- The foundation of value in economics:
  - the **preferences** of individuals

7

What is Environmental Economics?

- The preferences of an individual are a description of what she values and with what relative intensity.
- We take “preferences” to mean much more than a taste-based ranking of mundane items like chocolate ice cream and vanilla ice cream.

8

What is Environmental Economics?

- Preferences capture the values that an individual places on everything in her universe, including things as diverse as
  - material consumption
  - companionship
  - the well-being of other people and other creatures (both now and in the future)
  - etc.

9

What is Environmental Economics?

- Note too that we do not distinguish between what an individual "wants" and what that individual "needs".
- This binary distinction is neither useful nor measurable; it is better to think along a continuum in terms of her intensity of preference.

10

What is Environmental Economics?

- Why focus on individual preferences?
- There are two answers to this question:
  - it reflects a **liberal philosophical position**: an adult individual is the best judge of what makes him happy (as opposed to some authority figure who makes that judgment for him, as a parent might do for a child).

11

What is Environmental Economics?

  - it is a **pragmatic approach to policy** in the context of a liberal democracy, one that is broadly consistent with the political structure within which the policies are to be implemented.

12

What is Environmental Economics?

- How are preferences formed?
  - preferences are partly innate – a function of evolutionary history – and partly learned.
- This means that preferences are at least partly **endogenous***:*
  - they are determined partly by the influences of society itself.

13

What is Environmental Economics?

- This partial endogeneity of preferences introduces important dynamic forces through which current outcomes can affect future values.
- This adds complexity to a dynamic economic problem but it does not undermine the logic of preference-based values.

14

What is Environmental Economics?

- Some critics of economic thinking argue that economics is **anthropocentric** (human-centered), and is therefore fundamentally flawed as an intellectual framework for analyzing environmental issues.
- What is our response to this claim?

15

What is Environmental Economics?

- Economics is anthropocentric, pointedly and deliberately so.
- It puts people – and the values they place on the environment – at the centre of the methodology.
- This is a strength of economics, not a weakness, because government policy is ultimately judged by the people it affects.

16

What is Environmental Economics?

- Some alternative intellectual frameworks – such as "deep ecology" – purport to be non-anthropocentric.
- But is it truly possible for a person to divorce herself from her own human values in the assessment of any issue?

17

What is Environmental Economics?

- For example:
  - Suppose I believe that the survival of a whale population is more important than the human uses of whale meat.
  - Is this a "non-anthropocentric" position or simply a statement of my own particular human values?

18

What is Environmental Economics?

- Similarly, some critics argue that "ethics" or "morals" should transcend preferences in the determination of values.
- However, are "morals" or "ethics" anything more than a statement of the values of the individuals who subscribe to them?
- Are your "preferences" subordinate to my "ethics" simply because of the labeling?

19

What is Environmental Economics?

- These are issues on which reasonable people can disagree, and there is no single correct answer to the questions posed here.
- As interesting as these philosophical questions might be, we will not revisit them in this course.

20

What is Environmental Economics?

- Our purpose here is to present a rigorous analytical framework for thinking about **environmental policy design** in a setting where policy choices must ultimately face the judgment of the people on whom those policies are imposed.

21

1.2 THE ENVIRONMENT AS A RESOURCE

The Environment as a Resource

- To an economist, the environment is a resource, the value of which derives from the services it provides in support of human happiness.
- Its contribution in this regard is clearly enormous; life itself would not be possible without it.

23

The Environment as a Resource

- Its contribution is also richly multi-faceted.
- From the food-providing service of fertile soil, to the power-generating capacity of uranium via nuclear fission, to the sense of joy created by a songbird in a quiet forest, the environment is essential to every aspect of human happiness.

24

The Environment as a Resource

- To reflect the importance of the environment in this regard, we often refer to the environment as **natural capital**.
- As such, it constitutes one of the key components of **economic capital**, the others being knowledge (or human capital), manufactured capital, and social capital.

25

The Environment as a Resource

- All these components of economic capital are essential to a well-functioning economy.
- It is therefore unhelpful to couch environmental policy issues in terms of "the environment vs. the economy".
- Economists instead think in terms of the **benefits** and **costs** of utilizing the services that natural capital provides.

26

The Environment as a Resource

- In what sense are there benefits and costs of using these services ?
- Depending on the rate of usage, the utilization of environmental services can deplete some elements of natural capital, or cause other consequences that we regard as harmful.

27

The Environment as a Resource

- The cost of these consequences must be weighed against the benefits of natural capital utilization when deciding what rate of usage is optimal.

28

The Environment as a Resource

- For example, consider a population of oceanic fish as an element of natural capital.
- If fish are harvested at a rate faster than the population can replenish itself, then the stock of fish will be depleted over time.

29

The Environment as a Resource

- Eventually the stock could be driven to economic extinction, where harvesting is no longer worth the effort, or even biological extinction, where the stock falls to zero.
- This outcome could be optimal despite the costs, but in many circumstances it will not be; the cost of extinction will outweigh the benefits of an initially rapid rate of harvest.

30

Assimilative Capacity

- Another key element of natural capital – and one on which this course will focus – is the capacity of the environment to assimilate waste.

31

Assimilative Capacity

- Most aspects of human interaction with the environment can be viewed in terms of a reconfiguration of material for the purpose of providing a valuable service.
- This reconfiguration necessarily generates waste as a by-product.

32

Assimilative Capacity

- The **assimilation** (or recycling) of this waste is one of the most important services that natural capital provides.

33

Assimilative Capacity

- For example, consider the **carbon cycle** in the context of growing sugar cane in a field.
- Plant-growth involves combining minerals from the soil with water and carbon-dioxide from the atmosphere, in a process powered by solar energy (photosynthesis), to produce biomass (carbon-based cellular material).

34

Assimilative Capacity

- This biomass can then be used as food or burned as a fuel (after distillation into ethanol).
- The plant growth itself generates a waste product:
  – oxygen is released into the atmosphere as a by-product of photosynthesis.

35

Assimilative Capacity

- Similarly, when the biomass is metabolized (after consumption as food), or ethanol is combusted as a fuel, other waste products are generated:
  – carbon-dioxide and water are released back into the atmosphere.

36

Assimilative Capacity

- The purpose of planting and consuming sugar cane is not to generate these waste products; the waste products are incidental to the primary activity.

37

Assimilative Capacity

- The ultimate purpose of the activity is to convert solar energy, via photosynthesis, into a more useful form that can provide energy to our living cells (via the metabolization of food), and make our lives more pleasant via the provision of heat, light and transportation services (from the combustion of fuel).

38

Assimilative Capacity

- Waste by-products are nonetheless an inevitable consequence of this process, since the reconfigured material does not simply disappear.

39

Example: Combustion of Carbon-Based Fuels

- Consider the combustion of ethanol.
- Ethanol comprises two atoms of carbon (*C*), six atoms of hydrogen (*H*) and one atom of oxygen (*O*), arranged as a molecular chain:

$$CH_3 - CH_2 - OH$$

40

Combustion of Carbon-Based Fuels

- These atoms were originally drawn from water (H_2O) and atmospheric carbon-dioxide (CO_2) during photosynthesis.
- Combustion involves the oxidization of the ethanol molecule:

$$CH_3 - CH_2 - OH \ + \ 3O_2 \ \overset{+}{\rightarrow} \ 2CO_2 + 3H_2O$$

41

Combustion of Carbon-Based Fuels

- This combustion process releases energy (indicated by the "+" over the arrow).
- That energy came from the sun during photosynthesis as the sugar cane grew, and was stored in the chemical bonds between the *C* and *H* atoms in the ethanol molecule.

42

Combustion of Carbon-Based Fuels

- It is in this sense that planting sugar cane allows us to convert solar energy into a more useful form.

43

Combustion of Carbon-Based Fuels

- However, the combustion process also releases CO_2 and H_2O in amounts that ensure that materials are <u>balanced</u>:

$$CH_3 - CH_2 - OH \; + \; 3O_2 \;\overset{+}{\rightarrow}\; 2CO_2 + 3H_2O$$

- The number of atoms of each element is the same before and after combustion.

44

Combustion of Carbon-Based Fuels

- An essential service provided by natural capital is the assimilation of the waste CO_2 and H_2O.
- That assimilation occurs through ongoing photosynthesis which turns the CO_2 and H_2O back into plant material.

45

Combustion of Carbon-Based Fuels

- The same assimilation process recycles the waste products from the combustion of fossil fuels (like coal, oil and natural gas).
- These substances are the decayed remains of plants that grew thousands of years ago, and they still contain the solar energy captured during photosynthesis when those plants were growing in the distant past.

46

Combustion of Carbon-Based Fuels

- This solar energy is released – along with CO_2 and H_2O – when those fossil fuels are burned today.
- For example, the combustion of natural gas (methane) is

$$CH_4 + 2O_2 \overset{+}{\rightarrow} CO_2 + 2H_2O$$

47

Combustion of Carbon-Based Fuels

- These waste materials are recycled via photosynthesis by plants living today.
- However, the rate at which assimilation occurs is limited.

48

Combustion of Carbon-Based Fuels

- The rate at which fossil fuels have been burned over the past 150 years exceeds the rate at which CO_2 has been assimilated.
- As a consequence, CO_2 has accumulated in the atmosphere, and that atmospheric stock is now causing the global climate to change (via a "greenhouse effect").

49

Putting a Price on Resource Use

- A similarly limited rate of assimilation applies to all waste products produced by human activity, though that rate varies dramatically across different types of waste.
- For example, radioactive waste from nuclear reactors takes thousands of years to be assimilated, while wood-smoke is typically rendered harmless very quickly.

50

Putting a Price on Resource Use

- During the period in which waste products remain unassimilated, they can cause very harmful consequences.
- Thus, there is a cost associated with waste disposal if the rate at which waste is generated exceeds the rate of assimilation.

51

Putting a Price on Resource Use

- In this regard, it is useful to think of a waste-source – such as a polluting factory – as using natural capital as a valuable input into production, in much the same way that it uses labour, machinery and knowledge.
- The factory could not operate without access to the waste assimilation services provided by natural capital.

52

Putting a Price on Resource Use

- In the same way that labour and machinery are not free, the use of natural capital is not free:
  - using up limited assimilative capacity has an opportunity cost, and generating waste beyond that capacity has damaging consequences.
- This is the economic argument for why policy should put a <u>price</u> on pollution.

53

Putting a Price on Resource Use

- If pollution is <u>not</u> priced, then it generates an **externality**:
  - a cost imposed on the rest of society that the polluting source does not have to account for when making resource-use decisions.

54

Putting a Price on Resource Use

- Our primary focus in this course is on how to put the right price on pollution.

55

1.3 ENVIRONMENTAL POLICY DESIGN: AN OVERVIEW

Environmental Policy Design

- We begin with an overview of public policy design in general, and then turn specifically to the design of environmental policy.

57

Environmental Policy Design

- It is useful to think of policy-design in general as a two-part process:
  – articulation of the **policy goal**
  – **implementation** of that goal via the application of policy instruments to induce a corrected equilibrium

- See Figure 1-1.

58

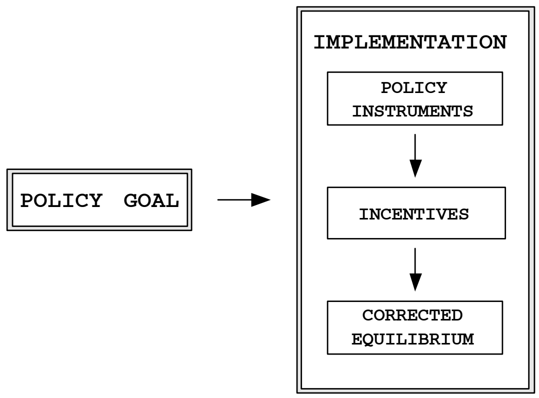

FIGURE 1-1

59

Environmental Policy Design

- Consider each of these two parts in turn.

60

Part 1: Policy Goals

- As we noted in section 1.1, economics puts individual preferences at the centre of value.
- However, preferences <u>differ</u> across the individuals that make up society.
- So how do we decide on <u>societal</u> goals?

61

Policy Goals

- Ideally, we would like to aggregate individual preferences into a set of <u>social preferences</u>, from which we could construct a measure of social welfare.

- It turns out that this is impossible.

62

Policy Goals

- **Arrow's impossibility theorem** proves that we cannot construct social preferences from individual preferences, except by appointing one individual as dictator.

63

Policy Goals

- The root of this result is the impossibility of interpersonal utility comparisons:
  - intensity of preference cannot be compared directly across individuals, so we cannot aggregate individual utilities into a measure of social welfare.
- See Appendix A1 for further discussion.

64

Policy Goals

- The impossibility theorem means that oft-stated terms like “the common good” or “the welfare of society as a whole” are ultimately vacuous, and useless for guiding practical policy design.

65

Policy Goals

- Policy goals in practice must ultimately be set through a **political process**, and that process if often messy, and inevitably has an element of arbitrariness to it.
- This is too often regarded as a bad thing.

66

Policy Goals

- For example, some people argue that "politics" should play no role in environmental policy, and that policy should be decided on the basis of "objective science".
- This is nonsense: there is no "objective" criterion for choosing best outcomes for society.

67

Policy Goals

- Science should clearly <u>inform</u> policy, but it cannot <u>decide</u> policy.
- If "science-based" policy means policy decided by a small group of people who call themselves "scientists" – unelected and unaccountable to broader society – then we should be very wary of such policy.

68

Policy Goals

- It is worth noting that the term "politics" is derived from the Greek word *politikos*, meaning "**of, for, or relating to citizens**".
- From an economic perspective, policy <u>should</u> be political, and ultimately must be decided through a political process.

69

Policy Goals

- Of course, if the political process simply fosters the fortunes of a privileged few, or if the process pays no heed to science and analysis, then it will produce very poor outcomes.
- The solution to that problem is to improve the process, not to appoint a group of learned individuals to dictate policy goals.

70

Policy Goals

- Economics has a vital role to play in the political process that sets policy goals, in two ways:
  - by providing a conceptual framework for articulating policy goals in a way that can instruct the implementation of those goals
  - by identifying tradeoffs that might underlie those goals

71

Policy Goals

- Our conceptual framework for articulating policy goals is based on two concepts:
  a) **redistribution** of property rights over resources; and
  b) **economic efficiency**

72

(a) Redistribution

- Public policy in practice is often concerned primarily with redistribution, or has significant distributional consequences.
- For example,
  - income taxation is often intended to redistribute wealth from the rich to the poor
  - health care policy often attempts to ensure access to care regardless of wealth

73

Redistribution

- Examples in an environmental context:
  - allocating a national GHG emissions target among provinces has significant implications for the distribution of national wealth
  - providing a subsidy to renewable energy producers redistributes wealth from tax-payers to energy users.

74

Redistribution

- Redistributive policy of any kind typically has significant **incentive effects**, and this introduces a potential tradeoff between redistribution and the creation of wealth.

75

Redistribution

- For example:
  - income taxes reduce the incentive to work and invest (through which wealth is created)
  - production subsidies for renewable energy reduce incentives to cut energy use

76

Redistribution

- Economic analysis is not able to identify the "optimal" distribution wealth (recall the impossibility theorem), but it nonetheless has an important role to play in the design of redistributive policy, in two ways.

77

Redistribution

- First, by identifying and quantifying crucial incentive effects, and the tradeoffs they introduce into the policy problem, economic analysis can help to design better ways of achieving redistributive goals.

78

Redistribution

- Second, economic analysis can also help to identify redistributive impacts – that are often unintended – of other types of policy, including policy designed to protect the environment.

79

(b) Economic Efficiency

- An allocation of resources is **Pareto efficient** if it is not possible to reallocate those resources in a way that makes at least one person better-off and no person worse-off.
- It is often more helpful to cast this definition in terms of a related concept.

80

Economic Efficiency

- A **Pareto improvement** (PI) is a reallocation of resources that makes at least one person better-off and no person worse off.
- Thus, an allocation of resources is Pareto efficient if and only if there are no PIs available.

81

Economic Efficiency

- What can we say about the efficiency of <u>market</u> outcomes?

82

Economic Efficiency

- The **first welfare theorem** states that the equilibrium of a perfectly competitive market economy is Pareto efficient.
- When markets are not perfectly competitive, or where markets do not exist at all, we have the potential for **market failure**.

83

Economic Efficiency

- There are three main sources of market failure:
  – imperfect competition
  – asymmetric information
  – externalities
- The most important of these in the context of environmental policy is **externalities**.

84

Economic Efficiency

- In some circumstances, the presence of an externality can lead to a market outcome that is Pareto inefficient.
- In particular, **reciprocal externalities** – where two parties each impose an external effect on each other – typically cause Pareto inefficiency.

85

Economic Efficiency

- Where the externality is **unilateral** – operating in only one direction – the outcome can be Pareto efficient.
- However, that outcome will typically fail a weaker efficiency test: the potential Pareto improvement criterion.

86

Economic Efficiency

- A **potential Pareto improvement** (PPI) is a reallocation of resources after which the winners could *in principle* fully compensate the losers, and still be better off.

87

Economic Efficiency

- For example,
  - suppose a reallocation of resources causes person A to gain $100, and person B to lose $60.
  - person A could fully compensate person B for his loss and still enjoy a net gain of $40
  - thus, the reallocation is a PPI.

88

Economic Efficiency

- Note that a reallocation can pass the PPI test but still create losers; compensation may not actually be paid.
- If the compensation is paid then the PPI becomes a PI (since no person is worse off once the compensation is made).
- It is in this sense that the PPI is a potential PI.

89

Economic Efficiency

- We will sometimes use alternative terms to describe a PPI, depending on the setting, but they all equivalent.
- In particular, if a reallocation of resources is a PPI then
  – the reallocation **creates social surplus**
  – the reallocation has a **positive net social benefit**

90

Economic Efficiency

- In our example where person A gains $100 and person B loses $60 from a reallocation, then
  - the reallocation is a PPI
  - the reallocation creates $40 in social surplus
  - the reallocation has a net social benefit of $40

91

Economic Efficiency

- The difference between a PI and a PPI is important in theory and in practice.
- In particular, if a policy creates a PI then no one is likely to object to that policy; the politics of implementation are relatively straightforward.

92

Economic Efficiency

- In contrast, if a policy creates a PPI but there is no actual compensation made to losers then there will be conflict: the winners will lobby for the policy, and the losers will lobby against the policy.
- The politics of implementation are much more difficult in this case.

93

Economic Efficiency

- Good policy design must be cognizant of these political considerations because they affect its implementation.
- A policy that looks good on a blackboard is useless if it cannot be implemented in a realistic political setting.

94

Economic Efficiency

- Articulating policy goals and policy consequences in terms of economic efficiency <u>and</u> redistribution helps to facilitate policy design that is politically astute.

95

Part 2: Implementation of Policy Goals

- Once a policy goal has been properly articulated, we can proceed to design policy tools that allow the implementation of that goal.

96

Implementation

- We begin the implementation process by asking
  - does the unregulated market equilibrium achieve the policy goal?
- If the answer is yes, there is no need for policy intervention.

97

Implementation

- If the answer is no, then our task is to design policy instruments that **change the incentives** – and hence, the behaviour – of economic agents such that the resulting "corrected equilibrium" achieves the policy goal.
- See Figure 1-2.

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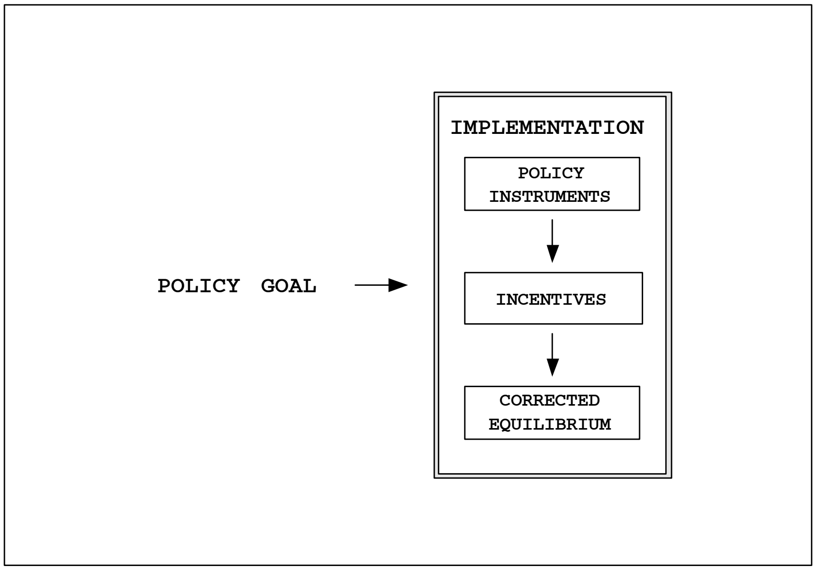

FIGURE 1-2

99

Implementation

- Example
  - Suppose the equilibrium in the unregulated energy market involves a greater use of fossil fuels than fits the policy goal
  - We could then impose a tax on fossil fuel usage (a carbon tax) that shifts consumption into alternative energy sources, and reduces energy consumption overall

100

Implementation

- Now let us look at environmental policy instruments more closely.

101

Implementation

- We can think of the application of policy instruments as a two-tier process:
  a) the choice of policy regime
  b) the choice of instrument values

- See Figure 1-3.

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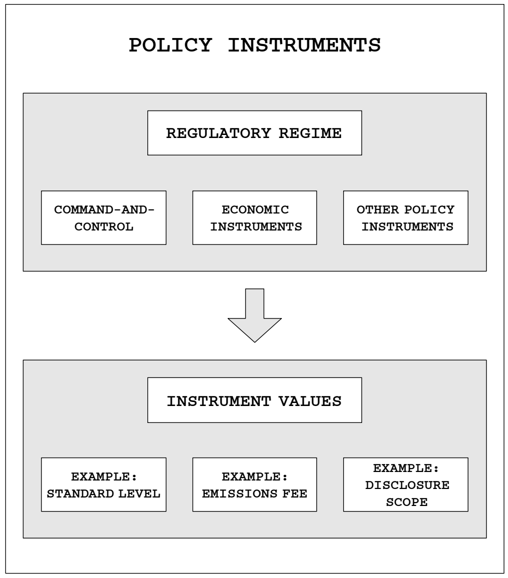

FIGURE 1-3

103

(a) The Choice of Regulatory Regime

- A **regulatory regime** refers to a class or category of policy instruments.
- We distinguish between three such regimes:
  - economic instruments
  - command-and-control
  - other policy instruments

104

Economic Instruments

- Economic instruments (sometimes called “market-based instruments”) attach an explicit **price** to pollution.
- Examples:
  - – a carbon tax
  - – an emissions trading (“cap-and-trade”) program

105

Command and Control

- Under a command and control regime, the regulator specifies what individual firms can and cannot do, enforced by the threat of penalties for non-compliance.
- Examples:
  - – specified pollution-control equipment must be installed
  - – emissions cannot exceed *x* tons per year

106

Other Policy Instruments

- This class of instruments refers to any policy instrument that does not fit neatly into the first two categories.
- Examples:
  - facilitating private law suits against polluting firms
  - providing information to the public about the environmental profiles of firms and products (disclosure policy)

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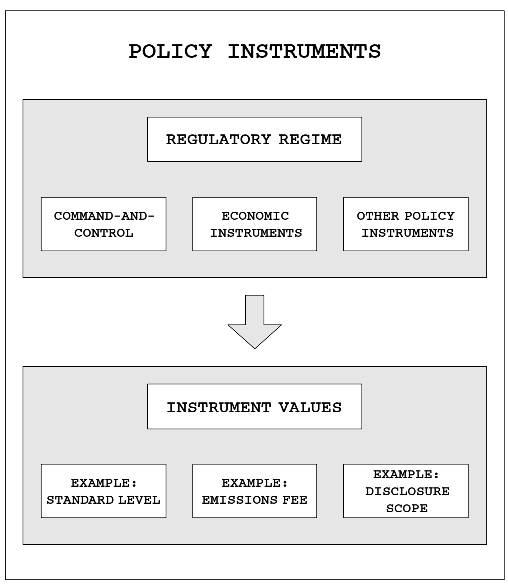

FIGURE 1-3 (repeat)

108

(b) The Choice of Instrument Values

- Once a regulatory regime has been chosen we must then choose the value of that instrument.
- Examples:
  - should a carbon tax be set at $10 per ton or $100 per ton?
  - should an annual emissions cap be 2m tons or 3m tons?

109

The Choice of Instrument Values

- This part of the instrument choice problem requires us to assign hard numbers to our policy recommendations, and that in turn requires extensive quantitative modeling of the regulated markets involved and the associated environmental impacts.

110

The Choice of Instrument Values

- Even with extensive quantitative modeling, the instrument value choice must typically be made under a lot of uncertainty about costs and benefits.
- This uncertainty is a key element of the policy design problem, and we will devote considerable attention to it in this course.

111

1.4 SUSTAINABILITY

Sustainability

- In our discussion of policy goals, we said nothing about "sustainability" despite the ubiquitous popular use of that term in reference to the environment.
- Why?

113

Sustainability

- There are two fundamental problems with "sustainability" as a criterion for assessing resource allocations:
  - incompleteness; and
  - immeasurability.
- Consider each of these in turn.

114

Incompleteness

- Consider a setting where a polluting activity today diminishes the future productivity of the economy.

115

Sustainability: Incompleteness

- Climate change is a good example:
  - – burning fossil fuels supports high living standards today but it also contributes to the stock of carbon-dioxide in the atmosphere
  - – that atmospheric stock disrupts the global climate with a generational lag, thereby reducing the productivity of future economic activity, causing an adverse effect on future living standards

116

Sustainability: Incompleteness

- Assume for the moment that we can measure living standards at date *t* by a single variable L_t.
- We will return to this measurement issue later.

117

Sustainability: Incompleteness

- Figures 1-4 and 1-5 depict two possible paths for L_t over time.
- Path 1 in Figure 1-4 charts a gradual increase in living standards that eventually levels off to a constant value $\bar{L}$ that is sustained from date $\bar{t}$ onwards.

118

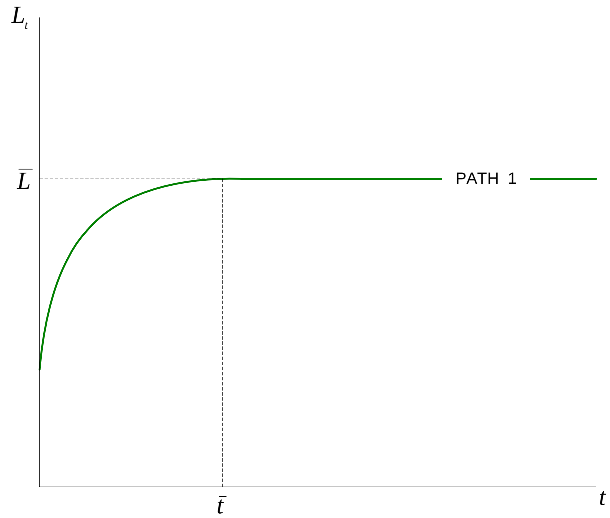

FIGURE 1-4

119

Sustainability: Incompleteness

- Suppose $\bar{L}$ is the maximum sustainable living standard (MSLS).
- That is, any growth in living standards beyond $\bar{L}$ puts so much stress on the environment that future productivity is diminished to the point that future living standards must eventually fall to a level below $\bar{L}$ for at least some time.

120

Sustainability: Incompleteness

- Path 2 in Figure 1-5 plots a path that carries living standards beyond the MSLS.
- Exceeding MSLS is possible in the short term because the consequences are not felt until a later date (for example, via the lagged impact on climate).

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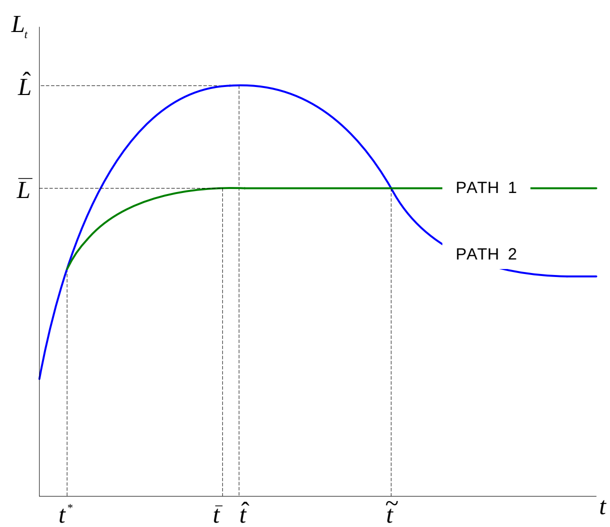

FIGURE 1-5

122

Sustainability: Incompleteness

- Along path 2, living standards peak at $\hat{L}$ at date $\hat{t}$.
- At that date, L_t starts to fall as a consequence of the earlier growth beyond MSLS.

123

Sustainability: Incompleteness

- Note that L_t must eventually fall below $\bar{L}$ because $\bar{L}$ is by definition the maximum SLS:
  - if it was possible to exceed $\bar{L}$ and then stay above $\bar{L}$, then $\bar{L}$ could not have been the maximum SLS.

124

Sustainability: Incompleteness

- After date $\tilde{t}$ along Path 2, L_t might continue to fall indefinitely or it might eventually level off, depending on the extent of the excessive earlier growth.

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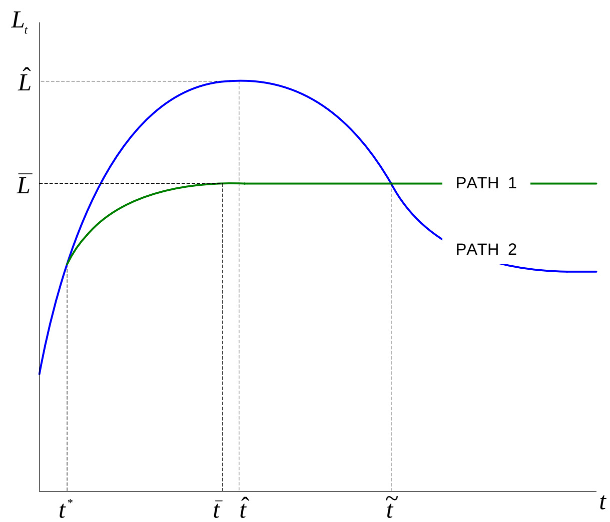

FIGURE 1-5 (repeat)

126

Sustainability: Incompleteness

- The crucial difference between Paths 1 and 2 is how they trade off current versus future living standards:
  - Path 1 gives up some early growth in living standards but supports higher living standards in the future relative to Path 2

127

Sustainability: Incompleteness

- Suppose we judge these paths according to a “sustainability” requirement.

128

Sustainability: Incompleteness

- In particular, suppose our sustainability requirement has two parts:
  - we must choose a growth path along which living standards do not fall at any point along that path; and
  - among these sustainable growth paths, the best path is the path that achieves the MSLS.

129

Sustainability: Incompleteness

- Path 1 satisfies this criterion and Path 2 does not, so the sustainability requirement provides a clear ranking over these two paths as viewed from date zero.

130

Sustainability: Incompleteness

- However, now suppose we have in fact followed Path 2 (possibly by mistake), and we are currently at date t_C, (beyond date t^*).
- Thus, it is too late to switch to Path 1 since we have already exceeded the MSLS.
- See Figure 1-6.

131

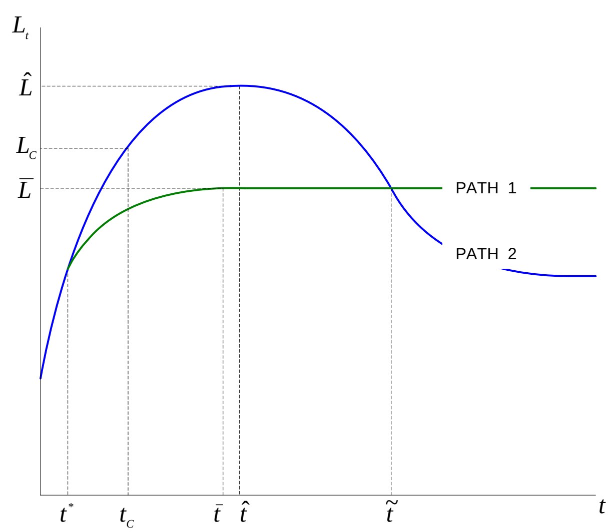

FIGURE 1-6

132

Sustainability: Incompleteness

- What do we do?
- The sustainability requirement tells us to pick a path that is sustainable, but once we pass t^* along Path 2, there <u>does not exist</u> a sustainable path from that point forward.

133

Sustainability: Incompleteness

- By definition of $\bar{L}$ as the MSLS, any growth beyond $\bar{L}$ <u>must</u> be followed by a period during which L_t falls.
- There are many possible paths that might bring living standards back down to a sustainable level.
- Figure 1-7 depicts two such paths.

134

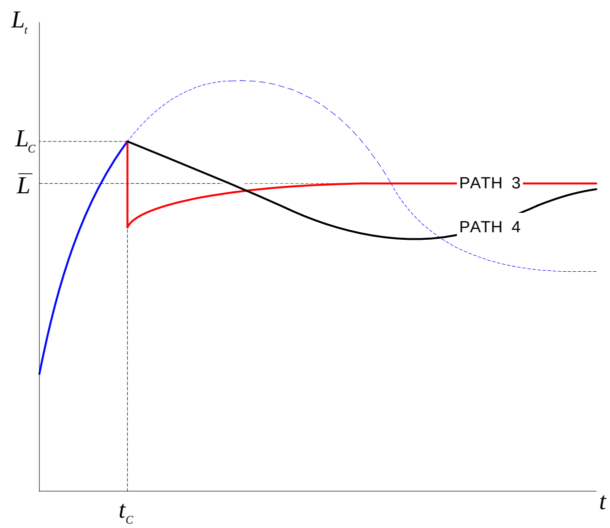

FIGURE 1-7

135

Sustainability: Incompleteness

- Path 3 is a "cold turkey" path: an immediate and drastic reduction in L_t followed by a period of gradual growth back towards $\bar{L}$.
- Path 4 involves a more gradual reduction but it requires a longer period over which L_t must remain below $\bar{L}$.
- Which path is better?

136

Sustainability: Incompleteness

- They cannot be ranked using our sustainability criterion; both violate the requirement that living standards must not fall over time.
- Our sustainability requirement demands a path that is not possible because we have already gone beyond $\bar{L}$.

137

Sustainability: Incompleteness

- Thus, our sustainability requirement tells us nothing about how we should proceed if we find ourselves at $L_C > \bar{L}$.

138

Sustainability: Incompleteness

- Consider an analogy:

What do you do to survive if you are lost in the mountains?

139

Sustainability: Incompleteness

Don't get lost in the mountains.

140

Sustainability: Incompleteness

- Our sustainability criterion – and any variation on it that retains a sustainability requirement – is inherently incomplete: it cannot provide any guidance when sustainability is not feasible.

141

Sustainability: Incompleteness

- If we could be sure that society is currently at a point before the point of no return (before t^* in Figure 1-5) then we might be inclined to dismiss this critique of sustainability as a theoretical quibble.

142

Sustainability: Incompleteness

- However, one can reasonably argue – especially in the context of the climate change problem – that society has already passed t^*, and that sustainability of current living standards is no longer an option.
- In that case the sustainability criterion is useless for guiding actions from this point forward.

143

Immeasurability

- The second problem with sustainability relates to the measurement of the entity we are trying to sustain.
- In our graphical example, we treated "living standards" as a single, measurable variable, L_t.

144

Sustainability: Immeasurability

- In an imaginary world with a single, infinitely-lived individual, L_t would simply measure the utility of that individual.

145

Sustainability: Immeasurability

- In a real setting we need to be able to measure the aggregate utility of all individuals alive at any date in time in order to construct a single variable for "living standards".
- That is, we need to construct a measure of social welfare, and straight away we run into the impossibility theorem.

146

Sustainability: Immeasurability

- How do we assess sustainability if there is no single measure of what it is we are trying to sustain?
- In the face of this obvious problem, “sustainability” means different things to different people, and is typically defined in terms of vacuous platitudes that provide no serious guidance for policy.

147

Sustainability: Immeasurability

- In summary, “sustainability” is not something we can measure at a societal level, nor is it necessarily feasible even if we could construct a suitable measure.

148

The Importance of the Future

- This does <u>not</u> mean that the concerns underlying "sustainability issues" are unimportant.
- On the contrary, the implications of resource use today for the lives of future individuals is a pressing issue for many people, and central to environmental policy design.

149

The Importance of the Future

- However, these issues are best addressed in the context of a standard economic framework based on efficiency and distributional considerations, rather than with a vague – and ultimately inadequate – alternative framework founded on "sustainability".

150

The Importance of the Future

- In particular, concern for the well-being of future individuals can and should be modeled as an aspect of preferences, and is fully consistent with a standard economic approach to value.

151

The Importance of the Future

- Similarly, disagreement among current individuals over the appropriate weight that should be given to future damage from current decisions can be couched in terms of intragenerational distributional considerations.

152

APPENDIX A1: SOCIAL CHOICE RULES AND ARROW'S IMPOSSIBILITY THEOREM

A Social Choice Rule?

- Ideally we would like to construct "social preferences" from individual preferences to derive a **social choice rule** for ranking different Pareto efficient allocations.

154

Arrow's Impossibility Theorem

- Kenneth Arrow (1951, 1963).
- Suppose we have a society of at least two individuals, and at least three different allocations (or outcomes) among which that society has to choose.

155

Arrow's Impossibility Theorem

- Can we construct a **social choice rule** – or social welfare function – that aggregates the preference orderings of those individuals into a unique social ranking over the available outcomes?

156

Arrow's Impossibility Theorem

- Example
  - three available allocations: *A*, *B* and *C*
  - preference ordering for person 1: $A > B > C$
  - preference ordering for person 2: $B > C > A$
  - preference ordering for person 3: $C > A > B$

  - what is the social preference ordering?

157

Arrow's Impossibility Theorem

- Arrow argues that a social choice rule should satisfy the following four properties:
- **1. Non-dictatorship**
  - the social choice rule cannot simply mimic the preferences of a single voter
- **2. Unrestricted domain**
  - the social choice rule must account for all preferences among all voters

158

Arrow's Impossibility Theorem

- **3. Pareto efficiency**
  - if every individual prefers a particular allocation to another, then so must the resulting social preference ordering

159

Arrow's Impossibility Theorem

- **4. Independence of irrelevant alternatives**
  - the social choice rule should provide the same ranking over a subset of options as it would for the complete set of options
  - that is, a change in any individual's ranking of *irrelevant* alternatives (ones outside the subset) should have no impact on the societal ranking of the *relevant* subset

160

Arrow's Impossibility Theorem

- Explanation of Property 4:
  - recall our example with allocations *A*, *B* and *C*
  - consider the preference ordering for person 3:
    - scenario 1: *C* > *A* > *B*
    - scenario 2: *A* > *C* > *B*
  - we should obtain the same social ranking over *A* and *B* under both scenarios (since option *C* is irrelevant to the social choice between *A* and *B*)

161

Arrow's Impossibility Theorem

- **Arrow's Impossibility Theorem**
  - there does not exist a social choice rule that satisfies properties 1 – 4.

162

Arrow’s Impossibility Theorem

- For a non-technical “proof”:
  - Valentino Dardanoni (2001), “A pedagogical proof of Arrow's Impossibility Theorem”, *Social Choice and Welfare*, 18(1), 107-112.
- For a rigorous technical proof:
  - John Geanakoplos (2005), “Three Brief Proofs of Arrow’s Impossibility Theorem,” *Economic Theory*, 26(1), 211-215.

163

Arrow’s Impossibility Theorem

- Recall the example:
  - three available allocations: A, B and C
  - preference ordering for person 1: $A > B > C$
  - preference ordering for person 2: $B > C > A$
  - preference ordering for person 3: $C > A > B$
  - what is the social preference ordering?

164

Arrow's Impossibility Theorem

- Consider a candidate social choice rule:
  - a two-step pair-wise majority voting rule
- Step 1: *A* vs. *B*
  - *A* wins by 2 votes to 1 and *B* is eliminated
- Step 2: *A* vs. *C*
  - *C* wins by 2 votes to 1
- Implied social ranking: *C* > *A* > *B*

165

Arrow's Impossibility Theorem

- But this outcome is **agenda-dependent.**
- In particular, suppose we reverse the steps.
- Step 1: *A* vs. *C*
  - *C* wins by 2 votes to 1 and *A* is eliminated
- Step 2: *C* vs. *B*
  - *B* wins by 2 votes to 1
- Implied social ranking: *B* > *C* > *A*

166

Arrow's Impossibility Theorem

- Thus, the social ranking over *A* and *B* is reversed depending on which agenda we choose.
- What is the problem?
- This voting rule violates property 4:
  - the social ranking over *A* and *B* depends on individual rankings of an irrelevant alternative (outcome *C*).

167

Arrow's Impossibility Theorem

- So why not simply vote over the agenda?
- Because a third agenda is also possible:
- Step 1: *B* vs. *C*
  - *B* wins by 2 votes to 1 and *C* is eliminated
- Step 2: *B* vs. *A*
  - *A* wins by 2 votes to 1
- Implied social ranking: $A > B > C$

168

Arrow's Impossibility Theorem

- Thus, the three different agenda yield three different social rankings (each one corresponding to the preference ordering of one of the three voters).
- This means that voting over the different agenda is equivalent to voting over the outcomes obtained under those agenda, and so we face the same problem all over again.

169

Arrow's Impossibility Theorem

- Q: What is the ultimate source of the impossibility in Arrow's theorem?
- A: The impossibility of identifying **relative intensity of preference**.
- That is, preference orderings do not reveal how *strongly* one person prefers one allocation, relative to how strongly another person prefers an alternative allocation.

170

Arrow's Impossibility Theorem

- Example: I prefer red wine, you prefer white wine; how do we choose?
- My argument:
  – "I *strongly* prefer red wine, so we should have red".
- Your response:
  – "Well, I *really strongly* prefer white wine, so we should have white".

171

Arrow's Impossibility Theorem

- Clearly, we could keep adding adverbs forever and still get no closer to a resolution.
- This relative intensity of preference problem is sometimes called the "impossibility of inter-personal utility (or happiness) comparisons".

172

Arrow's Impossibility Theorem

- The bottom line:
  - there is no way to construct a satisfactory social choice rule, derived from individual preferences, for ranking different allocations.
- Social welfare cannot be measured.

END

173

TOPIC 1 REVIEW QUESTIONS

1. The **foundation of value** in economics is

A. individual preferences.

B. social preferences.

C. social welfare.

D. efficiency.

2. In contrast to **anthropocentric value systems**, economics uses an objective analytical approach to assess resource allocations.

A. True.

B. False.

3. Which of the following is <u>not</u> a key component of **economic capital**?

A. social capital.

B. knowledge.

C. natural capital.

D. financial capital.

.

4. The energy released from the **combustion of fossil fuels** is derived from sunlight.

A. True.

B. False.

5. **Arrow's Impossibility Theorem** tells us that

A. social preferences cannot be constructed from individual preferences (except dictatorship).

B. intensity of preference cannot be compared across individuals.

C. interpersonal utility comparisons are impossible.

D. All of the above.

6. Economic analysis can help to design **redistributive policy** by

A. identifying the optimal distribution of wealth.

B. replacing political considerations with objective cost-benefit analysis.

C. identifying and quantifying incentive effects.

D. eliminating policies that are not Pareto improving.

7. A **Pareto improvement** is a reallocation of resources that makes everyone better-off.

A. True.

B. False.

8. The **first welfare theorem** states that

A. social welfare is maximized when marginal social cost is equated to marginal social benefit.

B. the equilibrium of a perfectly competitive market economy is Pareto efficient.

C. policy intervention is required to correct market failure if and only if the associated increase in social surplus exceeds the cost of the transaction.

D. any Pareto-efficient allocation can be supported as a competitive equilibrium with appropriate lump-sum transfers.

9. A **potential Pareto improvement** is a reallocation of resources after which the winners could *in principle* fully compensate the losers, and still be better off.

A. True.

B. False.

10. A potential Pareto improvement creates **social surplus** if and only if the social benefit from the reallocation is positive.

A. True.

B. False.

11. Under a **command and control** regulatory regime,

A. an explicit price is attached to pollution.

B. the regulator specifies what individual firms can and cannot do, enforced by the threat of penalties for non-compliance.

C. the primary role of the regulator is to facilitate private law suits against polluting firms.

D. the government regulates polluting industry via the operation of state-owned enterprises.

12. In reference to **sustainability**, "incompleteness" means that

A. alternative sustainable paths cannot be ranked.

B. the criterion cannot provide any guidance when sustainability is not feasible.

C. an unsustainable path does not satisfy the requirements of a socially optimal resource allocation.

D. not all Pareto-efficient paths are sustainable.

ANSWER KEY

1. A.

2. B.

3. D.

4. A.

5. A.

Options B and C are true statements in themselves – and they are the ultimate source of the impossibility of social preferences – but they are not what the theorem tells us.

6. C.

7. B.

A Pareto improvement is a reallocation of resources that makes at least one person better-off and no person worse off.

8. B.

As an aside, response D is a rough statement of the second welfare theorem.

9. A.

10. B.

11. B.

12. B.

2. EXTERNALITIES

OUTLINE

2

3

2.1 INTRODUCTION

Introduction

- An **externality** (or external effect) is an impact associated with an action, that is not felt by the agent taking that action.
- Externalities can be positive (an external benefit) or negative (an external cost).

5

Introduction

- A **source agent** undertakes the action that has the associated externality.
- An **external agent** is an agent impacted by the externality.

6

Introduction

- Externalities can be unilateral or reciprocal.
- **unilateral externalities**:
  – the externality operates in only one direction: from source agent to external agents
  – example: a firm discharges a pollutant that flows downstream, to the detriment of downstream water-users.

7

Introduction

- **reciprocal externalities**:
  – the externality operates in both directions: source agents are also external agents, and external agents are also source agents.
  – examples: GHG emissions; traffic congestion.

8

Introduction

- In this topic we will focus exclusively on unilateral externalities.
- Important reciprocal externalities arise in the context of transboundary pollution, and we will need to develop some game-theoretic tools for the analysis of that issue.
- We pursue this in Topic 11.

9

Introduction

- We begin our treatment of a unilateral externality with a simple setting in which there is a single source agent and a single external agent.

10

Introduction

- The source agent undertakes some continuously variable activity, the amount of which is denoted y.
- We first characterize the **private optimum** and then examine how and why it diverges from the **social optimum**.

11

A Note on Correct Terminology

- The private optimum is a particular allocation of resources; thus, “private optimum” is a noun.
- The defining characteristic of that allocation is that is it privately optimal; thus, “privately optimal” is a compound adjective.

12

A Note on Correct Terminology

- Similarly, "social optimum" is a noun; "socially optimal", a compound adjective.

13

2.2 THE PRIVATE OPTIMUM

The Private Optimum

- Let $PB(y)$ and $PC(y)$ denote the **private benefit** and **private cost** respectively to the source agent from an activity undertaken in the amount y.

- Consider two examples that we will use throughout this topic.

15

The Private Optimum

- Example A: y is the number of hectares that a land-owner protects as wildlife habitat.
- $PB(y)$ measures the personal enjoyment the land-owner gets from wildlife.
- $PC(y)$ measures the foregone revenue from leaving the land undeveloped (for farming or suburban housing).

16

The Private Optimum

- Example B: *y* is the amount of output produced in a factory.
- *PB*(*y*) measures revenue from the sale of that output
- *PC*(*y*) measures the cost to the factory of the materials and labour used in production.

17

The Private Optimum

- We assume that *PB*(*y*) is increasing in *y* at a decreasing rate, and that *PC*(*y*) is increasing in *y* at an increasing rate, as depicted in Figure 2-1.

18

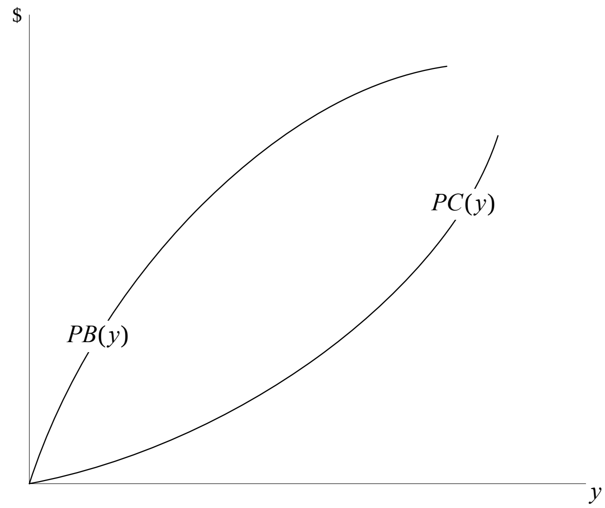

FIGURE 2-1

19

The Private Optimum

- **Net private benefit** (or private surplus) is

$$NPB(y) = PB(y) - PC(y)$$

- This is the vertical distance between the two curves in Figure 2-1.

20

The Private Optimum

- The **private optimum** is the value of *y* at which *NPB*(*y*) is maximized; it is denoted $\hat{y}$.
- The private optimum occurs where the rate of change of *PB*(*y*) is just equal to rate of change of *PC*(*y*); see Figure 2-2.

21

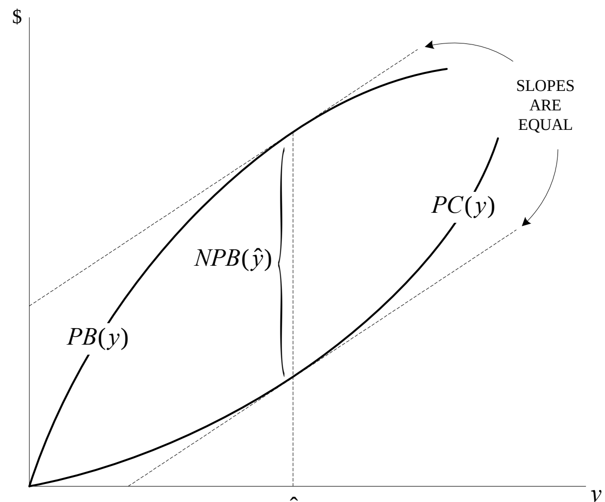

FIGURE 2-2

22

The Private Optimum

- Let us now characterize the private optimum directly in terms of the slopes of *PB*(*y*) and *PC*(*y*).

23

The Private Optimum

- Let *MPB*(*y*) denote the **marginal private benefit** *y*, defined as the rate of change (or slope) of *PB*(*y*).

- Let *MPC*(*y*) denote the **marginal private cost** of *y*, defined as the rate of change (or slope) of *PC*(*y*).

24

The Private Optimum

- Given our assumptions on *PB*(*y*) and *PC*(*y*), *MPB*(*y*) is negatively-sloped while *MPC*(*y*) is positively-sloped, as depicted in Figure 2-3.

25

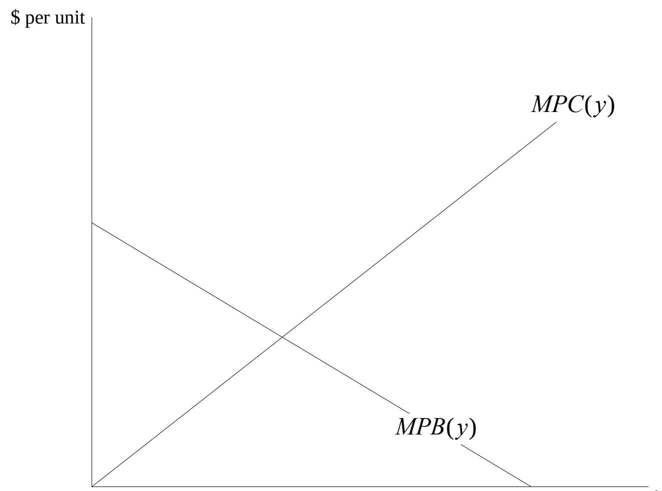

FIGURE 2-3

26

The Private Optimum

- The schedules depicted in Figure 2-3 are linear – and we will often work with examples that make this assumption for the sake of simplicity – but our general analysis does not depend on this assumption in any way.

27

The Private Optimum

- The **private optimum** is $\hat{y}$, where,

$$MPB(\hat{y}) = MPC(\hat{y})$$

- See Figure 2-4.

28

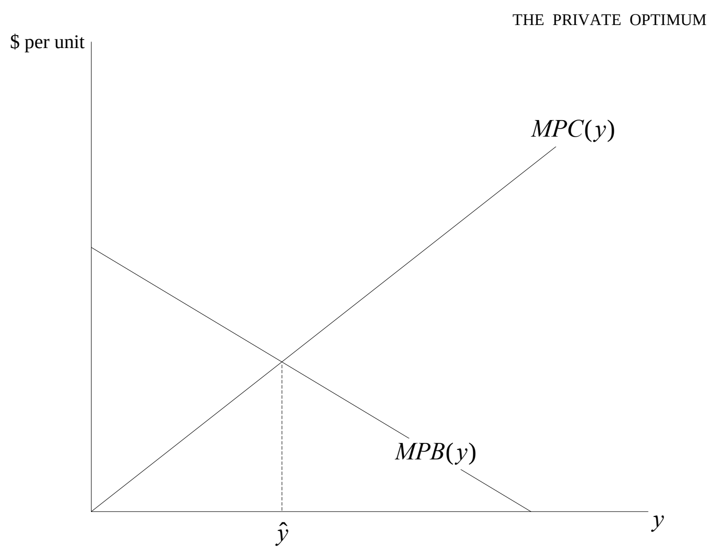

FIGURE 2-4

29

The Private Optimum

- The private benefit at the private optimum is the area (or definite integral),

$$PB(\hat{y}) = \int_0^{\hat{y}} MPB(y)dy$$

- See Figure 2-5.

30

PRIVATE BENEFIT AT THE PRIVATE OPTIMUM

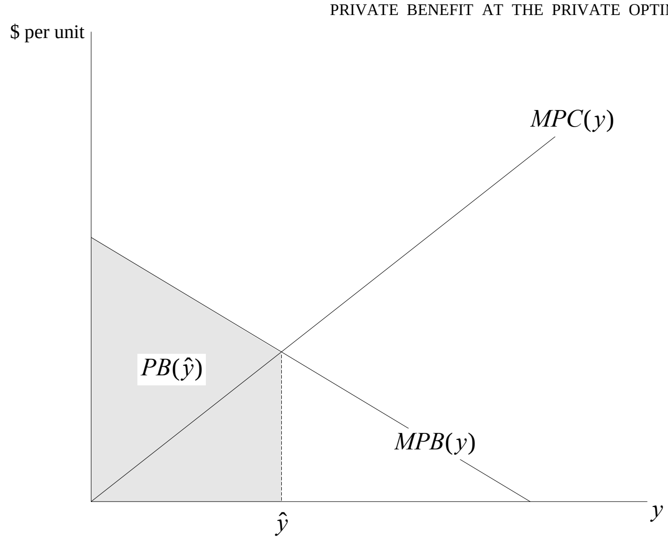

FIGURE 2-5

31

The Private Optimum

- The private cost at the private optimum is the definite integral

$$PC(\hat{y}) = \int_0^{\hat{y}} MPC(y)dy$$

- See Figure 2-6.

32

PRIVATE COST AT THE PRIVATE OPTIMUM

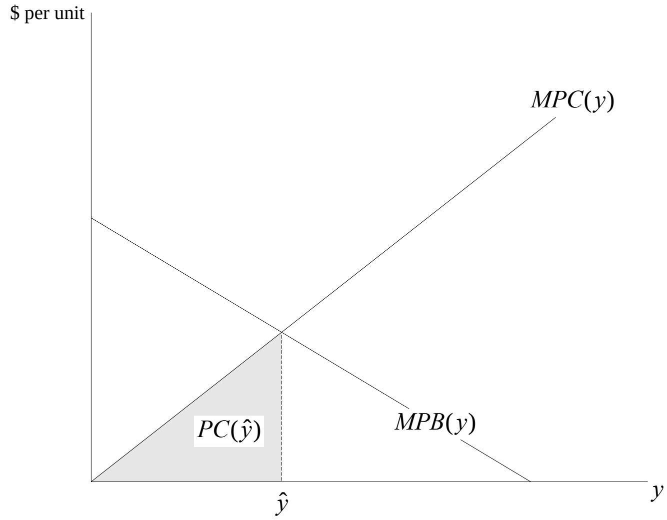

FIGURE 2-6

33

The Private Optimum

- The net private benefit at the private optimum is

$$NPB(\hat{y}) = \int_0^{\hat{y}} MPB(y)dy - \int_0^{\hat{y}} MPC(y)dy$$

- See Figure 2-7.

34

NET PRIVATE BENEFIT AT THE PRIVATE OPTIMUM

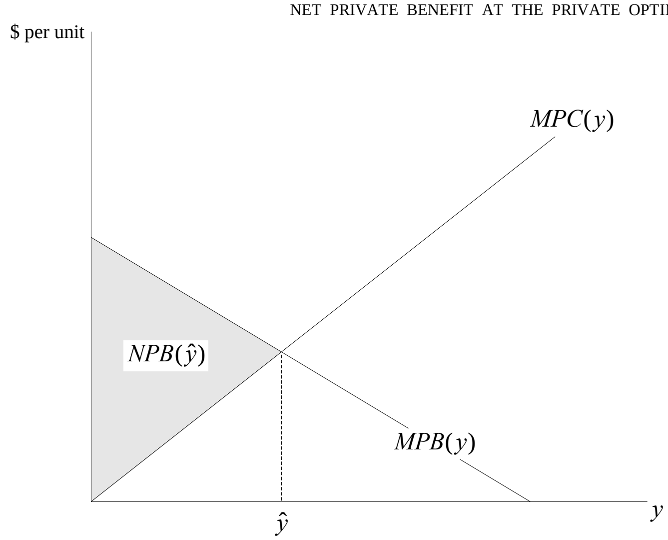

FIGURE 2-7

35

2.3 THE PRIVATE OPTIMUM:
AN ALTERNATIVE PRESENTATION

The Private Optimum: An Alternative Presentation

- The private optimum is at $\hat{y}$, where

$$MPB(\hat{y}) = MPC(\hat{y})$$

or equivalently, where

$$MPB(\hat{y}) - MPC(\hat{y}) = 0$$

37

The Private Optimum: An Alternative Presentation

- Define the **marginal net private benefit**

$$MNPB(y) = MPB(y) - MPC(y)$$

- Then the private optimum is $\hat{y}$, where

$$MNPB(\hat{y}) = 0$$

38

The Private Optimum: An Alternative Presentation

- Graphically, MNPB is constructed as the <u>vertical difference</u> between MPB and MPC.
- See Figure 2-8.

39

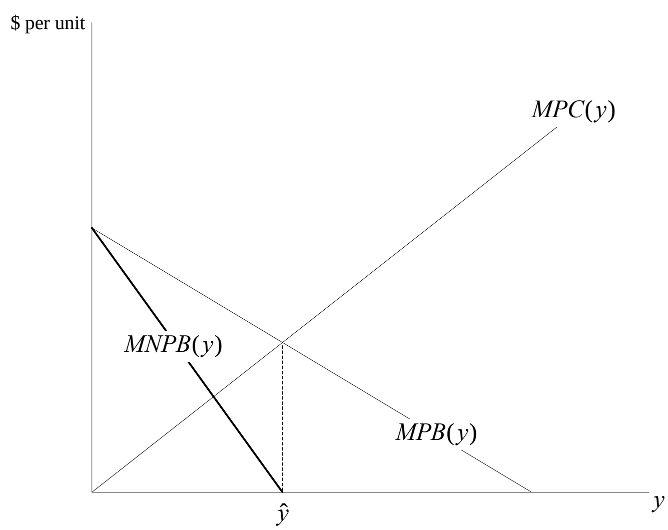

FIGURE 2-8

40

The Private Optimum: An Alternative Presentation

- Net private benefit at the private optimum is

$$NPB(\hat{y}) = \int_0^{\hat{y}} MNPB(y)dy$$

- See Figure 2-9.

41

NET PRIVATE BENEFIT AT THE PRIVATE OPTIMUM

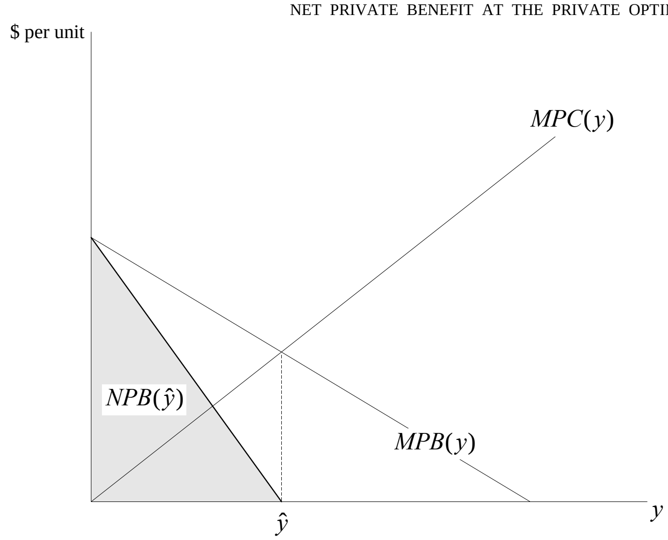

FIGURE 2-9

42

The Private Optimum: An Alternative Presentation

- Note that the shaded area in Figure 2-9 is necessarily equal to the shaded area in Figure 2-7.
- This can be verified from the figures themselves using basic geometry if the MPB and MPC schedules are linear.

43

NET PRIVATE BENEFIT AT THE PRIVATE OPTIMUM

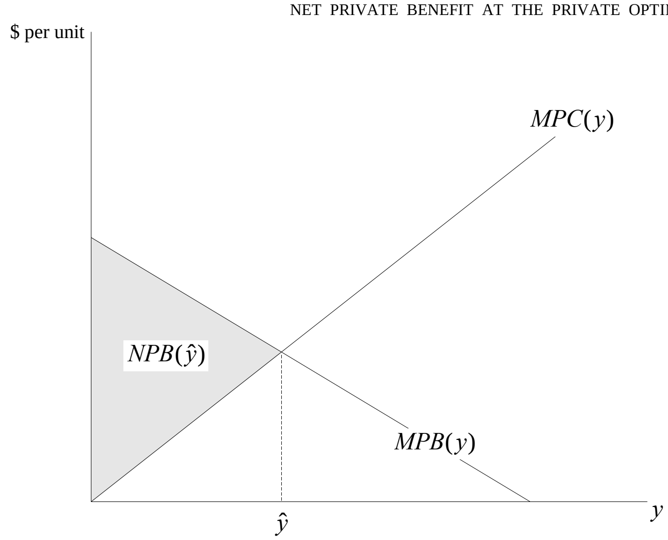

FIGURE 2-7 (repeat)

44

2.4 THE SOCIAL OPTIMUM

The Social Optimum

- If the activity bestows an **external benefit** $G(y)$ then the **social benefit** at y is the sum of the private benefit and the external benefit:

$$SB(y) = PB(y) + G(y)$$

46

The Social Optimum

- We assume that $G(y)$ is increasing at a decreasing rate, so given our assumption on $PB(y)$ from s.18, it follows that $SB(y)$ is increasing at a decreasing rate.

47

The Social Optimum

- If the activity imposes an **external cost** $D(y)$ then the **social cost** at y is the sum of the private cost and the external cost:

$$SC(y) = PC(y) + D(y)$$

48

The Social Optimum

- We assume that $D(y)$ is increasing at an increasing rate, so given our assumption on PC(y) from s.18, it follows that $SC(y)$ is increasing at an increasing rate.

49

The Social Optimum

- The **net social benefit** (or **social surplus**) from the activity is the difference between social benefit and social cost:

$$NSB(y) = SB(y) - SC(y)$$

50

The Social Optimum

- The **social optimum** is the value of *y* at which net social benefit is maximized; it is denoted y^*.

- We can characterize this social optimum in terms of marginal social benefit and marginal social cost.

51

The Social Optimum

- Let *MSB*(*y*) denote the **marginal social benefit** at *y*. This is defined as the rate of change of *SB*(*y*).

- Let *MSC*(*y*) denote the **marginal social cost** at *y*. This is defined as the rate of change of *SC*(*y*).

52

The Social Optimum

- The **social optimum** is y^*, where,

$$MSB(y^*) = MSC(y^*)$$

53

The Social Optimum

- Net social benefit at the social optimum is

$$NSB(y^*) = \int_0^{y^*} MSB(y)dy - \int_0^{y^*} MSC(y)dy$$

- See Figure 2-10.

54

NET SOCIAL BENEFIT AT THE SOCIAL OPTIMUM

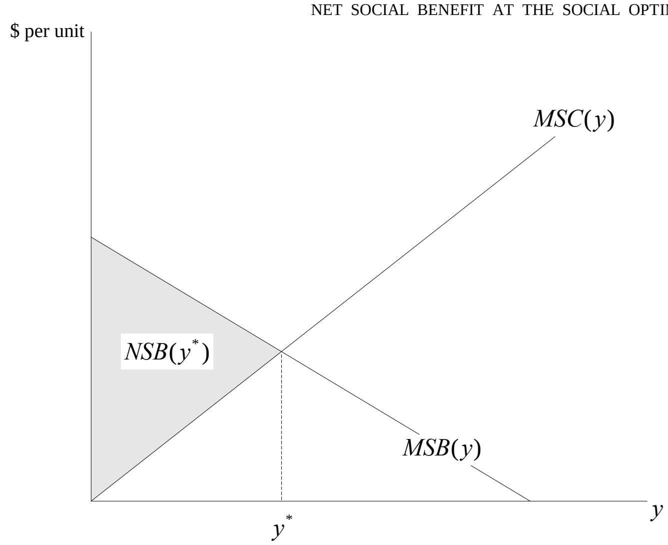

FIGURE 2-10

55

The Social Optimum

- If an activity has no external benefit <u>and</u> no external cost (that is, if $G(y) = 0$ at all values of y, and $D(y) = 0$ at all values of y) then the private optimum and the social optimum coincide.

56

The Social Optimum

- Conversely, if $G(y) \neq 0$ or $D(y) \neq 0$ at some values of y then the social optimum and the private optimum will typically not coincide.

- Let us consider each case in turn.

57

2.5 A POSITIVE EXTERNALITY

A Positive Externality

- Consider a setting where the activity has an external benefit but no external cost: that is, $G(y)>0$ but $D(y)=0$.

- For example, if y is hectares of protected wildlife habitat then $G(y)$ might be the enjoyment that local residents get from wildlife viewing in the area.

59

A Positive Externality

- Since there is no external cost,

$$SC(y) = PC(y)$$

and it follows that

$$MSC(y) = MPC(y)$$

60

A Positive Externality

- Conversely, social benefit is

$$SB(y) = PB(y) + G(y)$$

61

A Positive Externality

- We can decompose the rate of change of *SB*(*y*) into two components:

$$MSB(y) = MPB(y) + MEB(y)$$

where *MEB*(*y*) is the **marginal external benefit** of the activity at *y*.

62

A Positive Externality

- Graphically, *MEB*(y) at any given value of y (say y^0), is the vertical distance between *MSB*(y) and *MPB*(y) at y^0.

- See Figure 2-11.

63

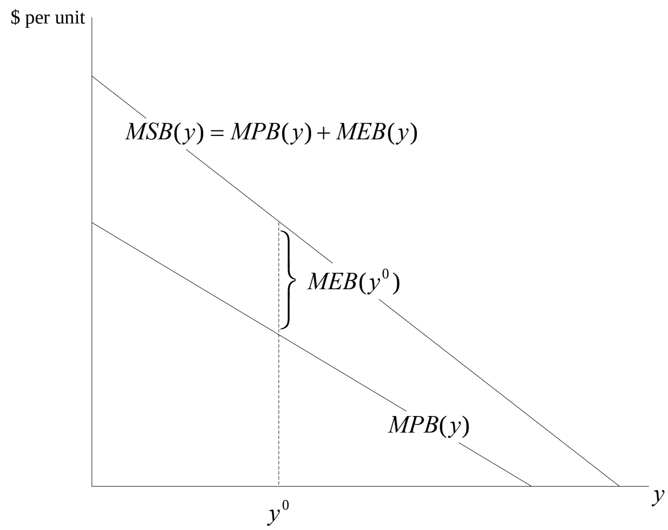

FIGURE 2-11

64

A Positive Externality

- Recall from s.47 our assumption that $G(y)$ is increasing at a decreasing rate.

- This is reflected in Figure 2-11: $MEB(y)$ declines as y rises; the gap between $MSB(y)$ and $MPB(y)$ becomes smaller.

65

A Positive Externality

- We will later find it useful to depict $MEB(y)$ as a separate graph.
- It is constructed by graphing the vertical distance between $MSB(y)$ and $MPB(y)$ at every value of y.

- See Figure 2-12.

66

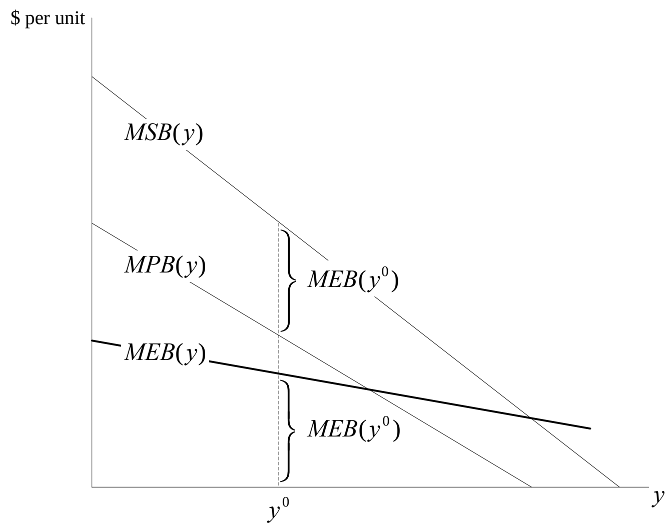

FIGURE 2-12

67

A Positive Externality

- Now let us consider the impact of this external benefit on the relationship between the private and social optima.

- See Figure 2-13.

68

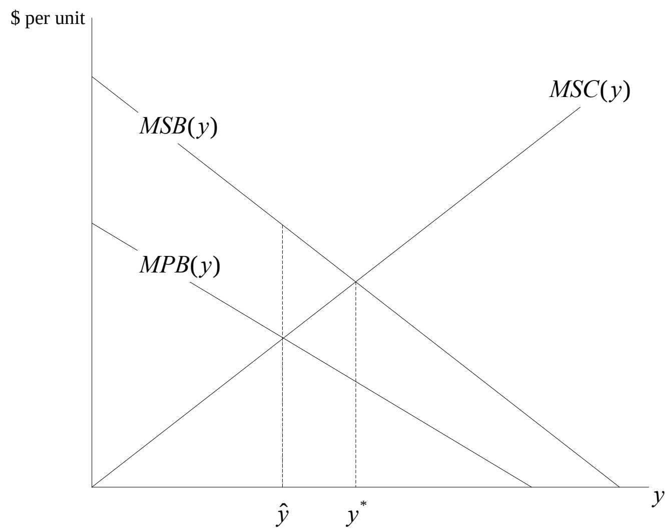

FIGURE 2-13

69

A Positive Externality

- The presence of the external benefit means:

$$\hat{y} < y^*$$

  – *ie*. the privately optimal level of activity is lower than the socially optimal level.

70

A Positive Externality

- Intuition:
  - the source agent does not take into account the benefit she bestows on the external agent when she chooses her action, and so her chosen level of the action is too low from a social perspective.

71

A Positive Externality: A Numerical Example

- Consider a numerical example. Suppose

$$MPB(y) = 50 - 2y$$

$$MPC(y) = 3y$$

$$MEB(y) = 28 - y$$

72

A Positive Externality: A Numerical Example

- First find the private optimum, given by $\hat{y}$ such that

$$50 - 2\hat{y} = 3\hat{y}$$

which solves for

$$\hat{y} = 10$$

73

A Positive Externality: A Numerical Example

- Next find the social optimum.
- *MSB*(*y*) is the sum of *MPB*(*y*) and *MEB*(*y*):

$$MSB(y) = (50 - 2y) + (28 - y) = 78 - 3y$$

- Since there is no external cost here, *MSC*(*y*) is simply equal to *MPC*(*y*).

74

A Positive Externality: A Numerical Example

- Thus, the social optimum is y^* such that

$$78 - 3y^* = 3y^*$$

which solves for

$$y^* = 13$$

- See Figure 2-14.

75

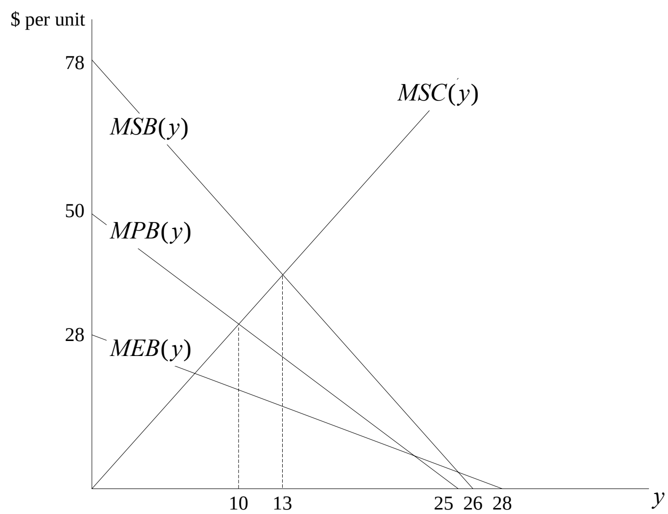

FIGURE 2-14

76

The Impact of a Regulated Increase in *y*

- Now suppose a third party (such as a government regulator) could force the source agent to raise her activity level from $\hat{y}$ to y^*.

77

The Impact of a Regulated Increase in *y*

- We will show that this forced increase yields a potential Pareto improvement:
  - the source agent loses but the external agent gains by more than enough to compensate for that loss.

78

The Impact of a Regulated Increase in *y*

- Consider first the gain to the external agent (the increase in external benefit).
- We derive this by first calculating the external benefit at the social optimum, and then the external benefit at the private optimum, and then we take the difference.

79

The Impact of a Regulated Increase in *y*

- External benefit at the private optimum is the area under *MEB*(*y*) from zero to $\hat{y}$:

$$G(\hat{y}) = \int_0^{\hat{y}} MEB(y)dy$$

80

The Impact of a Regulated Increase in *y*

- Since

$$MEB(y) = MSB(y) - MPB(y)$$

we can find the area under *MEB*(*y*) as the difference between the area under *MSB*(*y*) and the area under *MPB*(*y*).

81

The Impact of a Regulated Increase in *y*

- Thus,

$$G(\hat{y}) = \int_0^{\hat{y}} MEB(y)dy$$
$$= \int_0^{\hat{y}} MSB(y)dy - \int_0^{\hat{y}} MPB(y)dy$$

- See Figure 2-15.

82

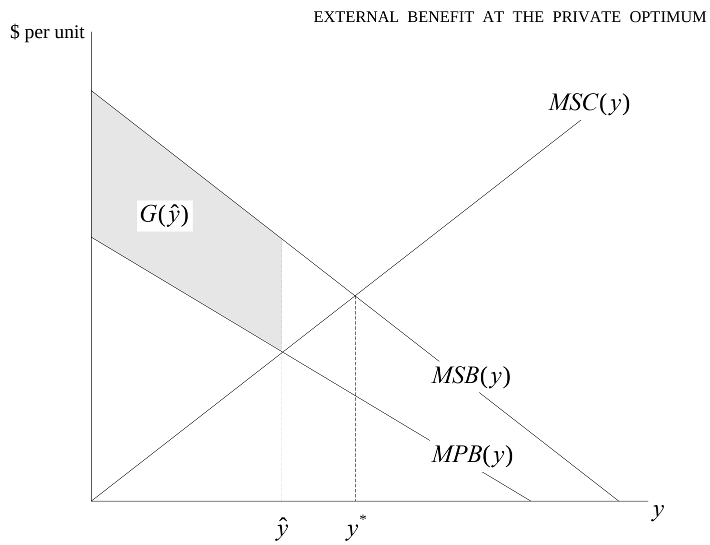

FIGURE 2-15

83

The Impact of a Regulated Increase in *y*

- External benefit at the social optimum is

$$G(y^*) = \int_0^{y^*} MEB(y)dy$$

$$= \int_0^{y^*} MSB(y)dy - \int_0^{y^*} MPB(y)dy$$

- See Figure 2-16.

84

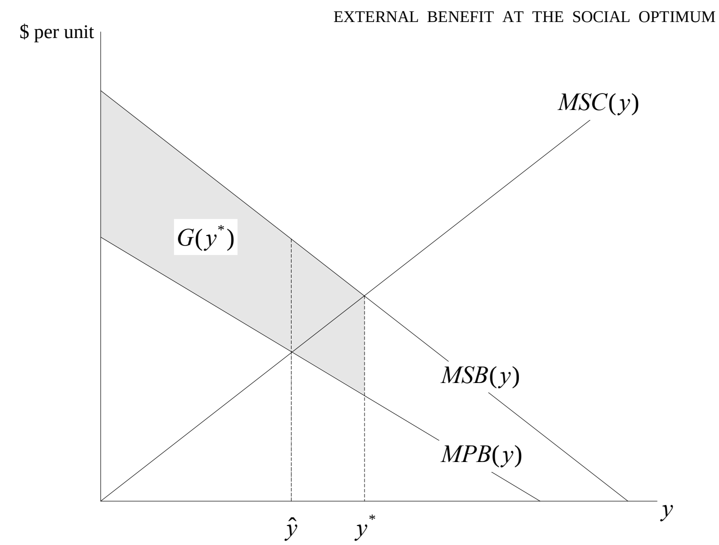

FIGURE 2-16

85

The Impact of a Regulated Increase in *y*

- Hence, the <u>increase</u> in external benefit is

$$G(y^*) - G(\hat{y}) = \int_0^{y^*} MEB(y)dy - \int_0^{\hat{y}} MEB(y)dy$$

- This is the **gain to the external agent**.

- See *area*(*abcd*) in Figure 2-17.

86

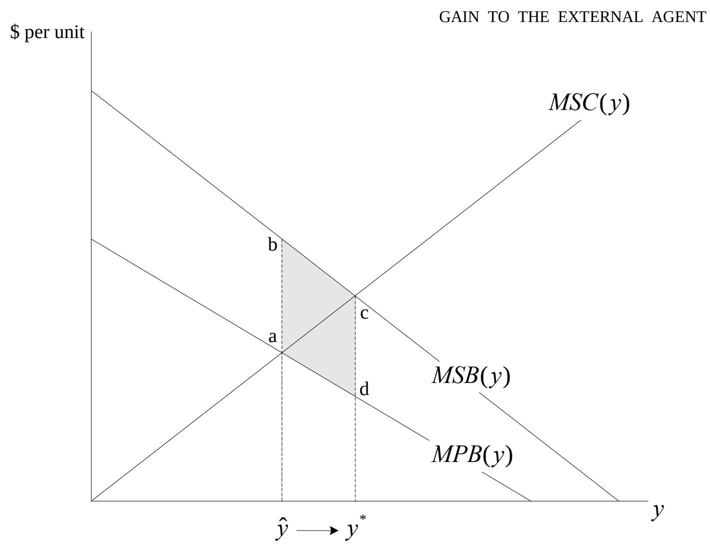

FIGURE 2-17 87

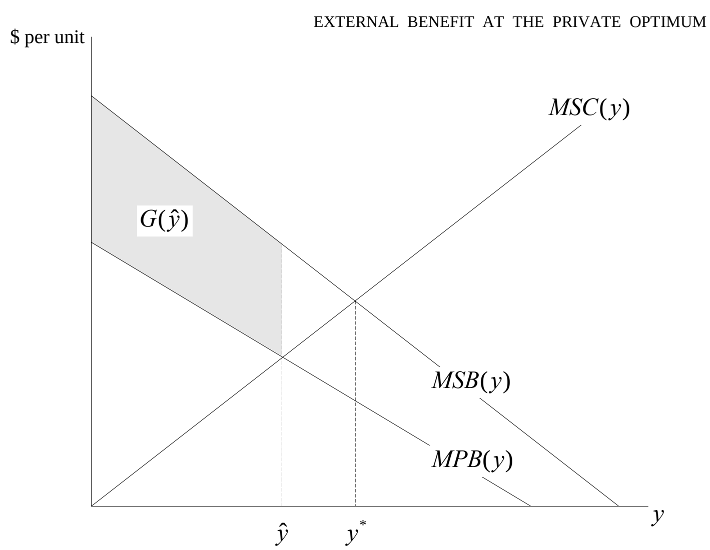

FIGURE 2-15 (repeat) 88

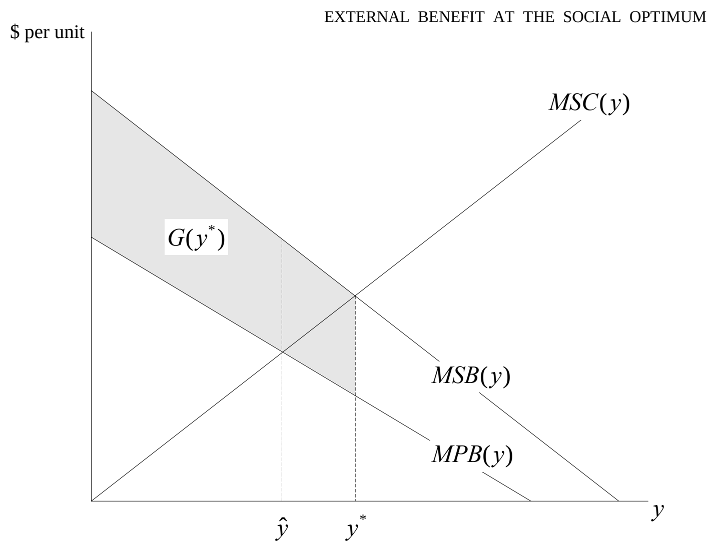

FIGURE 2-16 (repeat) 89

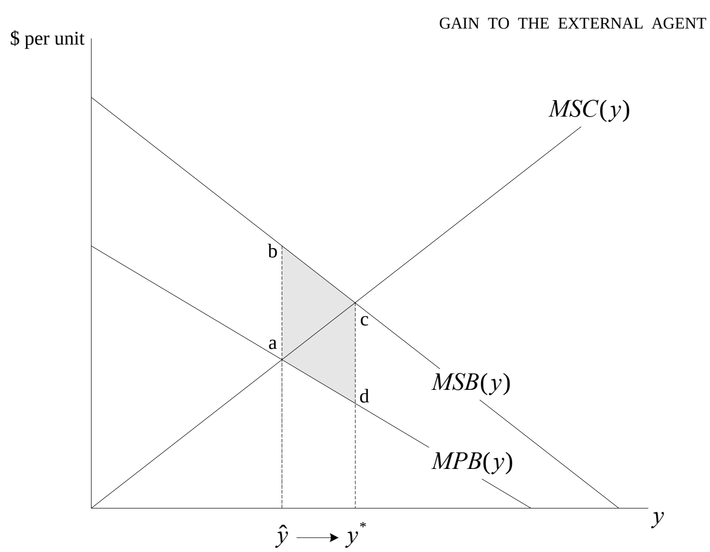

FIGURE 2-17 (repeat) 90

The Impact of a Regulated Increase in *y*

- Note that this gain to the external agent can also be written as

$$G(y^*) - G(\hat{y}) = \int_{\hat{y}}^{y^*} MEB(y)dy$$

- This definite integral is the area under $MEB(y)$ between $\hat{y}$ and y^*; see Figure 2-18.

91

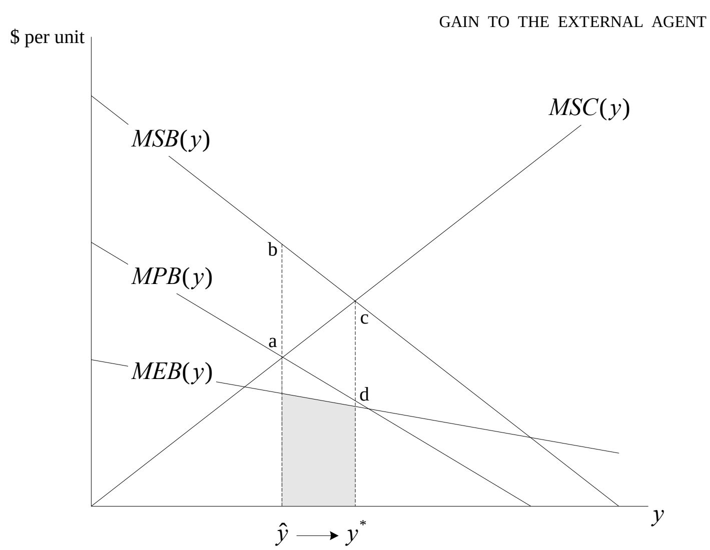

FIGURE 2-18

92

The Impact of a Regulated Increase in *y*

- The shaded areas in Figures 2-17 and 2-18 are necessarily equal; they are alternative graphical representations of the gain to the external agent.
- It is important to understand both representations.

93

The Impact of a Regulated Increase in *y*

- Next consider the reduction in net private benefit for the source agent.

94

The Impact of a Regulated Increase in *y*

- Recall that the private benefit to the source agent at the <u>private</u> optimum is the area under $MPB(y)$ between zero and $\hat{y}$:

$$PB(\hat{y}) = \int_0^{\hat{y}} MPB(y)dy$$

95

The Impact of a Regulated Increase in *y*

- In comparison, private benefit to the source agent at the <u>social</u> optimum is

$$PB(y^*) = \int_0^{y^*} MPB(y)dy$$

96

The Impact of a Regulated Increase in *y*

- Thus, the <u>increase</u> in private benefit to the source agent is

$$PB(y^*) - PB(\hat{y}) = \int_{\hat{y}}^{y^*} MPB(y)dy$$

- See Figure 2-19.

97

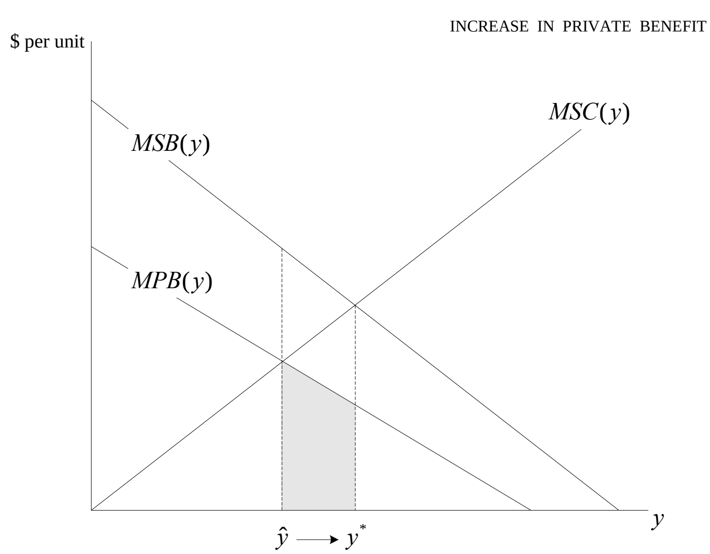

FIGURE 2-19

98

The Impact of a Regulated Increase in *y*

- By the same logic, the increase in private cost to the source agent is

$$PC(y^*) - PC(\hat{y}) = \int_{\hat{y}}^{y^*} MPC(y)dy$$

- See Figure 2-20.

99

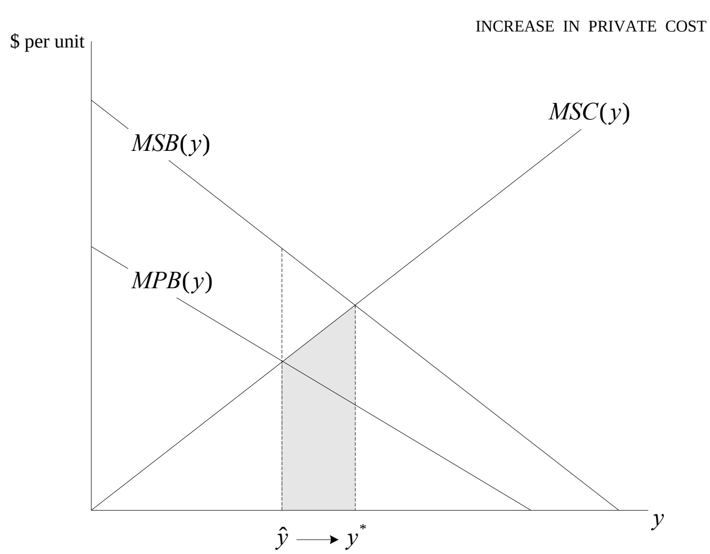

FIGURE 2-20

100

The Impact of a Regulated Increase in *y*

- It is clear from Figures 2-19 and 2-20 that the increase in private cost exceeds the increase in private benefit.
- Thus, the overall change in net private benefit for the source agent is negative.
- See Figure 2-21.

101

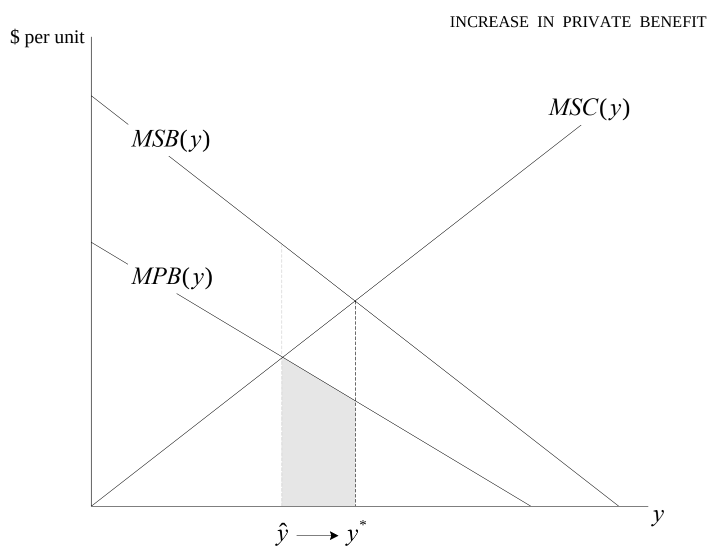

FIGURE 2-19 (repeat)

102

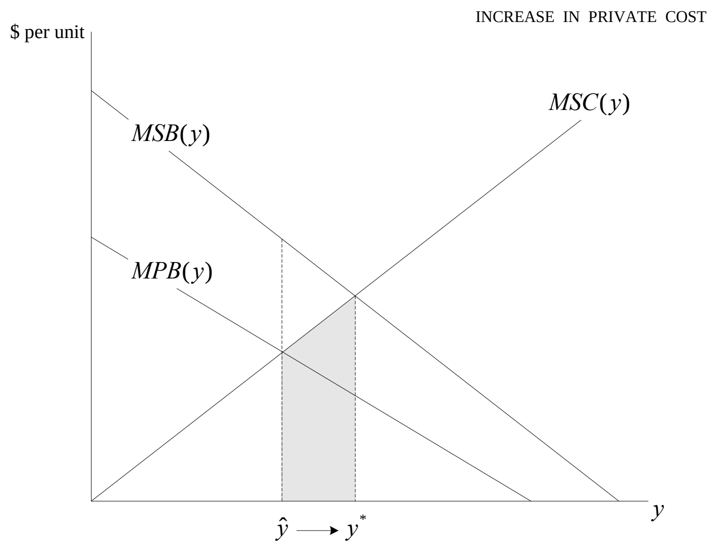

FIGURE 2-20 (repeat) 103

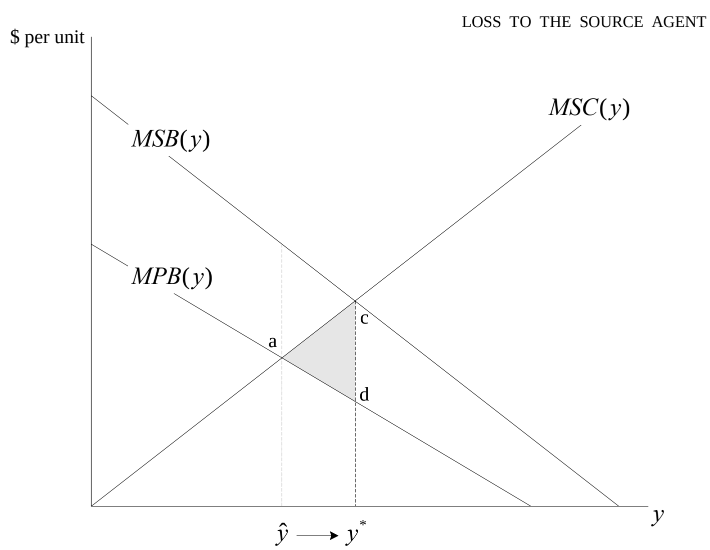

FIGURE 2-21 104

The Impact of a Regulated Increase in *y*

- The source agent is made worse-off because she is forced to move away from her private optimum, and there is no offsetting compensation.

105

The Impact of a Regulated Increase in *y*

- In summary, from Figures 2-17 and 2-21:
  - the gain to the external agent = *area*(*abcd*)
  - the loss to the source agent = *area*(*acd*)
- Thus, the overall **gain in social surplus** = *area*(*abc*)
- See Figure 2-22.

106

GAIN TO THE EXTERNAL AGENT

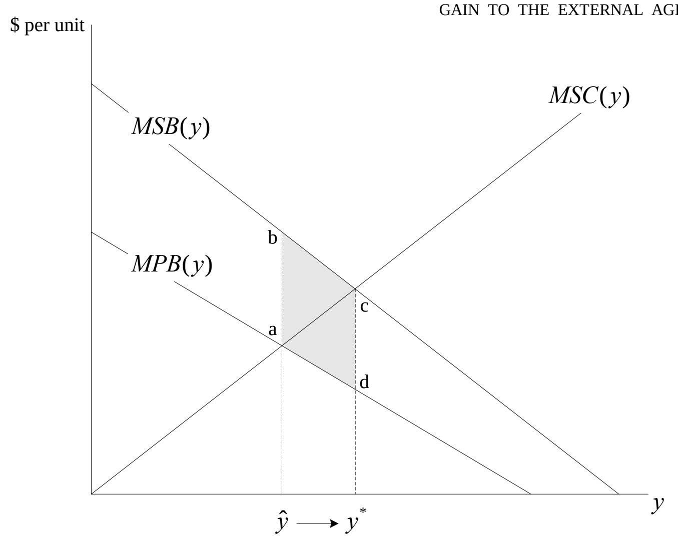

FIGURE 2-17 (repeat)

107

LOSS TO THE SOURCE AGENT

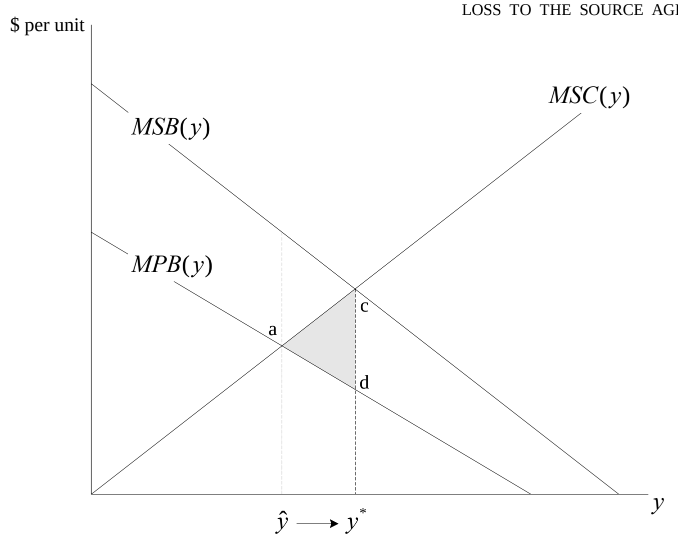

FIGURE 2-21 (repeat)

108

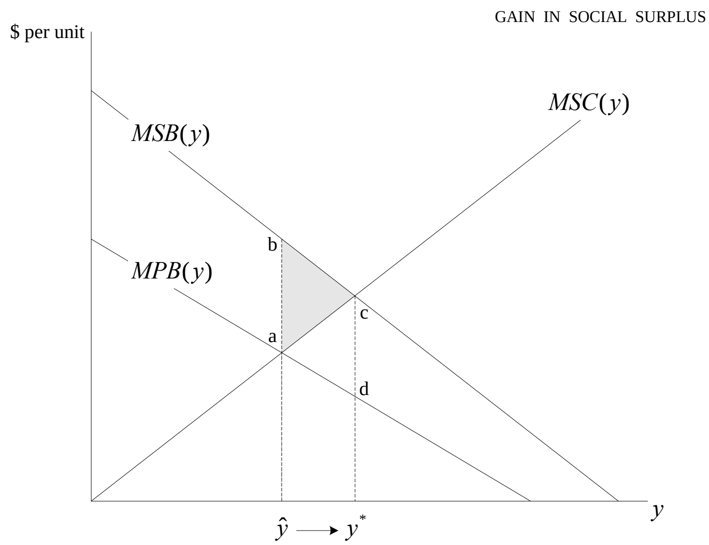

FIGURE 2-22

109

The Impact of a Regulated Increase in *y*

- What can we say about welfare overall?
- Recall the definition of a **Pareto improvement**:
  - a reallocation of resources that makes at least one person better-off and leaves no person worse-off.

110

The Impact of a Regulated Increase in *y*

- The forced move from $\hat{y}$ to y^* is <u>not</u> a Pareto improvement; the source agent is made worse-off.

111

The Impact of a Regulated Increase in *y*

- In contrast, recall the definition of a **potential Pareto improvement**:
  - a reallocation of resources under which the winners could *in principle* fully compensate the losers and still be better-off

112

The Impact of a Regulated Increase in y

- The forced move from $\hat{y}$ to y^* is a potential Pareto improvement:
  - the winner (the external agent) could in principle fully compensate the loser (the source agent) and still be better-off, by *area*(*abc*)

113

The Impact of a Regulated Increase in y

- Recall from Topic 1 that the distinction between a Pareto improvement (PI) and a potential Pareto improvement (PPI) is important for policy.
- In particular, a PI is relatively easy to implement since the absence of losers means that no one will oppose the policy.

114

The Impact of a Regulated Increase in *y*

- In contrast, a policy that creates only a PPI will be opposed by the losers unless those losers receive actual compensation.
- This makes the politics of the policy much more complicated.
- We will encounter these important distributional issues throughout the course.

115

2.6 A NEGATIVE EXTERNALITY

A Negative Externality

- Now consider a setting where the activity has an external cost but no external benefit: $D(y)>0$ but $G(y)=0$.

- For example, if y is output from a factory then $D(y)$ might be the damage associated with the pollution produced as a by-product.

117

A Negative Externality

- Since there is no external benefit,

$$SB(y) = PB(y)$$

and it follows that

$$MSB(y) = MPB(y)$$

118

A Negative Externality

- Conversely, social cost is

$$SC(y) = PC(y) + D(y)$$

119

A Negative Externality

- Accordingly, we can decompose the rate of change of $SC(y)$ into two components:

$$MSC(y) = MPC(y) + MEC(y)$$

where $MEC(y)$ is the **marginal external cost** of y.

120

A Negative Externality

- Graphically, $MEC(y)$ is the vertical distance between $MSC(y)$ and $MPC(y)$.
- See Figure 2-23.

121

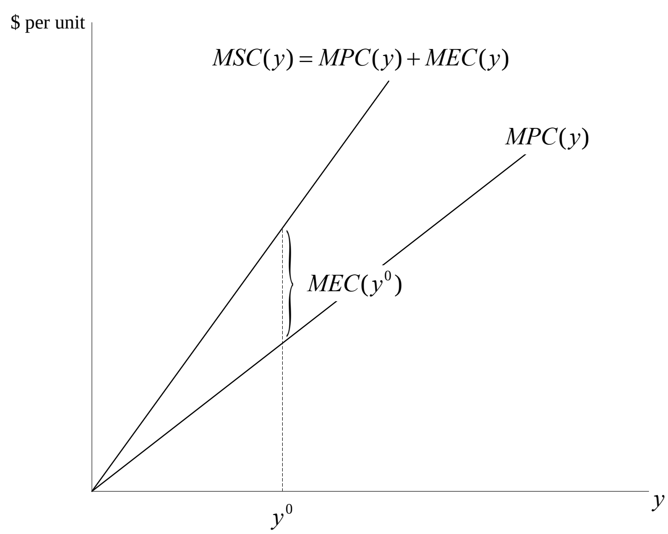

FIGURE 2-23

122

A Negative Externality

- Recall from s.47 our assumption that $D(y)$ is increasing at an increasing rate.

- This is reflected in Figure 2-23: $MEC(y)$ rises as y rises; the gap between $MSC(y)$ and $MPC(y)$ becomes larger.

123

A Negative Externality

- We will later find it useful to depict $MEC(y)$ as a separate graph.
- It is constructed by graphing the vertical distance between $MSC(y)$ and $MPC(y)$ at every value of y.

- See Figure 2-24.

124

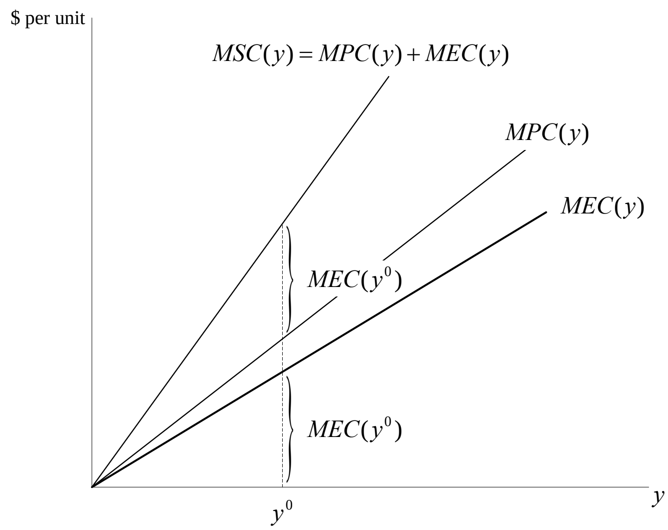

FIGURE 2-24

125

A Negative Externality

- Now let us consider the impact of this external cost on the relationship between the private and social optima.
- See Figure 2-25.

126

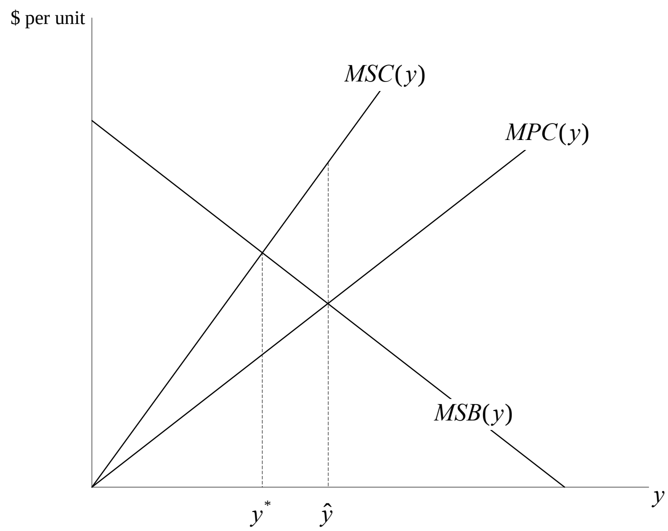

FIGURE 2-25

127

A Negative Externality

- The presence of the external cost means:

$$\hat{y} > y^*$$

– *ie*. the privately optimal level of activity is greater than the socially optimal level.

128

A Negative Externality

- Intuition:
  - the source agent does not take into account the cost she imposes on the external agent when she chooses her action, and so her chosen level of the action is too high from a social perspective.

129

A Negative Externality: A Numerical Example

- Consider a numerical example. Suppose

$$MPB(y) = 30 - y$$

$$MPC(y) = \frac{y}{2}$$

$$MEC(y) = y$$

130

A Negative Externality: A Numerical Example

- First derive the <u>private optimum</u>, given by $\hat{y}$ such that

$$30 - \hat{y} = \frac{\hat{y}}{2}$$

which solves for

$$\hat{y} = 20$$

131

A Negative Externality: A Numerical Example

- Next consider the <u>social optimum</u>.
- *MSC*(*y*) is the sum of *MPC*(*y*) and *MEC*(*y*):

$$MSC(y) = \frac{y}{2} + y = \frac{3y}{2}$$

- Since there is no external benefit here, *MSB*(*y*) is simply equal to *MPB*(*y*).

132

A Negative Externality: A Numerical Example

- Thus, the social optimum is y^* such that

$$30 - y^* = \frac{3y^*}{2}$$

which solves for

$$y^* = 12$$

- See Figure 2-26.

133

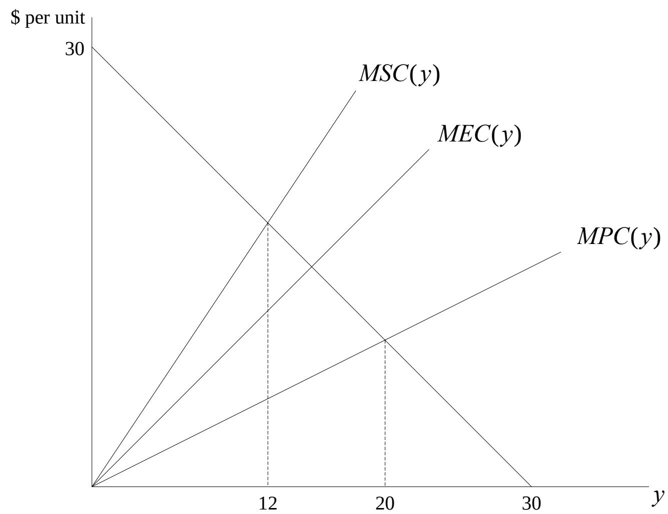

FIGURE 2-26

134

The Impact of a Regulated Reduction in *y*

- Now suppose a third party (such as a government regulator) could <u>force</u> the source agent to reduce her activity level from $\hat{y}$ to y^*.

135

The Impact of a Regulated Reduction in *y*

- Using the same methodology we used in Section 2.4, we will show that this forced increase yields a potential Pareto improvement:
  - the source agent loses but the external agent gains by more than enough to compensate for that loss.

136

The Impact of a Regulated Reduction in *y*

- Consider first the gain to the external agent (the <u>reduction</u> in external cost).
- External cost at the <u>private optimum</u> is

$$D(\hat{y}) = \int_0^{\hat{y}} MEC(y)dy$$

137

The Impact of a Regulated Reduction in *y*

- Since

$$MEC(y) = MSC(y) - MPC(y)$$

we can find the area under $MEC(y)$ as the difference between the area under $MSC(y)$ and the area under $MPC(y)$.

138

The Impact of a Regulated Reduction in *y*

- Thus,

$$D(\hat{y}) = \int_0^{\hat{y}} MEC(y)dy$$
$$= \int_0^{\hat{y}} MSC(y)dy - \int_0^{\hat{y}} MPC(y)dy$$

- See Figure 2-27.

139

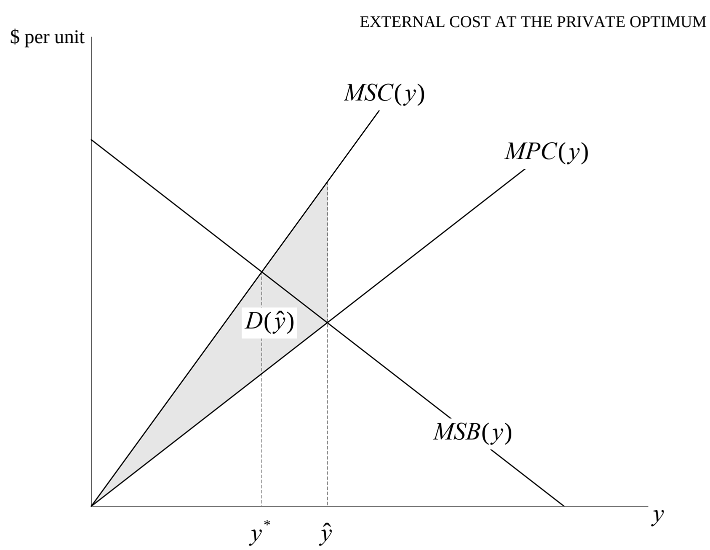

FIGURE 2-27

140

The Impact of a Regulated Reduction in y

- External cost at the social optimum is

$$D(y^*) = \int_0^{y^*} MEC(y)dy$$

$$= \int_0^{y^*} MSC(y)dy - \int_0^{y^*} MPC(y)dy$$

- See Figure 2-28.

141

EXTERNAL COST AT THE SOCIAL OPTIMUM

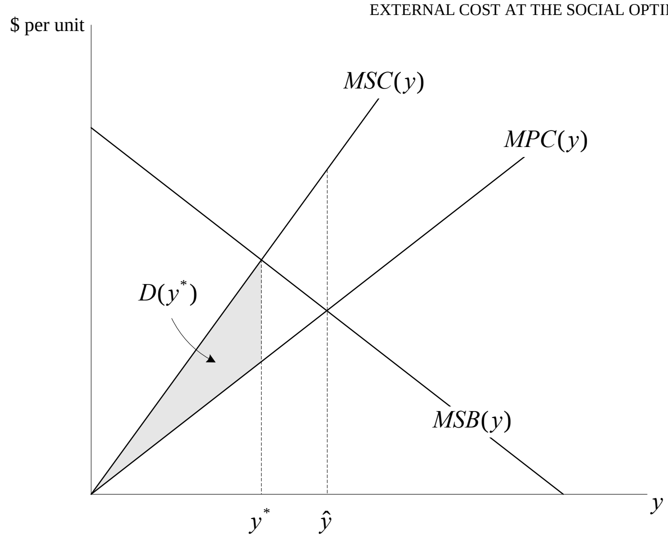

FIGURE 2-28

142

The Impact of a Regulated Reduction in *y*

- Hence, the reduction in external cost is

$$D(\hat{y}) - D(y^*) = \int_0^{\hat{y}} MEC(y)dy - \int_0^{y^*} MEC(y)dy$$

- This is the **gain to the external agent**; it is area *area*(*abcd*) Figure 2-29 (the difference in areas in Figures 2-27 and 2-28).

143

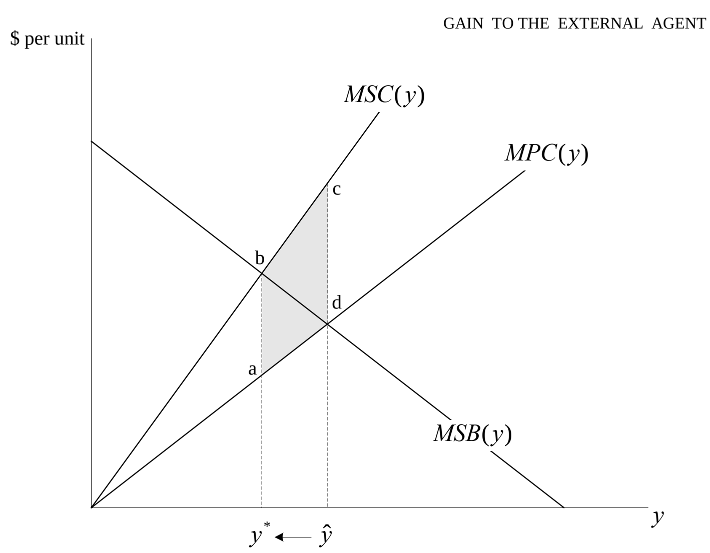

FIGURE 2-29

144

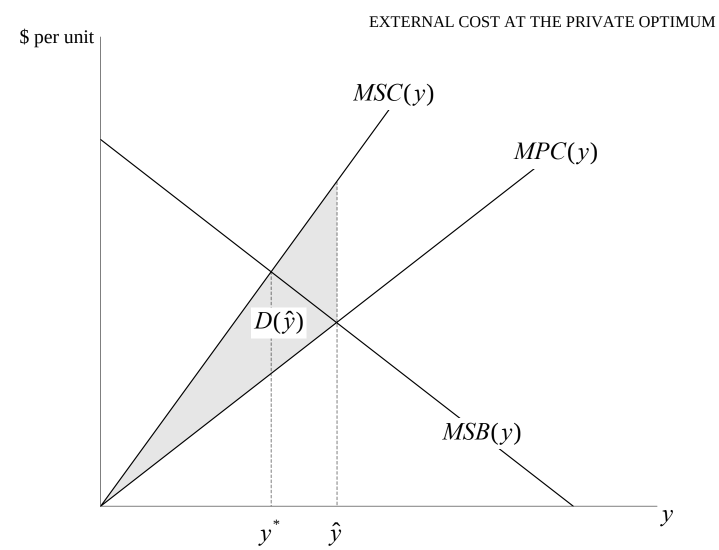

FIGURE 2-27 (repeat)

145

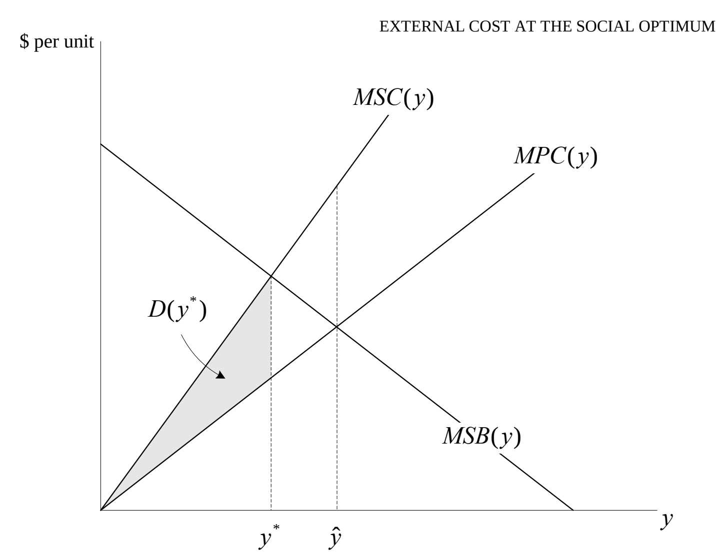

FIGURE 2-28 (repeat)

146

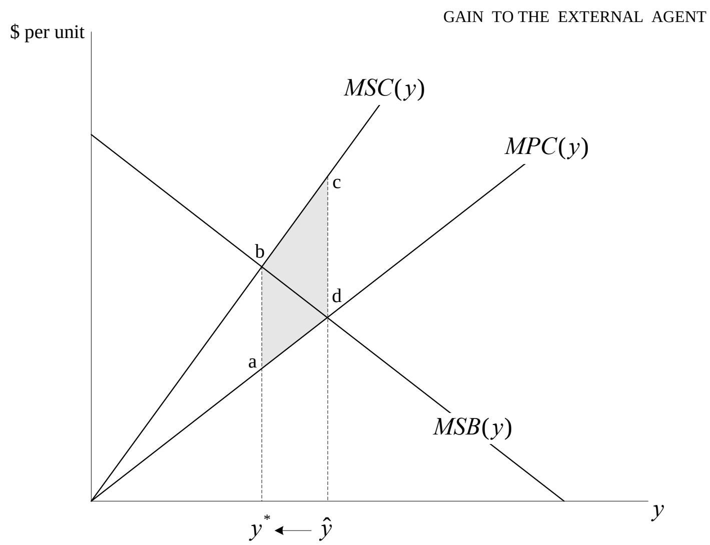

FIGURE 2-29 (repeat)

147

The Impact of a Regulated Reduction in *y*

- Note that this reduction in external cost can also be written as

$$D(\hat{y}) - D(y^*) = \int_{y^*}^{\hat{y}} MEC(y)dy$$

- This definite integral is the area under $MEC(y)$ between y^* and $\hat{y}$; see Figure 2-30.

148

GAIN TO THE EXTERNAL AGENT

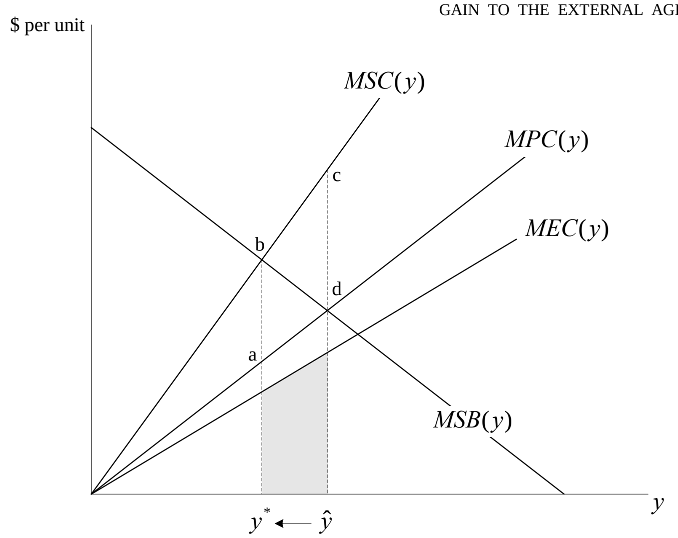

FIGURE 2-30

149

The Impact of a Regulated Reduction in *y*

- Note that the shaded areas in Figures 2-29 and 2-30 are necessarily equal; they are alternative graphical representations of the gain to the external agent.

150

The Impact of a Regulated Reduction in *y*

- Next consider the reduction in net private benefit for the source agent.
- Recall that private benefit to the source agent at the private optimum is

$$PB(\hat{y}) = \int_0^{\hat{y}} MPB(y)dy$$

151

The Impact of a Regulated Reduction in *y*

- In comparison, private benefit to the source agent at the social optimum is

$$PB(y^*) = \int_0^{y^*} MPB(y)dy$$

152

The Impact of a Regulated Reduction in *y*

- Thus, the reduction in private benefit to the source agent is

$$PB(\hat{y}) - PB(y^*) = \int_{y^*}^{\hat{y}} MPB(y)dy$$

- See Figure 2-31.

153

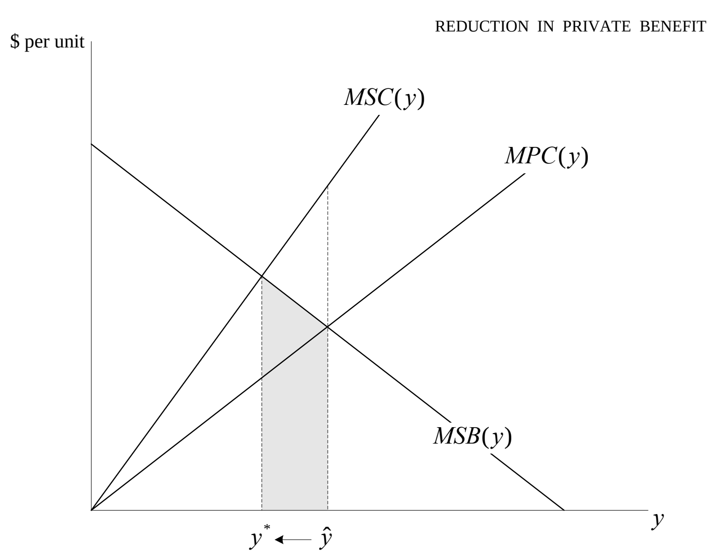

FIGURE 2-31

154

The Impact of a Regulated Reduction in *y*

- By the same logic, the reduction in private cost to the source agent is

$$PC(\hat{y}) - PC(y^*) = \int_{y^*}^{\hat{y}} MPC(y)dy$$

- See Figure 2-32.

155

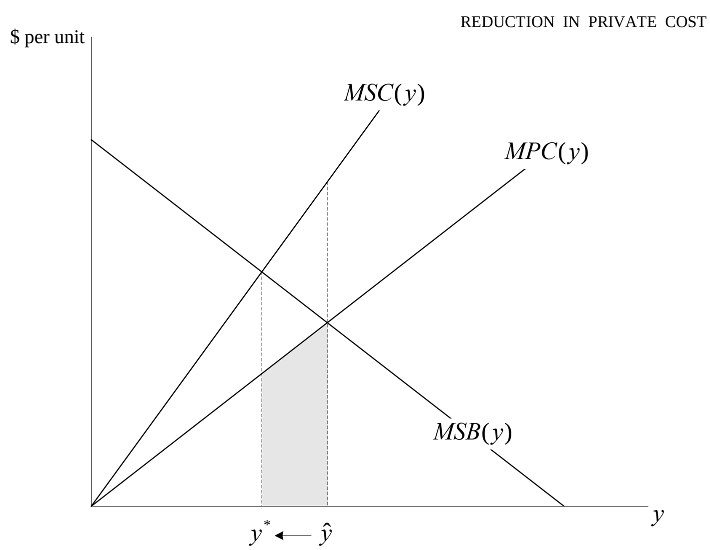

FIGURE 2-32

156

The Impact of a Regulated Reduction in *y*

- It is clear from Figures 2-31 and 2-32 that the reduction in private benefit exceeds the reduction in private cost.
- Thus, the overall change in <u>net</u> private benefit for the source agent is negative.
- See Figure 2-33.

157

LOSS TO THE SOURCE AGENT

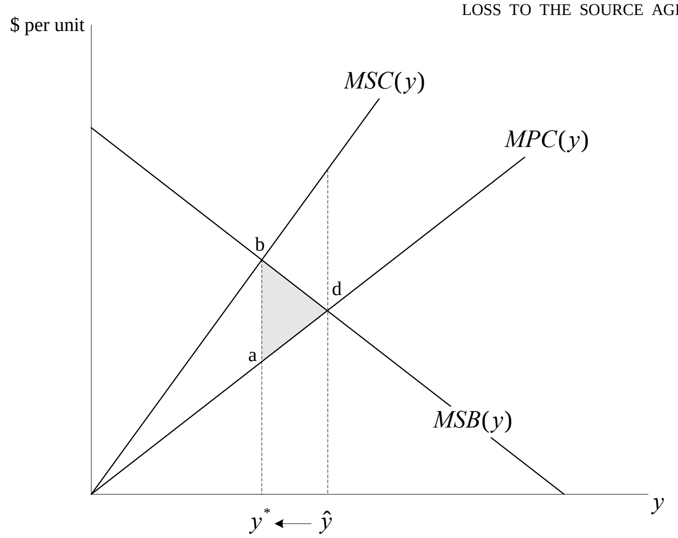

FIGURE 2-33

158

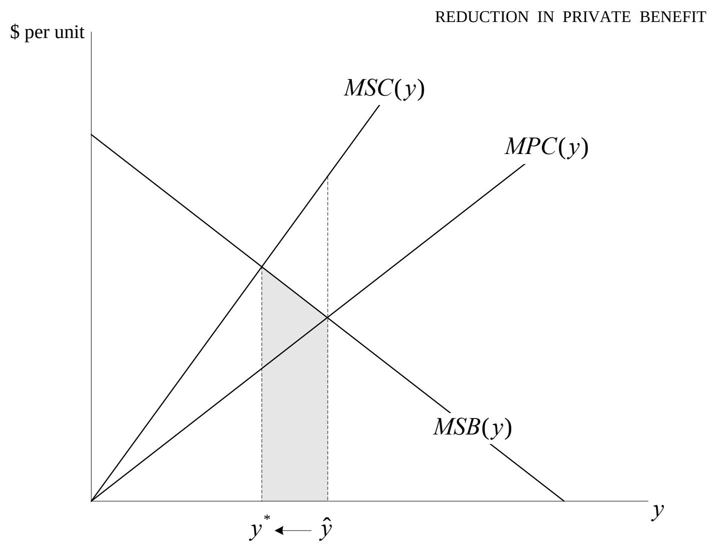

FIGURE 2-31 (repeat) 159

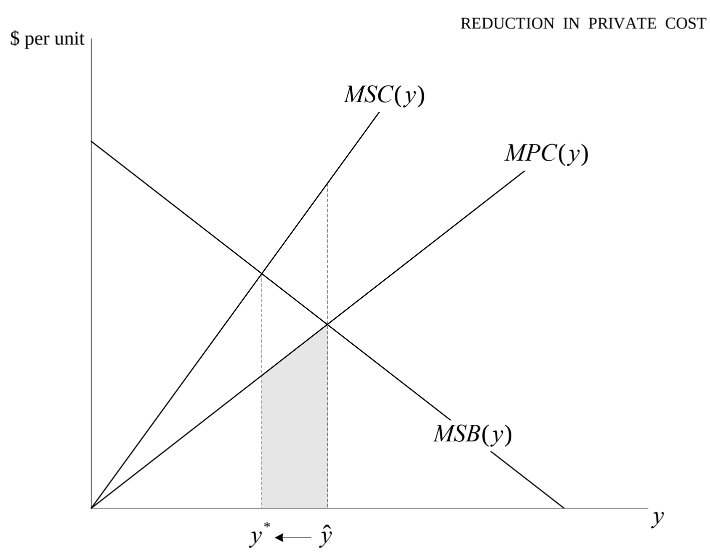

FIGURE 2-32 (repeat) 160

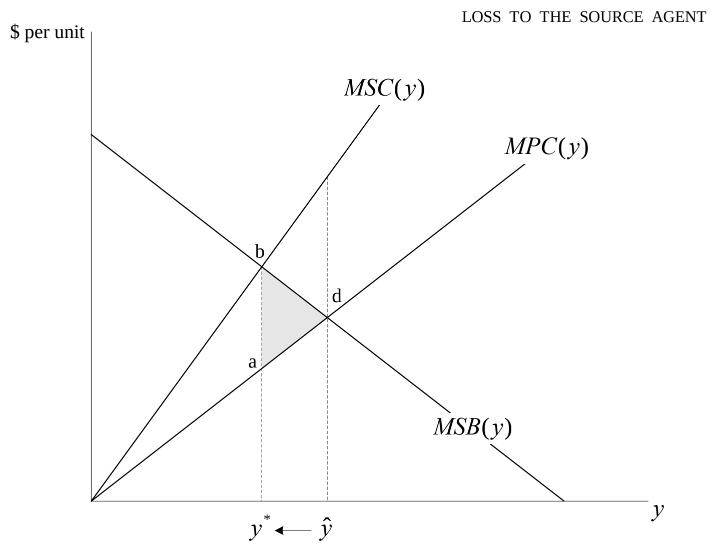

FIGURE 2-33 (repeat)

161

The Impact of a Regulated Reduction in *y*

- The source agent is made worse-off because she is forced to move away from her private optimum, and there is no offsetting compensation.

162

The Impact of a Regulated Reduction in *y*

- In summary, from Figures 2-29 and 2-33:
  - the gain to the external agent = *area*(*abcd*)
  - the loss to the source agent = *area*(*abd*)
- Thus, the overall **gain in social surplus** = *area*(*bcd*)
- See Figure 2-34.

163

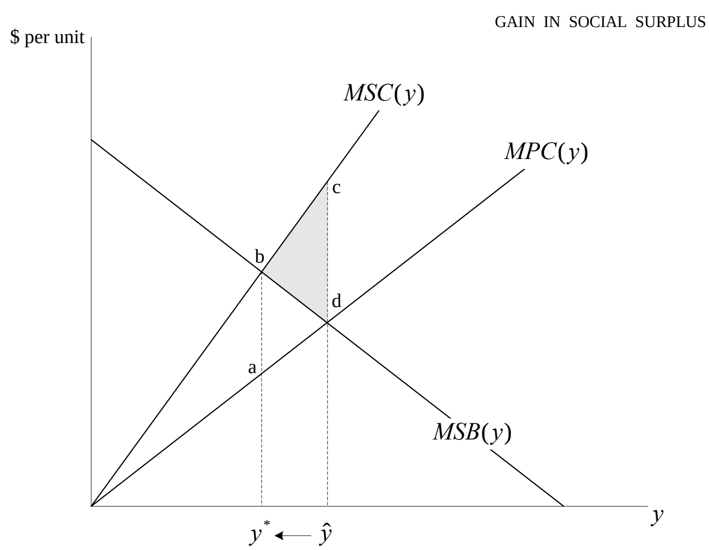

FIGURE 2-34

164

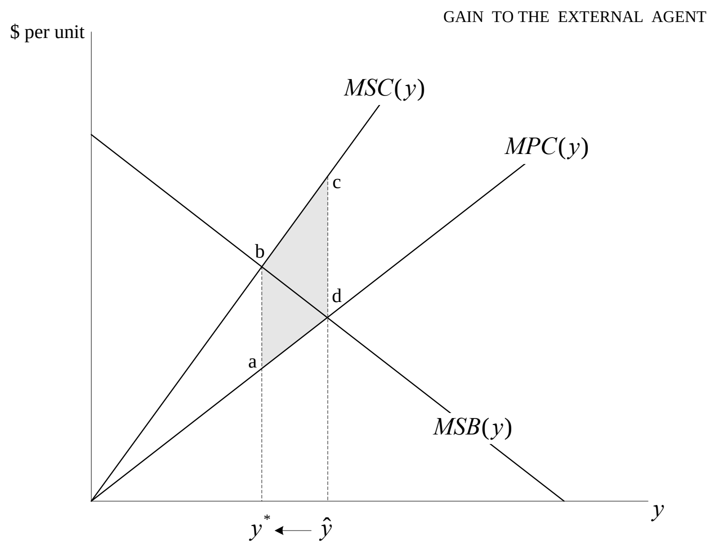

FIGURE 2-29 (repeat) 165

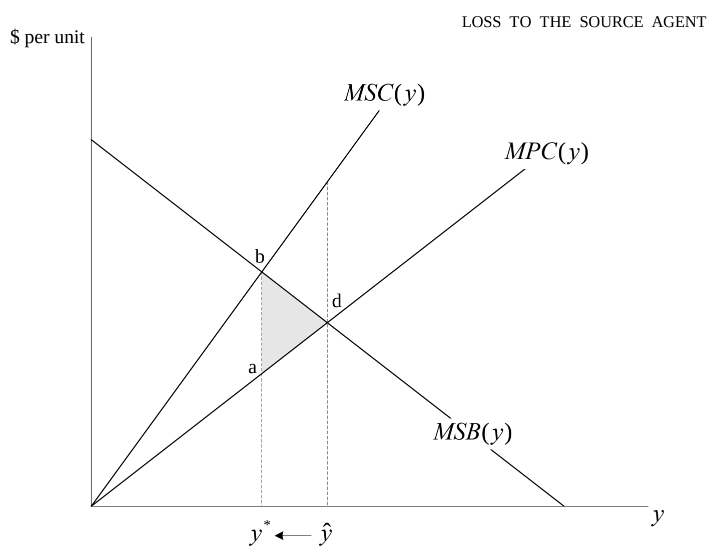

FIGURE 2-33 (repeat) 166

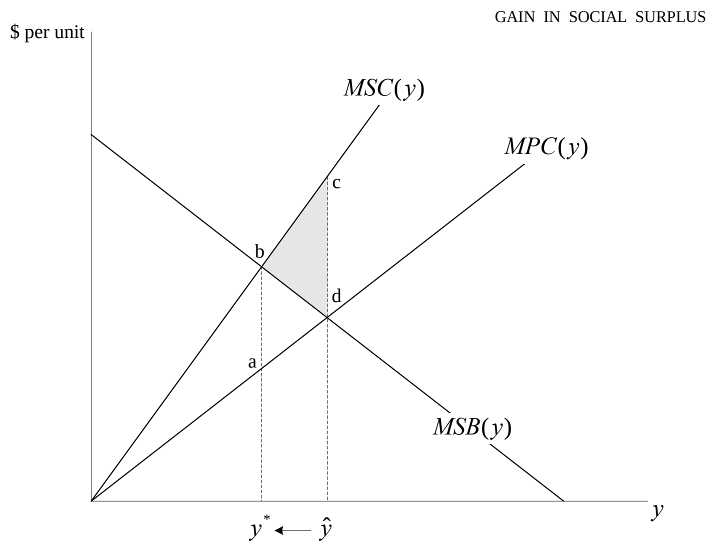

FIGURE 2-34 (repeat)

167

The Impact of a Regulated Reduction in *y*

- What can we say about welfare overall?

168

The Impact of a Regulated Reduction in y

- The forced move from $\hat{y}$ to y^* is not a Pareto improvement; the source agent is made worse-off.
- However, it is a potential Pareto improvement:
  - the winner (the external agent) could in principle fully compensate the loser (the source agent) and still be better-off, by *area*(*bcd*)

169

2.7 AN ALTERNATIVE PRESENTATION OF A NEGATIVE EXTERNALITY

A Negative Externality: An Alternative Presentation

- Recall from Section 2.1 that the private optimum can be characterized by

$$MNPB(\hat{y}) = 0$$

where

$$MNPB(y) \equiv MPB(y) - MPC(y)$$

171

A Negative Externality: An Alternative Presentation

- We can use the same approach to characterize the social optimum in the presence of a negative externality.
- In particular, recall that the social optimum is y^* such that

$$MPB(y^*) = MPC(y^*) + MEC(y^*)$$

172

A Negative Externality: An Alternative Presentation

- Subtract $MPC(y^*)$ from both sides to yield

$$MPB(y^*) - MPC(y^*) = MEC(y^*)$$

- The LHS of this equation is $MNPB(y)$ evaluated at the <u>social</u> optimum.

173

A Negative Externality: An Alternative Presentation

- Thus, at the social optimum

$$MNPB(y^*) = MEC(y^*)$$

- In comparison, at the private optimum

$$MNPB(\hat{y}) = 0$$

- See Figure 2-35.

174

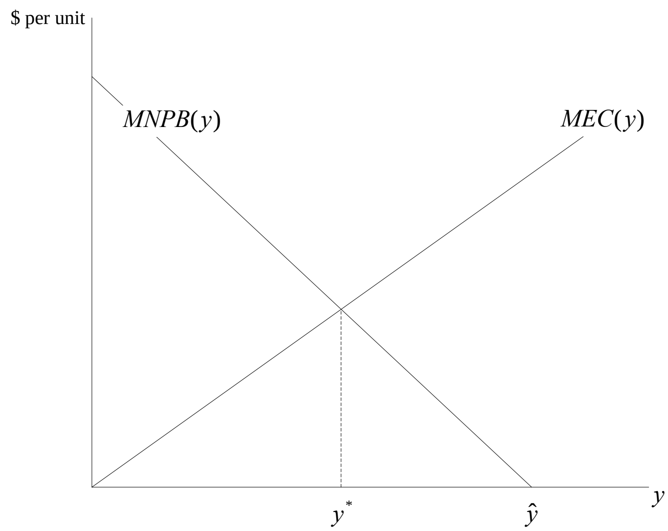

FIGURE 2-35

175

A Negative Externality: An Alternative Presentation

- Now consider a forced move from $\hat{y}$ to y^*.
- The loss to the source agent is

$$\int_{y^*}^{\hat{y}} MNPB(y)dy = area(abd)$$

in Figure 2-36.

176

LOSS TO THE SOURCE AGENT

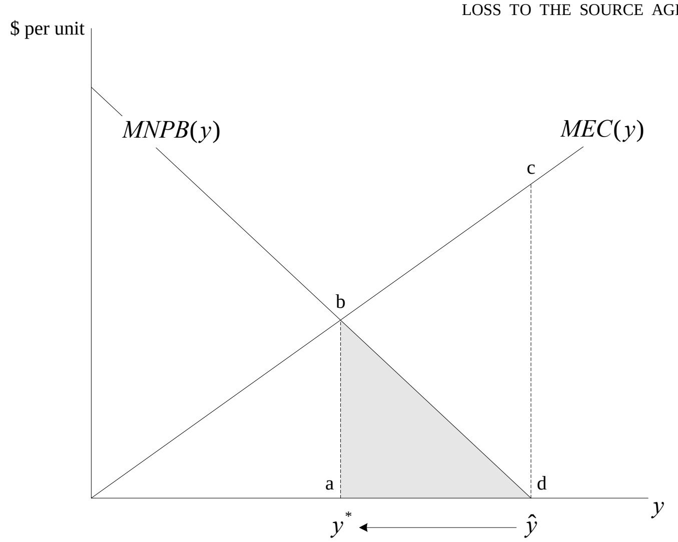

FIGURE 2-36

177

A Negative Externality: An Alternative Presentation

- The gain to the external agent is

$$\int_{y^*}^{\hat{y}} MEC(y)dy = area(abcd)$$

in Figure 2-37.

178

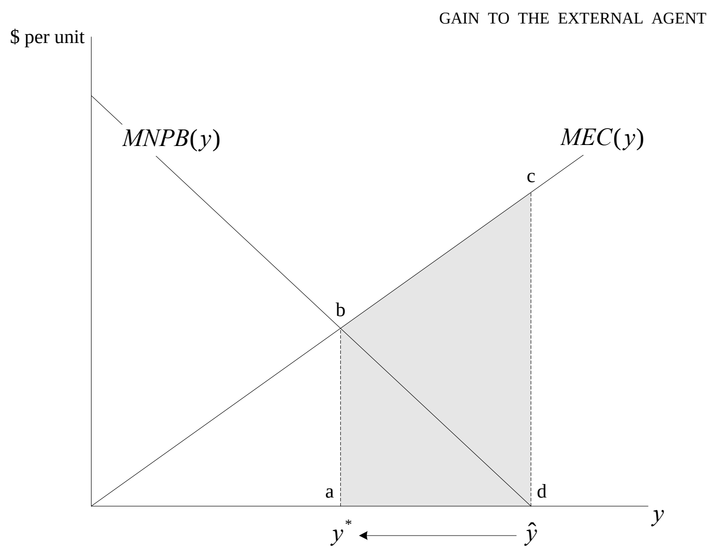

FIGURE 2-37

179

A Negative Externality: An Alternative Presentation

- In summary, from Figures 2-36 and 2-37:
  - the gain to the external agent = *area*(*abcd*)
  - the loss to the source agent = *area*(*abd*)
- Thus, the overall **gain in social surplus** = *area*(*bcd*)
- See Figure 2-38.

180

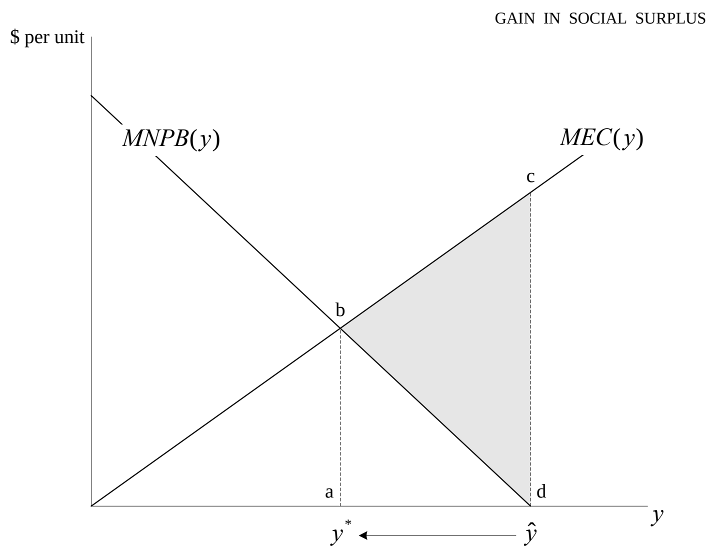

FIGURE 2-38 181

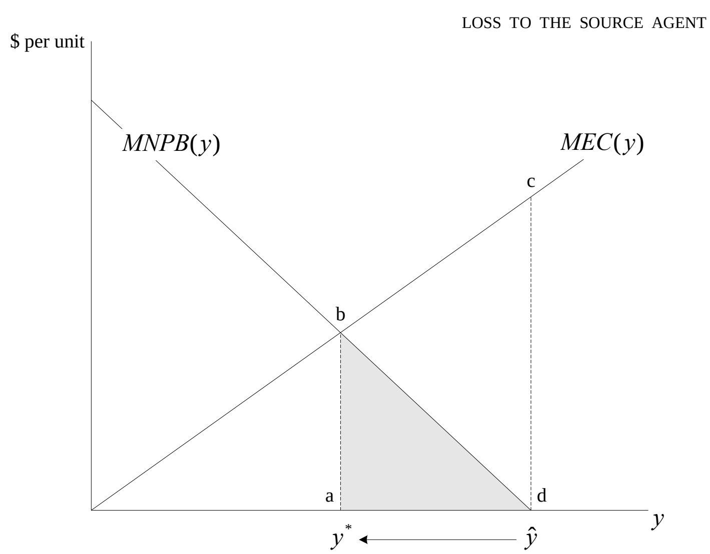

FIGURE 2-36 (repeat) 182

GAIN TO THE EXTERNAL AGENT

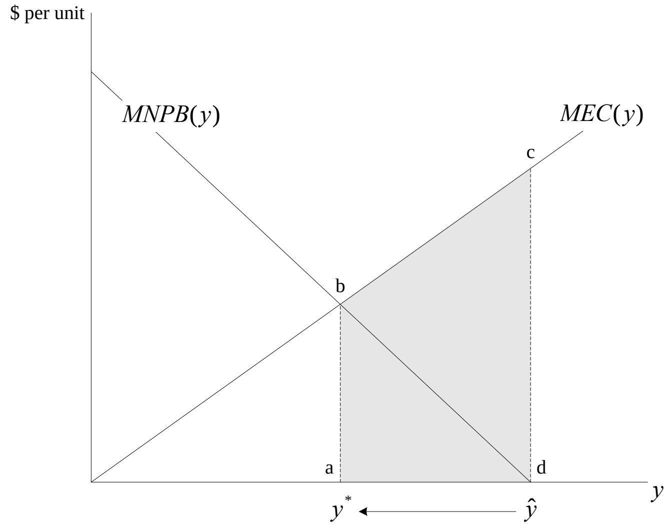

FIGURE 2-37 (repeat) 183

GAIN IN SOCIAL SURPLUS

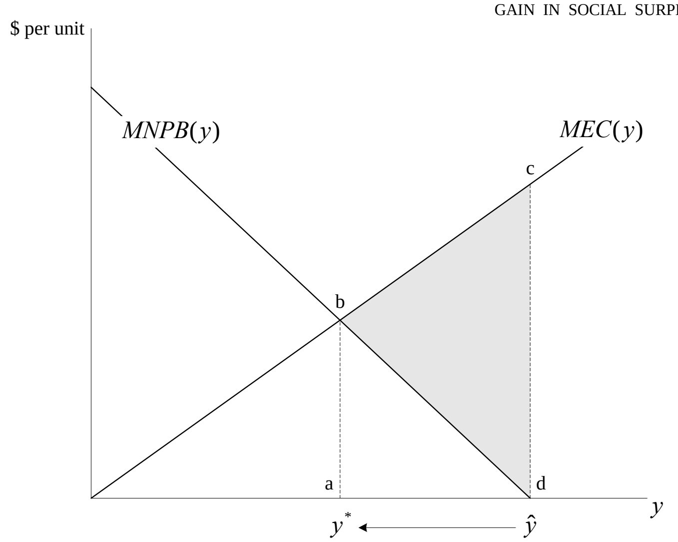

FIGURE 2-38 (repeat) 184

A Negative Externality: An Alternative Presentation

- If all our figures were drawn to the same scale, the shaded area in Figure 2-38 must be equal to the shaded area in Figure 2-34; they are alternative graphical representations of the gain in social surplus when y is reduced from $\hat{y}$ to y^*.

185

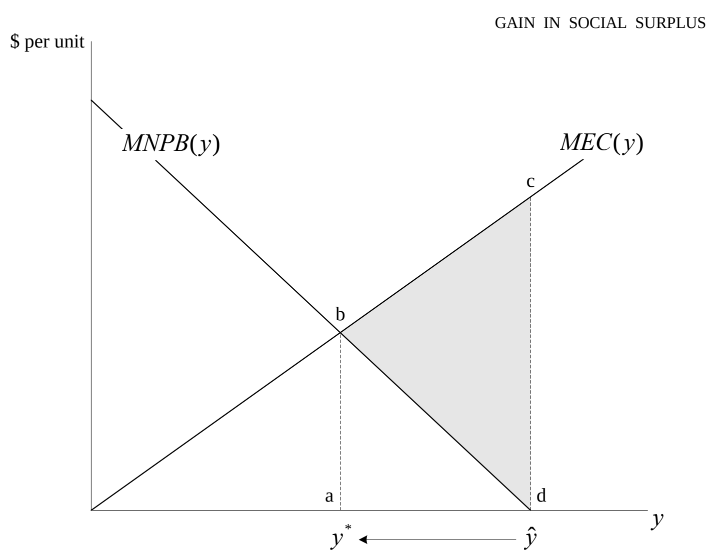

FIGURE 2-38 (repeat)

186

GAIN IN SOCIAL SURPLUS

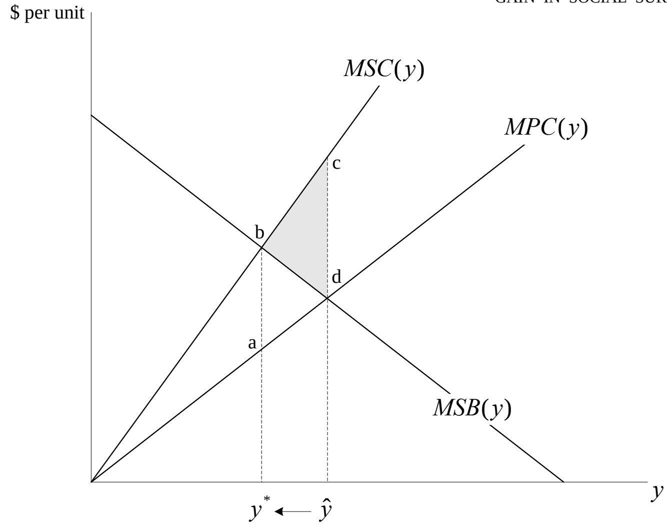

FIGURE 2-34 (repeat)

187

2.8 MULTIPLE EXTERNAL AGENTS

Multiple External Agents

- We have so far restricted attention to a scenario where there is only one external agent.
- Our framework extends easily to a setting with multiple external agents.
- (We can also extend consideration to multiple source agents but we will delay treatment of that issue until Topic 3).

189

Multiple External Agents

- Let $D_i(y)$ denote the external cost to external agent i, and suppose there are a total of n external agents affected by the activity.

- In addition, suppose that the activity is a **pure public bad** for the external agents.

190

Multiple External Agents

- This means that the impact of the activity on any one external agent does not depend on how other external agents are affected.

- It does not mean that all external agents suffer the same cost.

191

Multiple External Agents

- For example, oil discharged into a drinking-water source (such as a lake) has an impact on all users of the water.
- The effect of the oil is not eliminated when one user drinks polluted water; the remaining water is still polluted.
- The impact on each user might nonetheless be different, depending on their usage.

192

Multiple External Agents

- Of course, one can envisage settings where the impact on any one external agent does depend on how many agents are affected, as when the physical impact is partly "diluted" when spread across many people.
- In this case, the activity is an **impure** public bad.
- We will not consider such cases here.

193

Multiple External Agents

- If the activity is a pure public bad then the **aggregate external cost** of the activity is

$$D(y) = \sum_{i=1}^{n} D_i(y)$$

194

Multiple External Agents

- The same logic applies to the construction of **aggregate marginal external cost**:

$$MEC(y) = \sum_{i=1}^{n} MEC_i(y)$$

195

Multiple External Agents

- In graphical terms, we take the <u>vertical</u> summation of all the individual MEC schedules to obtain the aggregate MEC schedule.

- See Figure 2-39 for the case of two external agents.

196

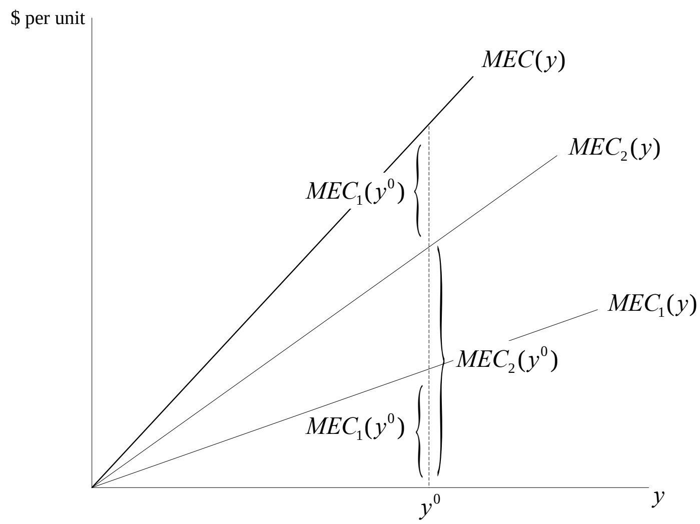

FIGURE 2-39

197

Multiple External Agents

- The aggregate external cost at some level of the activity y^0 is simply the sum of the areas under the two individual MEC schedules.
- See Figures 2-40 through 2-42.

198

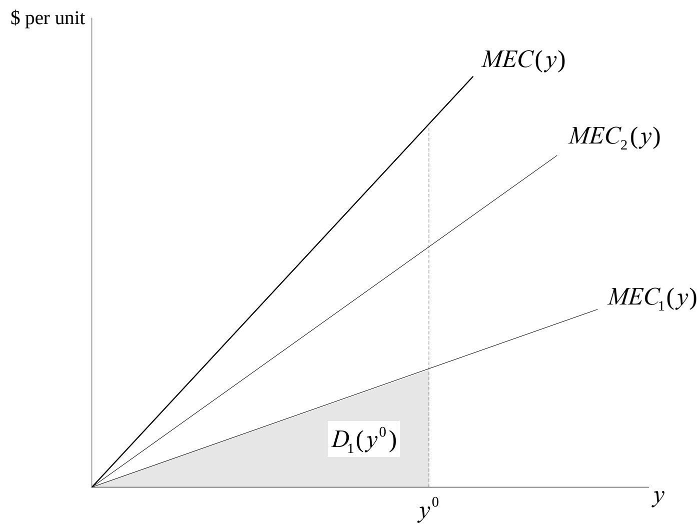

FIGURE 2-40

199

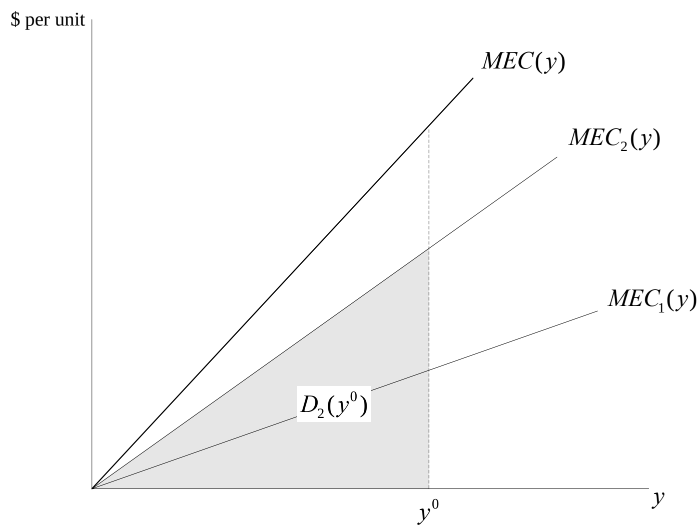

FIGURE 2-41

200

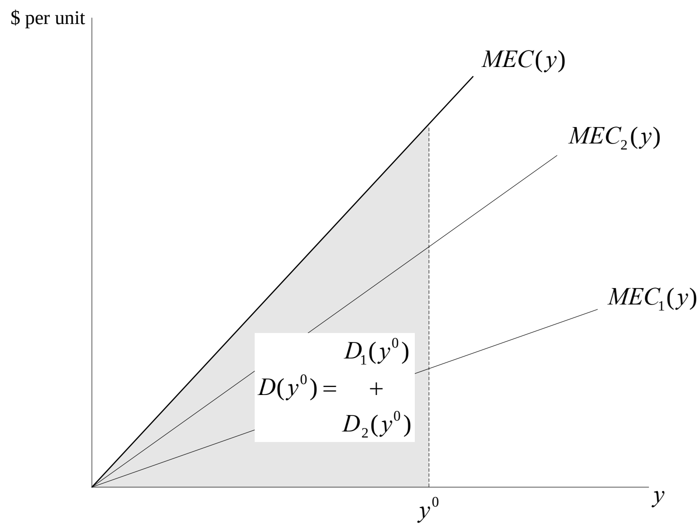

FIGURE 2-42

201

Multiple External Agents

- Consider an example. Suppose

$$MEC_1(y) = 10 + 3y$$

$$MEC_2(y) = 5y$$

- Then

$$MEC(y) = 10 + 8y$$

202

Multiple External Agents

- In a setting with multiple external agents, the social optimum is based on the aggregate marginal external cost:

$$MNPB(y^*) = MEC(y^*)$$

203

Multiple External Agents

- Thus, all of our previous analysis applies equally well to the multiple-agent setting, where MEC is now assumed to mean the aggregate marginal external cost, regardless of how many external agents there are.

204

2.9 WHERE IS THE MARKET FAILURE?

Where is the Market Failure?

- Recall that a shift from the private optimum to the social optimum is a PPI.
- In the case of a negative externality, this requires a reduction in the activity concerned.
- Why is this PPI not realized via a contract among the parties involved?

206

Where is the Market Failure?

- In particular, why don't the source agent and the external agents write a contract under which the source agent reduces her activity to y^* voluntarily, in exchange for a payment from the external agents?

207

Where is the Market Failure?

- There are two reasons in practice:
  - an absence of explicit property rights
  - transaction costs

- Let us consider each in turn.

208

Property Rights

- Trade requires that property rights be defined over the traded good.
- In the case of many externalities, property rights are not well-defined.
- For example, suppose a firm discharges pollution into a lake, and farmers draw water from that lake for irrigation.

209

Property Rights

- Who has property rights over the lake?
- There are two possible extremes:
  - the firm has an unlimited right to use the lake for disposal of its waste
  - farmers have an unlimited right to have access to unpolluted water

210

Property Rights

- In our analysis so far, we have assumed that the firm has an implicit unlimited right to pollute:
  - it is polluting at its private optimum without any requirement to consider the external costs
- The right is implicit in the sense that it reflects existing practice (even though the right may not be written down in law).

211

Property Rights

- Suppose we now make this right explicit, and allow the firm to trade some of those rights to the farmers if it so chooses.

212

Property Rights

- The starting point for negotiations with farmers is $\hat{y}$, the private optimum for the firm.
- Any contract between the farmers and the firm would require that farmers pay the firm to reduce its pollution below $\hat{y}$.
- We will henceforth refer to this reduction in pollution as **abatement**.

213

Property Rights

- What would the contract look like?
- Let us frame the contract negotiations in terms of a **price** per unit of abatement.
- In particular, suppose the contract specifies that the farmers must pay the firm a price p for each unit of pollution reduced.

214

Property Rights

- Faced with a price *p*, how many units of abatement will the farmers "buy" from the firm?
- We can describe the decision for the farmers in terms of their marginal costs and marginal benefits.

215

Property Rights

- The marginal cost of purchasing abatement is simply the price that must be paid to the firm for that abatement, *p*.
- The marginal benefit of abatement is the marginal external cost avoided.

216

Property Rights

- Thus, the farmers will buy abatement from the firm up to the point where

$$MEC(y) = p$$

- See Figure 2-43.

217

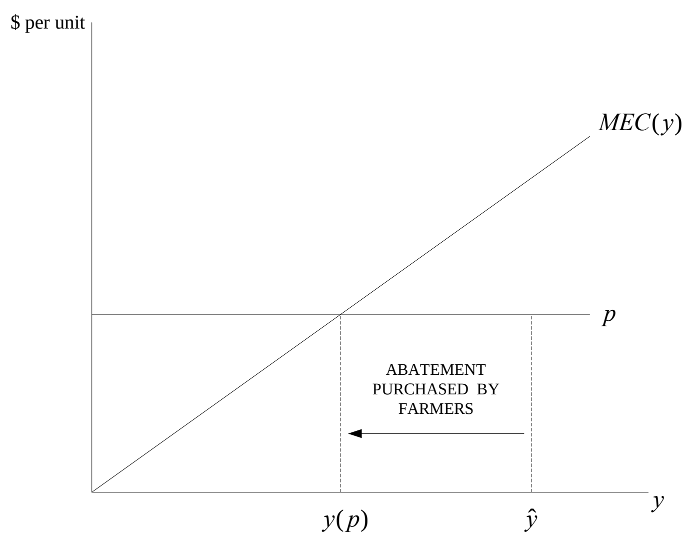

FIGURE 2-43

218

Property Rights

- How many units of abatement is the firm willing to "sell" at price *p*?
- Again, we can frame this decision in terms of marginal costs and marginal benefits.
- The marginal benefit of selling abatement is simply the price received, *p*.

219

Property Rights

- The marginal cost of selling abatement is the marginal net private benefit foregone when the firm cuts pollution.

220

Property Rights

- Thus, the firm will sell abatement to farmers up to the point where

$$MNPB(y) = p$$

- See Figure 2-44.

221

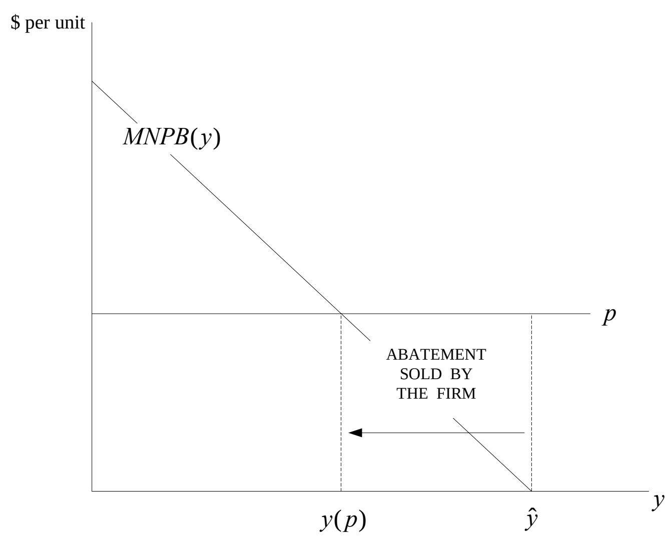

FIGURE 2-44

222

Property Rights

- To reach an agreed price, the amount of abatement that farmers are willing to buy at that price must be equal to the amount of abatement that the firm is willing to sell at that price.
- What is this **equilibrium price**?
- See Figure 2-45.

223

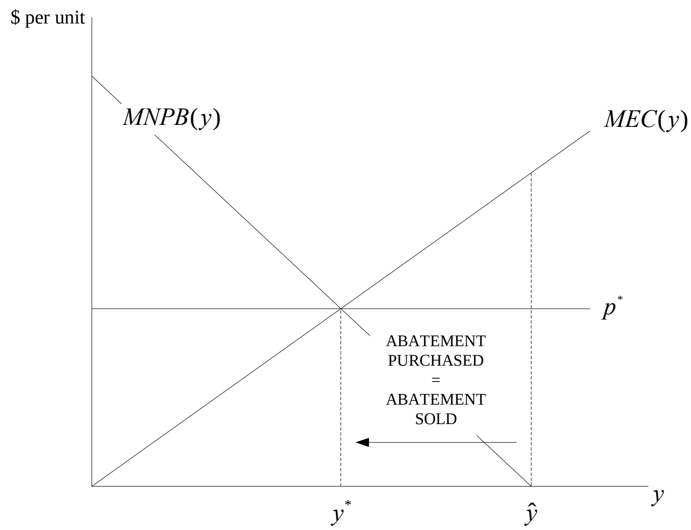

FIGURE 2-45

224

Property Rights

- It is clear from Figure 2-45 that equilibrium is reached at price p^* where

$$MEC(y) = p^* = MNPB(y)$$

225

Property Rights

- The abatement traded in equilibrium is

$$a(p^*) = \hat{y} - y^*$$

- Thus, the contract between the firm and the farmers reduces pollution from $\hat{y}$ to y^*; the contract achieves the social optimum.

226

Property Rights

- How is the associated gain in net social benefit split between the firm and the farmers?

227

Property Rights

- The **gains from trade for the firm**
  = (the revenue from the sale of abatement)
  – (the net private benefit foregone due to that abatement)

$$= p^*(\hat{y} - y^*) - \int_{y^*}^{\hat{y}} MNPB(y)dy$$

- See Figure 2-46.

228

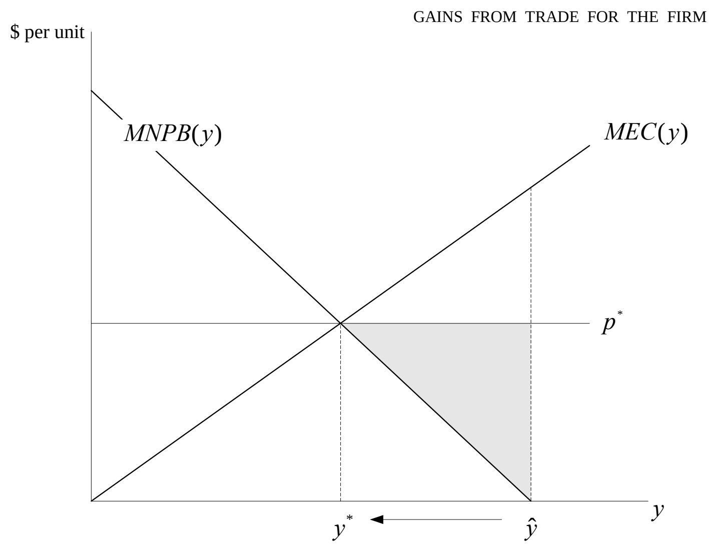

FIGURE 2-46

229

Property Rights

- The **gains from trade for the farmers**
  = (the external cost avoided)
  – (the total payment to the firm)

$$= \int_{y^*}^{\hat{y}} MEC(y)dy - p^*(\hat{y} - y^*)$$

- See Figure 2-47.

230

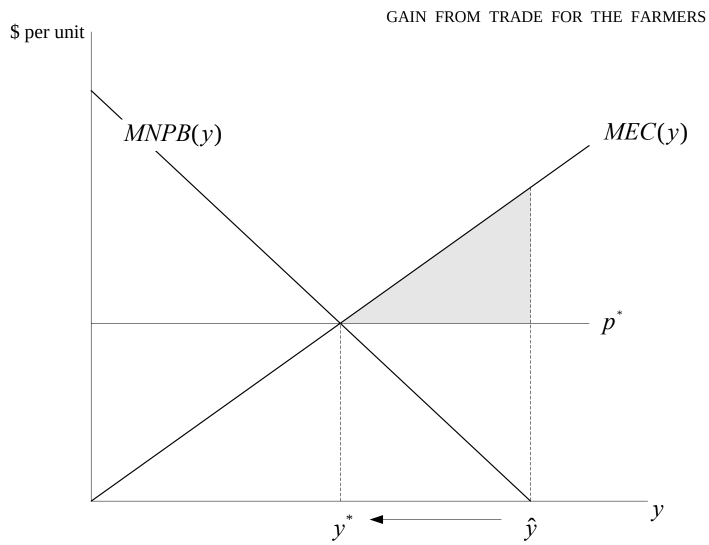

FIGURE 2-47

231

Property Rights

- Comparing Figures 2-46 and 2-47 with Figure 2-38 (repeated next slide) we see that the <u>total</u> gains from trade are exactly equal to the gain in social surplus from a forced reduction in *y*.

232

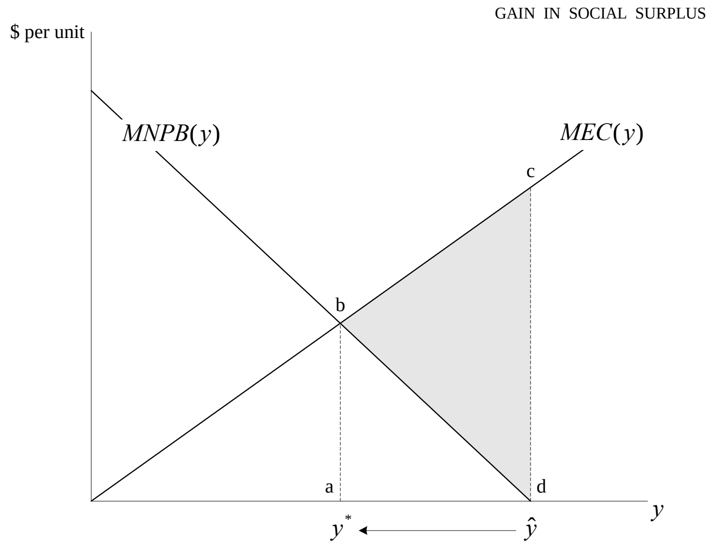

FIGURE 2-38 (repeat)

233

Property Rights

- Thus, the potential Pareto improvement available at the private optimum is fully realized through trade when property rights are made explicit.

234

Property Rights

- This appears to be a straightforward solution to the externality problem; why is policy intervention needed?

235

Property Rights

- There are two potential problems with the "property rights solution":
  - Policy-makers may not be willing to assign explicit pollution rights to polluters even when those rights are currently implicit
  - Transaction costs may create an obstacle to trade
- Let us now explore the second of these.

236

Transaction Costs

- Contracts are costly to construct.
- Significant resources are required to bargain towards an agreement, and then write down that agreement in legally robust terms that cover all possible contingencies.
- These **transaction costs** can be large enough to prevent an otherwise mutually beneficial trade from occurring.

237

Transaction Costs

- Do these transaction costs necessarily justify policy intervention?
- No. If real resources must be used up to capture gains from trade through a contract then the true gains from trade are less than they appear; the potential Pareto improvement associated with an externality may be an illusion.

238

Transaction Costs

- However, private trade is not the only mechanism through which resources can be reallocated, and in some settings it may not be the best one.
- In some settings, policy intervention may be a better mechanism in the sense that fewer resources are required to achieve the reallocation than are required via trade.

239

Transaction Costs

- Policy intervention is most likely to have an advantage over trade when there are large numbers of external agents.
- Why?
- Let us explore the answer in the context of the firm versus the farmers.

240

Transaction Costs

- Abatement by the firm is a **public good** from the perspective of the farmers: each farmer benefits even if he does not participate in the bargaining.
- However, a farmer must incur transaction costs in order to participate.
- Thus, each farmer has an incentive to **free-ride** on the bargaining efforts of the others.

241

Transaction Costs

- This free-riding can mean that an abatement agreement is not reached even though it would be to the mutual benefit of all parties involved.
- A better solution might involve direct policy intervention by government.

242

Transaction Costs

- In general, the existence of an externality is not enough on its own to justify policy intervention.
- Any intervention (on efficiency grounds) must be argued on the basis of policy being able to achieve a net social benefit when the market cannot.

243

Transaction Costs

- This is most often true when there are large numbers of external agents, and this is very often true in environmental policy settings.
- The rest of the course focuses on policy design under the premise that a good case for intervention has already been established in the particular setting examined.

END

244

TOPIC 2 REVIEW QUESTIONS

These review questions begin with a solved example. It is recommended that you work through this example before commencing the questions.

A SOLVED EXAMPLE

Consider the following setting with a negative externality.

$$MPB(y) = 1600 - 200y$$

$$MPC(y) = 120y$$

$$MEC(y) = 80y$$

Our goal is characterize the private and social optima, and compare all relevant private and social surplus measures under the two optima.

Method 1

This method uses with the MPC and MPB functions. (Method 2 following is simpler; it uses the MNPB function).

Find marginal social cost

$$MSC(y) = MPC(y) + MEC(y) = 120y + 80y = 200y$$

See Figure R2-1. In order to illustrate all key values clearly, the figures here are to scale.

The private optimum

Solve the following for $\hat{y}$:

$$MPB(\hat{y}) = MPC(\hat{y})$$

$$1600 - 200\hat{y} = 120\hat{y}$$

$$\hat{y} = 5$$

The social optimum

Solve the following for y^*:

$$MSB(y^*) = MSC(y^*)$$

$$1600 - 200y^* = 200y^*$$

$$y^* = 4$$

External cost at the private optimum

$$D(\hat{y}) = \int_0^5 MEC(y)dy = \int_0^5 MSC(y)dy - \int_0^5 MPC(y)dy$$

which is equal to the shaded area in Figure R2-2 and in Figure R2-3. It can be calculated without calculus – using the geometry of triangles – from Figure R2-2 as follows:

$$D(\hat{y}) = \int_0^5 MEC(y)dy = \frac{5*400}{2} = 1000$$

Alternatively, it can be calculated using the geometry of triangles from Figure R2-3 as follows:

$$D(\hat{y}) = \int_0^5 MSC(y)dy - \int_0^5 MPC(y)dy = \frac{5*1000}{2} - \frac{5*600}{2} = 1000$$

External cost at the social optimum

$$D(y^*) = \int_0^4 MEC(y)dy = \int_0^4 MSC(y)dy - \int_0^4 MPC(y)dy$$

which is equal to the shaded area in Figure R2-4 and in Figure R2-5. It can be calculated using the geometry of triangles from Figure R2-4 as follows:

$$D(y^*) = \int_0^4 MEC(y)dy = \frac{4*320}{2} = 640$$

Alternatively, it can be calculated using the geometry of triangles from Figure R2-5 as follows:

$$D(y^*) = \int_0^4 MSC(y)dy - \int_0^4 MPC(y)dy = \frac{4*800}{2} - \frac{4*480}{2} = 640$$

Gain to external agents from a regulated move from private to social optimum

$$G_{EXT}(\hat{y} \to y^*) = \int_4^5 MEC(y)dy = D(\hat{y}) - D(y^*) = 100 - 640 = 360$$

See Figures R2-6 and R2-7. (The shaded areas are equal).

Loss to the source agent from a regulated move from private to social optimum

$$L_S(\hat{y} \to y^*) = \int_4^5 MPB(y)dy - \int_4^5 MPC(y)dy$$

This is the shaded area in Figure R2-8. It can be calculated using the geometry of triangles as follows:

$$L_S(\hat{y} \to y^*) = \frac{(5-4)*(800-480)}{2} = 160$$

Gain in social surplus from a regulated move from private to social optimum

$$\Delta SS(\hat{y} \to y^*) = G_{EXT}(\hat{y} \to y^*) - L_S(\hat{y} \to y^*) = 360 - 160 = 200$$

See Figure R2-9. To confirm the calculations, we can calculate the shaded area in Figure R2-9 directly:

$$\Delta SS(\hat{y} \to y^*) = \frac{(5-4)*(1000-600)}{2} = 200$$

Method 2

Find the marginal net private benefit function

$$MNPB(y) = MPB(y) - MPC(y) = (1600 - 200y) - 120y = 1600 - 320y$$

See Figure R2-10.

The private optimum

Solve the following for $\hat{y}$:

$$MNPB(\hat{y}) = 0$$

$$1600 - 320\hat{y} = 0$$

$$\hat{y} = 5$$

The social optimum

Solve the following for y^* :

$$MNPB(y^*) = MEC(y^*)$$

$$1600 - 320y^* = 80y^*$$

$$y^* = 4$$

External cost at the private optimum

$$D(\hat{y}) = \int_0^5 MEC(y)dy$$

which is equal to the shaded area in Figure R2-11. It can be calculated using the geometry of triangles from Figure R2-11 as follows:

$$D(\hat{y}) = \int_0^5 MEC(y)dy = \frac{5*400}{2} = 1000$$

External cost at the social optimum

$$D(y^*) = \int_0^4 MEC(y)dy$$

which is equal to the shaded area in Figure R2-12. It can be calculated using the geometry of triangles from Figure R2-12 as follows:

$$D(y^*) = \int_0^4 MEC(y)dy = \frac{4*320}{2} = 640$$

Gain to external agents from a regulated move from private to social optimum

$$G_{EXT}(\hat{y} \to y^*) = \int_4^5 MEC(y)dy = D(\hat{y}) - D(y^*) = 100 - 640 = 360$$

See Figure R2-13.

Loss to the source agent from a regulated move from private to social optimum

$$L_S(\hat{y} \to y^*) = \int_4^5 MNPB(y)dy$$

This is the shaded area in Figure R2-14. It can be calculated using the geometry of triangles as follows:

$$L_S(\hat{y} \to y^*) = \frac{(5-4)*320}{2} = 160$$

Gain in social surplus from a regulated move from private to social optimum

$$\Delta SS(\hat{y} \to y^*) = G_{EXT}(\hat{y} \to y^*) - L_S(\hat{y} \to y^*) = 360 - 160 = 200$$

See Figure R2-15. To confirm the calculations, we can calculate the shaded area in Figure R2-15 directly:

$$\Delta SS(\hat{y} \to y^*) = \frac{(5-4)*400}{2} = 200$$

We now also easily calculate gains from trade if property rights are assigned.

The equilibrium price

$$p^* = MEC(y^*) = MNPB(y^*) = 320$$

See Figure R2-16.

Gains from trade to source agent if rights assigned to source agent

$$GFT_S(\hat{y} \to y^*) = \text{reciepts} - \text{cost} = p^*(\hat{y} - y^*) - \int_4^5 MNPB(y)dy = 320 - 160 = 160$$

See Figure R2-17.

<u>Gains from trade to external agents if rights assigned to source agent</u>

$$GFT_{EXT}(\hat{y} \to y^*) = \text{benefit} - \text{payment} = \int_4^5 MEC(y)dy - p^*(\hat{y} - y^*) = 360 - 320 = 40$$

See Figure R2-18.

Check that total GFT are equal to $\Delta SS(\hat{y} \to y^*)$:

$$GFT_S(\hat{y} \to y^*) + GFT_{EXT}(\hat{y} \to y^*) = 160 + 140 = 200$$

<u>Gains from trade to source agent if rights assigned to external agents</u>

$$GFT_S(0 \to y^*) = \text{benefit} - \text{payment}$$

$$= \int_0^4 MNPB(y)dy - p^* y^* = \frac{4*(1600-320)}{2} = 2560$$

See Figure R2-19.

<u>Gains from trade to external agents if rights assigned to external agents</u>

$$GFT_{EXT}(0 \to y^*) = \text{receipts} - \text{cost} = p^* y^* - \int_0^4 MEC(y)dy = \frac{4*320}{2} = 640$$

See Figure R2-20.

Note that total GFT are higher if property rights are assigned to the external agents in this example. This does <u>not</u> means that property rights <u>should</u> be assigned to the external agents. It does mean that if trade is impossible for some reason then the net social cost of that absence of trade would be larger when external agents hold the rights than when the source agent holds the rights.

REVIEW QUESTIONS

Answer Questions 1 – 8 as a set and then grade yourself on those. Once you have resolved any problems with that first set, do Questions 9 – 19 as a set and grade those. Then do Questions 20 – 28 as a set, followed by Questions 29 – 38 as a set.

Questions 1 – 8 relate to the following data.

$$MPB(y) = 318 - 23y$$

$$MPC(y) = 30y$$

$$MEB(y) = 330 - y$$

1. Marginal social benefit is

A. $MSB(y) = 618 - 11y$

B. $MSB(y) = 648 - 24y$

C. $MSB(y) = 198 - 33y$

D. $MSB(y) = 420 - 22y$

2. The private optimum is

A. 4

B. 5

C. 6

D. 7

3. The social optimum is

A. 7

B. 9

C. 10

D. 12

4. External benefit at the private optimum is

A. 1962

B. 2024

C. 2456

D. 2784

5. External benefit at the social optimum is

A. 2764

B. 3888

C. 4240

D. 6260

Now suppose that a regulation forces the source agent to increase his level of the activity to the social optimum. Consider Questions 6 – 8 under this scenario.

6. The gain to the external agents is

A. 986

B. 1160

C. 1926

D. 2456

7. The loss to the source agent is

A. 954

B. 1644

C. 1756

D. 2132

8. The gain in social surplus is

A. 788

B. 972

C. 1358

D. 1674

Questions 9 – 19 relate to the following data.

$$MPB(y) = 192 - 60y$$

$$MPC(y) = 4y$$

$$MEC(y) = 192y$$

9. Marginal social cost is

A. $MSC(y) = 192 - 64y$

B. $MSC(y) = 196y$

C. $MSC(y) = 192 + 52y$

D. $MSC(y) = 188y$

10. The private optimum is

A. 4

B. 5

C. 3

D. 8

11. The social optimum is

A. 0.75

B. 1.5

C. 2.5

D. 1

12. External cost at the private optimum is

A. 812

B. 636

C. 1234

D. 864

13. External cost at the social optimum is

A. 728

B. 54

C. 142

D. 256

Now suppose that a regulation forces the source agent to reduce his level of the activity to the social optimum. Consider Questions 14 – 17 under this scenario.

14. The gain to the external agents is

A. 84

B. 978

C. 1092

D. 810

15. The loss to the source agent is

A. 384

B. 454

C. 162

D. 26

16. The gain in social surplus is

A. 594

B. 648

C. 794

D. 58

Now return to the unregulated scenario, and suppose that the source agent is assigned the explicit right to undertake the activity at his private optimum. He may trade that right in whole or in part if he wishes. Suppose a costless contract can be written between the source agent and the external agents that specifies the price for each unit of activity reduced by the source agent. Consider Questions 17 – 19 under this scenario.

17. The equilibrium price at which trade will occur is

A. 62

B. 144

C. 176

D. 224

18. The gains from trade for the source agent are

A. 228

B. 326

C. 162

D. 208

19. The gains from trade for the external agents are

A. 486

B. 366

C. 586

D. 420

Questions 20 – 27 relate to the following data.

$$MPB(y) = 72 - 6y$$

$$MPC(y) = 6y$$

$$MEB(y) = 84 - y$$

20. Marginal social benefit is

A. $MSB(y) = 180 - 30y$

B. $MSB(y) = 192 - 7y$

C. $MSB(y) = 156 - 7y$

D. $MSB(y) = 192 + 5y$

21. The private optimum is

A. 10

B. 6

C. 8

D. 12

22. The social optimum is

A. 14

B. 8

C. 12

D. 14

23. External benefit at the private optimum is

A. 1096

B. 896

C. 524

D. 486

24. External benefit at the social optimum is

A. 936
B. 1096
C. 1160
D. 1400

Now suppose that a regulation forces the source agent to increase his level of the activity to the social optimum. Consider Questions 25 – 27 under this scenario.

25. The gain to the external agents is
A. 524
B. 450
C. 372
D. 38

26. The loss to the source agent is
A. 328
B. 312
C. 252
D. 216

27. The gain in social surplus is
A. 80
B. 174
C. 234
D. 256

Questions 28 – 38 relate to the following data.

$$MPB(y) = 80 - 19y$$

$$MPC(y) = y$$

$$MEC(y) = 60y$$

28. Marginal social cost is

A. $MSC(y) = 59y$

B. $MSC(y) = 80 - 18y$

C. $MSC(y) = 80 - 79y$

D. $MSC(y) = 61y$

29. The private optimum is

A. 4

B. 5

C. 3

D. 8

30. The social optimum is

A. 3

B. 2

C. 2.5

D. 1

31. External cost at the private optimum is

A. 526

B. 480

C. 328

D. 412

32. External cost at the social optimum is
A. 30
B. 324
C. 148
D. 226

Now suppose that a regulation forces the source agent to reduce his level of the activity to the social optimum. Consider Questions 33 – 35 under this scenario.

33. The gain to the external agents is
A. 300
B. 264
C. 450
D. 102

34. The loss to the source agent is
A. 90
B. 112
C. 146
D. 84

35. The gain in social surplus is
A. 360
B. 18
C. 188
D. 152

Now return to the unregulated scenario, and suppose that the source agent is assigned the explicit right to undertake the activity at his private optimum. He may trade that right in whole or in part if he wishes. Suppose a costless contract can be written between the source agent and the external agents that specifies the price for each unit of activity reduced by the source agent. Consider Questions 36 – 38 under this scenario.

36. The equilibrium price at which trade will occur is

A. 60

B. 180

C. 120

D. 30

37. The gains from trade for the source agent are

A. 120

B. 90

C. 76

D. 212

38. The gains from trade for the external agents are

A. 32

B. 240

C. 98

D. 270

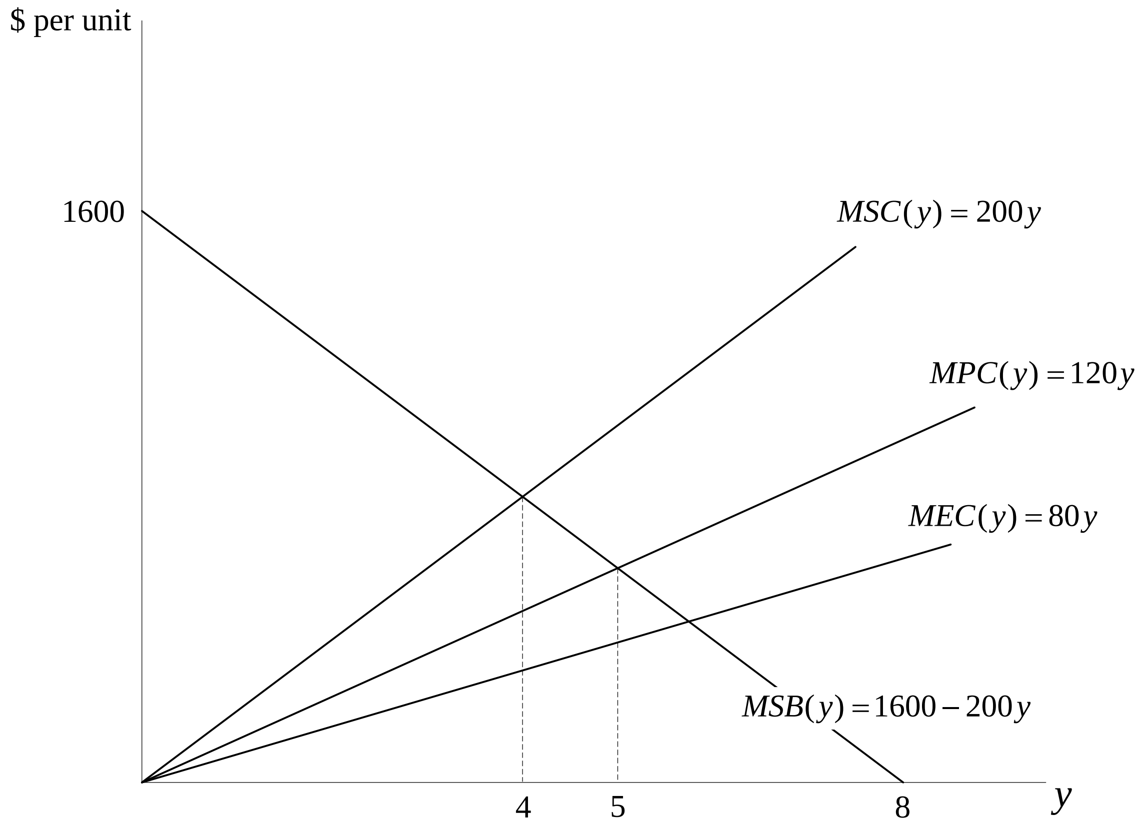
$ per unit
1600
MSC(y) = 200y
MPC(y) = 120y
MEC(y) = 80y
MSB(y) = 1600 − 200y
4
5
8
y

Figure R2-1

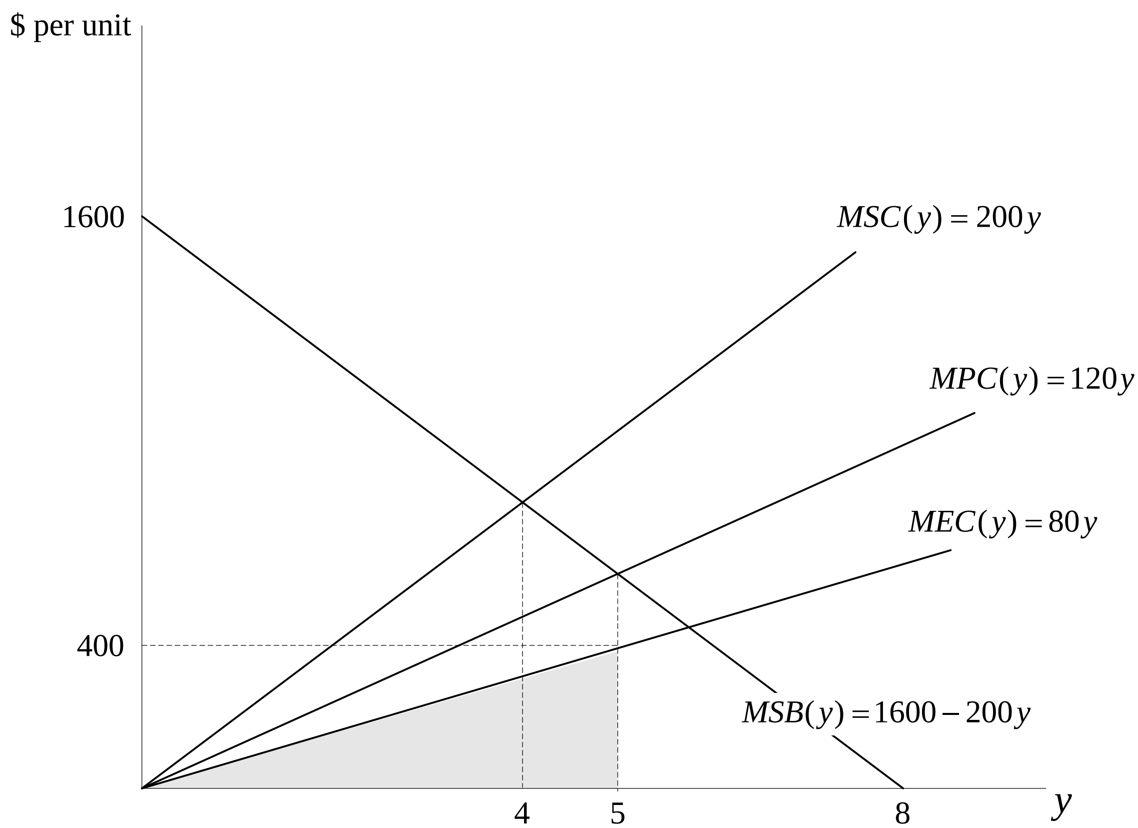
$ per unit
1600
400
$MSC(y) = 200y$
$MPC(y) = 120y$
$MEC(y) = 80y$
$MSB(y) = 1600 - 200y$
4
5
8
y

Figure R2-2

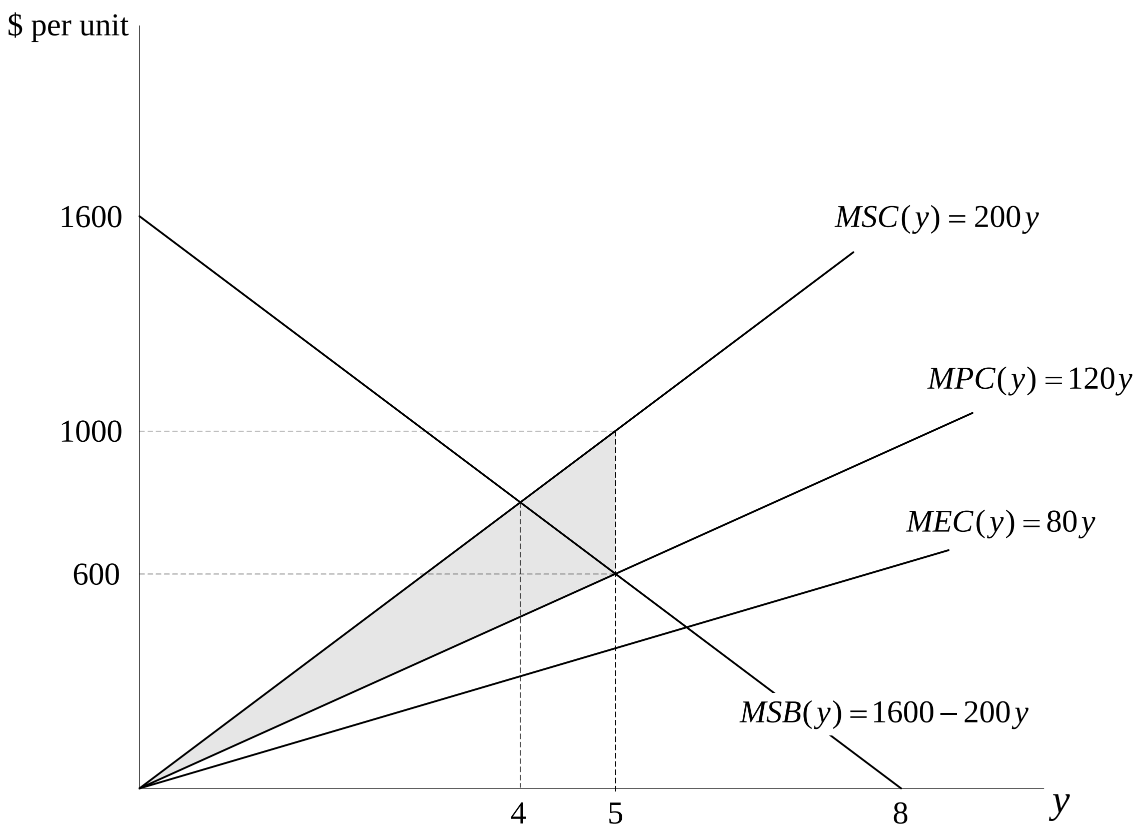
$ per unit
1600
1000
600
MSC(y) = 200y
MPC(y) = 120y
MEC(y) = 80y
MSB(y) = 1600 − 200y
4
5
8
y

Figure R2-3

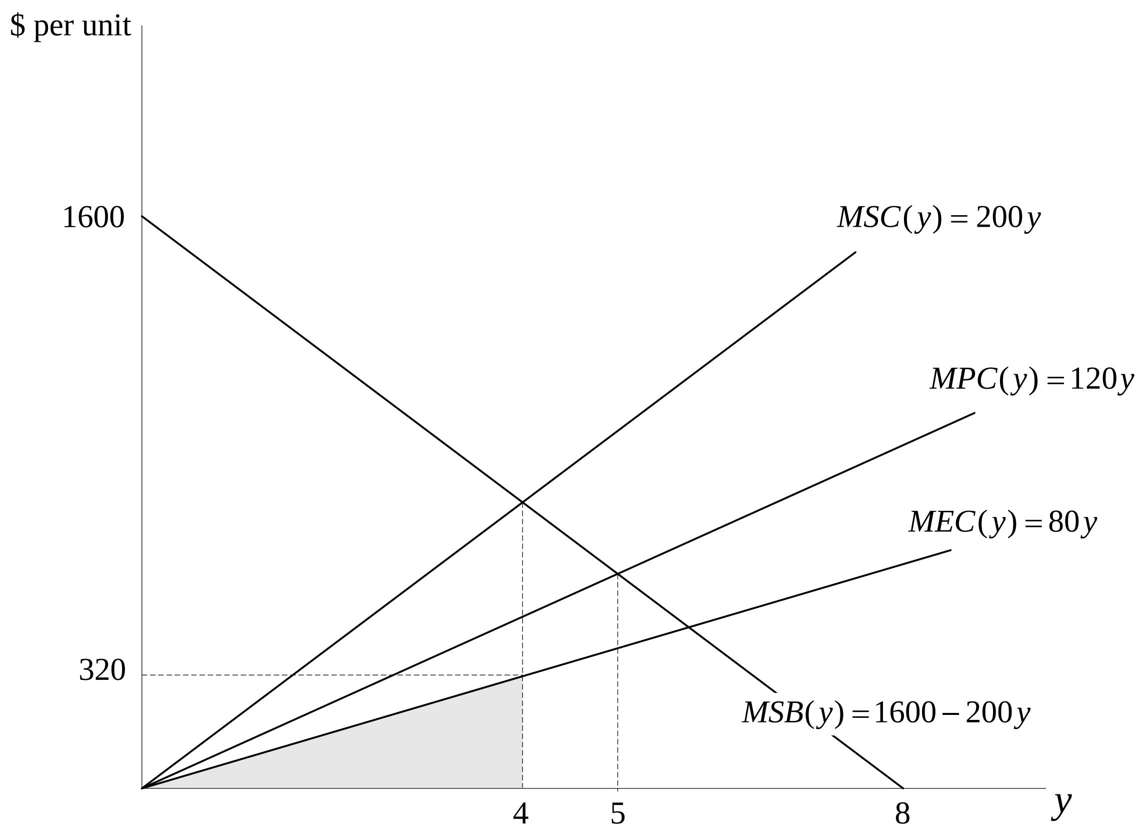
$ per unit
1600
320
MSC(y) = 200y
MPC(y) = 120y
MEC(y) = 80y
MSB(y) = 1600 − 200y
4
5
8
y

Figure R2-4

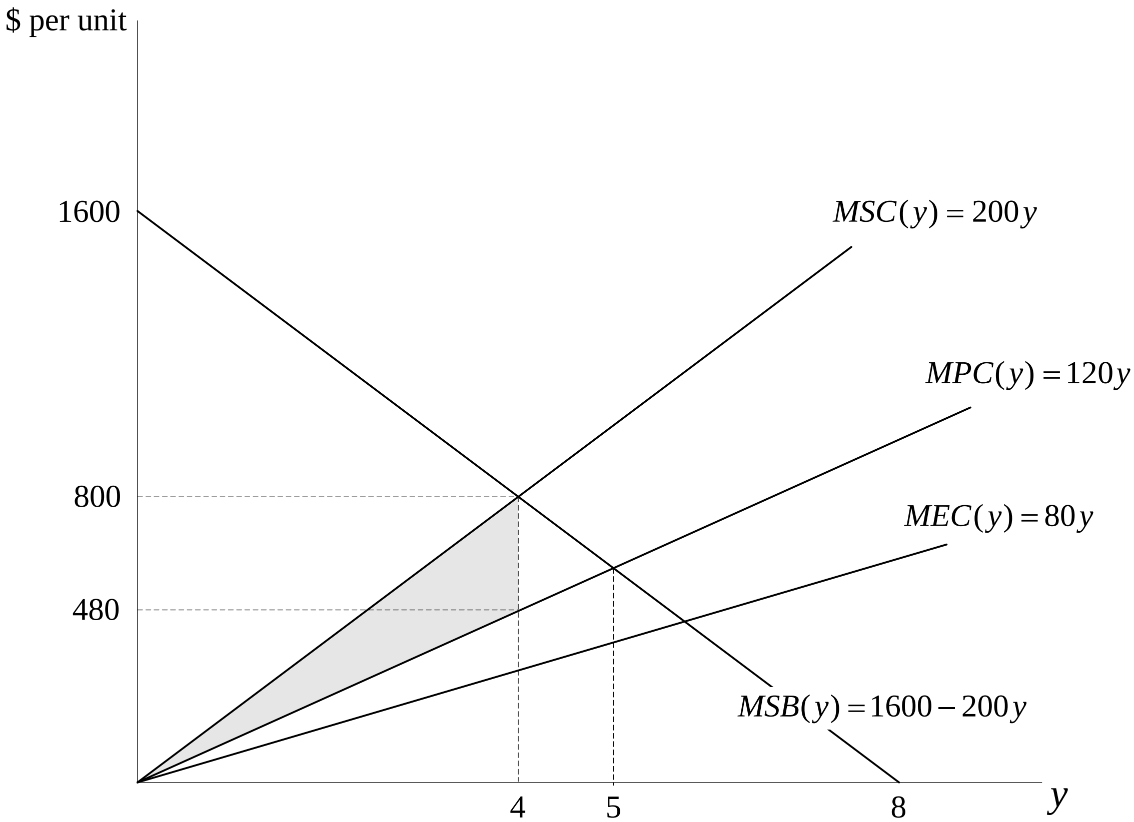
$ per unit
1600
800
480
MSC(y) = 200y
MPC(y) = 120y
MEC(y) = 80y
MSB(y) = 1600 − 200y
4
5
8
y

Figure R2-5

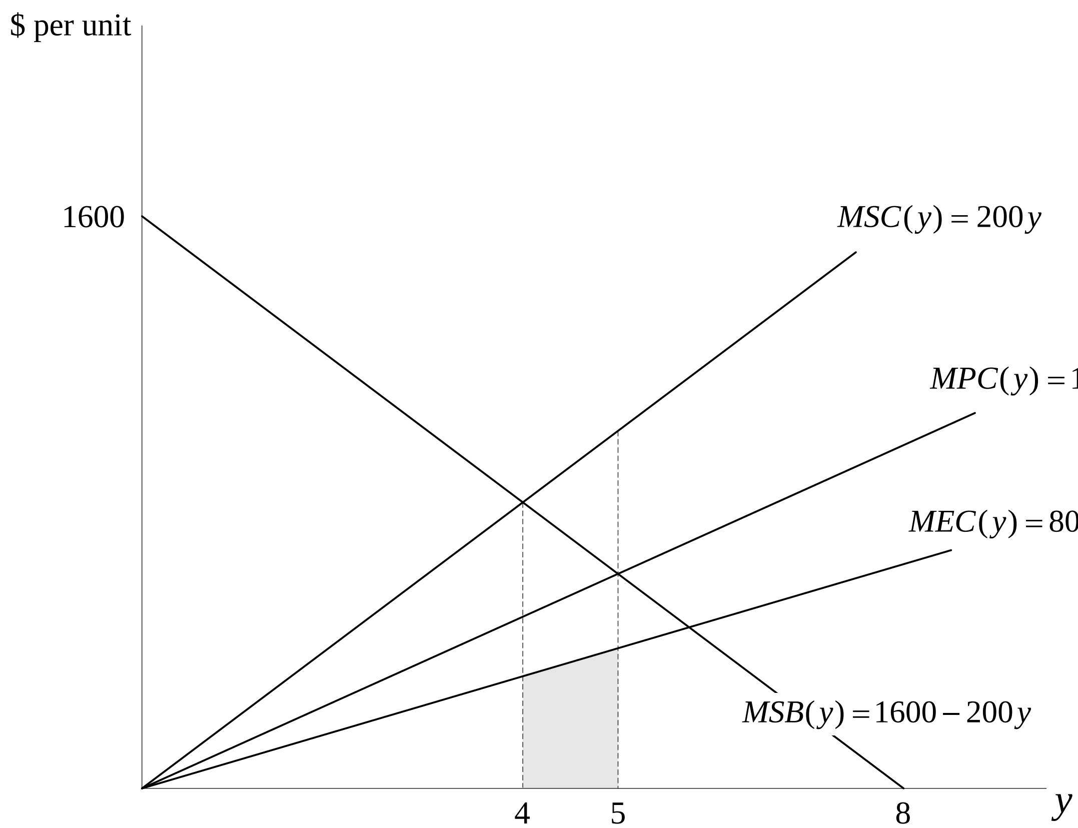
$ per unit
1600
MSC(y) = 200y
MPC(y) = 120y
MEC(y) = 80y
MSB(y) = 1600 − 200y
4
5
8
y

Figure R2-6

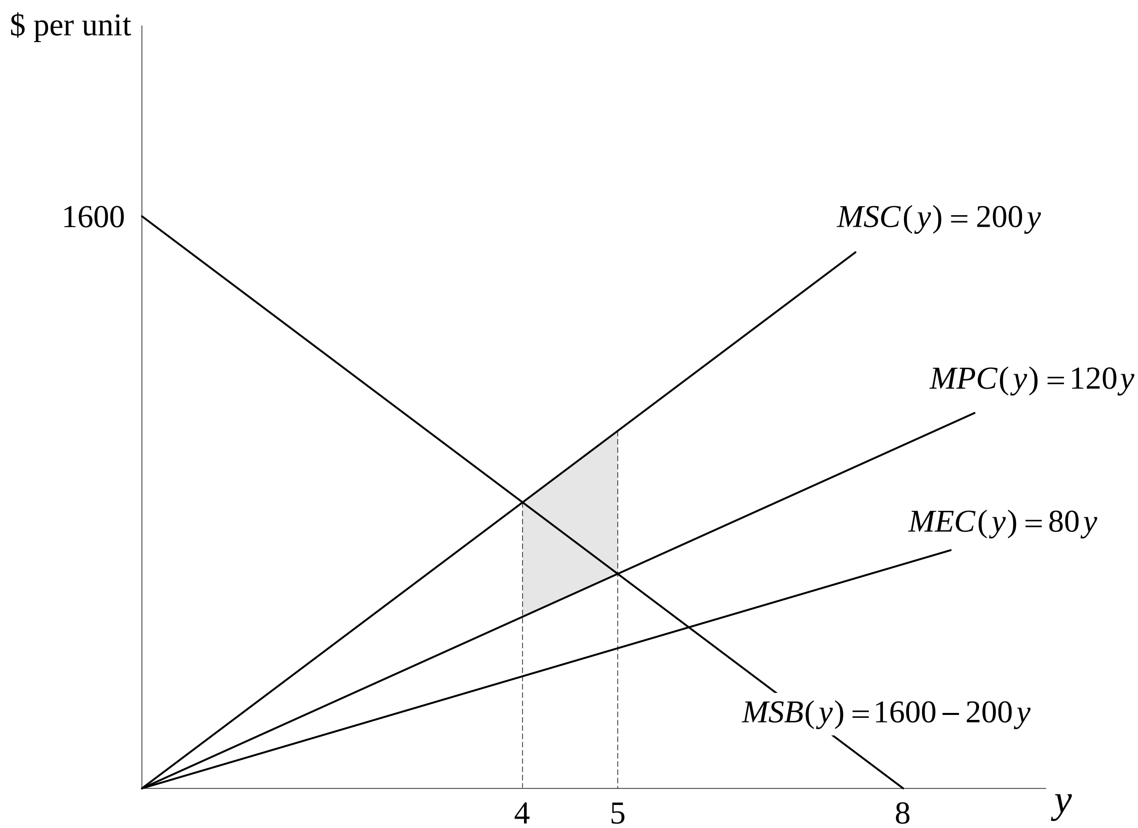
$ per unit
1600
MSC(y) = 200y
MPC(y) = 120y
MEC(y) = 80y
MSB(y) = 1600 − 200y
4
5
8
y

Figure R2-7

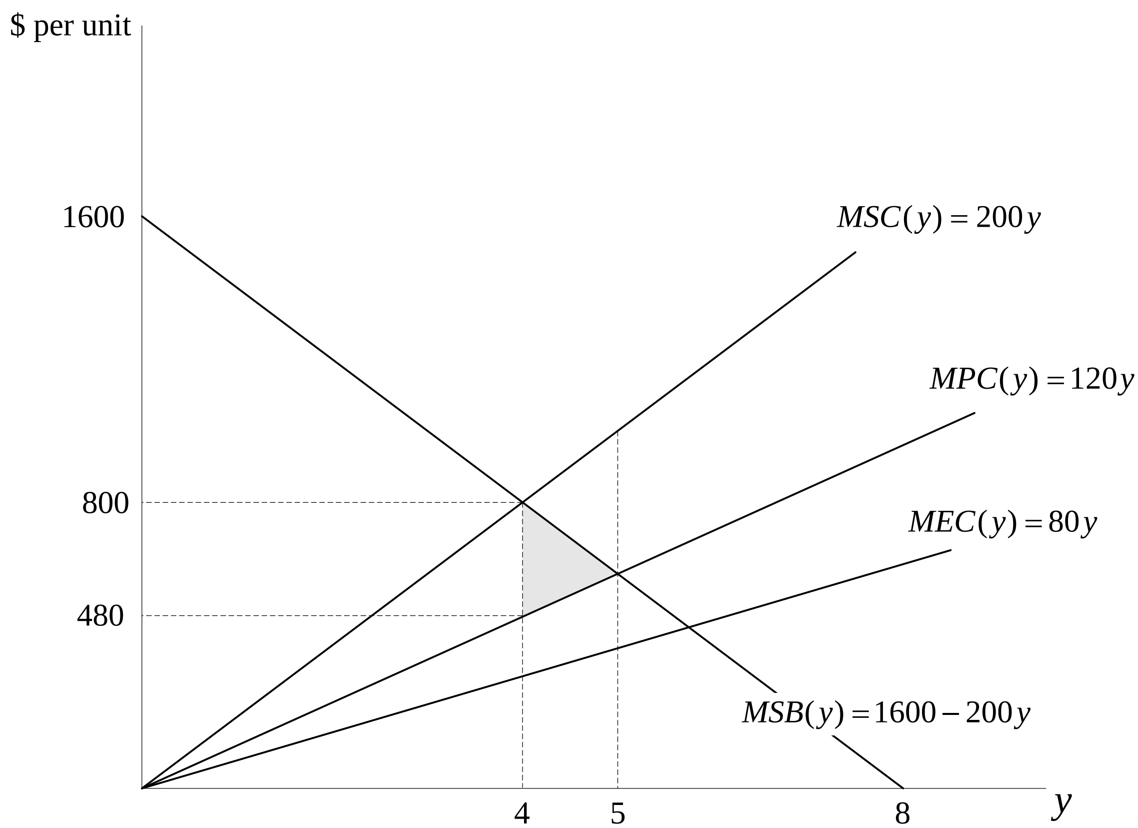
$ per unit
1600
800
480
MSC(y) = 200y
MPC(y) = 120y
MEC(y) = 80y
MSB(y) = 1600 − 200y
4
5
8
y

Figure R2-8

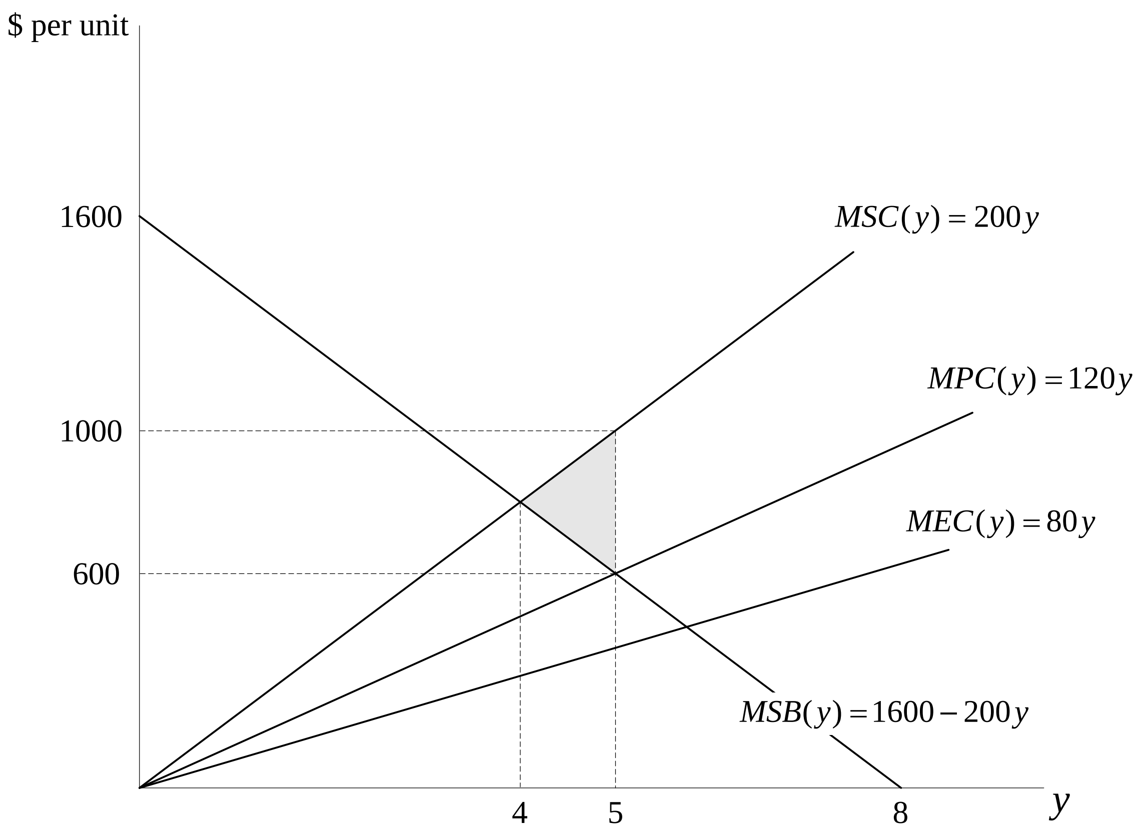
$ per unit
1600
1000
600
MSC(y) = 200y
MPC(y) = 120y
MEC(y) = 80y
MSB(y) = 1600 − 200y
4
5
8
y

Figure R2-9

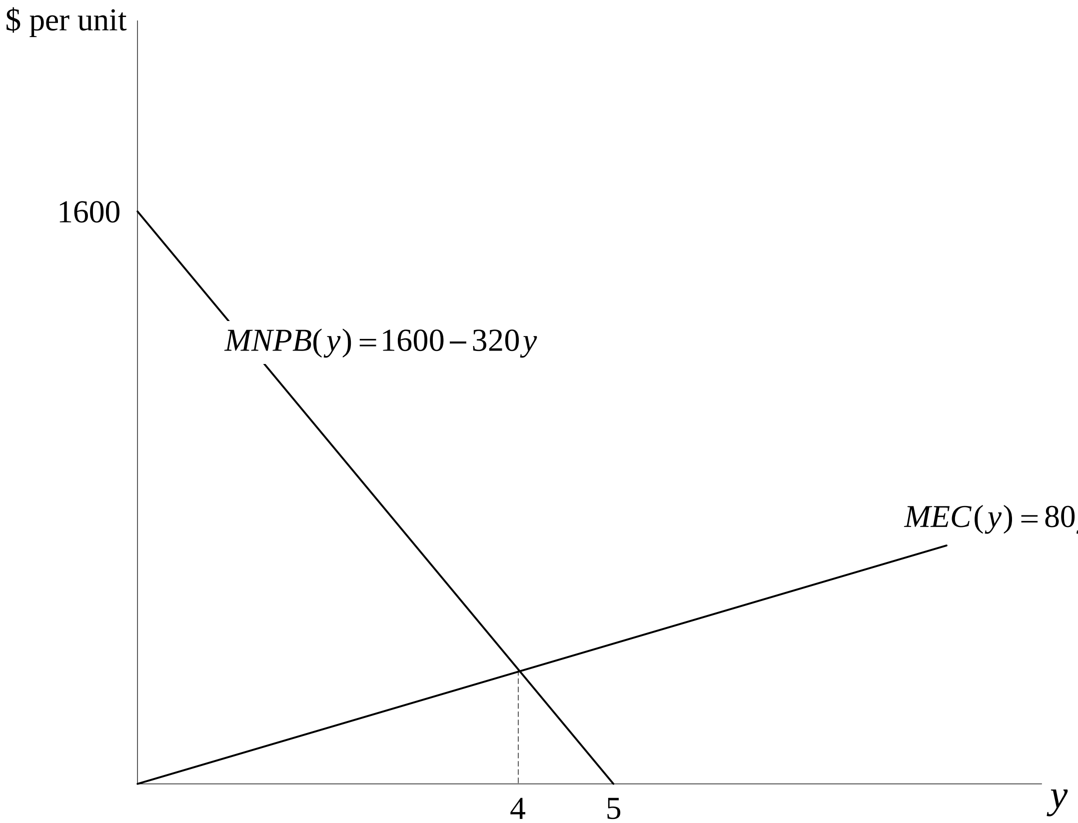

Figure R2-10

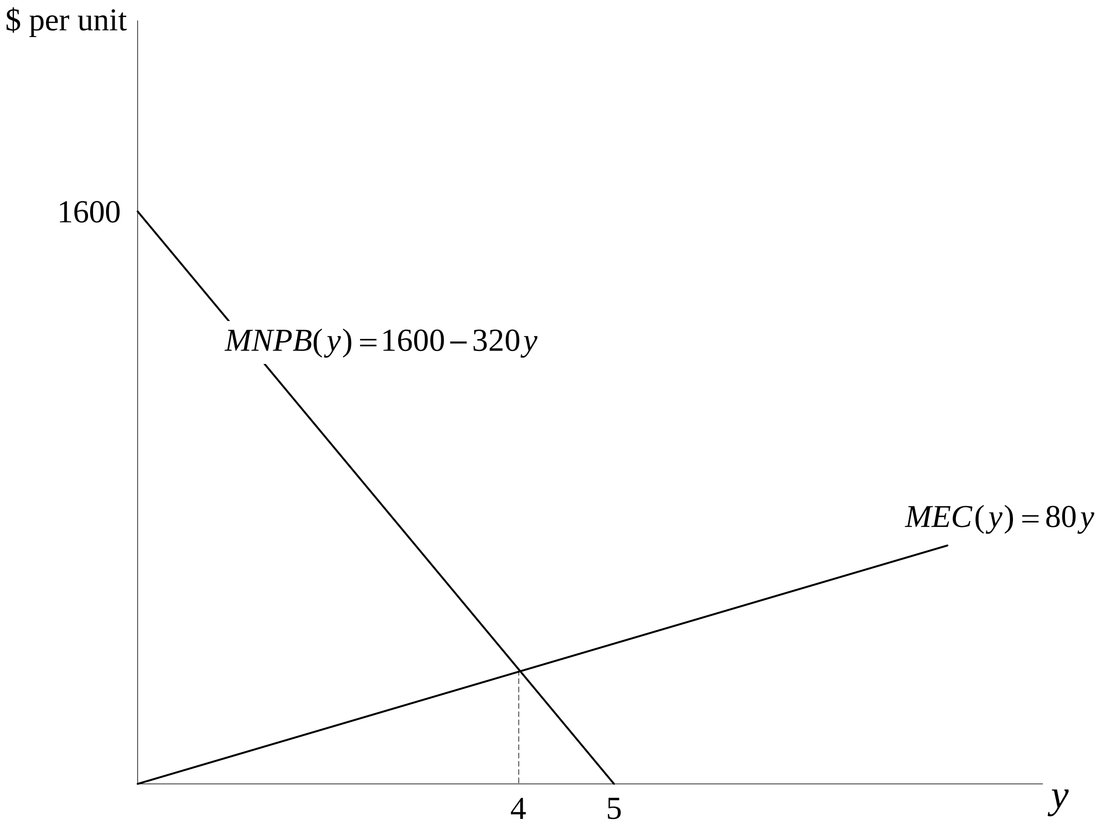

Figure R2-11

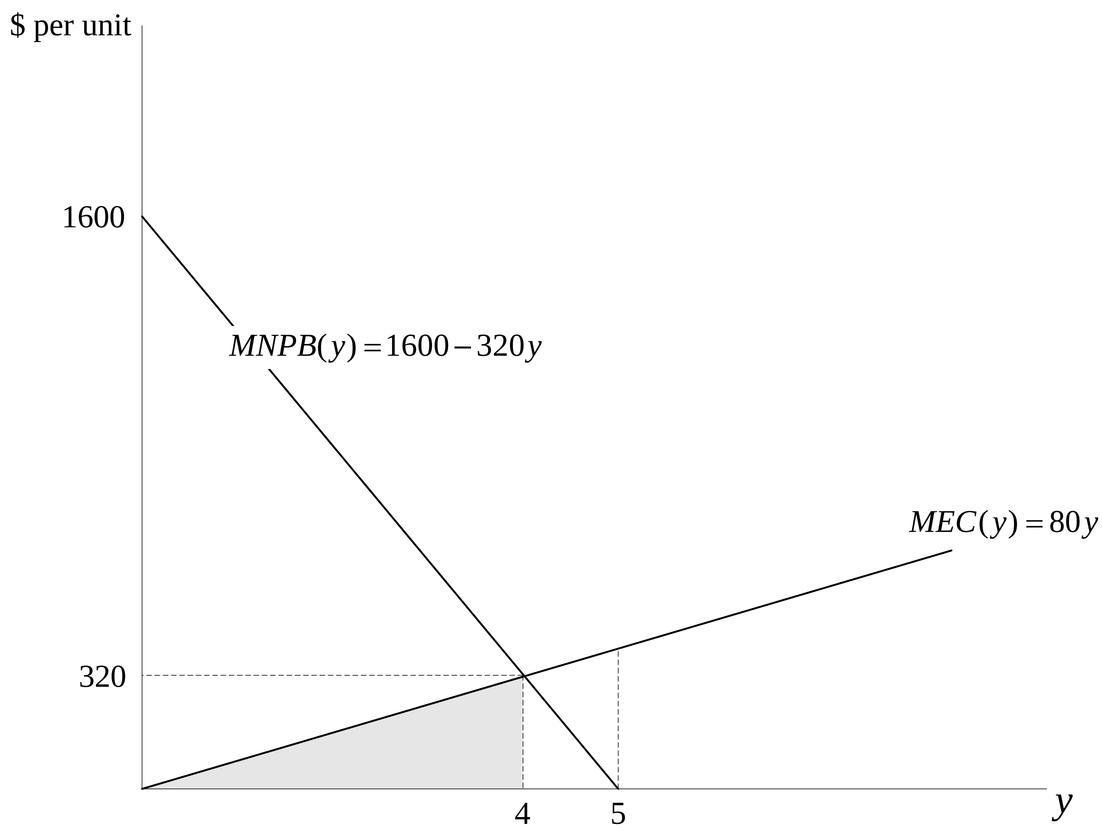

Figure R2-12

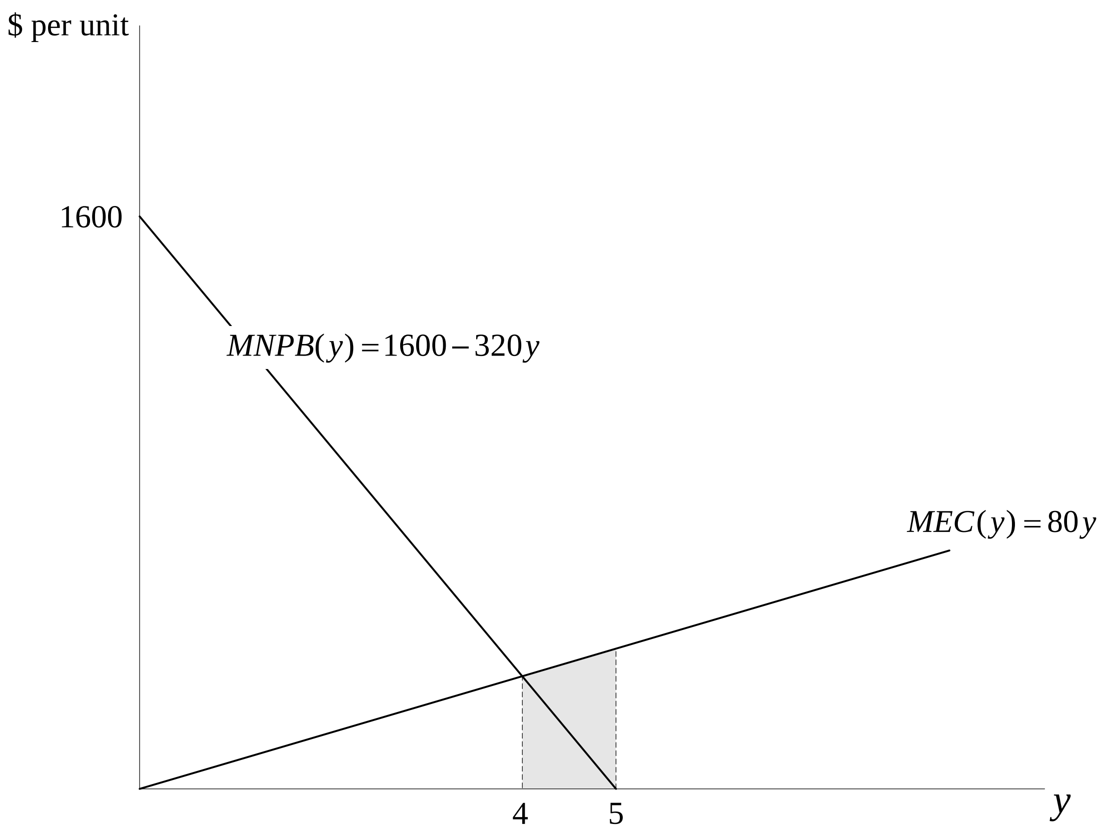

Figure R2-13

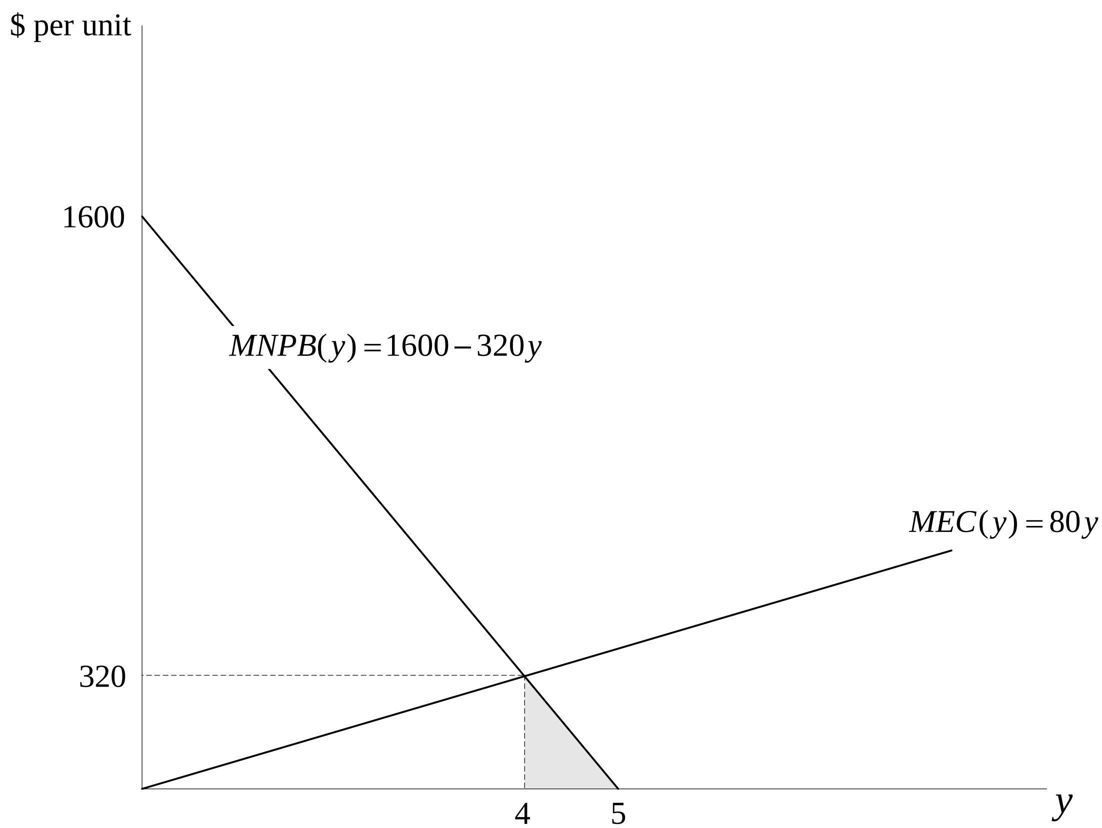

Figure R2-14

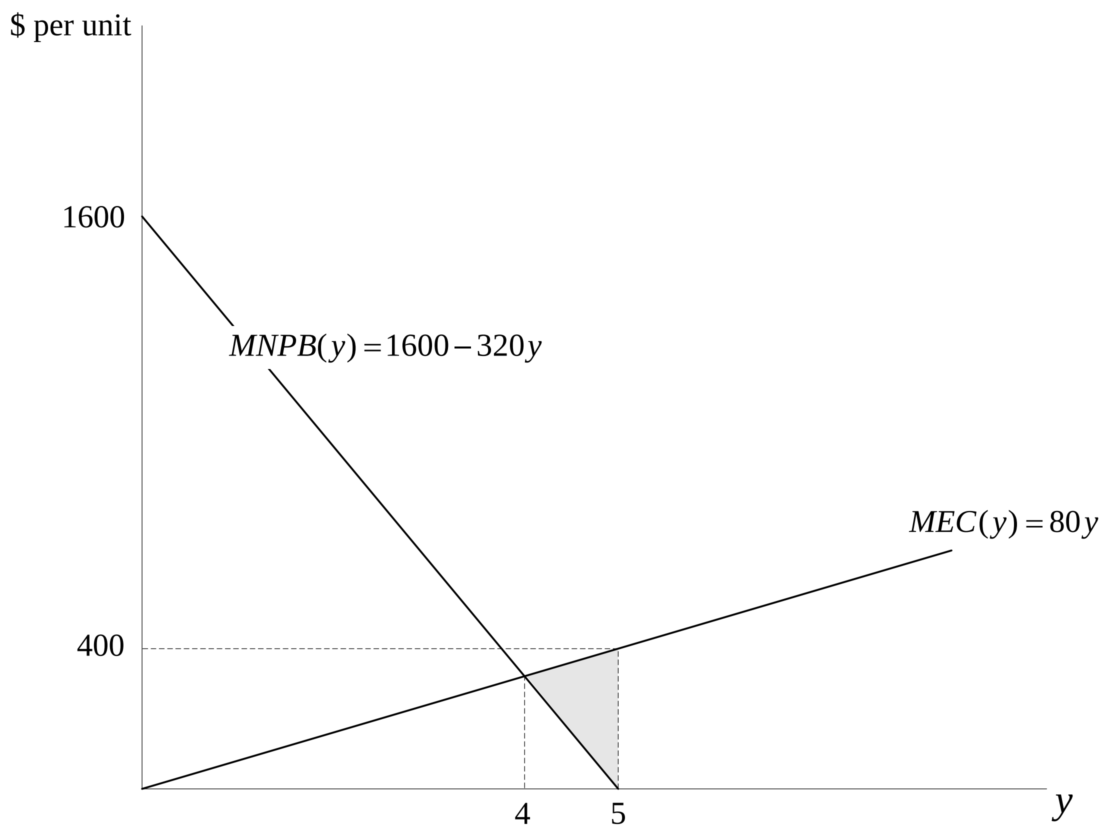

Figure R2-15

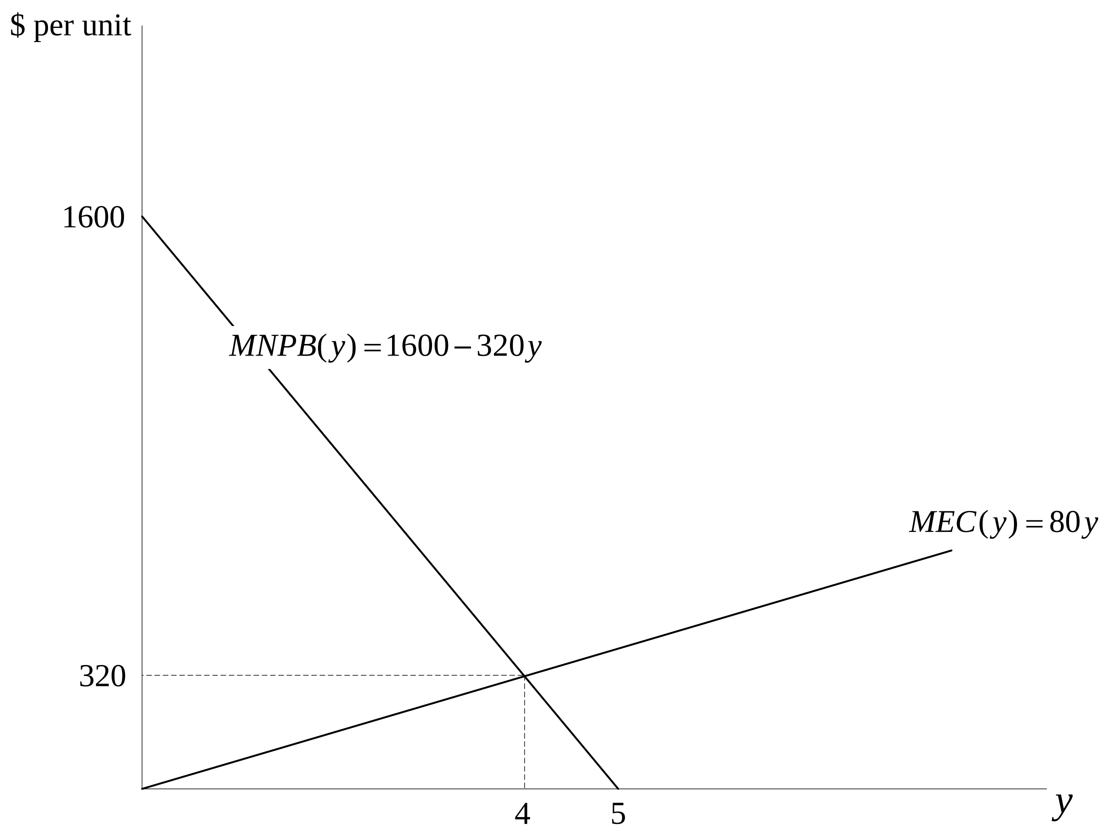

Figure R2-16

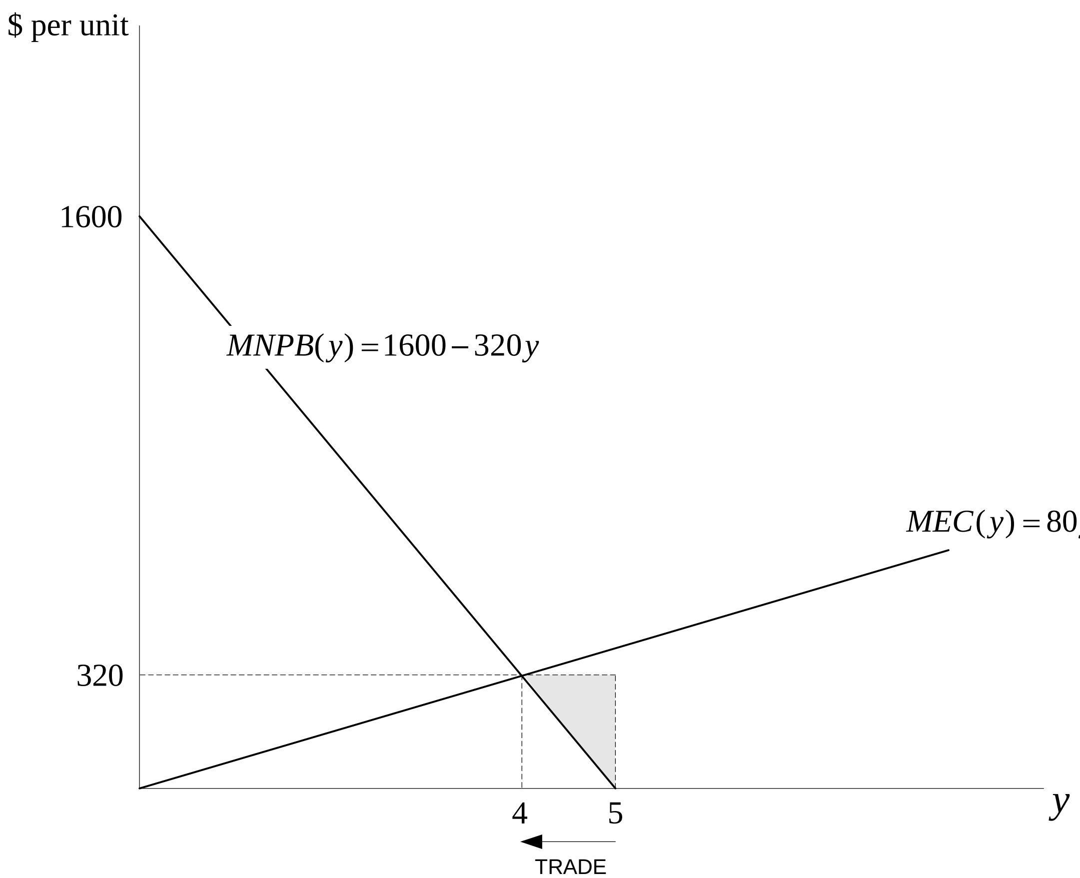

Figure R2-17

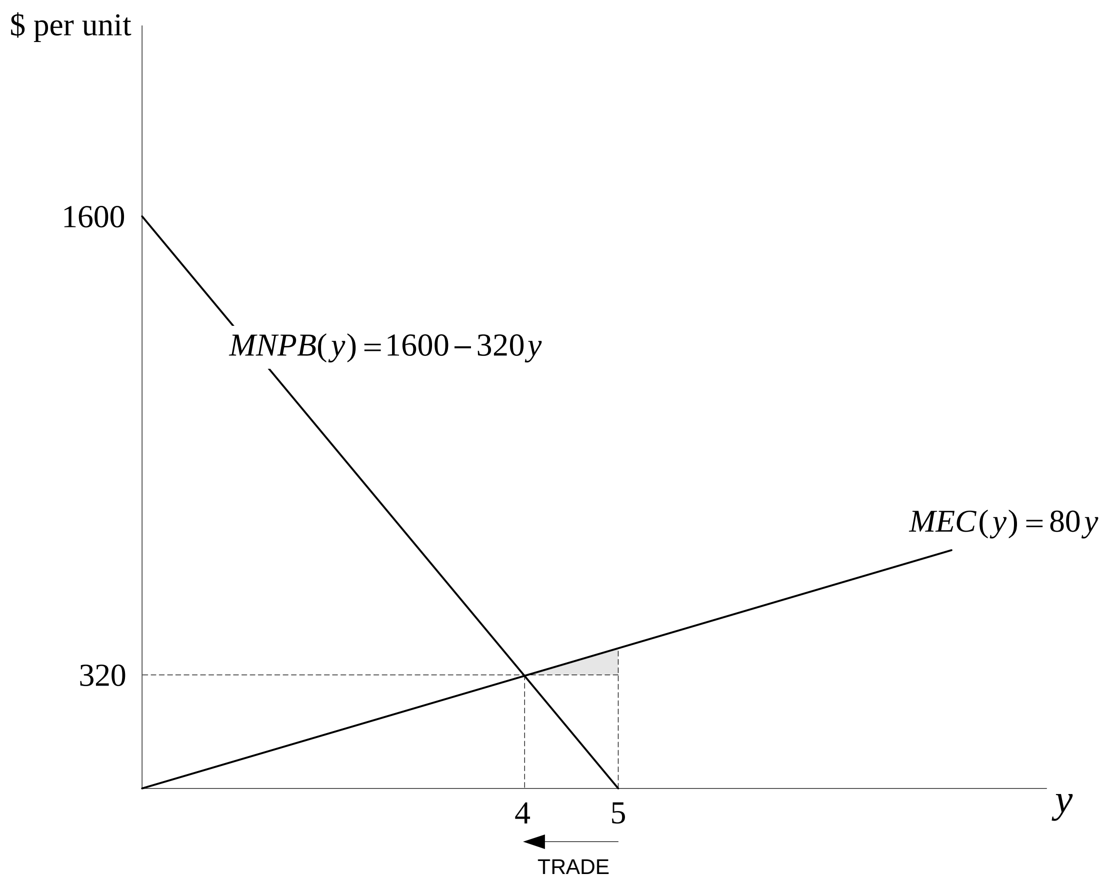

Figure R2-18

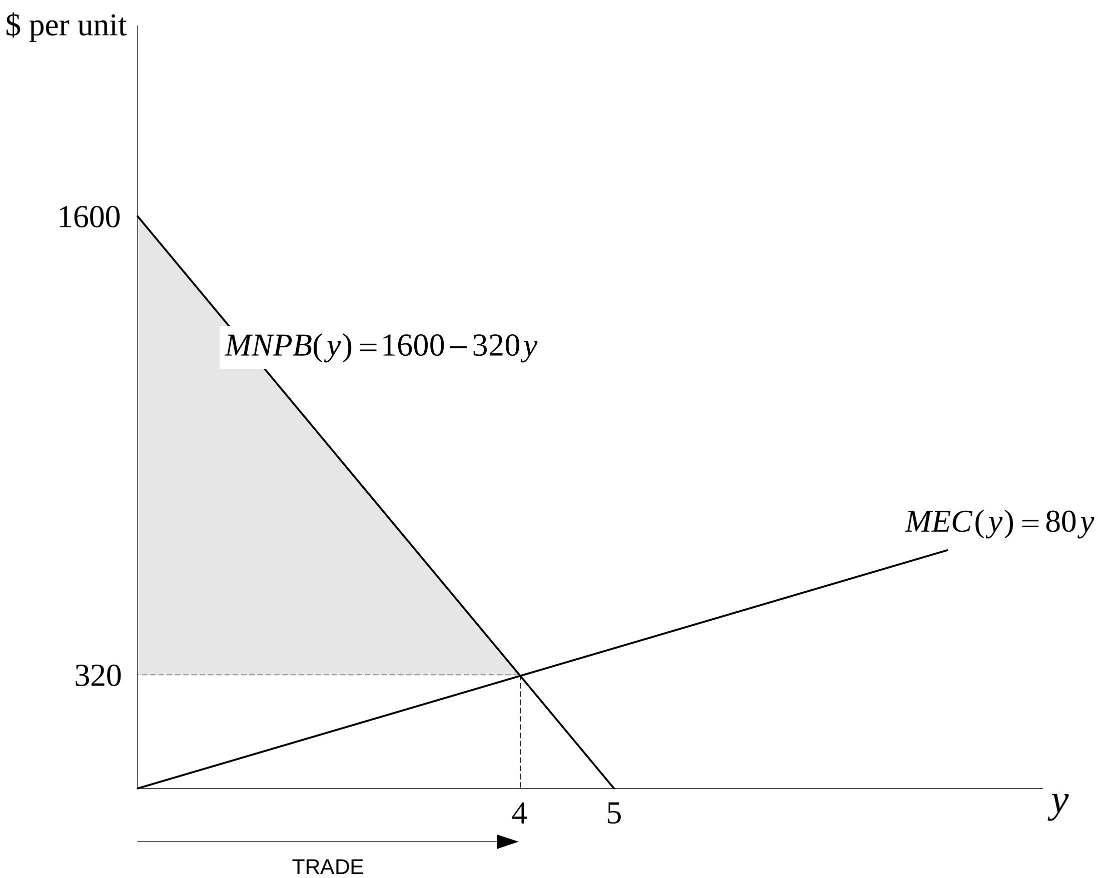

Figure R2-19

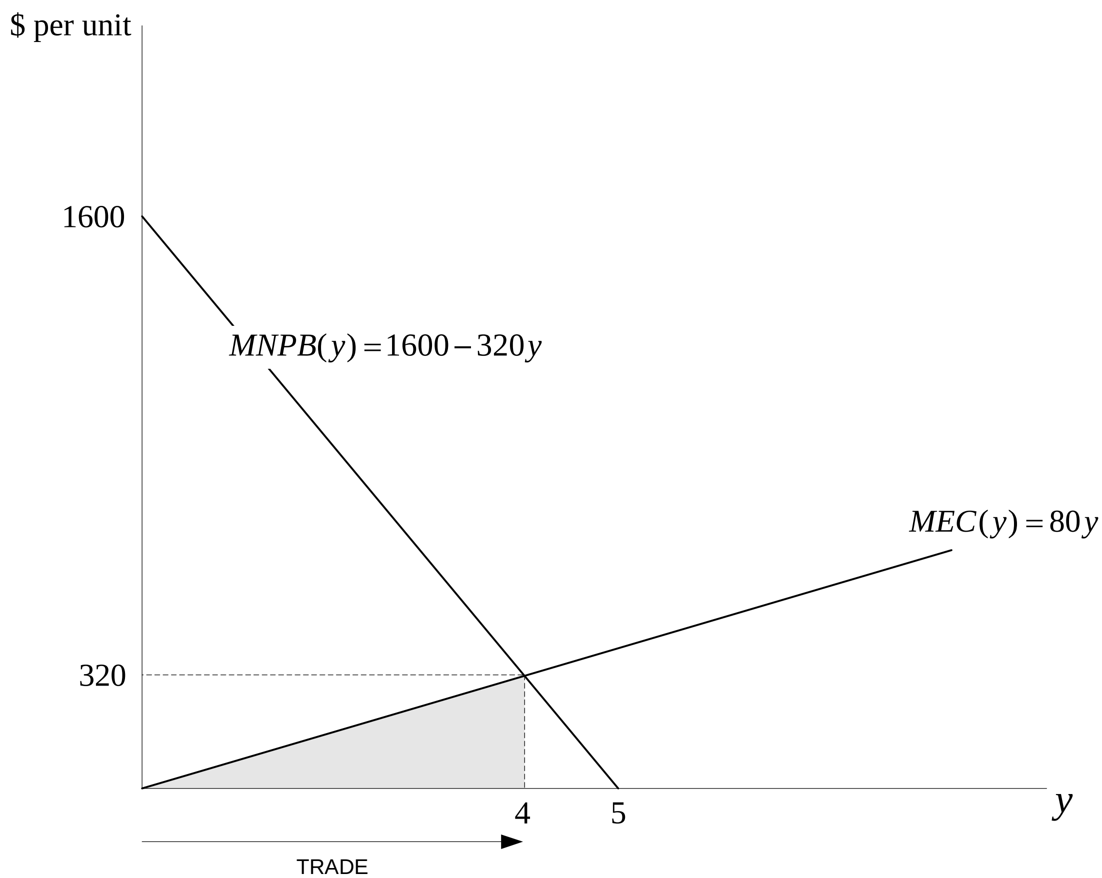

Figure R2-20

ANSWER KEY

1. B
2. C
3. D
4. A
5. B
6. C
7. A
8. B
9. B
10. C
11. A
12. D
13. B
14. D
15. C
16. B
17. B
18. C
19. A
20. C
21. B
22. C
23. D
24. A
25. B
26. D
27. C
28. D
29. A
30. D
31. B
32. A
33. C
34. A
35. A
36. A
37. B
38. D

3. OPTIMAL POLLUTION CONTROL

OUTLINE

2

3.1 INTRODUCTION

Introduction

- Our purpose in this topic is to address the specific question that underlies pollution control policy:
  - **what is the optimal level of pollution?**
- We will cast this question in terms of social costs and social benefits using the same framework we developed for the analysis of externalities in Topic 2.

4

3.2 BALANCING COSTS AND BENEFITS

Balancing Benefits and Costs

- We begin with a setting in which there is a single polluter (the source agent) and multiple parties who are adversely affected by the pollution (the external agents).

6

Balancing Benefits and Costs

- Recall from Topic 2.6 that we can characterize the socially optimal level of any activity in terms of **marginal net private benefit** to the source agent and **marginal external cost**.
- See Figure 3-1.

7

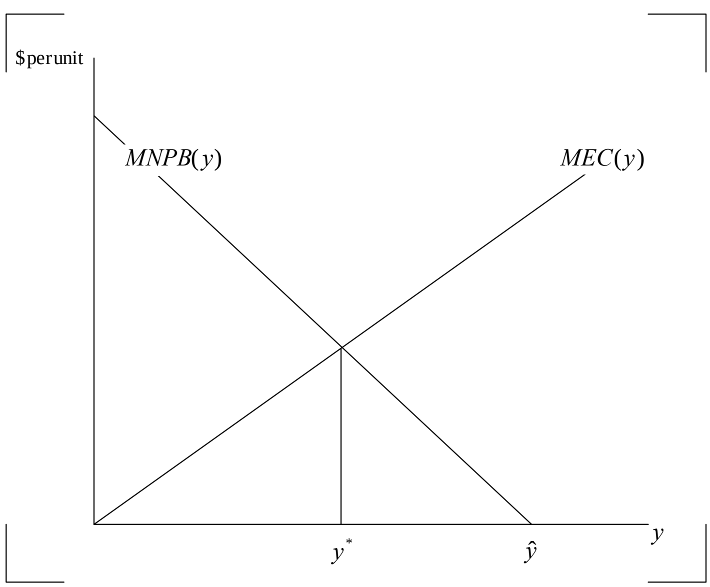

FIGURE 3-1

8

Balancing Benefits and Costs

- The **social optimum** is y^* such that

$$MNPB(y^*) = MEC(y^*)$$

- In contrast, the source agent will choose the **private optimum**, $\hat{y}$:

$$MNPB(\hat{y}) = 0$$

9

Balancing Benefits and Costs

- We now want to consider in more detail the meaning of MEC and MNPB in the specific context of a polluting activity.
- In particular, we will recast the MEC from a polluting activity in terms of **marginal damage**, and the MNPB in terms of **marginal abatement cost**.

10

3.3 MARGINAL DAMAGE

Marginal Damage

- The way we model environmental damage typically depends on the type of pollutant under consideration.
- Let us introduce some terminology with respect to pollutant types.

12

Pollutant Types

- Let *e* denote the quantity of the pollutant discharged annually from given source.

- In general, we refer to *e* as **emissions** if the pollutant is airborne, and as **effluent** if the pollutant is waterborne.

13

Pollutant Types

- Emissions are typically measured in tons; effluent is typically measured in liters or cubic meters (of a given pollutant concentration).
- We will treat the pollutant as airborne for the purposes of our discussion, but the analysis applies equally to a waterborne pollutant.

14

Pollutant Types

- Initially, we will focus on a **dissipative pollutant** (one that does not persist in the environment for more than a year after it is discharged).
- We will extend consideration to a **stock pollutant** (one that accumulates in the environment over time) in Topic 12.

15

Pollutant Types

- We will also restrict attention here to a **local pollutant** (one that causes no damage beyond the jurisdiction of the policy-maker).
- We extend our analysis to **transboundary pollutants** (that flow across jurisdictional boundaries) in Topic 11.

16

Environmental Damage

- The external cost of the pollutant is called **environmental damage**.
- It is represented by the **damage function**, denoted $D(e)$.
- $D(e)$ measures the total dollar value of the **harm** imposed by e on living humans.
- This harm could be **direct** or **indirect**.

17

Environmental Damage

- **Direct harm** includes impacts like
  - adverse health effects
  - damage to crops and other food sources
  - reduced productivity of industrial processes
  - damage to building materials
  - noxious odors
  - reduced visibility
  - aesthetic impacts

18

Environmental Damage

- **Indirect harm** refers to impacts on sentient beings whose harm cannot be measured directly but whose welfare is important to living humans.
- These sentient beings include
  – non-human animals
  – potential future humans

19

Environmental Damage

- The harm done to these two groups <u>cannot</u> be measured directly.
- It is included in our measure of harm only to the extent that existing humans place value on the welfare of these groups.

20

Environmental Damage

- In an extreme scenario under which no living human cares about the welfare of these groups, then any physical impact on these groups has no cost.
- In reality, many living humans do care deeply about these groups so indirect harm can be at least as important as direct harm in the measurement of environmental damage.

21

Environmental Damage

- This distinction between direct and indirect harm is conceptually useful – it highlights the recognition that economics gives to the breadth of human values – but we do not need to separate the two for most practical purposes.
- Henceforth, we will simply refer to "harm"; the inclusion of indirect harm is implied.

22

The Valuation of Harm: WTP vs. WTA

- Of much greater practical importance is the question of how harm is measured in dollar terms.
- There are two alternative ways to assign a dollar value to harm:
  - willingness-to-pay (WTP) to avoid it
  - willingness-to-accept (WTA) to tolerate it

23

The Valuation of Harm: WTP vs. WTA

- **WTP** is the amount of money that an agent would have to pay in order to make him just indifferent (equally happy) between paying the money and avoiding the harm, and not paying the money but incurring the harm.

24

The Valuation of Harm: WTP vs. WTA

- **WTA** is the amount of money that the agent would have to receive in order to make him just indifferent (equally happy) between receiving the money and incurring the harm, and not receiving the money but not incurring the harm.

25

The Valuation of Harm: WTP vs. WTA

- These two measures are not necessarily equal:
  - the difference depends critically on the **substitutability** between the environmental loss and goods that can be purchased with money.
- Consider two examples to illustrate the point.

26

The Valuation of Harm: WTP vs. WTA

- Example 1: high substitutability
- A commercial farmer suffers crop damage that reduces her revenues by $5000.
  - her WTP to avoid this loss is $5000
  - her WTA to tolerate this loss is also $5000
- The loss in this case is money; thus, there is perfect substitutability between the loss and goods that can be purchased with money.

27

The Valuation of Harm: WTP vs. WTA

- Example 2: low substitutability
- An individual will die within a year if his exposure to the pollutant continues
  - his WTP to avoid this loss might be close to his entire wealth but it is necessarily finite
  - his WTA might be infinite
- Money-purchased goods are a poor substitute for a life lost.

28

The Valuation of Harm: WTP vs. WTA

- In the first example, it does not matter whether we measure the harm by WTP or WTA, but in the second example it does.
- Which is the correct measure?

29

The Valuation of Harm: WTP vs. WTA

- The answer cannot be separated from the political issue of <u>property rights</u>:
  - if the harmed agent has a right not to be harmed, then WTA is the correct measure of harm
  - if the harmed agent has no right not to be harmed, then WTP is the correct measure of harm

30

The Valuation of Harm: WTP vs. WTA

- In most settings where environmental harm is incurred, property rights are not well-defined.
- For example, a steel mill may have an established practice of discharging pollution – and hence has an historically-based implicit right to pollute – but it may not have an explicit legal right to do so.

31

The Valuation of Harm: WTP vs. WTA

- Similarly, communities proximate to the steel mill may have historically suffered harm from the pollution without compensation, but this does not establish in law that they have no right not to be harmed.

32

The Valuation of Harm: WTP vs. WTA

- Environmental policy is often very contentious in practice because the decision to intervene often requires that the assignment of property rights be clarified in settings where those rights were previously undetermined.

33

The Valuation of Harm: WTP vs. WTA

- In practice, the choice of policy instrument – taxes versus subsidies, for example – is affected as much by political considerations regarding property rights as it is by hard cost-benefit analysis.

34

The Valuation of Harm: WTP vs. WTA

- Nonetheless, as analysts we still need to decide which measure we should use when property rights have not been assigned.

- Textbooks are often surprisingly fuzzy on this issue, and the question has not even been fully answered at a theoretical level.

35

The Valuation of Harm: WTP vs. WTA

- The choice of measure should ultimately depend on the **purpose** of our measurement.

- If our purpose is to calculate the payment that will actually be made to compensate an individual for a loss, then we should use her WTA as the measure of that loss.

36

The Valuation of Harm: WTP vs. WTA

- Similarly, if our purpose is to calculate the payment that a beneficiary will actually make in return for a gain, then we should use his WTP as the measure of that gain.

37

The Valuation of Harm: WTP vs. WTA

- Conversely, if our purpose is to calculate the loss imposed on an individual who will not actually be compensated for that loss, then we should use her WTP to avoid that loss as the measure of that loss.

38

The Valuation of Harm: WTP vs. WTA

- The underlying logic is that if she will not be compensated then by implication she has no right not to be damaged.
- In the absence of that right, if she wishes not to be damaged then she must pay for the privilege (as she would to obtain anything that she does not currently own).

39

The Valuation of Harm: WTP vs. WTA

- Similarly, if our purpose is to calculate the gain to an individual who will not actually have to pay for that gain then we use his WTA to forgo that gain as the measure of its value.

40

The Valuation of Harm: WTP vs. WTA

- The logic here is that if he will not actually have to pay for the gain, then by implication he has a right to that gain.

41

The Valuation of Harm: WTP vs. WTA

- It should be stressed that there is still much debate on this issue in the economics literature, and that no consensus currently exists.
- Nonetheless, in practice we need to adopt a particular approach and use it consistently.

42

Marginal Damage

- We can now introduce the meaning of <u>marginal</u> damage.
- **Marginal damage** measures the rate of change of $D(e)$; it is denoted $MD(e)$.

43

Marginal Damage

- In the language of Topic 2, $MD(e)$ is the <u>marginal external cost</u> of the pollutant, and is measured in \$ per unit of emissions.

44

Marginal Damage

- The area under the $MD(e)$ schedule, to a given level of emissions e^0, measures the damage associated with that level of emissions:

$$D(e^0) = \int_0^{e^0} MD(e)de$$

- See Figure 3-2.

45

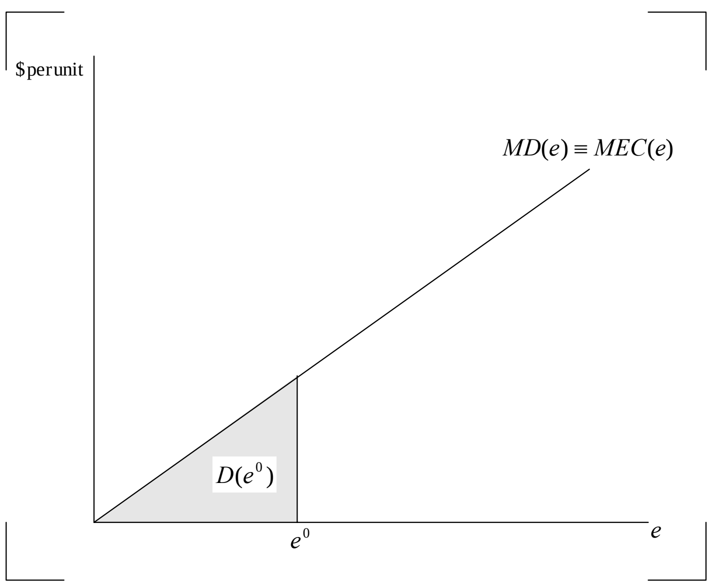

FIGURE 3-2

46

Marginal Damage

- Figure 3-2 assumes a linear form for *MD*(*e*), and we will often work with that example, but *MD*(*e*) could of course be non-linear.

- The more important property reflected in Figure 3-2 is the positive slope of *MD*(*e*), a property we call **increasing marginal damage**.

47

Marginal Damage

- A positively-sloped *MD*(*e*) function means that large doses of a pollutant are proportionately more damaging than small doses of that pollutant.

- For example, if 100 tons of the pollutant causes $1000 in damage, then 200 tons causes more than $2000 in damage.

48

Marginal Damage

- More generally, compare the damage done by $2e^0$ tons of the pollutant – depicted in Figure 3-3 – with the damage done by e^0 tones (from Figure 3-2).

49

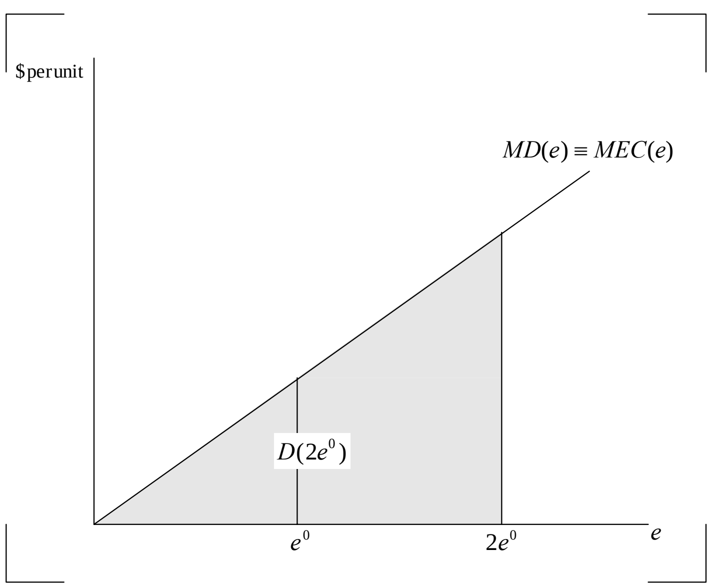

FIGURE 3-3

50

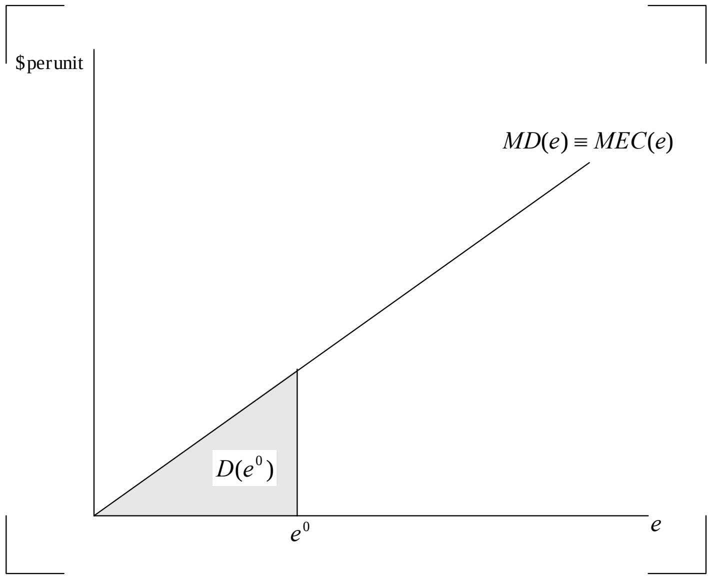

FIGURE 3-2 (repeat)

51

Marginal Damage

- Consider an example of why $MD(e)$ typically has this property.
- Particulate pollution – small particles that stay suspended in the air – is a serious problem in many urban areas.
- It is caused mainly from the combustion of fossil fuels.

52

Marginal Damage

- In low doses it is unpleasant but causes few immediate health effects (though repeated exposure to even low doses can cause long-term lung damage).

53

Marginal Damage

- In higher doses (as caused by higher concentrations in the air), it causes discomforting eye and lung irritation to healthy people, and the potential for serious asthma attacks among those with existing lung ailments.

54

Marginal Damage

- At still higher doses, most people suffer serious discomfort, and asthma attacks can be fatal.

- People are often warned to stay indoors, schools are be closed, and airline flights are cancelled due to poor visibility.

55

Marginal Damage

- Thus, a doubling of particulate pollution from e^0 to 2 e^0 can mean the difference between mild discomfort (at e^0) and extremely serious health and productivity impacts (at 2 e^0): the damage is more than proportionately higher at the higher dose level.

56

Marginal Damage

- Not all pollutants cause increasing marginal damage in all settings, but most do.
- In some cases, marginal damage might be constant at a given level – as depicted in Figure 3-4 – at least over some range, and we will cover those cases as we proceed, but these cases are not typical of most pollution problems in practice.

57

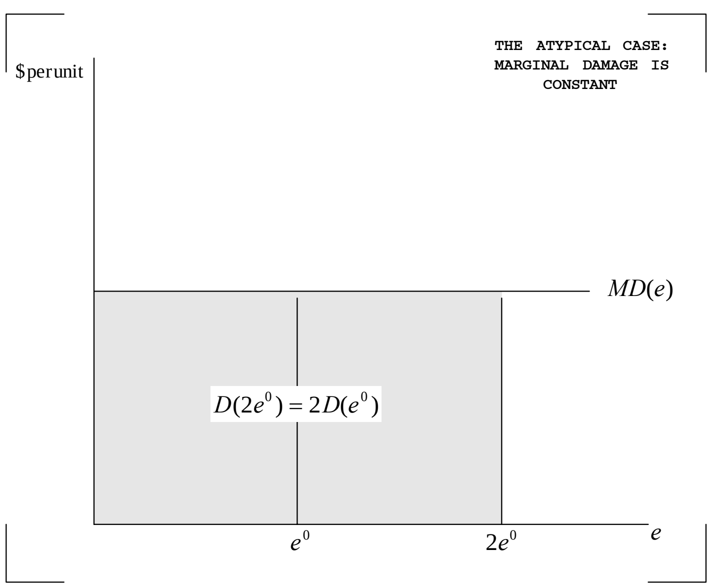

FIGURE 3-4

58

3.4 MARGINAL ABATEMENT COST

The Net Private Benefit from Emissions

- It is useful to think of emissions from a source as creating a net private benefit to that source.

60

The Net Private Benefit from Emissions

- We do not mean that emissions create a direct net benefit to the source in the sense that emissions *per se* are valuable.

- The net benefit from emitting derives from some valuable activity that produces those emissions.

61

The Net Private Benefit from Emissions

- For example, emissions may create a net private benefit to a firm in the form of profits from the production process that creates those emissions.

62

The Net Private Benefit from Emissions

- Similarly, emissions may create a net private benefit to an individual in the form of a valuable transportation service that creates those emissions.

63

The Net Private Benefit from Emissions

- In both cases, we can reinterpret the marginal net private benefit from the polluting activity itself in terms of the marginal net private benefit from the emissions it creates.
- Thus, we reinterpret $MNPB(y)$ from Figure 3-1 in terms of $MNPB(e)$; see Figure 3-5.

64

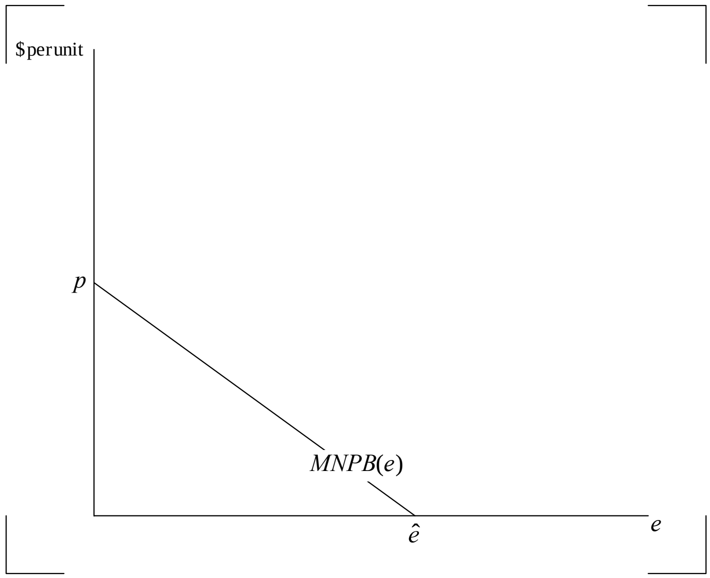

FIGURE 3-5

65

The Net Private Benefit from Emissions

- The privately optimal level of emissions is $\hat{e}$, where $MNPB(\hat{e})=0$.
- We can measure the net private benefit for the source from emitting $\hat{e}$ tons of the pollutant as the area under $MNPB(e)$; see Figure 3-6.

66

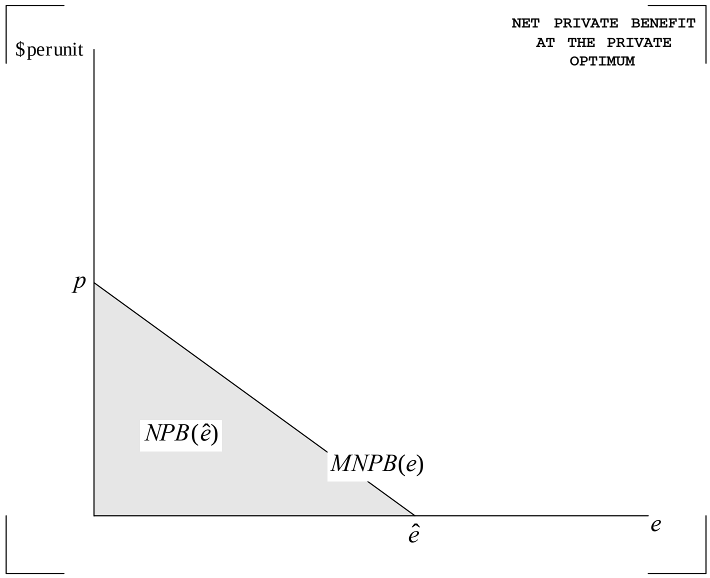

FIGURE 3-6

67

Marginal Abatement Cost

- Now suppose the source is required to reduce emissions from its current private optimum $\hat{e}$ to some lower level e^0.

- The cost of this **abatement** (the reduction in emissions) is the associated loss in net private benefit; see Figure 3-7.

68

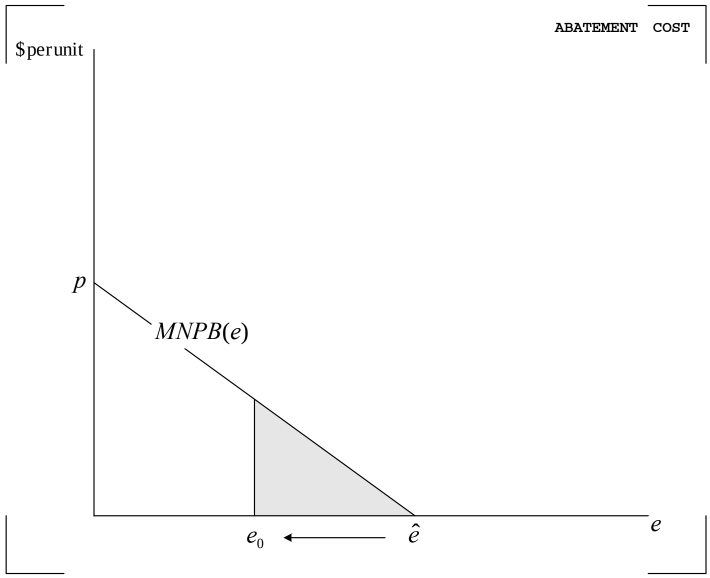

FIGURE 3-7

69

Marginal Abatement Cost

- In a typical policy setting, we usually work explicitly in terms of abatement from the existing level of emissions.
- We therefore rename the *MNPB*(*e*) schedule as the **marginal abatement cost schedule**, denoted *MAC*(*e*).
- See Figure 3-8.

70

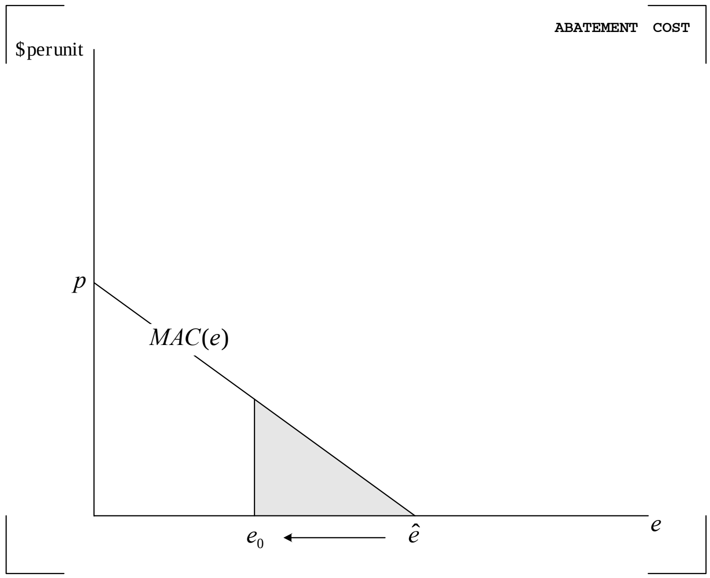

FIGURE 3-8

71

Marginal Abatement Cost

- Note that we write $MAC(e)$ as a function of e but abatement is measured as the reduction in emissions required to get from $\hat{e}$ down to some level e, as indicated by the arrow in Figure 3-8.

72

Marginal Abatement Cost

- The way we measure abatement cost depends on the nature of the source.

73

Marginal Abatement Cost

- If the source is a commercial enterprise then abatement cost for that enterprise is measured as the **loss of profit** associated with the abatement undertaken.

74

Marginal Abatement Cost

- If the source agent is an individual then abatement cost for that individual is measured as the amount of compensation needed to restore the individual to her pre-abatement level of utility.
- This money amount is called the **compensating variation**.

75

Marginal Abatement Cost

- A source will typically choose a <u>mix</u> of methods to reduce emissions in response to a requirement to cut emissions.
- That might include some reduction in the activity itself and some reduction in the **emissions-intensity** of the activity, via the adoption of less-polluting methods.

76

Marginal Abatement Cost

- For example, a polluting firm might reduce emissions by cutting production or by adopting cleaner production methods, or by using a mix of both measures.

77

Marginal Abatement Cost

- Similarly, an individual might cut her emissions from personal transportation by reducing the number of kilometers traveled, or by adopting less-polluting transport modes, or by using a mix of both measures.

78

Marginal Abatement Cost

- What mix of measures will the source choose if given the freedom to choose?
- The source will choose the mix of abatement methods that minimizes its own overall abatement cost for the required abatement.

79

Marginal Abatement Cost

- The true MAC schedule for the source measures marginal abatement cost based on this cost-minimizing mix.
- Henceforth, we will always work with the true abatement cost, and $MAC(e)$ will always be interpreted accordingly.

80

Increasing Marginal Abatement Cost

- Our assumption that sources choose the cost-minimizing approach to abatement implies that marginal abatement cost is increasing, as depicted in Figure 3-8.

81

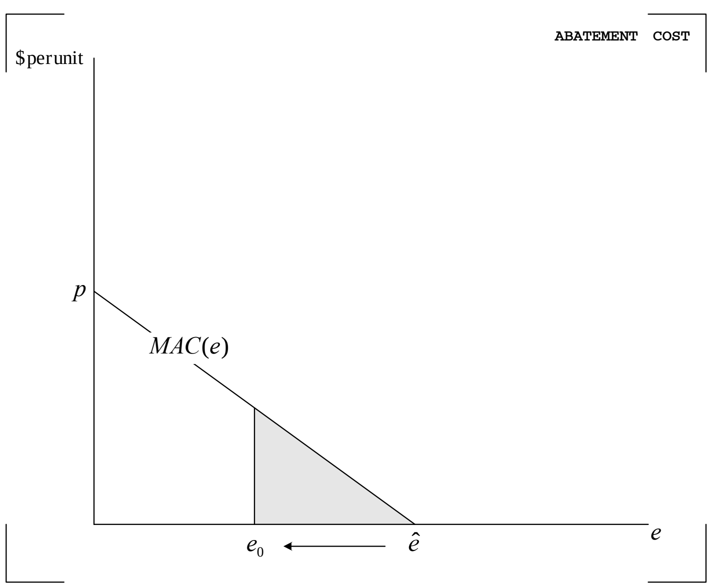

FIGURE 3-8 (repeat)

82

Increasing Marginal Abatement Cost

- Why is marginal abatement cost increasing?
- Consider the "low-hanging fruit" analogy:
  - Imagine that that you have a quota of apples to pick from a tree whose apples are all of equal quality.
  - The low-hanging fruit is the easiest to pick, so you pick that first.

83

Increasing Marginal Abatement Cost

- The higher the quota, the further you have to climb, with increasingly more effort required.
- Thus, the last apple is more costly to pick than the first apple because the last apple picked is higher up the tree.
- The cost function for apples picked would therefore look a lot like the cost function in Figure 3-8.

84

3.5 OPTIMAL ABATEMENT

Optimal Abatement

- We have now constructed both sides of the cost-benefit analysis needed to characterize the socially optimal level of emissions (or equivalently, the socially optimal level of abatement).
- Let us now put these two pieces together; see Figure 3-9.

86

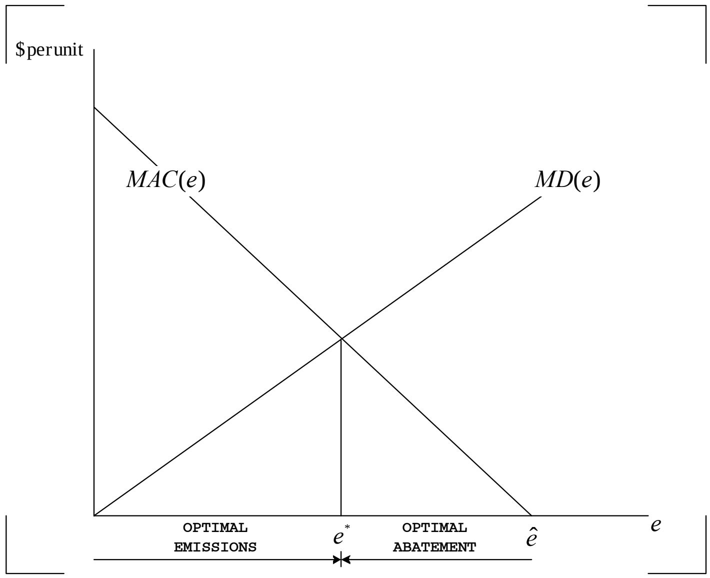

FIGURE 3-9

87

Optimal Abatement

- The **optimal level of emissions** is e^*, where

$$MD(e^*) = MAC(e^*)$$

and this also characterizes the **optimal level abatement**, measured as $\hat{e}$-e^*.

88

Optimal Abatement

- We can interpret this optimality condition in two equivalent ways.

Interpretation 1

- Starting at e = 0, construct the cost and benefit of a small increase in emissions to some level e^0>0.

89

Optimal Abatement

- The cost to society of this increase in emissions is the associated increase in environmental damage:

$$\int_0^{e^0} MD(e)de$$

- See Figure 3-10.

90

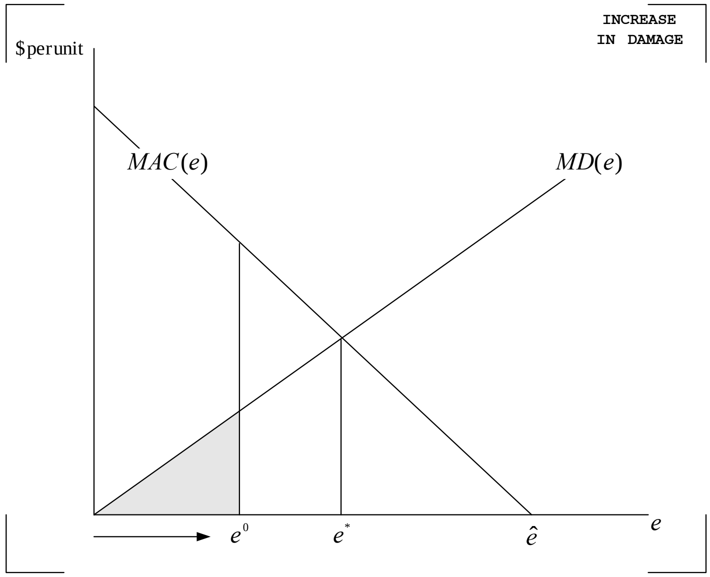

FIGURE 3-10

91

Optimal Abatement

- The benefit to society is the reduction in abatement cost:

$$\int_{0}^{e^0} MAC(e)de$$

- See Figure 3-11.

92

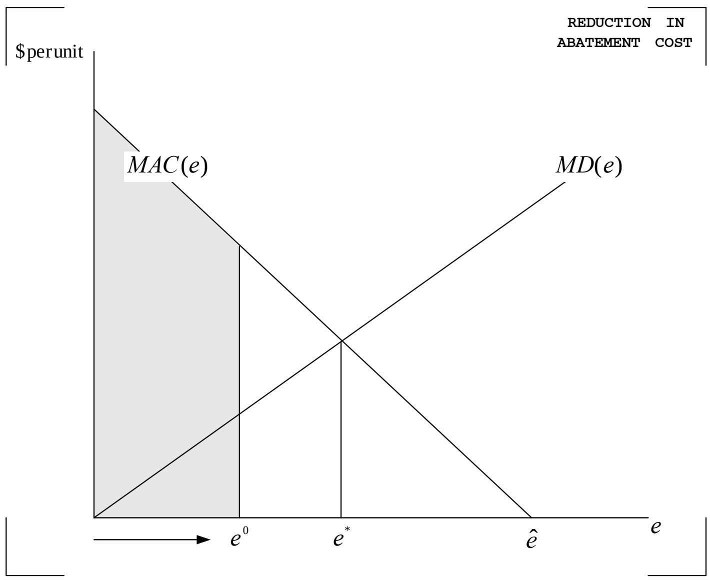

FIGURE 3-11

93

Optimal Abatement

- It is clear from Figures 3-10 and 3-11 that the reduction in abatement cost more than offsets the increase in damage; social surplus rises.
- See Figure 3-12.

94

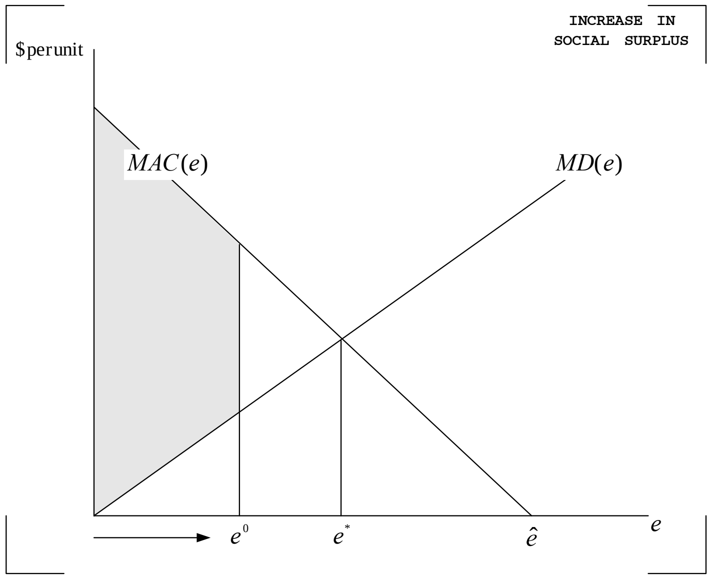

FIGURE 3-12

95

Optimal Abatement

- This comparison of costs and benefits tells us that for any $e < e^*$, an increase in the level of emissions allowed by the source creates an increase in net social benefit.

96

Optimal Abatement

Interpretation 2

- Starting at $e = \hat{e}$, construct the cost and benefit of a small reduction in emissions to some level $e^0 < \hat{e}$.

97

Optimal Abatement

- The benefit from this abatement is the reduction in environmental damage:

$$\int_{e^0}^{\hat{e}} MD(e)\,de$$

- See Figure 3-13.

98

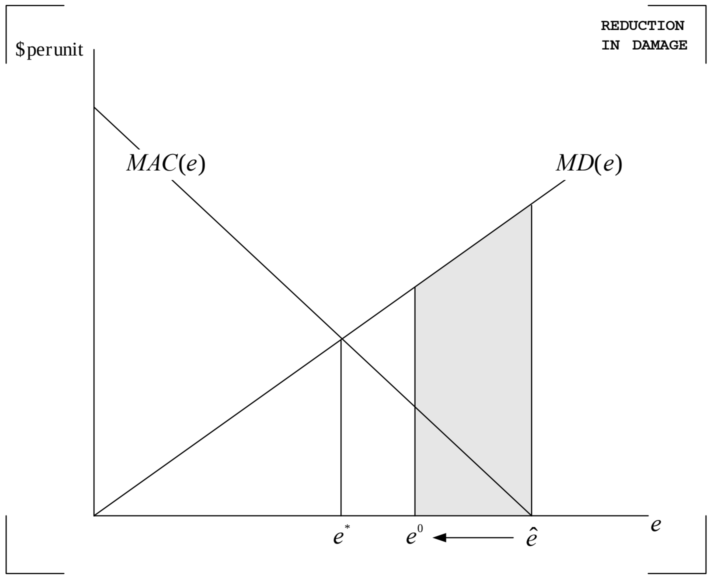

FIGURE 3-13

99

Optimal Abatement

- The <u>cost</u> of this abatement is the increase in abatement cost:

$$\int_{e^0}^{\hat{e}} MAC(e)de$$

- See Figure 3-14.

100

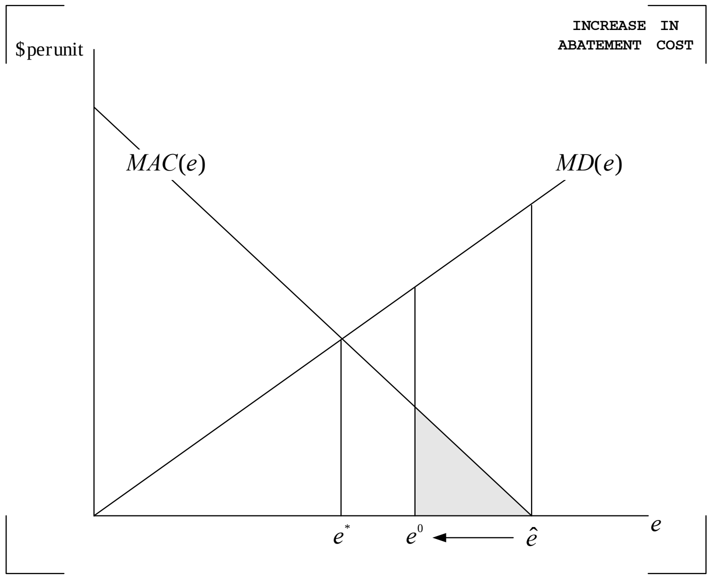

FIGURE 3-14

101

Optimal Abatement

- It is clear from Figures 3-13 and 3-14 that the reduction in damage more than offsets the increase in abatement cost; social surplus rises.
- See Figure 3-15.

102

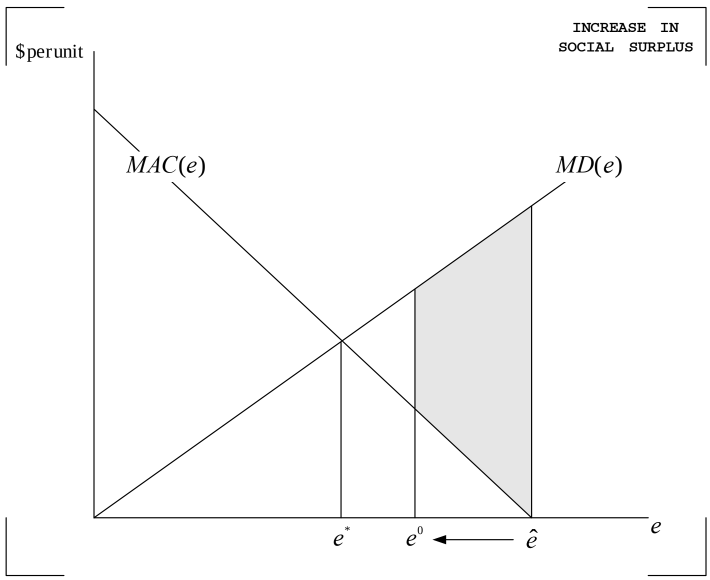

FIGURE 3-15

103

Optimal Abatement

- This comparison of costs and benefits tells us that for any $e > e^*$, a reduction in the level of emissions allowed by the source creates an increase in net social benefit.

104

Optimal Abatement

- The typical policy setting we face is one in which an existing polluter is subject to a new regulation, and so the policy question usually relates to how much abatement is optimal.
- Thus, we will usually frame our analysis in terms of interpretation 2.

105

Minimizing Total Social Cost

- It is also often useful to recast the policy objective in terms of minimizing **total social cost**, defined as abatement cost plus damage.
- This is equivalent to maximizing net social benefit.
- Why?

106

Minimizing Total Social Cost

- Recall from Figure 3-12 that an increase in emissions (a reduction in abatement) leads to an increase in net social benefit if the reduction in abatement cost exceeds the increase in damage, or equivalently if the sum of abatement cost and damage falls.

107

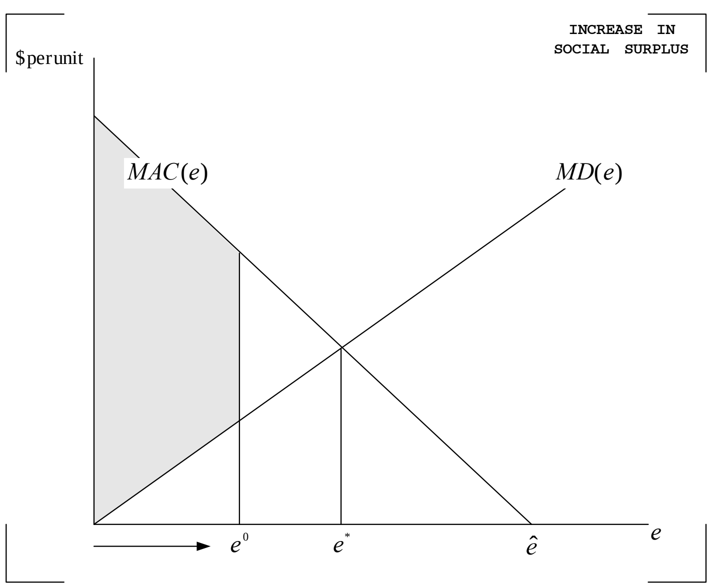

FIGURE 3-12 (repeat)

108

Minimizing Total Social Cost

- Conversely, recall from Figure 3-15 that a reduction in emissions (an increase in abatement) leads to an increase in net social benefit if the reduction in damage exceeds the increase in abatement cost, or equivalently if the sum of abatement cost and damage falls.

109

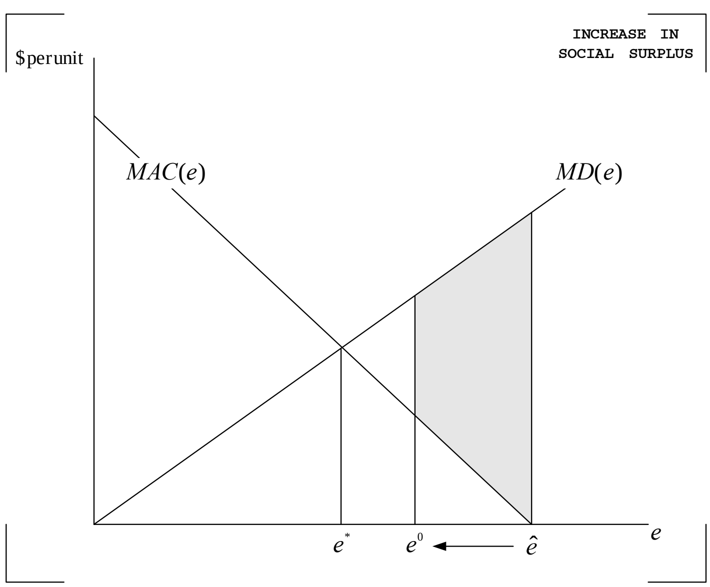

FIGURE 3-15 (repeat)

110

Minimizing Total Social Cost

- At e^*, any further change in emissions – an increase or reduction – must cause the sum of abatement cost and damage to rise; thus, total social cost is minimized at e^*.
- Figure 3-16 illustrates total social cost at the optimum, comprising damage at e^* (area D) and abatement cost at e^* (area AC).

111

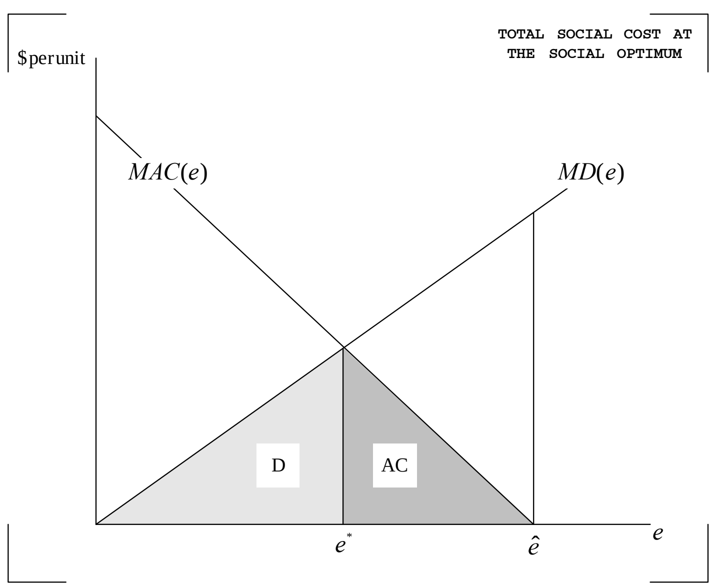

FIGURE 3-16

112

Minimizing Total Social Cost

- In comparison, Figures 3-17 and 3-18 illustrate total social cost at an emissions level $e^0 < e^*$ and $e^0 > e^*$ respectively.

113

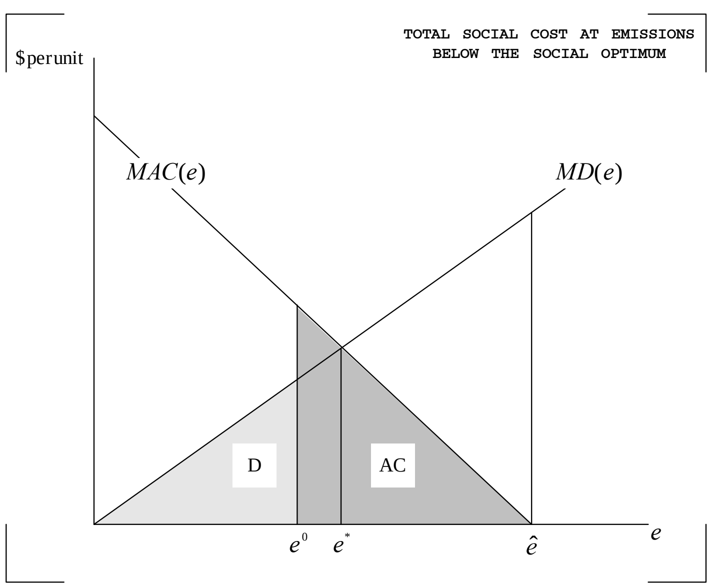

FIGURE 3-17

114

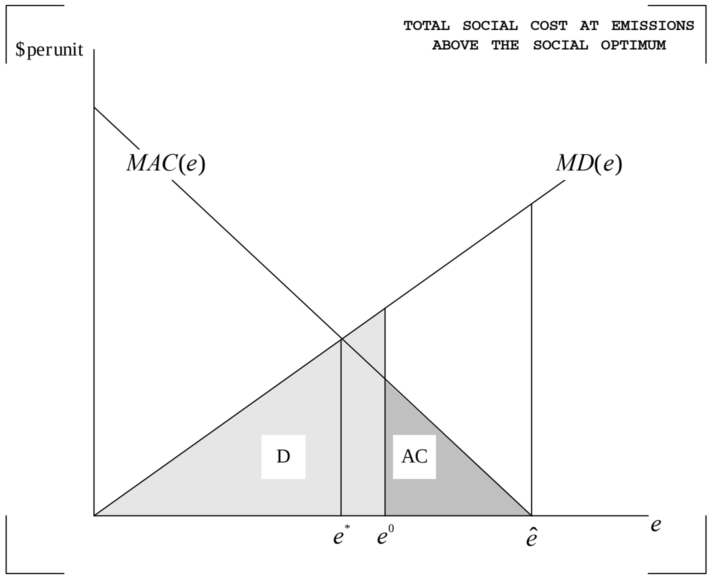

FIGURE 3-18

115

Minimizing Total Social Cost

- It is clear from Figures 3-16 – 3-18 that total social cost is lower at $e = e^*$ than at any level of emissions below or above e^*.

116

3.6 AN EXAMPLE WITH LINEAR MARGINAL COSTS

An Example with Linear Marginal Costs

- Throughout the course we will use a simple example with linear marginal costs to illustrate key concepts.
- We introduce the key elements of that example here.

118

An Example with Linear Marginal Costs

- Suppose

$$MAC(e) = \gamma(\hat{e} - e)$$

and

$$MD(e) = \delta e$$

- See Figure 3-19.

119

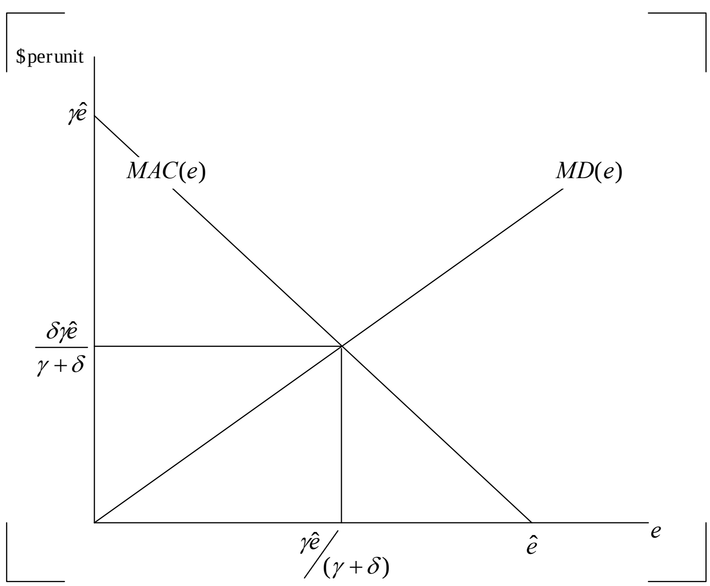

FIGURE 3-19

120

An Example with Linear Marginal Costs

- The social optimum is at e^*, where

$$\gamma(\hat{e} - e^*) = \delta e^*$$

- This solves for

$$e^* = \left(\frac{\gamma}{\gamma + \delta} \right) \hat{e}$$

121

An Example with Linear Marginal Costs

- It can be useful to express this optimum in terms of a **relative reduction requirement** (measured relative to the no-abatement emissions level):

$$r^* \equiv \frac{\hat{e} - e^*}{\hat{e}} = \left(\frac{\delta}{\gamma + \delta} \right)$$

122

An Example with Linear Marginal Costs

- Note that r^* is increasing in δ:
  - a steeper slope for MD means that the damage from any given quantity of emissions is higher, and so the optimal quantity of emissions is lower; thus, the relative reduction requirement is larger.

123

An Example with Linear Marginal Costs

- Conversely, r^* is decreasing in γ:
  - a steeper slope for MAC means that the cost of abatement for any given amount of abatement is higher, so the optimal amount of abatement is lower; thus, the relative reduction requirement is small.

124

An Example with Linear Marginal Costs

- Damage at the optimum is

$$D(e^*) = \int_0^{e^*} MD(e)de = \frac{\delta}{2}\left(\frac{\gamma\hat{e}}{\gamma+\delta}\right)^2$$

- See area D in Figure 3-20.

125

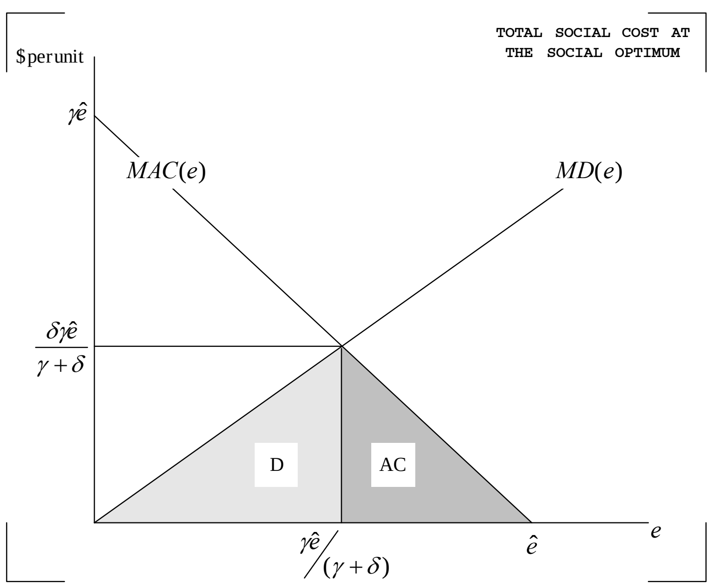

FIGURE 3-20

126

An Example with Linear Marginal Costs

- Abatement cost at the optimum is

$$AC(e^*) = \int_{e^*}^{\hat{e}} MAC(e)de = \frac{\gamma}{2}\left(\frac{\delta\hat{e}}{\gamma+\delta}\right)^2$$

- See area AC in Figure 3-20.

127

An Example with Linear Marginal Costs

- Total social cost at the optimum is

$$C(e^*) = D(e^*) + AC(e^*) = \frac{\gamma\delta\hat{e}^2}{2(\gamma+\delta)}$$

- This is the total shaded area in Figure 3-20.

128

3.7 THE IMPORTANCE OF SPECIFIC CONDITIONS

The Importance of Specific Conditions

- The linear cost example highlights a key general point:
  - the optimal level of abatement for the steel mill depends on the damage it causes (as reflected in δ in the example), and this could be different in different settings even for the same pollutant.

130

The Importance of Specific Conditions

- For example, a given airborne pollutant will likely cause higher damage in a densely populated region with a sheltered airshed than will the same pollutant discharged into a sparsely populated area subject to strong dispersing winds.

131

The Importance of Specific Conditions

- Thus, optimal abatement for two identical sources located in two different physical locations could be very different.
- In general, the optimal level of abatement depends on the specific conditions in place.

END

132

TOPIC 3 REVIEW QUESTIONS

These review questions begin with a solved example. It is recommended that you work through this example before commencing the questions.

A SOLVED EXAMPLE

Consider a setting where

$$MAC(e) = 243 - 3e$$

$$MD(e) = 6e$$

See Figure R3-1.

The unregulated private optimum solves

$$MAC(\hat{e}) = 0$$

$$243 - 3\hat{e} = 0$$

$$\hat{e} = 81$$

Damage at the unregulated private optimum:

$$D(\hat{e}) = \int_0^{81} MD(e)de = \frac{81*486}{2} = 19683$$

See Figure R3-2.

Abatement cost at the unregulated private optimum:

$$AC(\hat{e}) = 0$$

Total social cost at the unregulated private optimum:

$$SC(\hat{e}) = D(\hat{e}) + AC(\hat{e}) = 19683$$

The social optimum solves

$$MAC(e^*) = MAC(e^*)$$

$$243 - 3e^* = 6e^*$$

$$e^* = 27$$

Socially optimal abatement

$$\hat{e} - e^* = 81 - 27 = 54$$

Damage at the social optimum:

$$D(e^*) = \int_0^{27} MD(e)de = \frac{27*162}{2} = 2187$$

See Figure R3-3.

Abatement cost at the social optimum:

$$AC(e^*) = \int_{27}^{81} MAC(e)de = \frac{54*162}{2} = 4374$$

See Figure R3-4.

Total social cost at the social optimum:

$$SC(e^*) = D(e^*) + AC(e^*) = 6561$$

REVIEW QUESTIONS

1. A dissipative pollutant is one that

A. does not persist in the environment for more than a year after it is discharged.

B. causes no damage beyond the jurisdiction of the policy-maker.

C. accumulates in the environment over time.

D. None of the above.

2. The harm caused to potential future humans by a polluting activity today has no cost because it cannot measured directly.

A. True.

B. False.

3. Willingness-to-pay is the amount of money that an agent

A. would have to pay in order to make him just indifferent between not paying the money and avoiding the harm, and paying the money but incurring the harm.

B. would have to receive in order to make him just indifferent between receiving the money and incurring the harm, and not receiving the money but not incurring the harm.

C. would have to pay in order to make him just indifferent between paying the money and avoiding the harm, and not paying the money but incurring the harm.

D. would be happy to pay to avoid a harm.

4. In measuring value of an environmental loss, willingness-to-pay (WTP) and willingness-to-accept (WTA) are not necessarily equal because

A. income levels differ across individuals.
B. WTP is a function of wealth but WTA is not.
C. there may be little substitutability between the environmental loss and goods that can be purchased with money.
D. property rights are typically not clearly defined in settings where environmental losses are incurred.

5. The appropriate measure of an environmental loss is WTP because property rights are typically not defined.

A. True.
B. False.

6. The unit of measure for marginal damage is dollars.

A. True.
B. False.

7. If *MD*(*e*) is positive and increasing, then twice the pollution causes

A. twice the damage
B. more than twice the damage
C. less than twice the damage
D. more or less than twice the damage, depending on the type of pollutant.

8. Suppose an industrial plant is producing 12000 tons of steel per year, and 24000 tons of a particular pollutant per year. Then its emissions intensity for that pollutant is

A. 36000
B. 2
C. ½
D. None of the above.

Questions 9 – 13 relate to the following information. Suppose an industrial plant can reduce emissions only be reducing output (y), and that it generates one ton of pollutant for every unit of output produced. Suppose also that it sells its output in a competitive market, and currently faces a price equal to 100 dollars per unit. Its marginal production cost is

$$MC(y) = 10y$$

9. Its marginal net private benefit (MNPB) from emitting is

A. $100 + 10y$

B. $100 - 10y$

C. 1000

D. 500

10. In terms of emissions, its private optimum is

A. 10

B. 100

C. 5

D. 50

11. Its net private benefit at the private optimum is

A. 1000

B. 500

C. equal to producer surplus

D. Both B and C

12. Suppose the plant is required to reduce emissions by 50% from its private optimum. The associated abatement cost is

A. 500

B. 125

C. 250

D. 1000

13. The marginal abatement cost function for this plant is

A. $MAC(e) = 10(10 - e)$

B. $MAC(e) = 100 - e$

C. $MAC(e) = 100(e - 10)$

D. None of the above

14. In practice, polluting sources typically

A. hold output fixed and reduce emissions via intensity reduction measures (IRMs)

B. use a mix of abatement methods starting with the most costly methods so as to get the "hard part" done first

C. use a mix of abatement methods starting with the least costly methods and moving to increasingly costly methods as necessary

D. alternate between production cuts and IRMs

Questions 15 – 24 relate to the following information. A polluting source has a marginal abatement cost (MAC) schedule given by

$$MAC(e) = 400 - 20e$$

15. This MAC schedule can be re-expressed as

A. $MAC(e) = 40(\hat{e} - e)$ where $\hat{e} = 20$

B. $MAC(e) = 400(\hat{e} - e)$ where $\hat{e} = 40$

C. $MAC(e) = 20(\hat{e} - e)$ where $\hat{e} = 20$

D. $MAC(e) = 20(e - \hat{e})$ where $\hat{e} = 20$

The marginal damage (MD) schedule is

$$MD(e) = 5e$$

16. This MD schedule exhibits

A. increasing marginal damage

B. constant marginal damage

C. increasing damage

D. Both A and C

17. Damage at the private optimum is

A. 1000

B. 2300

C. 640

D. 200

18. Abatement cost at the private optimum is

A. 300

B. 1200

C. 0

D. 400

19. Total social cost at the private optimum is

A. 2600

B. 1840

C. 1000

D. 0

20. The socially optimal level of emissions is

A. 20

B. 16

C. 12

D. 0

21. The socially optimal level of abatement is

A. 0

B. 2

C. 4

D. 16

22. Damage at the social optimum is

A. 640

B. 480

C. 1080

D. 1200

23. Abatement cost at the social optimum is

A. 320

B. 640

C. 160

D. 120

24. Total social cost at the social optimum is

A. 1000

B. 1200

C. 800

D. 640

Questions 25 – 30 relate to the following information. A polluting source has a marginal abatement cost (MAC) schedule given by

$$MAC(e) = 2(360 - e)$$

The marginal damage (MD) schedule is

$$MD(e) = 10e$$

25. Damage at the private optimum is

A. 12000

B. 220000

C. 648000

D. 312400

26. The socially optimal level of emissions is

A. 20

B. 40

C. 80

D. 60

27. The socially optimal level of abatement is

A. 240

B. 300

C. 60

D. 120

28. Damage at the social optimum is

A. 12000

B. 360000

C. 18000

D. 78000

29. Abatement cost at the social optimum is

A. 28000

B. 36000

C. 16000

D. 90000

30. Total social cost at the social optimum is

A. 108000

B. 240000

C. 124000

D. 96000

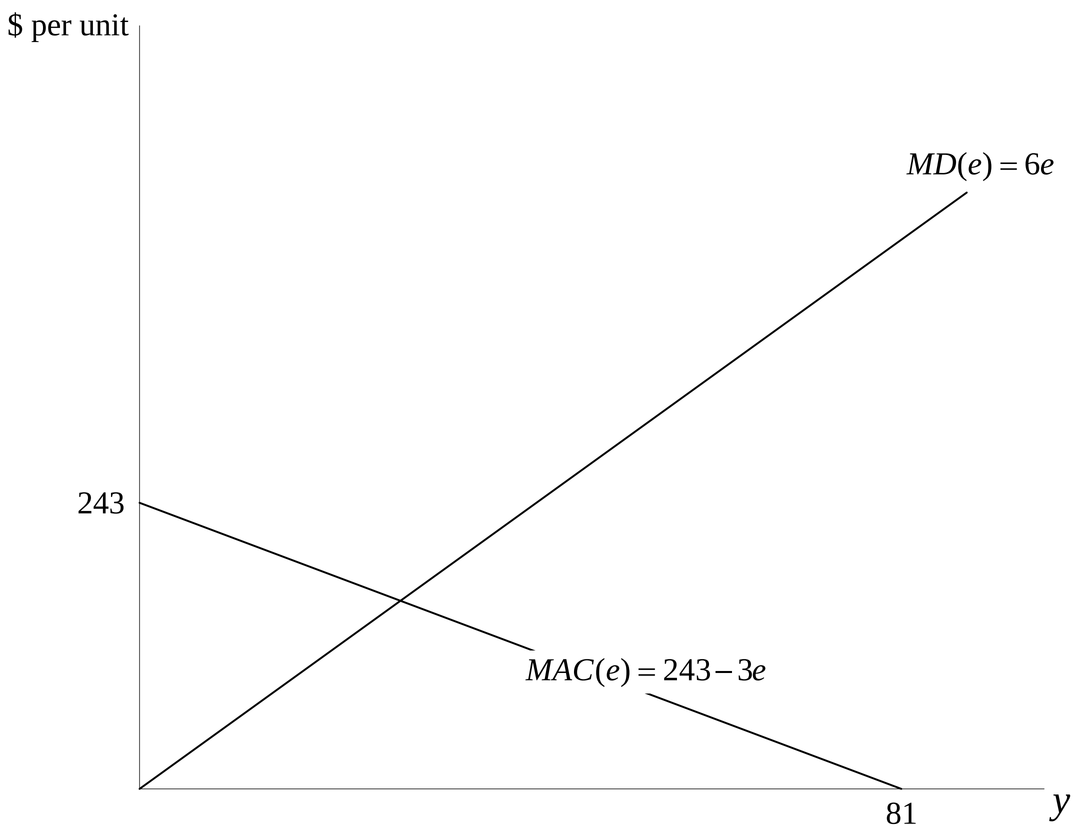

Figure R3-1

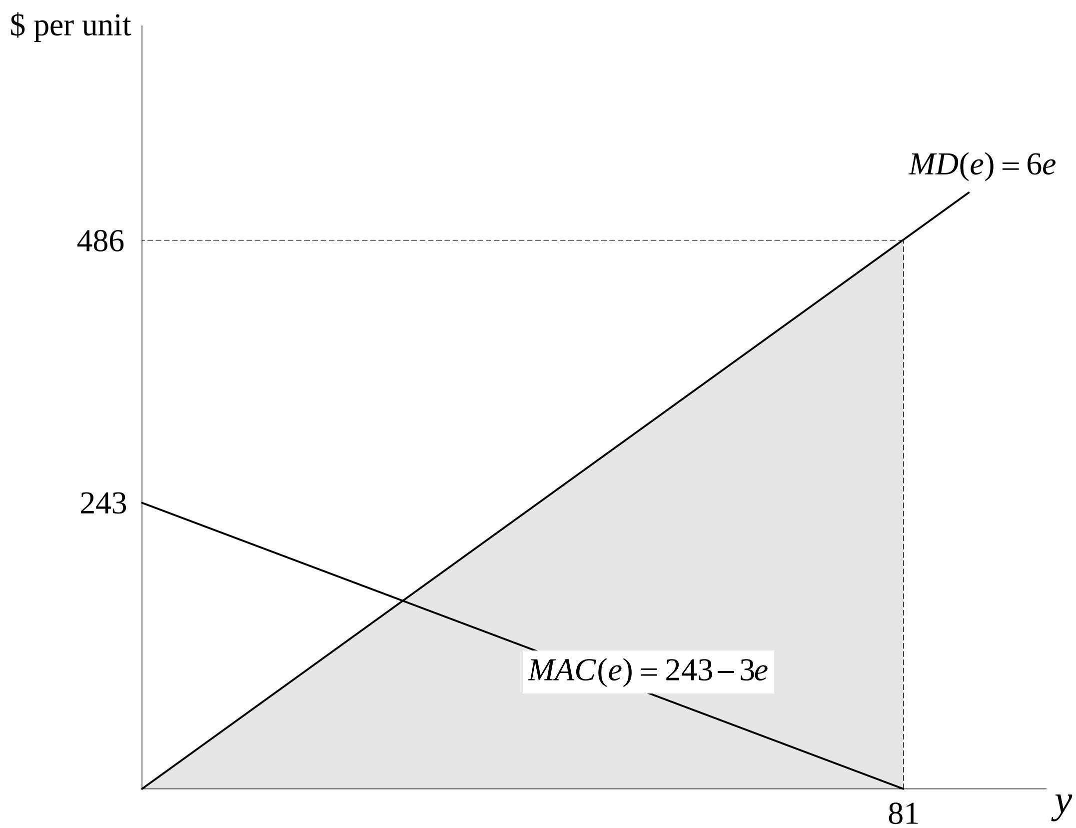

Figure R3-2

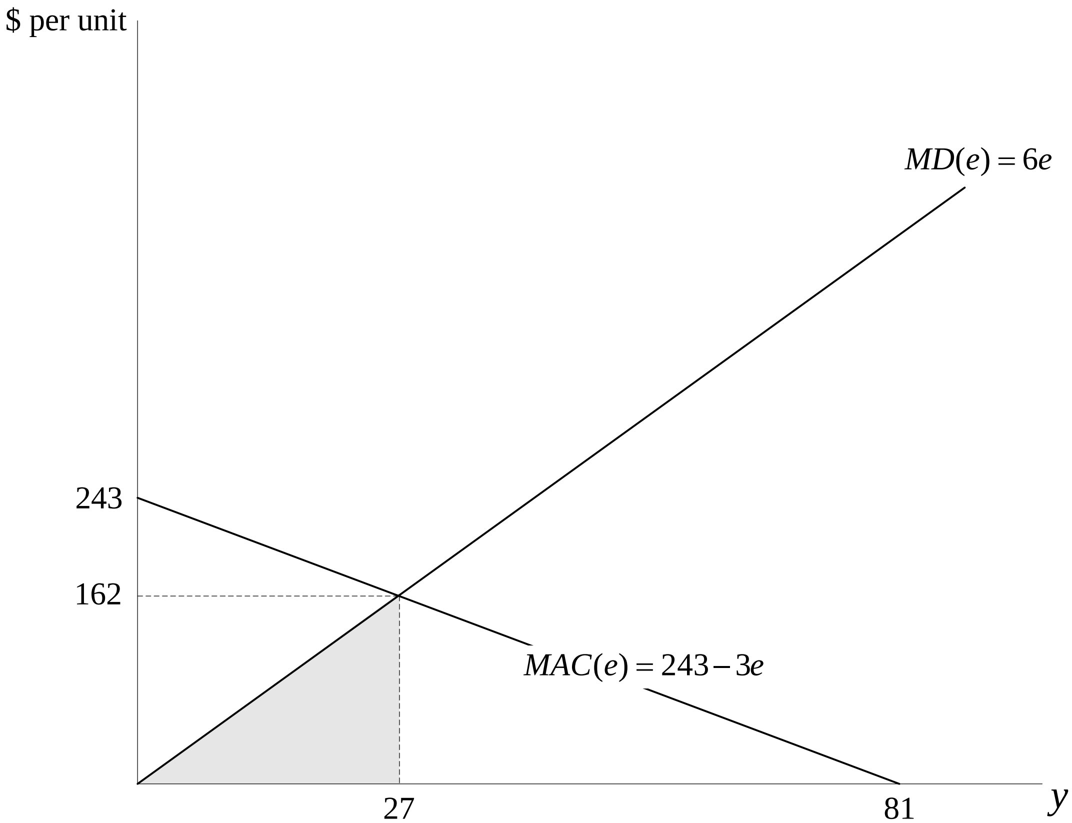

Figure R3-3

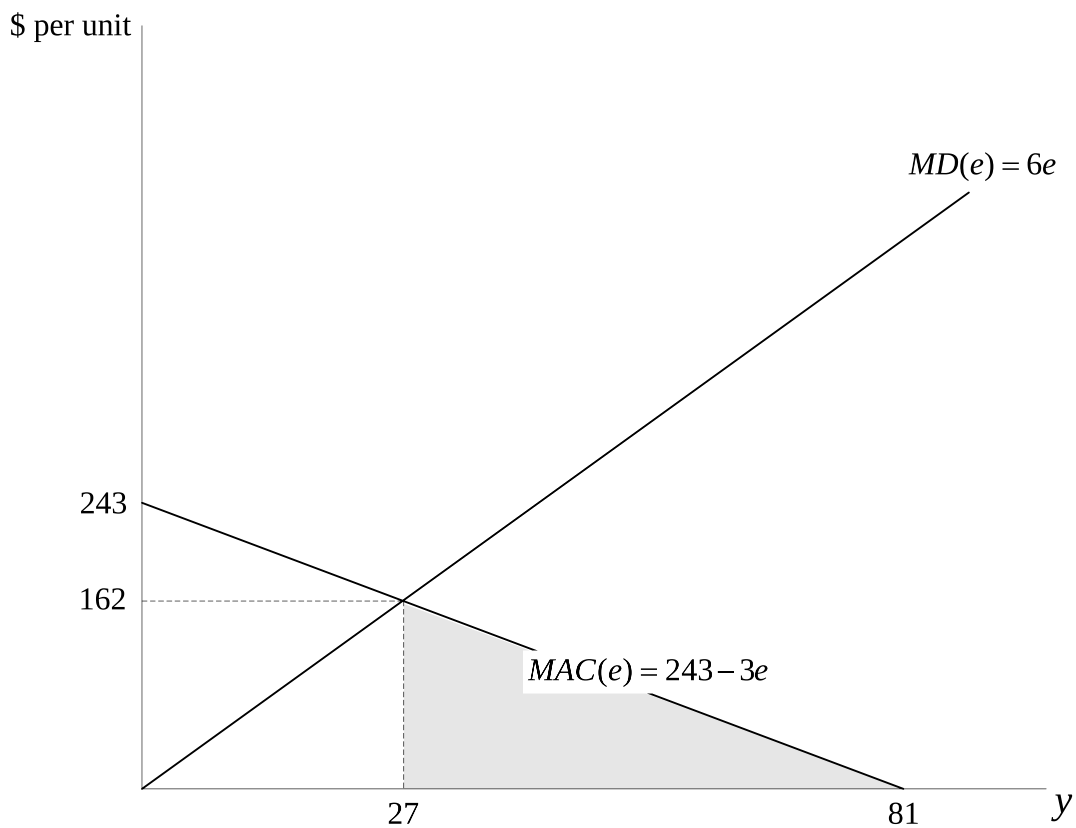

Figure R3-4

ANSWER KEY

1. A
2. B
3. C
4. C
5. B
6. B
7. B
8. B
9. B
10. A
11. D
12. B
13. A
14. C
15. C
16. D
17. A
18. C
19. C
20. B
21. C
22. A
23. C
24. C
25. C
26. D
27. B
28. C
29. D
30. A

4. DEFENSIVE ACTION

OUTLINE

4.1 Introduction

4.2 The Costs and Benefits of Defensive Action

4.3 Optimal Defensive Action

4.4 Coming to the Nuisance

2

4.1 INTRODUCTION

Introduction

- We have so far assumed that the harm suffered by external agents is determined solely by the quantity of pollution emitted.
- In practice, it is often possible to reduce that damage via defensive measures (sometimes called mitigation measures), including adaptation to a polluted environment.

4

Introduction

- For example, damage due to climate change will depend critically on the extent to which investments are made in building seawalls, developing new crop varieties, relocating people away from vulnerable areas, *etc*.

5

Introduction

- When defensive measures are available, the optimal level of pollution is still determined by balancing abatement cost and damage but where damage is now determined by the optimal level of defensive action.

- Let us explore this issue further.

6

Introduction

- Suppose MD is determined jointly by the quantity of emissions and the amount of defensive action undertaken by – or on behalf of – the damaged parties.

- Let v denote the amount of defensive action, measured in dollars invested.

7

Introduction

- Graphically, defensive action causes the MD schedule (plotted against e) to pivot downwards; see Figure 4-1 (where $v_1 > v_0$).

8

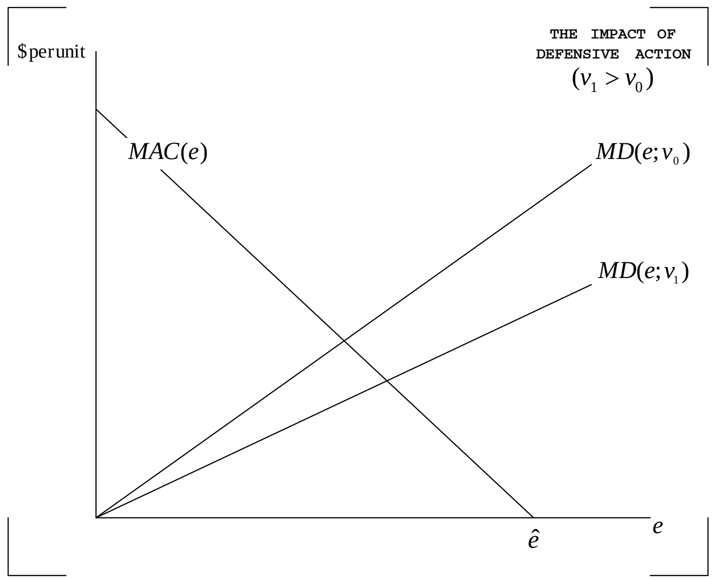

FIGURE 4-1

9

Introduction

- To determine the optimal amount of defensive action, we need to examine the costs and benefits of that defensive action.

10

4.2 THE COSTS AND BENEFITS OF DEFENSIVE ACTION

The Costs and Benefits of Defensive Action

- If defensive action can be taken in variable amounts (as opposed to an all-or-nothing action), then we can treat v as a continuous variable.

- Since we are measuring v in dollars, the marginal cost of defensive action is one.

12

The Costs and Benefits of Defensive Action

- The calculation of marginal benefit of defensive action is more complicated; we need to think in terms of damage avoided and abatement cost avoided.
- Consider Figure 4-2; it depicts optimal pollution under two different amounts of defensive action, v_0 and $v_1 > v_0$.

13

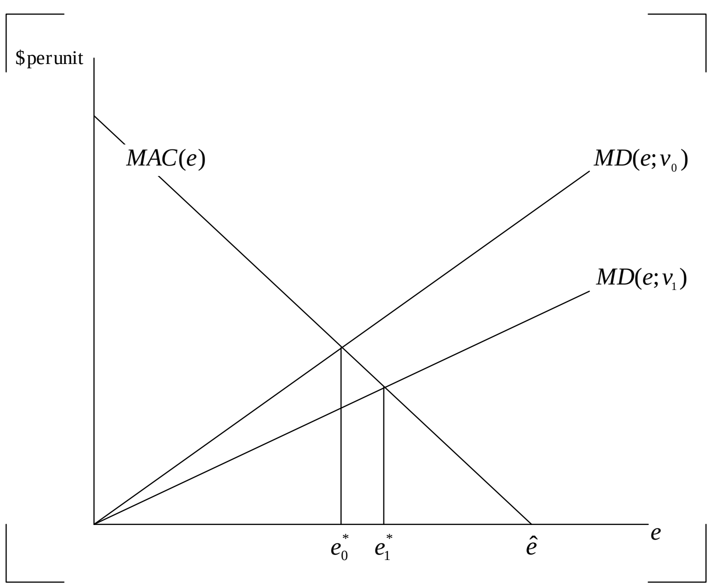

FIGURE 4-2

14

The Costs and Benefits of Defensive Action

- Note that optimal pollution when $v = v_1$ is higher than when $v = v_0$ because marginal damage is lower when $v = v_1$.
- In this respect, abatement and defensive action are substitutes.

15

The Costs and Benefits of Defensive Action

- The benefit derived from increasing v to v_1 from v_0 comprises two parts:
  – abatement cost avoided; and
  – net damage avoided
- Let us consider each part in turn.

16

The Costs and Benefits of Defensive Action

- The **abatement cost avoided** when optimal pollution rises from $e_0{}^*$ to $e_1{}^*$ is

$$\int_{e_0^*}^{e_1^*} MAC(e)de$$

- See Figure 4-3.

17

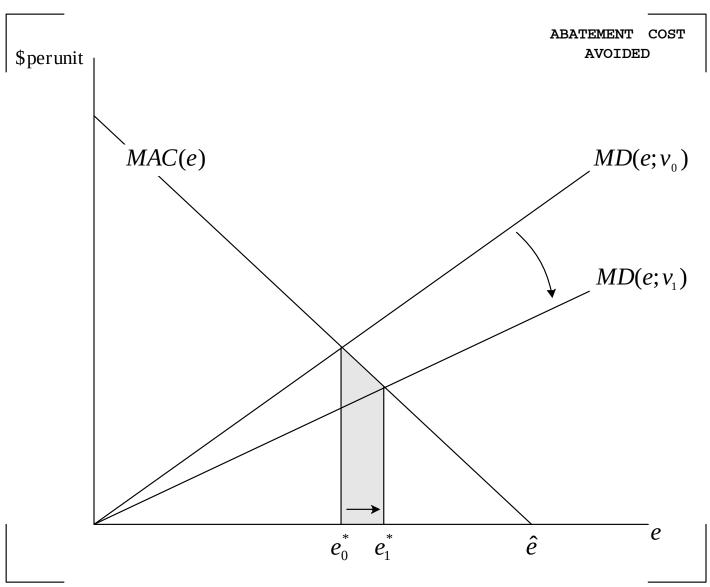

FIGURE 4-3

18

The Costs and Benefits of Defensive Action

- The **net damage avoided** is calculated as the difference between damage at e_1^* with v_1 , and damage at e_0^* with v_0 :

$$\int_0^{e_1^*} MD(e;v_1)de - \int_0^{e_0^*} MD(e;v_0)de$$

- See Figures 4-4 – 4-6.

19

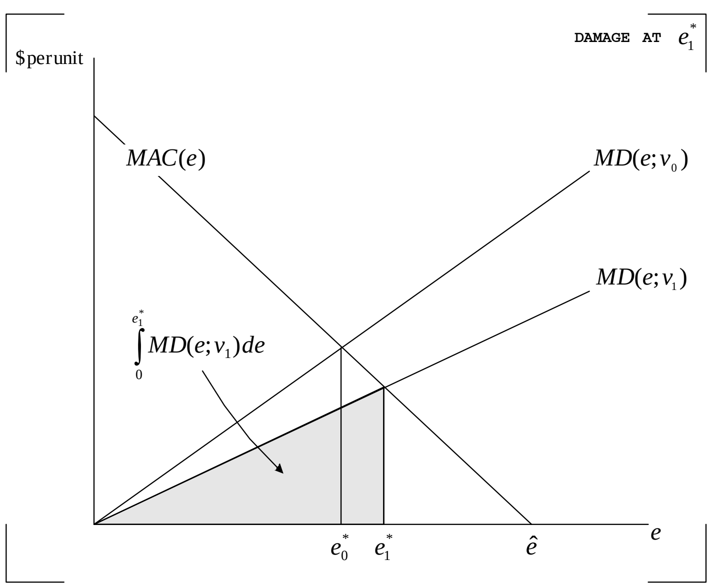

FIGURE 4-4

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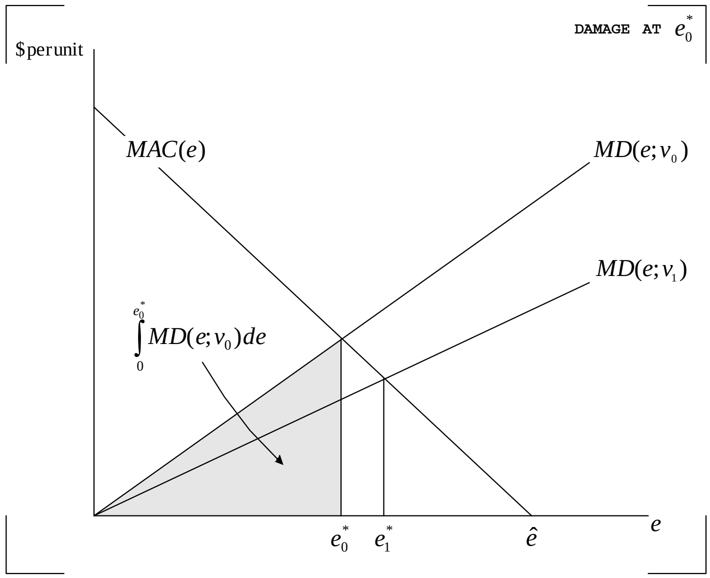


FIGURE 4-5 21

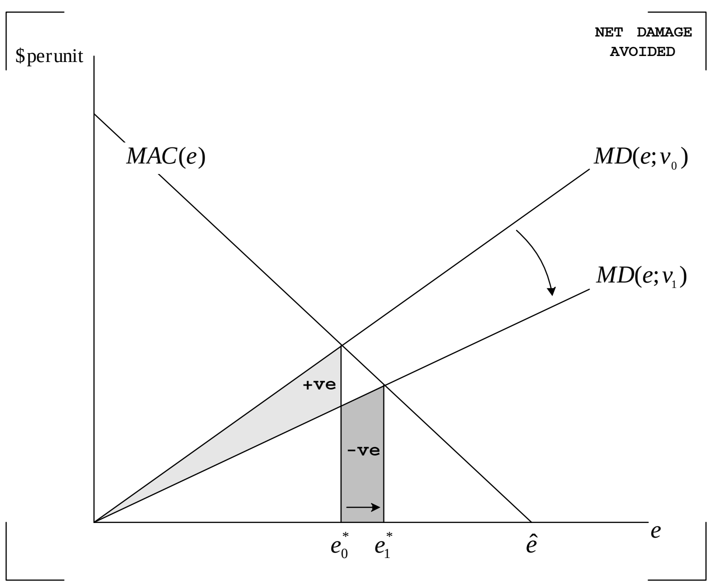


FIGURE 4-6 22

The Costs and Benefits of Defensive Action

- Note that net damage avoided comprises a positive area and a negative area.
  - The positive area measures the reduction in damage due to the increase in defensive action (from v_0 to v_1) when emissions remain unchanged at e_0^*.
  - The negative area measures the increase in damage due to the increase in optimal emissions (from e_0^* to e_1^*)

23

The Costs and Benefits of Defensive Action

- Can we be sure that the net damage avoided is positive?
- No; it is entirely possible that damage is actually higher at the $\{v_1, e_1^*\}$ combination that at the $\{v_0, e_0^*\}$ combination; that is, there is more damage overall when more defensive action is taken.

24

The Costs and Benefits of Defensive Action

- This might seem counter-intuitive but it is crucial to remember that the increase in emissions from e_0* to e_1* means that abatement costs fall; recall Figure 4-3.
- Once we take this into account, the overall benefit from the increase in defensive action must be positive; see Figure 4-7.

25

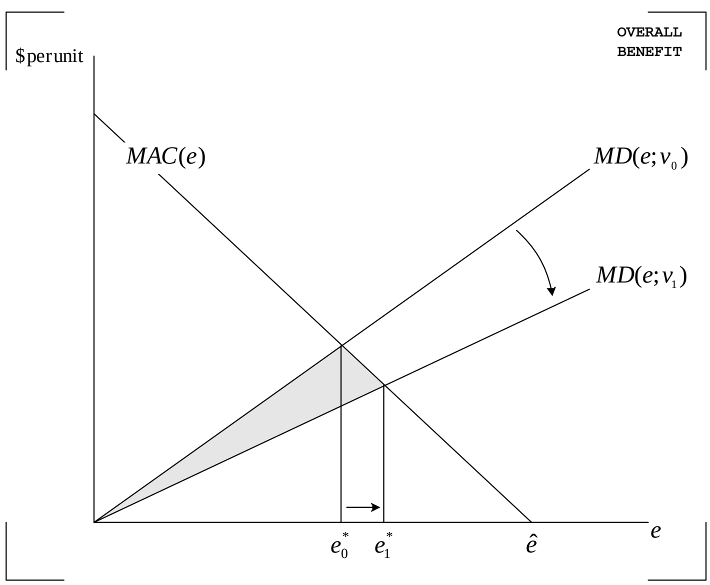


FIGURE 4-7

26

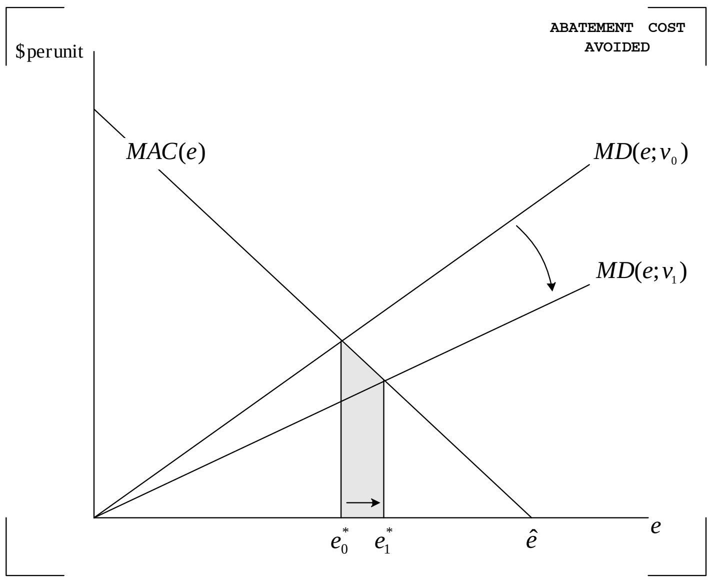


FIGURE 4-3 (repeat) 27

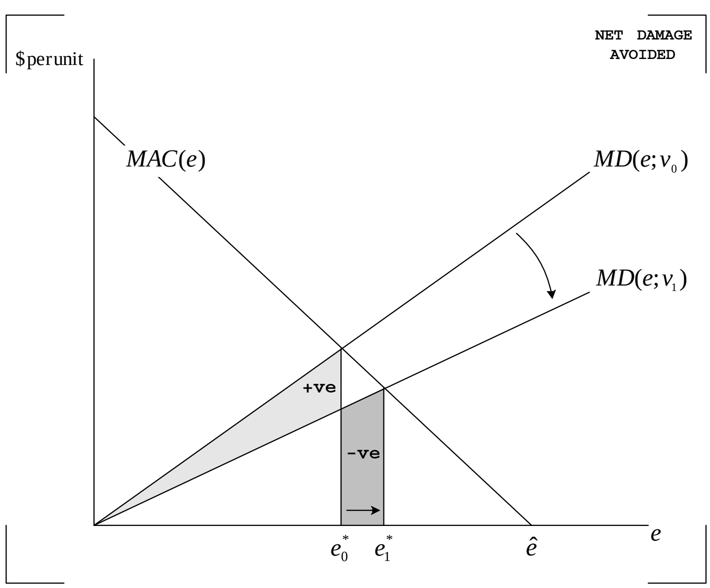


FIGURE 4-6 (repeat) 28

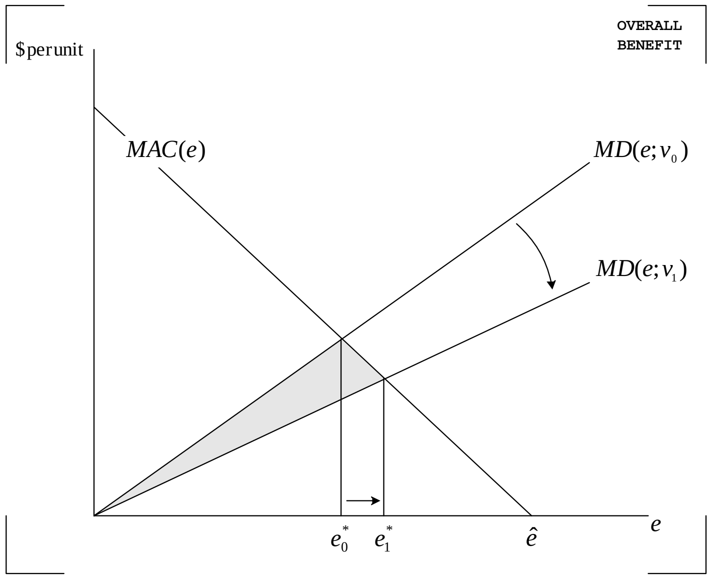


FIGURE 4-7 (repeat)

29

4.3 OPTIMAL DEFENSIVE ACTION

Optimal Defensive Action

- Let us now consider the <u>optimal</u> amount of defensive action.

31

Optimal Defensive Action

- Imagine that we gradually raise *v* and plot the resulting increase in benefit (as measured by the expansion of the area in Figure 4-7 as MD pivots down) on the vertical axis of a separate graph.
- The resulting graph is the **marginal benefit schedule** for *v*; see Figure 4-8.

32

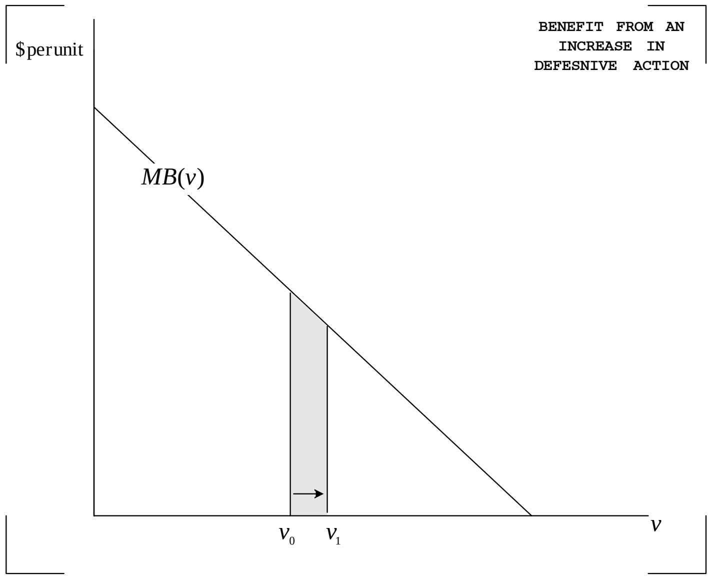


FIGURE 4-8

33

Optimal Defensive Action

- Note that the shaded area in Figure 4-8 must be equal to the shaded area in Figure 4-7.
- Both areas measure the total benefit from an increase in defensive action from v_0 to v_1.

34

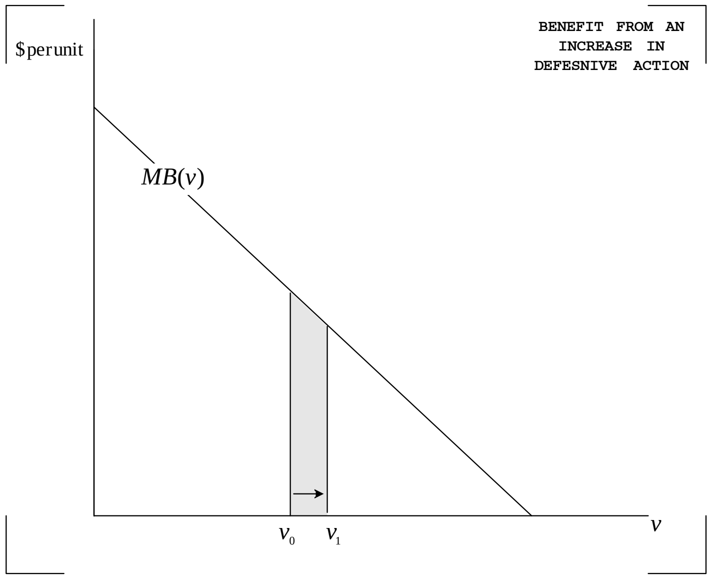


FIGURE 4-8 (repeat)

35

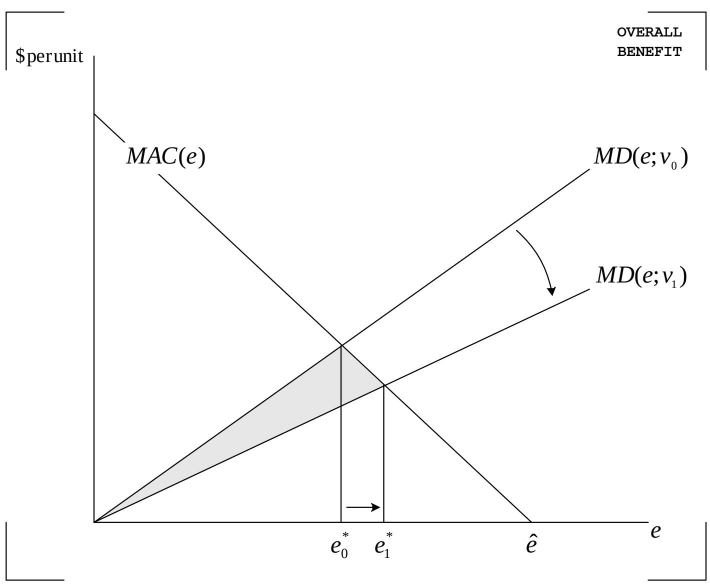


FIGURE 4-7 (repeat)

36

Optimal Defensive Action

- Recall from s.12 that we have chosen units to ensure that the marginal cost of defensive action is one.
- Thus, the <u>optimal</u> quantity of defensive action is v^*, where

$$MB(v^*) = 1$$

- See Figure 4-9.

37

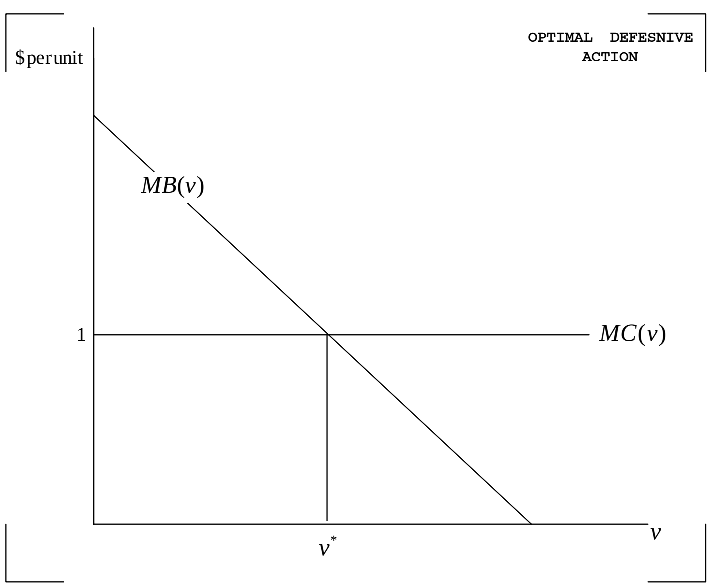


FIGURE 4-9

38

Optimal Defensive Action

- We can now define the optimal quantities for defensive action <u>and</u> emissions.
- The optimal pair $\{e^*, v^*\}$ is defined by

$$MAC(e^*) = MD(e^*; v^*)$$

$$MB(v^*) = 1$$

- See Figures 4-9 and 4-10.

39

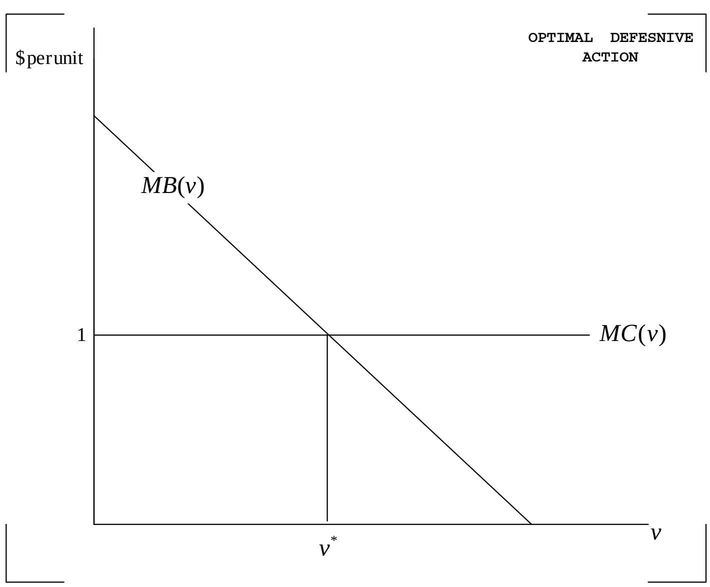


FIGURE 4-9 (repeat)

40

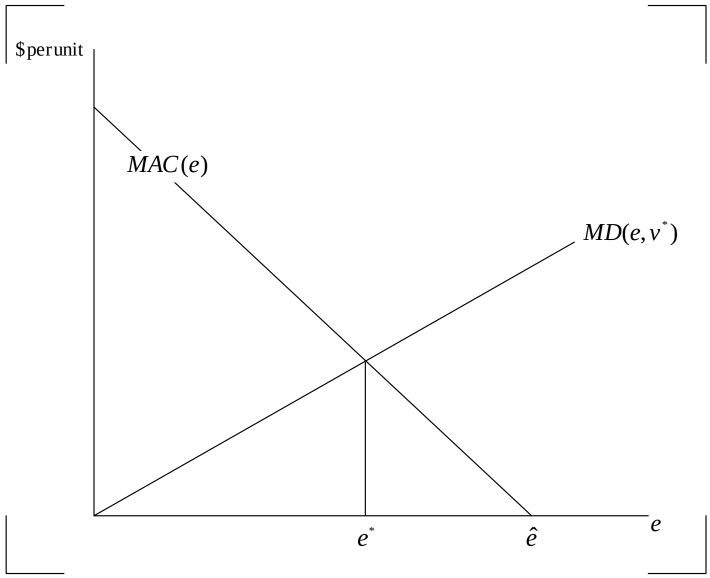


FIGURE 4-10 41

4.4 COMING TO THE NUISANCE

Coming to the Nuisance

- The antithesis of defensive action is "coming to the nuisance":
  - a deliberate decision by an external agent to locate near an existing polluting facility, and thereby create an external cost that did not previously exist.

43

Coming to the Nuisance

- Such a decision can be socially optimal, depending on the social costs and benefits, but the private costs and benefits will not necessarily create the right incentives.

- Let examine this issue more closely.

44

Coming to the Nuisance

- Let *B* denote the private benefit to the agent who locates near the polluting source, and assume that the private and social benefit are equal.

45

Coming to the Nuisance

- If the agent locates near the source, then she suffers damage from the pollution, as determined by a marginal damage function, $MD(e)$.
- Conversely, if she does not locate near the source, she foregoes *B*, but the polluting source causes no damage.

46

Coming to the Nuisance

- Suppose the polluting source has marginal abatement cost *MAC*(*e*).
- Thus, if the external agent locates near the polluting source, the socially optimal quantity of emissions is e^* such that

$$MAC(e^*) = MD(e^*)$$

47

Coming to the Nuisance

- Conversely, if the external agent does not locate near the polluting source then the socially optimal quantity of emissions is $\hat{e}$ such that

$$MAC(\hat{e}) = 0$$

48

Coming to the Nuisance

- Thus, the social cost of locating near the polluting source (given socially optimal emissions) is

$$C^* = \int_0^{e^*} MD(e)de + \int_{e^*}^{\hat{e}} MAC(e)de$$

- See Figure 4-11.

49

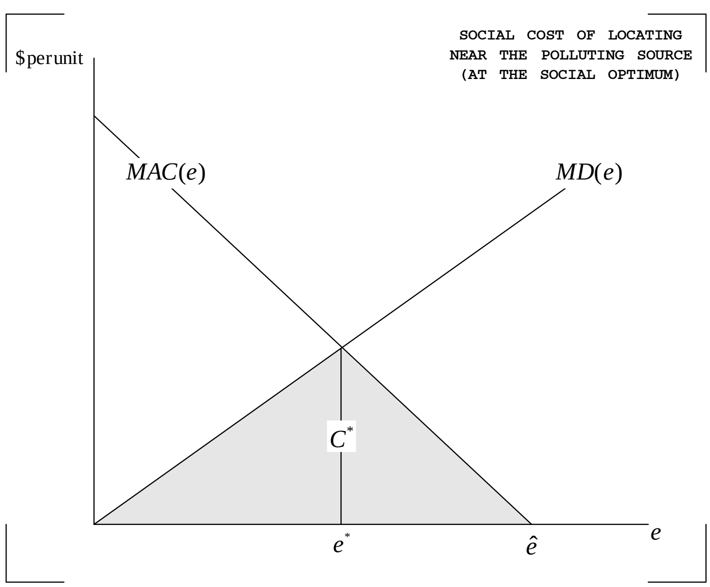


FIGURE 4-11

50

Coming to the Nuisance

- We can therefore conclude that locating near the polluting facility is socially optimal if and only if

$$B > C^*$$

51

Private Incentives

- Now consider the private incentives behind the location choice, under two different scenarios with respect to regulation and compensation.

52

Private Incentives

Scenario 1

- The polluting source is unregulated and continues to pollute at $\hat{e}$ even if the external agent locates near it.

53

Private Incentives

- Under this scenario, the private cost to the external agent of locating near the polluting source is

$$PC_1 = \int_0^{\hat{e}} MD(e)de > C^*$$

- See Figure 4-12.

54

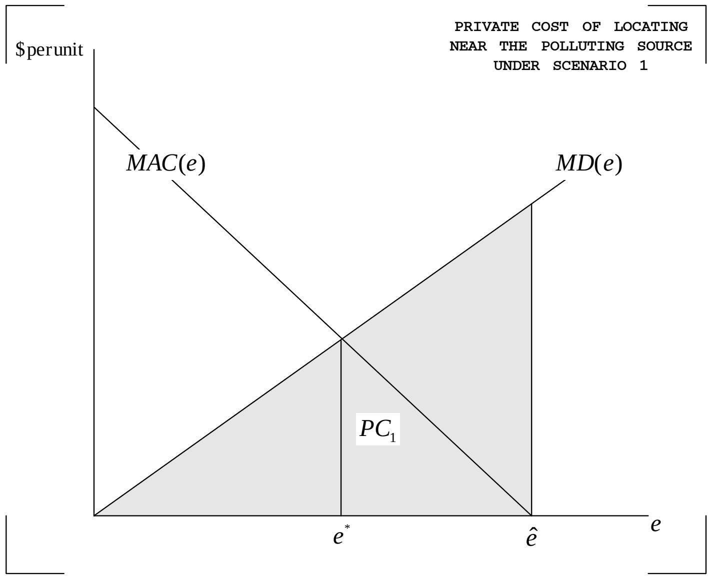


FIGURE 4-12 55

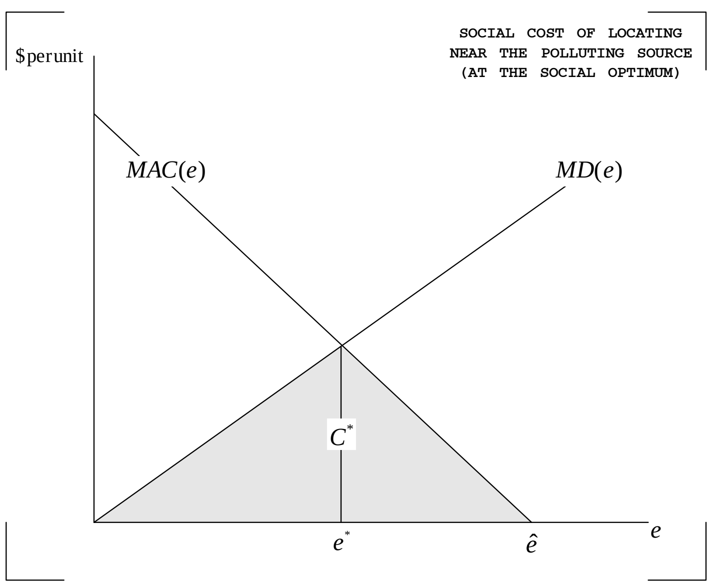


FIGURE 4-11 (repeat) 56

Private Incentives

- Since $PC_1 > C^*$, there exists the possibility under this scenario that $B > C^*$ but $B < PC_1$; that is, locating near the polluting source could be socially optimal but not privately optimal for the external agent.

57

Private Incentives

Scenario 2

- The polluting source is regulated and is required to reduce emissions to e^* if the external agent locates near it, but the external agent does not have to compensate the polluting source for the cost of that abatement.

58

Private Incentives

- Under this scenario, the private cost to the external agent of locating near the polluting source is

$$PC_2 = \int_0^{e^*} MD(e)de < C^*$$

- See Figure 4-13.

59

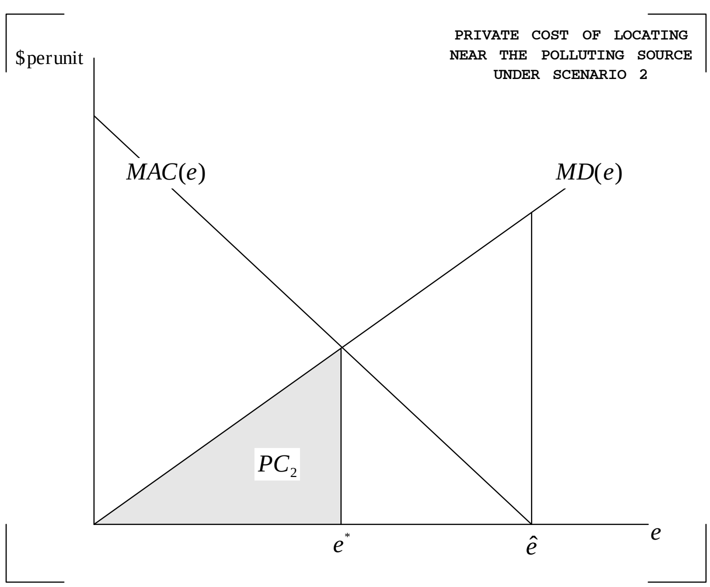


FIGURE 4-13

60

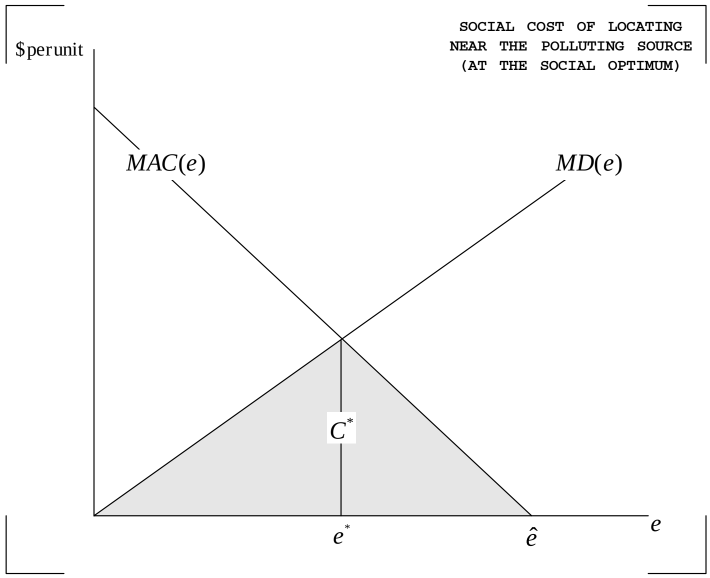


FIGURE 4-11 (repeat)

61

Private Incentives

- Since $PC_2 < C^*$, there exists the possibility under this scenario that $B < C^*$ but $B > PC_2$; that is, locating near the polluting source could be privately optimal for the external agent but not socially optimal.

62

Private Incentives

- The root of the problem under scenario 2 is that the external agent creates a social cost by locating near the polluting source.
- If the external agent does not have to account fully for that cost when making her location choice, then she does not face the right incentives from a social perspective.

63

Private Incentives

- The private and social cost of locating near the polluting source are equal if and only if
  - the polluting source is regulated at the socially-optimal emissions level; and
  - the external agent pays for the abatement required to achieve that socially-optimal emissions level

64

An Example with Linear Marginal Costs

- Recall the linear example from Topic 3.5, where

$$MAC(e) = \gamma(\hat{e} - e)$$

and

$$MD(e) = \delta e$$

An Example with Linear Marginal Costs

- If the external agent locates near the polluting source then the socially optimal quantity of emissions is

$$e^* = \left(\frac{\gamma}{\gamma + \delta}\right)\hat{e}$$

- See Figure 4-14.

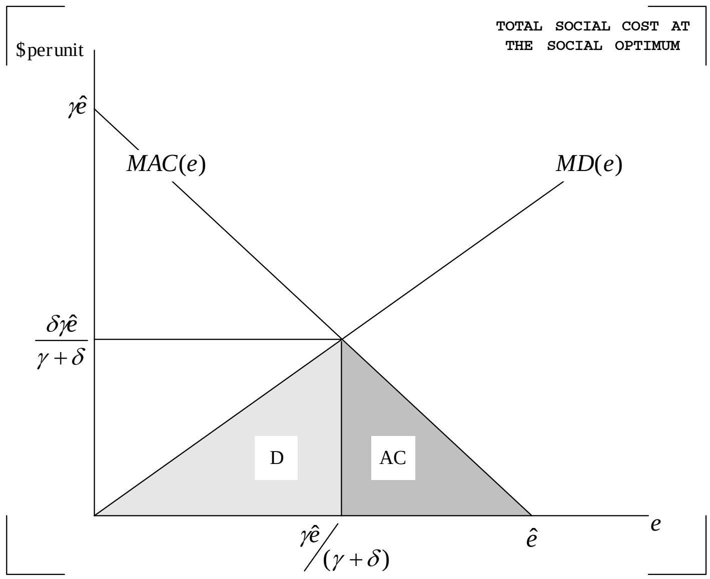


FIGURE 4-14

67

An Example with Linear Marginal Costs

- Damage at the optimum is

$$D(e^*) = \int_0^{e^*} MD(e)de = \frac{\delta}{2}\left(\frac{\gamma\hat{e}}{\gamma+\delta}\right)^2$$

- See area D in Figure 4-14.

68

An Example with Linear Marginal Costs

- Abatement cost at the optimum is

$$AC(e^*) = \int_{e^*}^{\hat{e}} MAC(e)de = \frac{\gamma}{2}\left(\frac{\delta\hat{e}}{\gamma+\delta}\right)^2$$

- See area AC in Figure 4-14.

69

An Example with Linear Marginal Costs

- Total social cost at the optimum is

$$C(e^*) = D(e^*) + AC(e^*) = \frac{\gamma\delta\hat{e}^2}{2(\gamma+\delta)}$$

- This is the total shaded area in Figure 4-14.

70

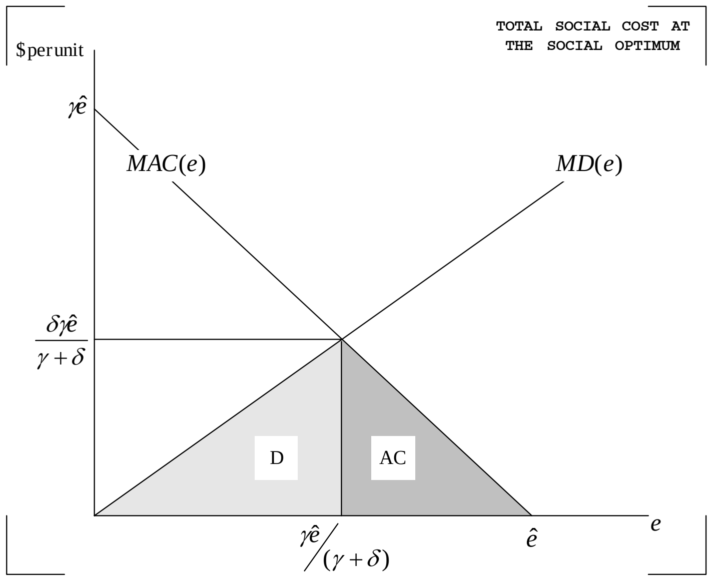


FIGURE 4-14 (insert)

71

An Example with Linear Marginal Costs

- Thus, locating near the polluting source is socially optimal if and only if

$$B > \frac{\gamma\delta\hat{e}^2}{2(\gamma+\delta)}$$

72

An Example with Linear Marginal Costs

- Under Scenario 1, the private cost to the external agent of locating near the polluting source is

$$PC_1 = \frac{\delta \hat{e}^2}{2}$$

- See Figure 4-15.

73

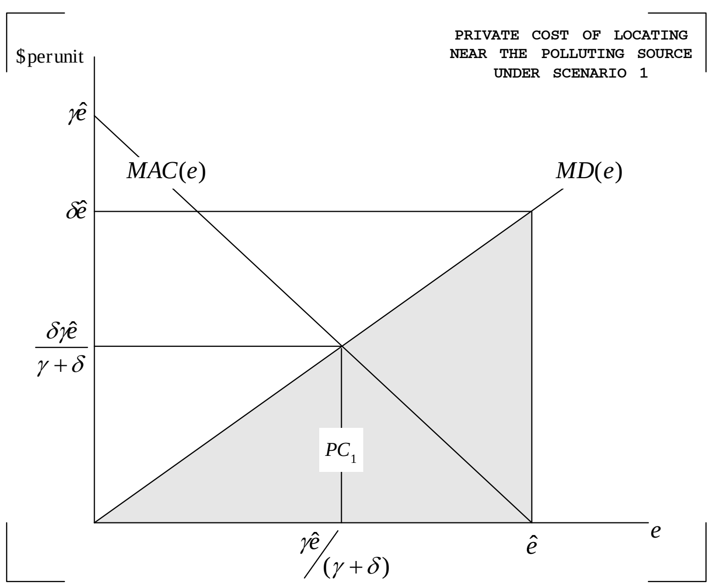


FIGURE 4-15

74

An Example with Linear Marginal Costs

- Under Scenario 2, the private cost to the external agent of locating near the polluting source is

$$PC_2 = D(e^*) = \frac{\delta}{2}\left(\frac{\gamma\hat{e}}{\gamma+\delta}\right)^2$$

- See area D Figure 4-14.

75

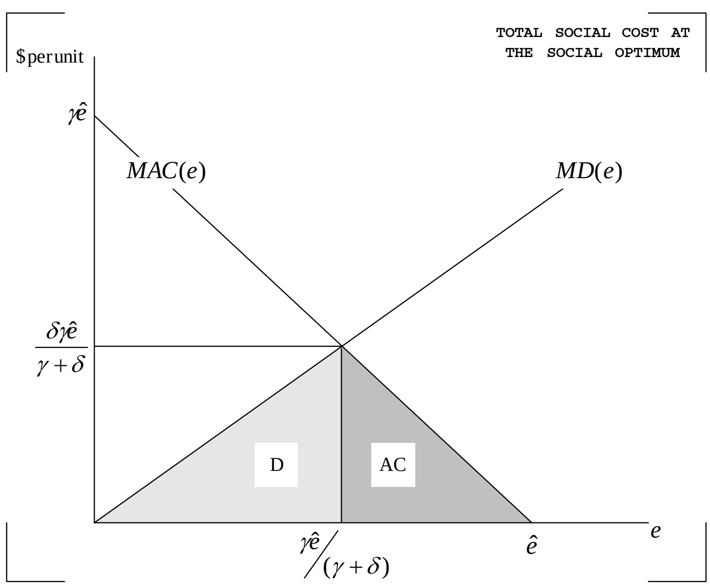


FIGURE 4-14 (insert) 76

TOPIC 4 REVIEW QUESTIONS

These review questions begin with a solved example. It is recommended that you work through this example before commencing the questions.

A SOLVED EXAMPLE

Consider a setting where

$$MAC(e) = 320 - 4e$$

$$MD(e) = \left(\frac{6}{1+v}\right)e$$

Suppose $v = 0$. The socially optimal level of emissions solves

$$MAC(e_0^*) = MD(e_0^*)\Big|_{v=0}$$

$$320 - 4e_0^* = 6e_0^*$$

$$e_0^* = 32$$

See Figure R4-1.

Damage at the social optimum is

$$D(e_0^*)\Big|_{v=0} = \int_0^{32} MD(e)\Big|_{v=0} de = \frac{32*192}{2} = 3072$$

See Figure R4-2.

Abatement cost at the social optimum is

$$AC(e_0^*) = \int_{32}^{80} MAC(e)de = \frac{(80-32)*192}{2} = 4608$$

See Figure R4-3.

Social cost at the social optimum is

$$SC(e_0^*) = D(e_0^*)\Big|_{v=0} + AC(e_0^*) = 7680$$

Now suppose $v = 5$. The socially optimal level of emissions solves

$$MAC(e_5^*) = MD(e_5^*)\Big|_{v=5}$$

$$320 - 4e_5^* = e_5^*$$

$$e_5^* = 64$$

See Figure R4-4.

Damage at the social optimum is

$$D(e_5^*)\Big|_{v=5} = \int_0^{64} MD(e)\Big|_{v=5} de = \frac{64*64}{2} = 2048$$

See Figure R4-5.

Abatement cost at the social optimum is

$$AC(e_5^*) = \int_{64}^{80} MAC(e)de = \frac{(80-64)*64}{2} = 512$$

See Figure R4-6.

Social cost at the social optimum is

$$SC(e_5^*) = D(e_5^*)\Big|_{v=5} + AC(e_5^*) = 2560$$

The benefit from defensive action at $v = 5$ is

$$B(v=5) = SC(e_0^*) - SC(e_5^*) = 7680 - 2560 = 5120$$

See Figure R4-7.

If the marginal cost of defensive action is one (as we have assumed) then the net benefit from defensive action at $v = 5$ is

$$NB(v = 5) = 5120 - 5 = 5115$$

REVIEW QUESTIONS

1. There are two components to the benefit from defensive action. They are

A. abatement cost avoided plus damage avoided, because the optimal level of emissions is lower.

B. abatement cost avoided because the optimal level of emissions is higher, plus net damage avoided (which could be negative).

C. abatement cost avoided because the optimal level of emissions is higher, plus damage avoided (which must be positive).

D. None of the above.

Questions 2 – 14 relate to the following information. Suppose MAC is given by

$$MAC(e) = 30(20 - e)$$

and suppose MD is given by

$$MD(e) = \left(\frac{10}{1+v}\right)e$$

where v is the dollar investment in defensive action.

2. The effect of v is to

A. reduce the slope of the MD function.

B. reduce the vertical intercept of the MD function.

C. to reduce both the slope of the vertical intercept of the MD function.

D. None of the above.

3. Damage can be eliminated entirely with a sufficiently large but finite investment in defensive action.

A. True.

B. False.

4. Suppose $v = 0$. Then the socially optimal level of emissions is

A. 10

B. 15

C. 5

D. 12

5. Suppose $v = 0$. Then damage at the social optimum is

A. 555

B. 1050

C. 1125

D. 875

6. Suppose $v = 0$. Then abatement cost at the social optimum is

A. 1125

B. 1200

C. 565

D. 375

7. If $v = 2$, the socially optimal level of emissions is

A. 15

B. 18

C. 12

D. 0

8. If $v = 2$, damage at the social optimum is

A. 540

B. 620

C. 1125

D. 455

9. In this example, damage at the social optimum is lower when $v > 0$ than when $v = 0$ because this must be true in *any* example; it is a fundamental effect of defensive action in any setting.

A. True.

B. False.

10. If $v = 2$, abatement cost at the social optimum is

A. 325

B. 180

C. 240

D. 60

11. The overall benefit from raising defensive action from $v = 0$ to $v = 2$ is

A. 900

B. 600

C. 1200

D. 300

12. The unit of measure for the answer to Q10 is dollars per unit of emissions reduced.

A. True.

B. False.

13. The net benefit from raising defensive action from $v = 0$ to $v = 2$ is

A. 760

B. 898

C. 1170

D. 230

14. The optimal level of defensive action is $v^* = 2$.

A. True.

B. False.

Hint: Evaluate the net benefit at $v = 1$ and $v = 3$.

Questions 15 – 22 relate to the following information. Suppose MAC is given by

$$MAC(e) = 10(30 - e)$$

and suppose MD is given by

$$MD(e) = \left(\frac{5}{1+v}\right)e$$

where *v* is the dollar investment in defensive action.

15. Suppose $v = 0$. Then the socially optimal level of emissions is

A. 20

B. 30

C. 24

D. 12

16. Suppose $v = 0$. Then damage at the social optimum is

A. 500

B. 1000

C. 1100

D. 600

17. Suppose $v = 0$. Then abatement cost at the social optimum is

A. 500

B. 1000

C. 1100

D. 600

18. If $v = 1$, the socially optimal level of emissions is

A. 20

B. 30

C. 18

D. 24

19. If $v = 1$, damage at the social optimum is

A. 1200

B. 860

C. 720

D. 480

20. If $v = 1$, abatement cost at the social optimum is

A. 360

B. 180

C. 1100

D. 760

21. The overall benefit from raising defensive action from $v = 0$ to $v = 1$ is

A. 400

B. 600

C. -200

D. -100

22. The net benefit from raising defensive action from $v = 0$ to $v = 1$ is

A. 599

B. 399

C. -101

D. -201

Questions 23 – 25 relate to the following information. An existing polluting plant has MAC given by

$$MAC(e) = 12(15 - e)$$

Agent Q decides to locate next to the plant based on the expectation that she will lobby successfully to have the plant regulated, with emissions set at the socially optimal level. She also expects that she will not have to pay for the abatement required of the plant. The MD function for agent Q is

$$MD(e) = 8e$$

23. Given that agent Q locates next to the polluting plant, the socially optimal level of emissions is

A. 12

B. 5

C. 8

D. 9

24. We know that the benefit to agent Q from locating next to the plant must be at least

A. 324

B. 522

C. 256

D. 312

25. It is socially optimal for agent Q to locate next to the plant if and only if the social benefit from her doing so is at least

A. 640

B. 720

C. 540

D. 580

Questions 26 – 30 relate to the following information. An existing polluting plant has MAC given by

$$MAC(e) = 40 - 8e$$

Agent K is deciding whether or not to locate next to the plant. If he does locate next to the plant, he will face a MD schedule given by

$$MD(e) = \left(\frac{32}{1+v}\right)e$$

where *v* is the dollar investment in defensive action that he can make if he does locate there. Suppose that this investment can only be made at one of three discrete levels: $v = 0$, $v = 5$ or $v = 15$.

26. If agent K locates next to the plant and emissions are set at the socially optimal level, then the net benefit from raising defensive action from $v = 0$ to $v = 5$ is

A. 20

B. 30

C. 35

D. 40

27. If agent K locates next to the plant and emissions are set at the socially optimal level, then the net benefit from raising defensive action from $v = 0$ to $v = 15$ is

A. 25

B. 30

C. 40

D. 45

28. If agent K locates next to the plant then the socially optimal level of defensive action is

A. 0

B. 5

C. 15

D. None of the above

29. If emissions are set at the socially optimal level, and defensive action is taken at the socially optimal level, then it is socially optimal for agent K to locate next to the polluting plant if the social benefit from him doing so is at least

A. 35

B. 20

C. 15

D. 45

30. In answering question 29, did you remember to add the cost of the defensive action when calculating the social cost of locating next to the plant?

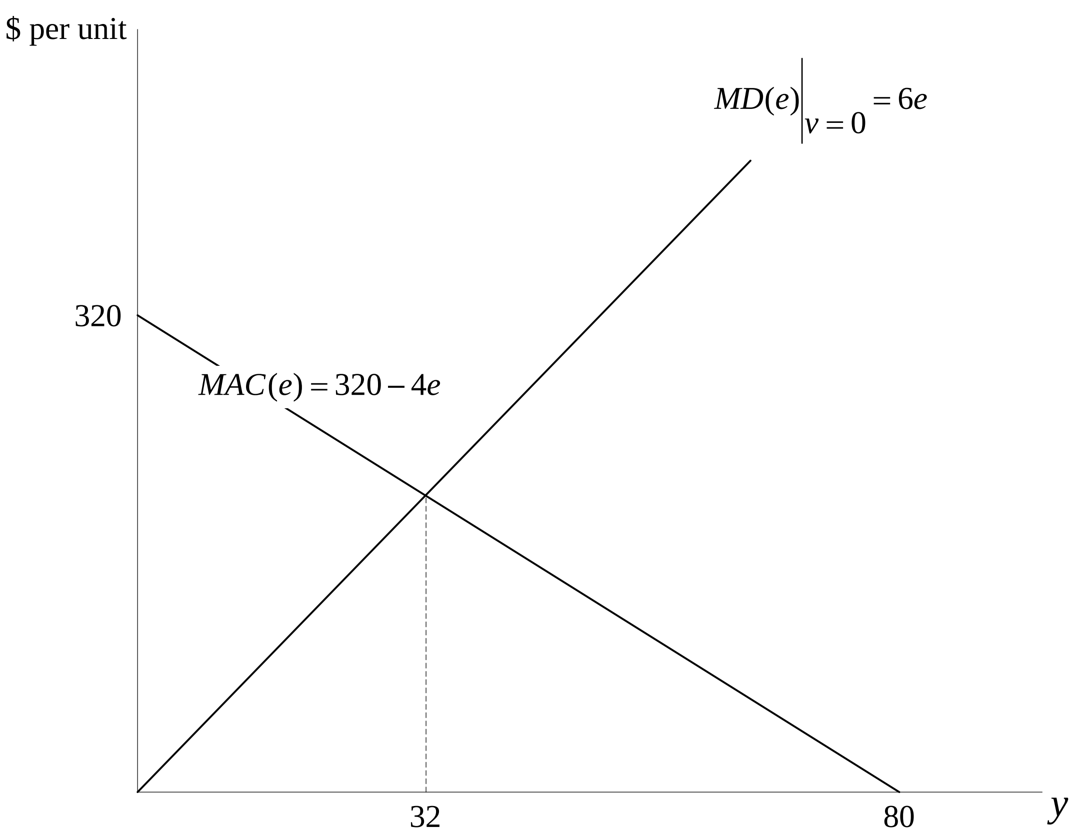


Figure R4-1

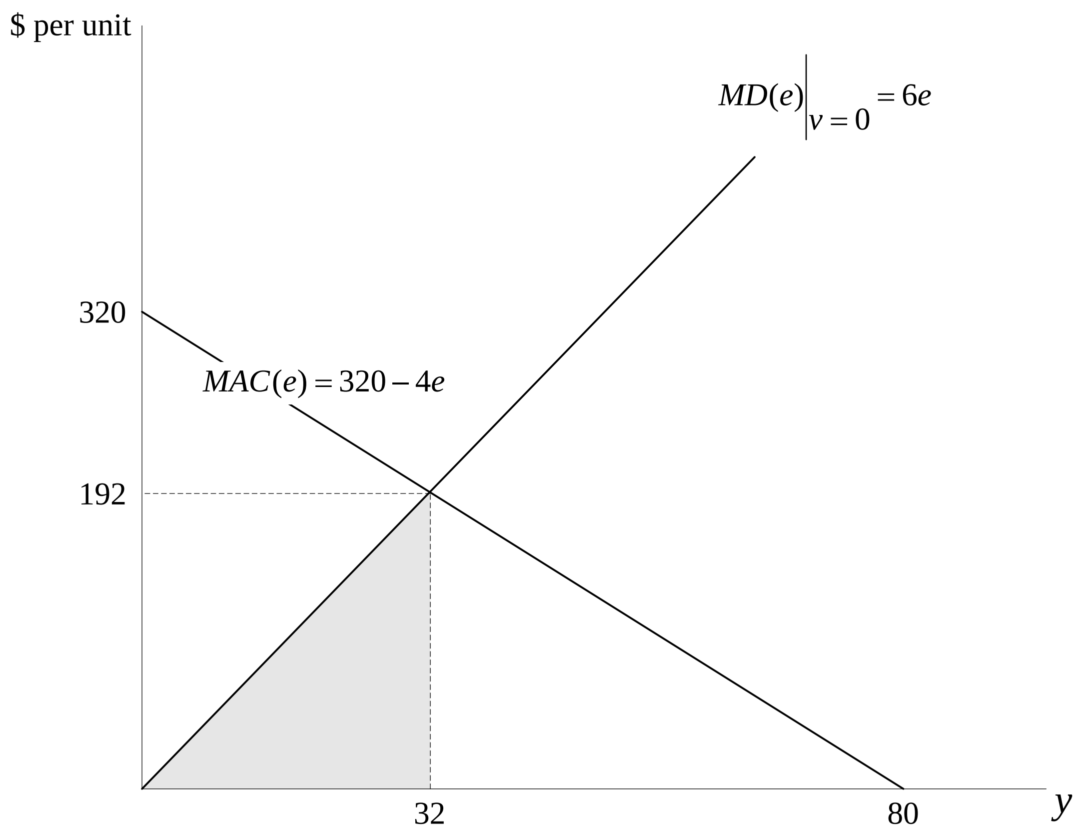


Figure R4-2

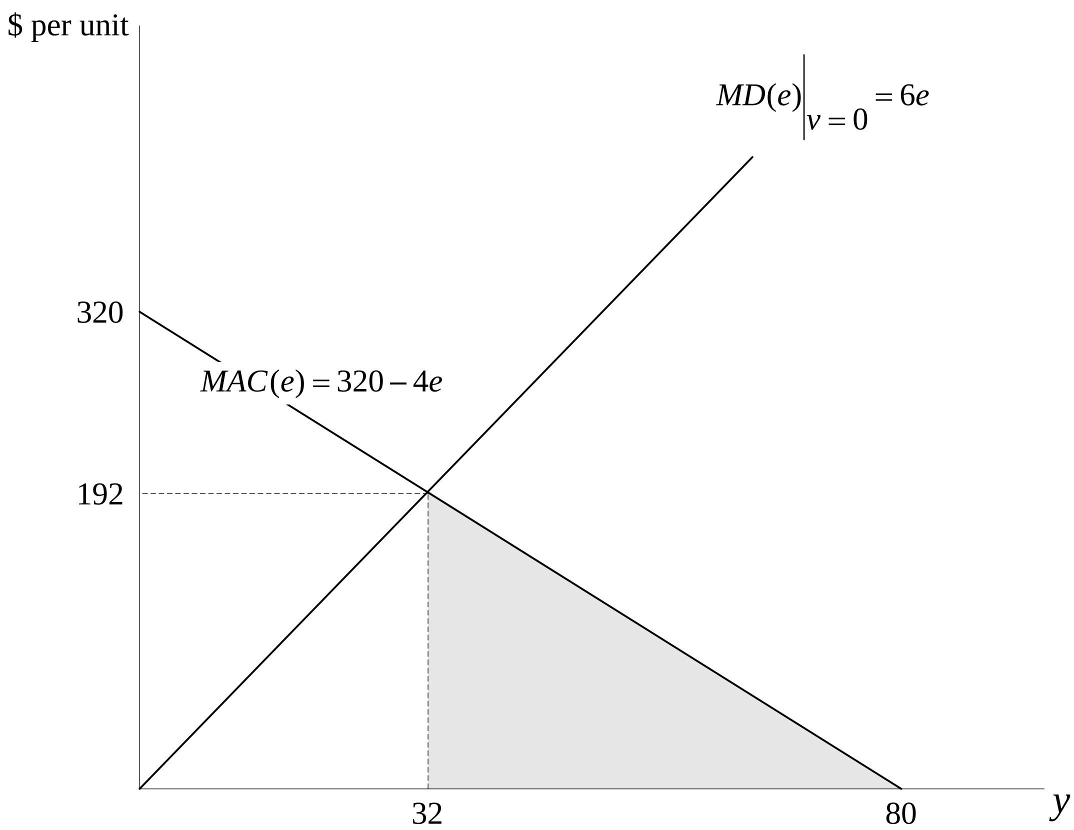


Figure R4-3

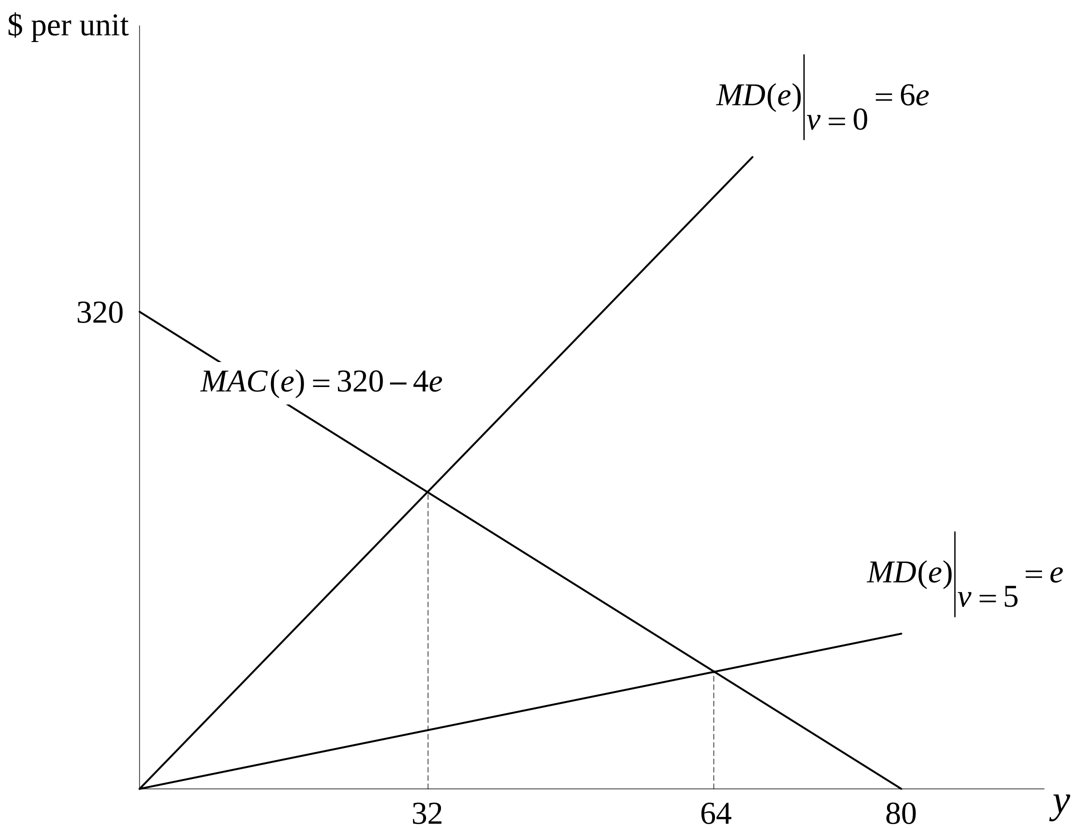


Figure R4-4

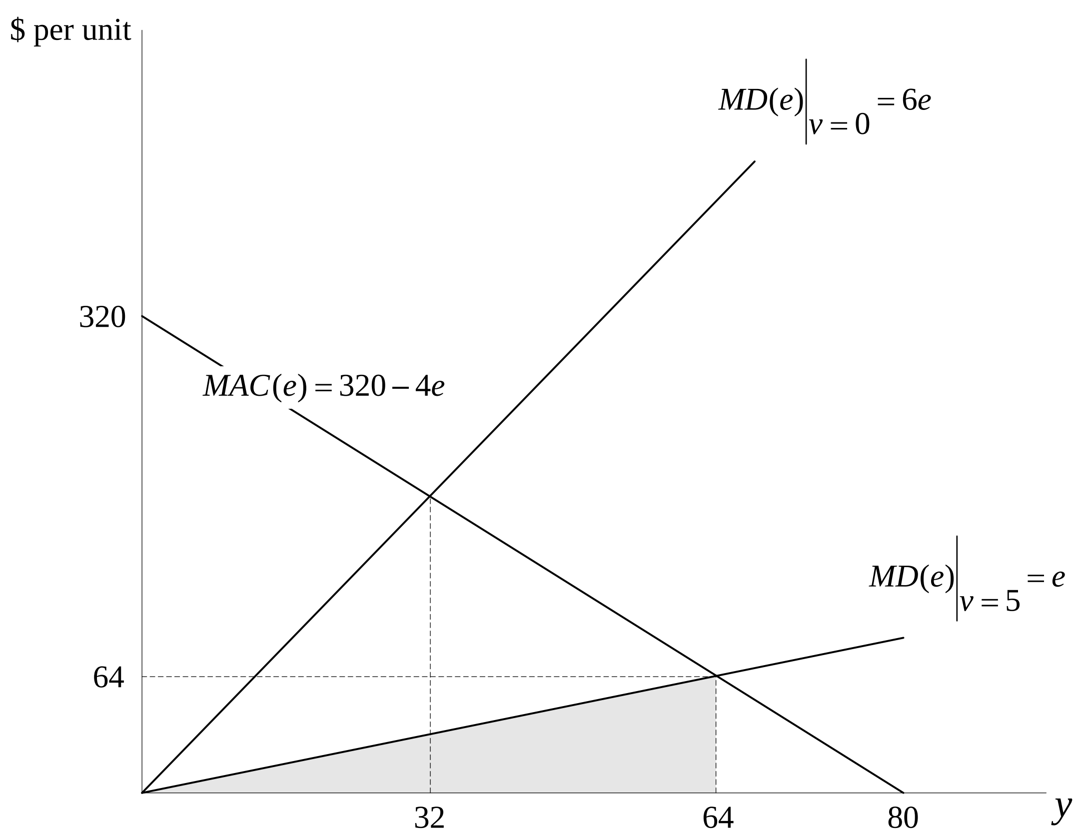


Figure R4-5

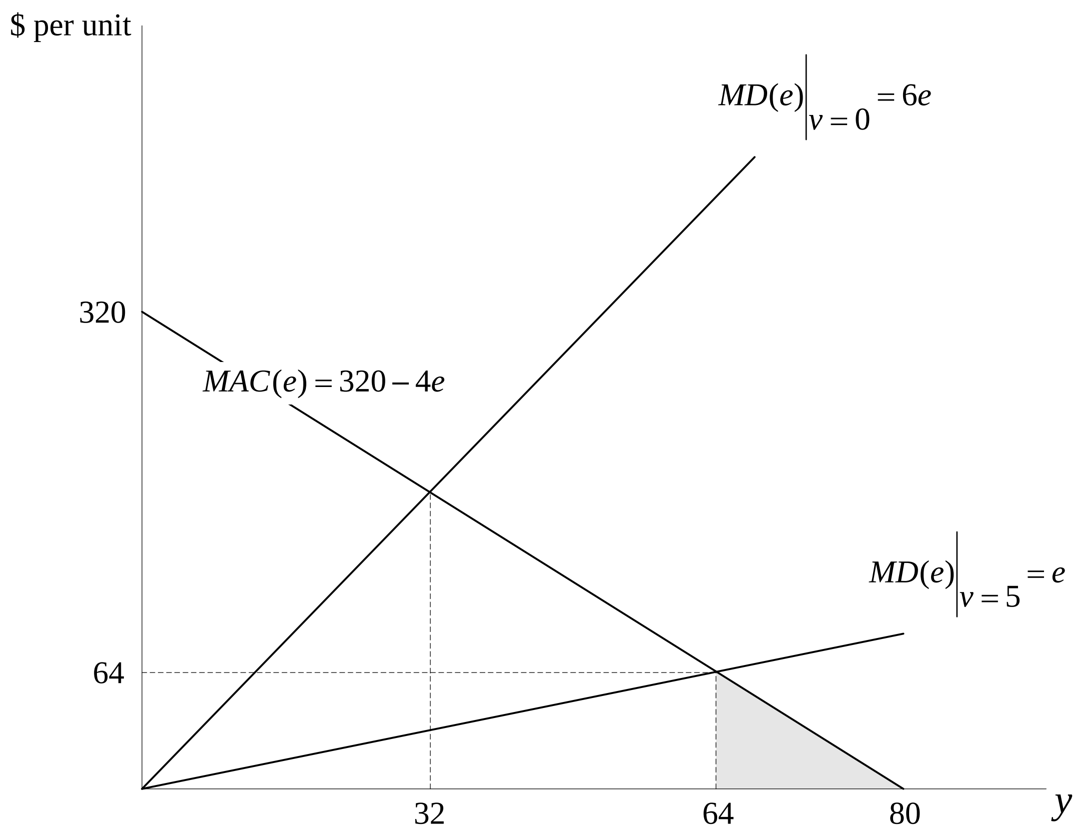


Figure R4-6

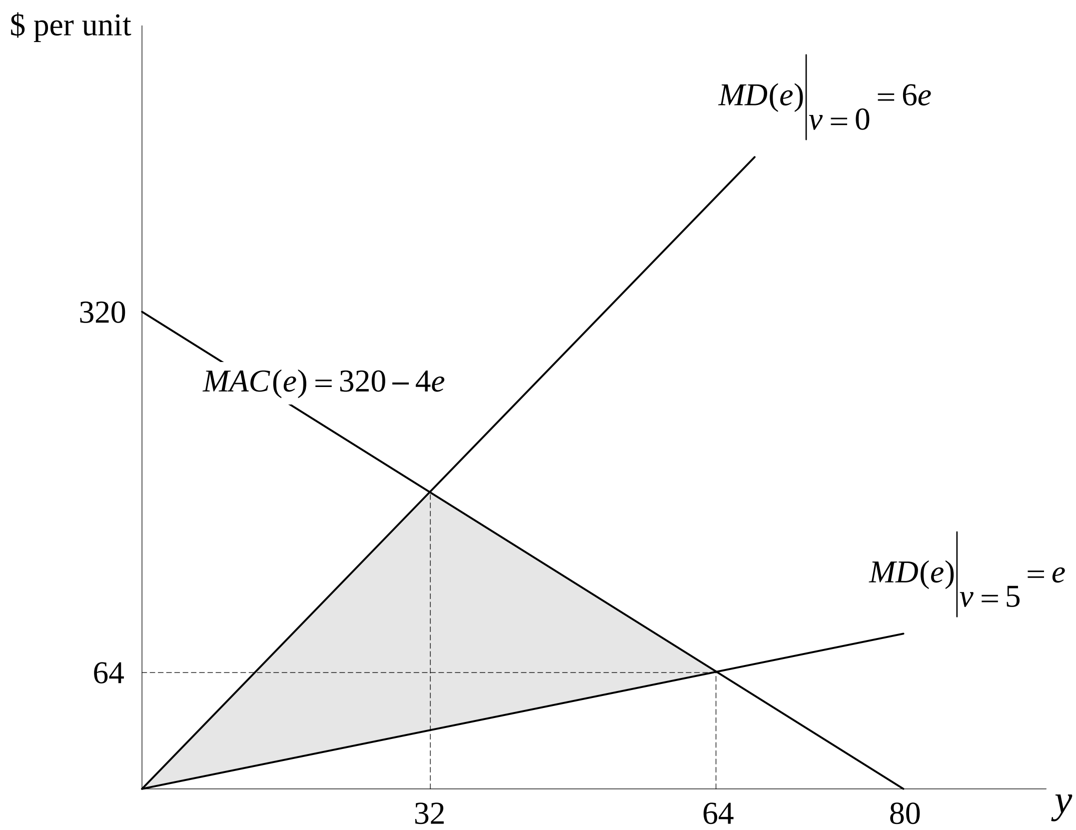


Figure R4-7

ANSWER KEY

1. B

2. A

3. B

4. B

5. C

6. D

7. B

8. A

9. B

10. D

11. A

12. B

13. B

14. B

If you set $v = 2$ you should find that the associated net benefit is 898 (see Q13). In comparison, setting $v = 1$ yields a net benefit of 641.9 (which is less than 898), while setting $v = 3$ yields a net benefit of 1035.5 (which is more than 898). Thus, net benefit is still rising as we move from $v = 1$ to $v = 2$ to $v = 3$. It follows that we cannot be at an optimum at $v = 2$. In fact, we can show that net benefit keeps rising until we hit $v = 43.39$, at which point net benefit is 1411.9. Beyond $v = 43.39$, net benefit starts to fall again. Thus, the optimal level of defensive action is $v = 43.39$. Discovering this by trial-and-error takes a lot of calculation but it is very simple if instead we use calculus.

15. A

16. B

17. A

18. D

19. C

20. B

21. B

22. A

23. D

24. A

25. C

26. C (see explanations below for 2 6 – 29)

27. D

28. C

29. A

Setting $v = 0$, you should be able to show that $e_0^* = 1$, and that associated values for damage and abatement cost are 24 and 16 respectively. So social cost is 16 + 64 = 80.

If instead you set $v = 5$, you should be able to show that $e_5^* = 3$, and that associated values for damage and abatement cost are 24 and 16 respectively. So social cost is 24 + 16 = 40. Thus, the net benefit of moving from $v = 0$ to $v = 5$ is the reduction in social cost (80 – 40) less the cost of the defensive action (5). Net benefit is 35 = (80 – 40) – 5.

Do the same for $v = 15$. You should find $e_{15}^* = 4$, and that the associated values for damage and abatement cost are 4 and 16 respectively. So social cost is 4 + 16 = 20. Thus, the net benefit of moving from $v = 0$ to $v = 15$ is the reduction in social cost (80 – 20) less the cost of the defensive action (15). So net benefit is 45 = (80 – 20) – 15.

So what is the optimal level of defensive action? We know that we can only choose between $v = 0$, $v = 5$ and $v = 15$, and we have just seen that $v = 5$ creates a positive net benefit over $v = 0$, and that $v = 15$ creates an even bigger positive net benefit over $v = 0$. So of the three available levels, the best level if $v = 15$.

So we now know that the optimal level of defensive action is $v = 15$ (with associated cost of 15), and we have already calculated that $e_{15}^* = 4$, and also calculated that the associated values for damage and abatement cost are 4 and 16 respectively. So if agent K locates next to the plant, and defensive action and emissions are both set at their socially optimal levels, then the total cost of locating there 4 + 16 + 15 = 35. None of these costs are incurred if agent K does not locate next to the plant. Thus, it is socially optimal for agent K to locate next to plant if and if the benefit to her from doing is at least great as 35.

5. OPTIMAL ABATEMENT WITH MULTIPLE SOURCES

1

OUTLINE

* Advanced Topic

2

5.1 INTRODUCTION

3

Introduction

- We have so far assumed that there is a single source of pollution.
- We now want to extend consideration to a setting with multiple sources.
- The way in which we approach that problem depends on the type of pollutant emitted.

4

Introduction

- In particular, we need to distinguish between uniformly mixed and non-uniformly mixed pollutants.

5

5.2 UNIFORMLY MIXED VS. NON-UNIFORMLY MIXED POLLUTANTS

6

Uniformly vs. Non-Uniformly Mixed Pollutants

- A **uniformly-mixed pollutant** is one that spreads out evenly within the receptive region, leading to measured pollutant concentrations that are approximately equal across the region.

7

Uniformly vs. Non-Uniformly Mixed Pollutants

- For example, consider a waterborne pollutant discharged from a single pipe into a lake.
- A uniformly-mixed pollutant will spread through the water, giving rise to equal concentrations in all parts of the lake, regardless of the specific location of the pipe.

8

Uniformly vs. Non-Uniformly Mixed Pollutants

- In contrast, a **non-uniformly mixed pollutant** will tend to pool in an area of concentration near the point of discharge, forming a "hot spot".
- In our lake example, pollutant concentrations around the lake will depend on the specific location of the pipe.

9

Uniformly vs. Non-Uniformly Mixed Pollutants

- The extent to which a pollutant is uniformly mixed or non-uniformly mixed depends on the physical properties of the pollutant and the geophysical properties of the receptive region.

10

Uniformly vs. Non-Uniformly Mixed Pollutants

- For example, wind patterns in an airshed may be important for the extent to which an airborne pollutant becomes uniformly mixed within that airshed.
- Similarly, the amount of tidal action is important for how waterborne pollutants become dispersed in a marine area.

11

Uniformly vs. Non-Uniformly Mixed Pollutants

- In contrast, some pollutants are uniformly-mixed regardless of geophysical conditions.
- Greenhouse gas emissions fall into this latter category; they are uniformly-mixed on a global scale.

12

Uniformly vs. Non-Uniformly Mixed Pollutants

- For our purposes, the key issue regarding the degree of mixing is whether or not we can reasonably measure the damage in a receptive region in terms of **aggregate emissions** of the pollutant, without reference to the locations of individual sources.

13

Uniformly vs. Non-Uniformly Mixed Pollutants

- If the pollutant is uniformly-mixed then all agents within the region are exposed to the same pollutant concentration, as determined by aggregate emissions within the region.
- We can therefore measure damage to each agent in terms of aggregate emissions, regardless of individual source locations.

14

Uniformly vs. Non-Uniformly Mixed Pollutants

- We can then also measure total damage – summing all of the individual damages – in terms of aggregate emissions within the region.
- Note that this does not imply that all damaged agents suffer the same damage.

15

Uniformly vs. Non-Uniformly Mixed Pollutants

- In contrast, if the pollutant is non-uniformly mixed then the damage to each external agent depends on their proximity to each individual source, and we cannot simply express aggregate damage in terms of aggregate emissions.
- Instead we need a detailed model of individual exposures based on location.

16

Uniformly vs. Non-Uniformly Mixed Pollutants

- Non-uniformly mixed pollutants are clearly more difficult to model for policy purposes, and we will delay consideration of this case. until Topic 7.7.
- Here we will restrict consideration to uniformly mixed, dissipative pollutants.

17

Uniformly vs. Non-Uniformly Mixed Pollutants

- We begin our treatment of multiple sources with the simplest possible case: there are two sources.
- The key results developed can then be extended easily to a setting with any number of sources.

18

5.3 THE POLICY PROBLEM WITH TWO SOURCES

19

The Policy Problem with Two Sources

- Suppose there are two sources of a uniformly-mixed pollutant within the receptive region (for example, a steel plant and a cement plant).
- Let e_1 denote the quantity of emissions from source 1, and let e_2 denote the quantity of emissions from source 2.
- Aggregate emissions are $E = e_1 + e_2$.

20

The Policy Problem with Two Sources

- Marginal abatement cost is $MAC_1(e_1)$ for source 1, and $MAC_2(e_2)$ for source 2.
- Current emissions levels are $\hat{e}_1$ and $\hat{e}_2$ for source 1 and source 2 respectively, where

$$MAC_i(\hat{e}_i) = 0 \qquad \text{for } i = 1 \text{ and } 2$$

- Figure 5-1 illustrates an example.

21

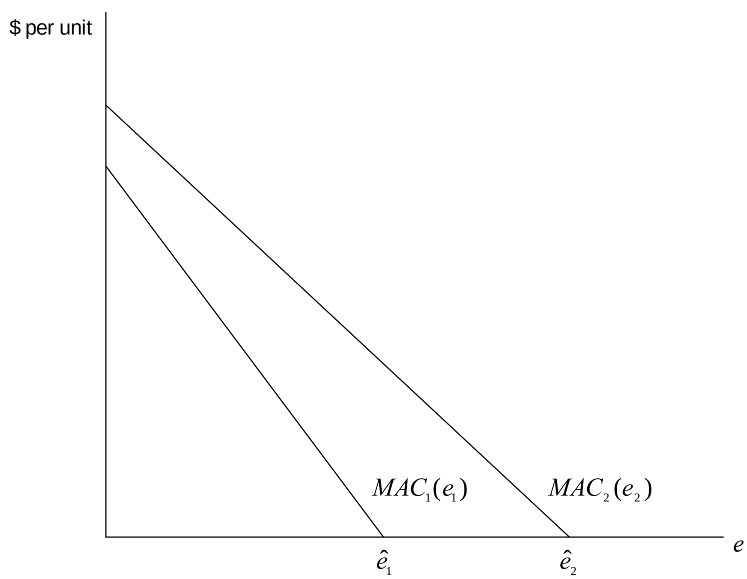


FIGURE 5-1

22

The Policy Problem with Two Sources

- As depicted in Figure 5-1, source 2 is currently the larger emitter, and has a higher marginal abatement cost than source 1.
- This could reflect the fact that source 2
  - is a bigger producer of output; and/or
  - produces a more valuable product; and/or
  - uses an older, more polluting production process.

23

The Policy Problem with Two Sources

- If the pollutant is uniformly mixed then we can measure damage in terms of E, and define the marginal damage function accordingly, henceforth denoted $MD(E)$.
- Recall from Topic 3 that this marginal damage function is the vertical summation of all individual marginal damage functions.
- See Figure 5-2.

24

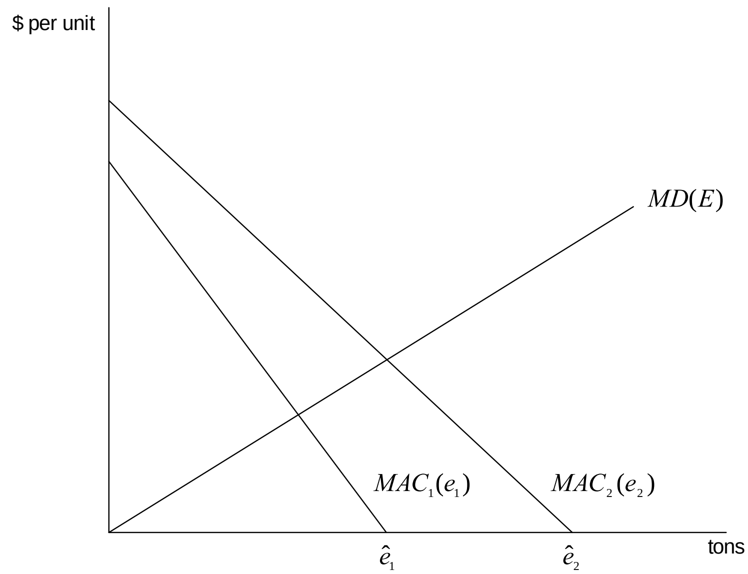


FIGURE 5-2

25

The Policy Problem with Two Sources

- Our **policy problem** is to determine the socially optimal level of aggregate emissions (denoted E^*), and the optimal allocation of E^* between the two sources.

- It is useful to solve this problem in two steps.

26

The Policy Problem with Two Sources

Step 1

a) for any given aggregate emissions *target*, we find the allocation between the two sources that minimizes aggregate abatement cost.

b) we use the result from 1(a) to construct a marginal aggregate-abatement cost function in terms of the aggregate emissions target.

27

The Policy Problem with Two Sources

Step 2

a) we bring marginal damage and marginal aggregate abatement cost together to determine the *optimal* target in terms of aggregate emissions.

b) we use the allocation rule from 1(a) to allocate the optimal aggregate emissions target between the two sources.

28

Step 1(a): Abatement Cost Minimization

- Consider some arbitrary target for aggregate emissions, E_0 and an allocation between the two sources such that

$$e_1 + e_2 = E_0$$

- What allocation minimizes aggregate abatement cost?

29

Step 1(a): Abatement Cost Minimization

- As a candidate solution, consider a 50/50 split between the two sources, such that

$$e_{10} = \frac{E_0}{2} \quad \text{and} \quad e_{20} = \frac{E_0}{2}$$

- See Figure 5-3.

30

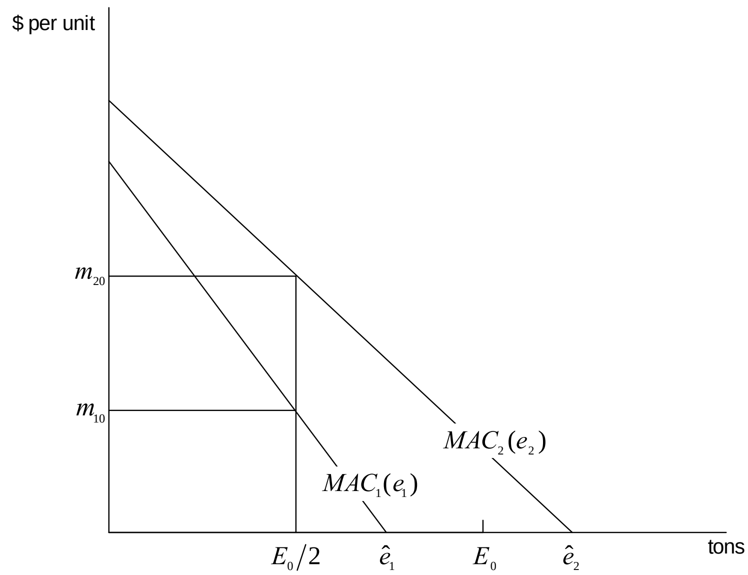


FIGURE 5-3

31

Step 1(a): Abatement Cost Minimization

- At this allocation, MAC is m_{10} for source 1 and m_{20} for source 2.

- Note that $m_{10} < m_{20}$; thus, any additional abatement is less costly for source 1 than for source 2.

32

Step 1(a): Abatement Cost Minimization

- Critically, this means that source 1 could abate a little more, and source 2 could abate less by an offsetting amount – thereby leaving aggregate emissions unchanged – and aggregate abatement cost would be lower.

- See Figure 5-4.

33

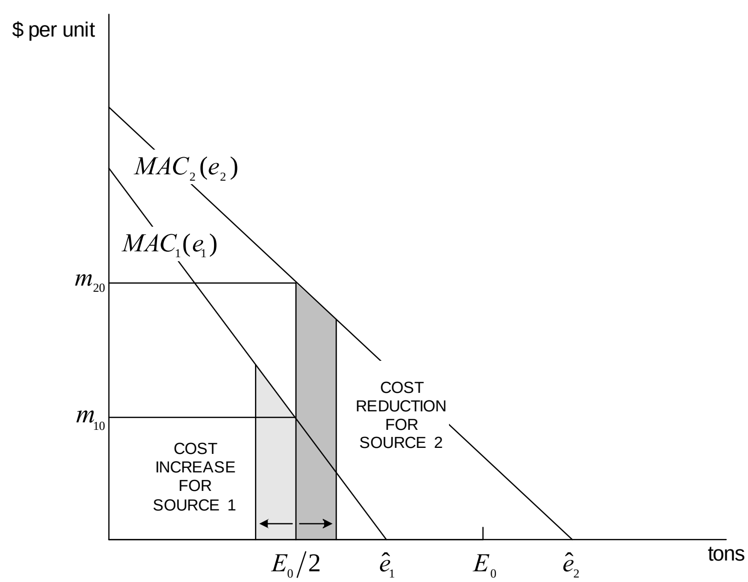


FIGURE 5-4

34

Step 1(a): Abatement Cost Minimization

- It is clear from Figure 5-4 that the cost reduction for source 2 exceeds the cost increase for source 1.
- Thus, aggregate abatement cost falls even though the aggregate emissions target is still met.

35

Step 1(a): Abatement Cost Minimization

- Now continue the experiment illustrated in Figure 5-4 – more abatement for source 1 with an offsetting reduction in abatement for source 2 – until we reach the point where the MACs are equated, at m_0 in Figure 5-5.

36

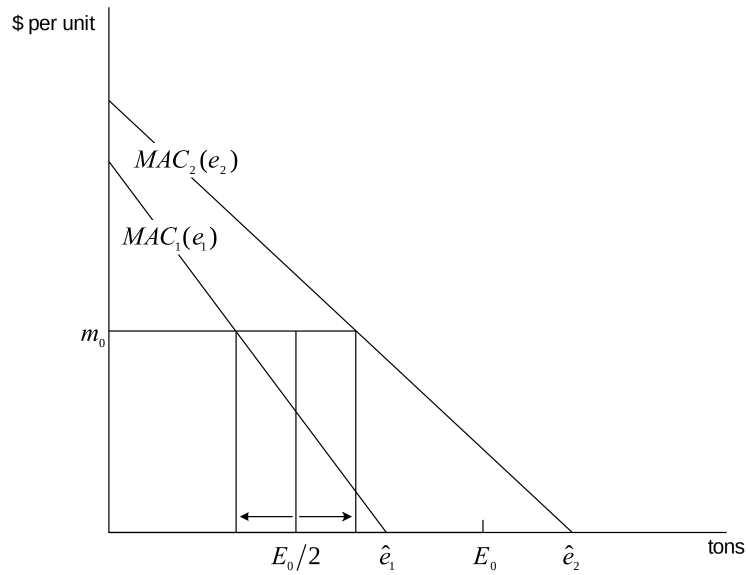


FIGURE 5-5

37

Step 1(a): Abatement Cost Minimization

- At this allocation of the aggregate target, any further shifting of abatement from source 2 to source 1 would cause aggregate abatement cost to rise; see Figure 5-6.

38

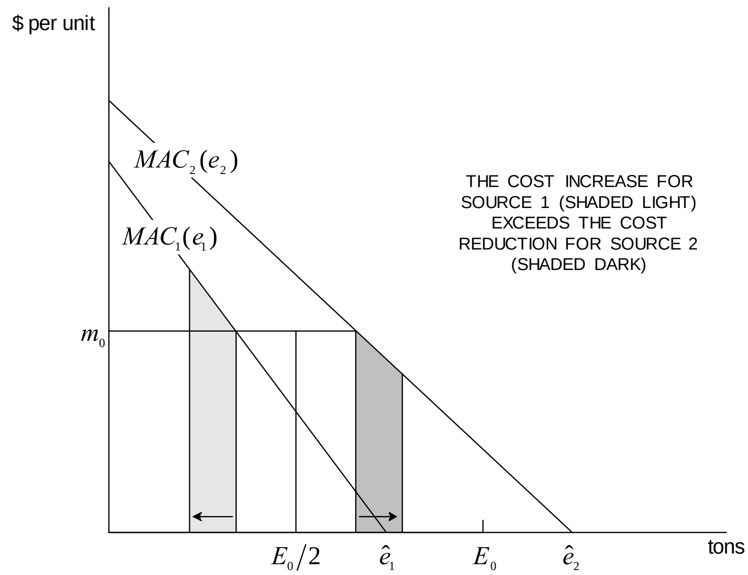


FIGURE 5-6

39

Step 1(a): Abatement Cost Minimization

- We can now summarize the results of these experiments as a **fundamental result** in environmental policy design.

40

A Fundamental Result

- The aggregate abatement cost of meeting an aggregate emissions target E_0 is minimized at the allocation e_{10} and e_{20} where

$$e_{10} + e_{20} = E_0$$

and

$$MAC_1(e_{10}) = MAC_2(e_{20})$$

41

A Fundamental Result

- We will henceforth refer to this cost-minimizing rule as the **M**arginal **A**batement **C**ost **E**qualization (**MACE**) rule.
- This cost-minimizing rule is depicted in Figure 5-7.

42

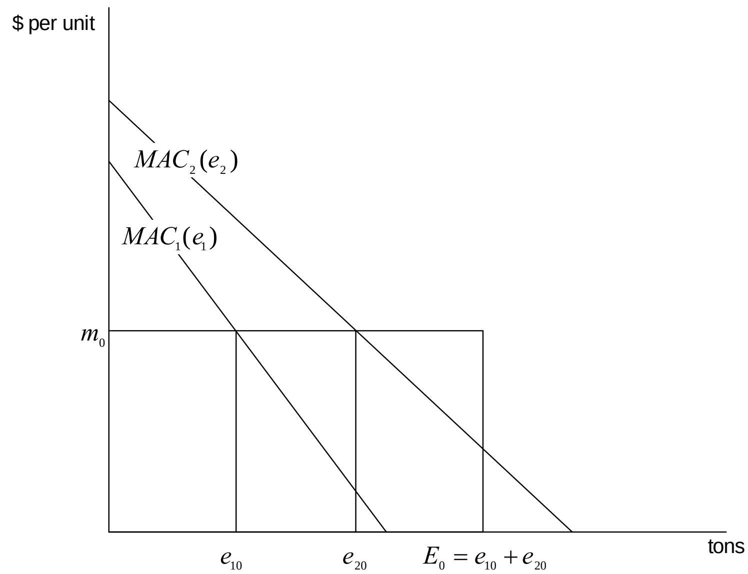


FIGURE 5-7

43

A Fundamental Result

- Note how Figure 5-7 is constructed:
  - E_0 is the horizontal summation of e_{10} and e_{20}, as highlighted in Figure 5-8 where the two marked distances are both equal to e_{10}

44

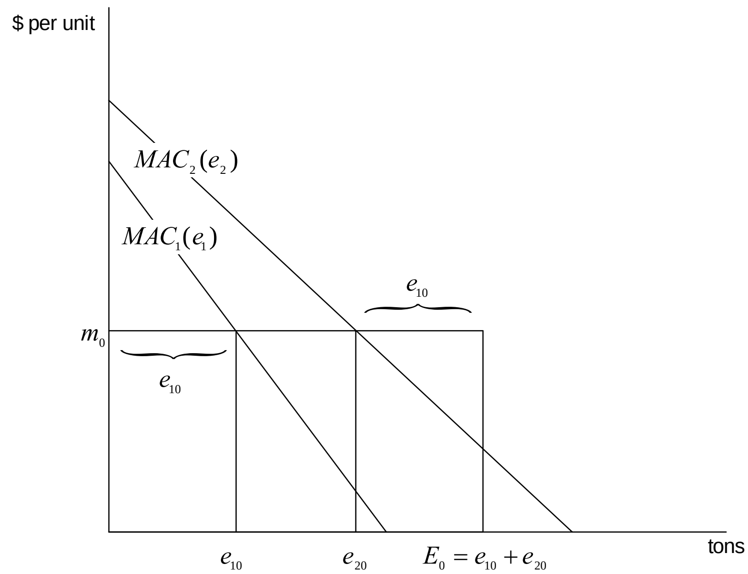


FIGURE 5-8

45

A Fundamental Result

- The intuition behind the **MACE** rule:
  - If one source can abate at lower cost than the other source, as determined by relative MACs, then the lower cost source should abate more until that cost advantage no longer exists (that is, until MACs are equated).

46

The Relative Cost Burden

- The asymmetric abatement required in the cost-minimizing solution for a given aggregate emission target may or may not result in one source incurring a higher overall abatement cost than the other; it depends on the particular circumstances.

47

The Relative Cost Burden

- In the case illustrated in Figure 5-7, source 1 undertakes less abatement, and at lower total cost, than source 2; see Figures 5-9 and 5-10.

48

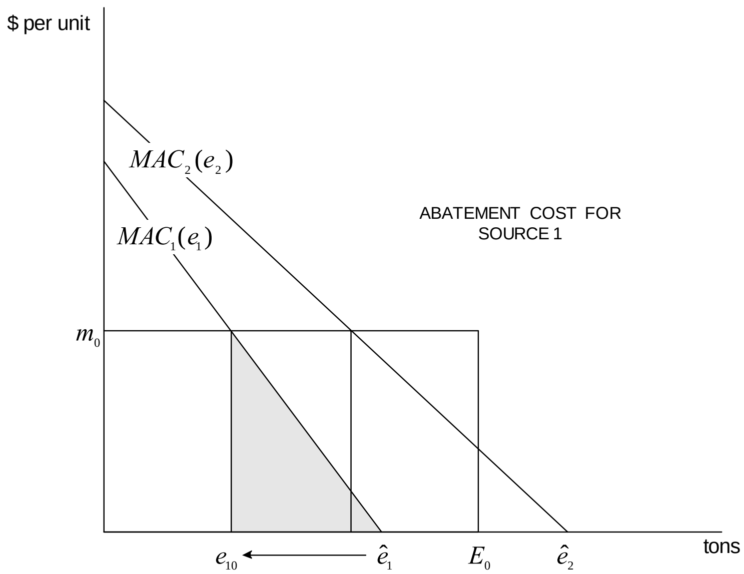


FIGURE 5-9

49

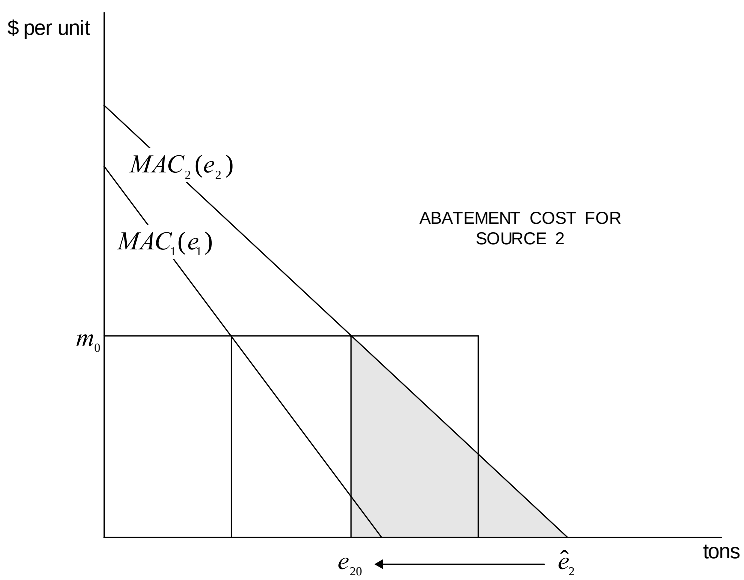


FIGURE 5-10

50

The Relative Cost Burden

- Under different conditions, the opposite outcome could arise (where the source with the higher allocation incurs a lower total abatement cost).
- In either case, the asymmetry of treatment can be contentious from a political perspective; we will later see that emissions pricing can help level the playing field.

51

Step 1(b): Marginal Aggregate-Abatement Cost

- Recall Step 1(b) from our two-step approach to the optimal abatement problem:
  - we use the result from 1(a) to construct a marginal aggregate-abatement cost function in terms of the aggregate emissions target

- Let us explore this part of the problem.

52

Step 1(b): Marginal Aggregate-Abatement Cost

- The marginal aggregate-abatement cost function measures the marginal cost of reducing aggregate emissions, given that emissions are always allocated between the two sources according to the **MACE** rule from slide 41.

53

Step 1(b): Marginal Aggregate-Abatement Cost

- Graphically, the marginal aggregate-abatement cost function – denoted $MAAC(E)$ – is the horizontal summation of $MAC_1(e_1)$ and $MAC_2(e_2)$.
- Figure 5-11 depicts this horizontal summation, highlighting that $e_{10}+e_{20}=E_0$ at any given value of E_0 on the $MAAC(E)$ schedule.

54

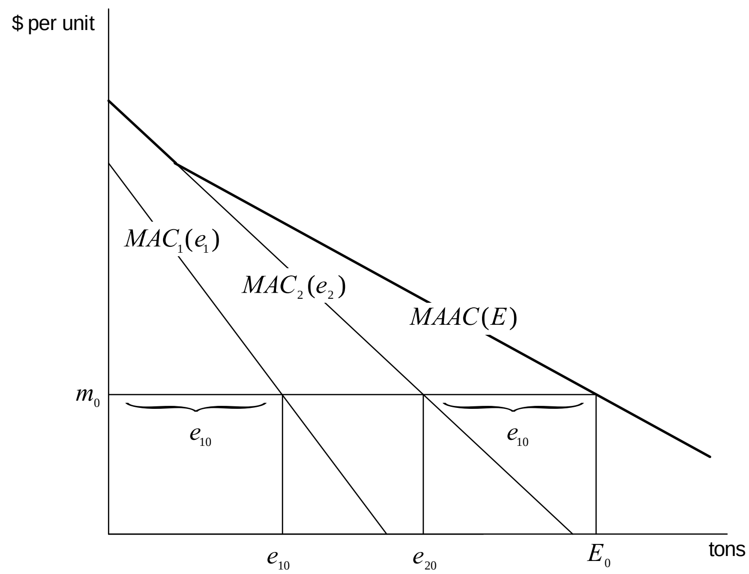


FIGURE 5-11

55

Step 1(b): Marginal Aggregate-Abatement Cost

- Why is $MAAC(E)$ equal to the horizontal summation of $MAC_1(e_1)$ and $MAC_2(e_2)$?
- Suppose we wish to reduce aggregate emissions by one more unit.
- Since $MAC_1(e_1)=MAC_2(e_2)$ at the **MACE** solution, the cost of one more unit of abatement is the same whether that abatement is made by source 1 or source 2.

56

Step 1(b): Marginal Aggregate-Abatement Cost

- Thus, the cost of reducing aggregate emissions by one more unit at E_0 is

$$MAC_1(e_{10}(E)) = MAC_2(e_{20}(E)) = m(E)$$

as depicted in Figure 5-11. It follows that

$$MAAC(E) = m(E)$$

57

A Linear Example (Step 1)

- Consider a linear example. Suppose

$$MAC_1(e_1) = \gamma_1(\hat{e}_1 - e_1)$$

$$MAC_2(e_2) = \gamma_2(\hat{e}_2 - e_2)$$

58

A Linear Example (Step 1)

- We will find it useful to derive solutions in terms of the aggregate no-abatement emissions level, defined as

$$\hat{E} = \hat{e}_1 + \hat{e}_2$$

- From this we can define the **aggregate abatement target**, $\hat{E}-E$.

59

A Linear Example (Step 1)

- We begin by solving the cost-minimization problem by applying the **MACE** rule from slide 41:

$$\gamma_1(\hat{e}_1 - e_1) = \gamma_2(\hat{e}_2 - e_2)$$

where $e_1+e_2 = E$.

60

A Linear Example (Step 1)

- Substituting $e_2 = E - e_1$ and $\hat{e}_2 = \hat{E} - \hat{e}_1$, we then have

$$\gamma_1(\hat{e}_1 - e_1) = \gamma_2((\hat{E} - \hat{e}_1) - (E - e_1))$$

61

A Linear Example (Step 1)

- Solving this equation for e_1 yields

$$e_1(E) = \hat{e}_1 - \frac{\gamma_2}{\gamma_1 + \gamma_2}(\hat{E} - E)$$

62

A Linear Example (Step 1)

- This solution for e_1 is the allocation rule for source 1:
  - it tells us the emissions allocation for source 1 in the cost-minimizing allocation of the aggregate target E between the two sources

63

A Linear Example (Step 1)

- We can then find the allocation rule for source 2 by constructing $e_2(E)=E-e_1(E)$, which yields

$$e_2(E) = \hat{e}_2 - \frac{\gamma_1}{\gamma_1 + \gamma_2}(\hat{E} - E)$$

64

A Linear Example (Step 1)

- Figure 5.12 depicts these cost-minimizing solutions for a particular target $E=E_0$.

65

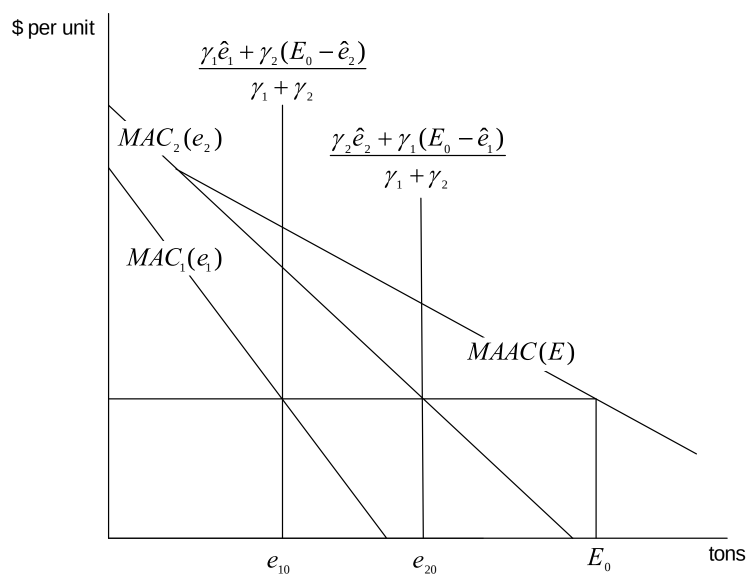


FIGURE 5-12

66

A Linear Example (Step 1)

- It is helpful to rearrange these allocation rules to express them terms of required abatement.
- For source 1:

$$\hat{e}_1 - e_1(E) = \frac{\gamma_2}{\gamma_1 + \gamma_2}(\hat{E} - E)$$

67

A Linear Example (Step 1)

- This tells us the abatement requirement for source 1 as a share of the aggregate abatement target.
- That share for source 1 is

$$\frac{\gamma_2}{\gamma_1 + \gamma_2}$$

- It increasing in γ_2 and decreasing in γ_1.

68

A Linear Example (Step 1)

- Similarly, the abatement requirement for source 2 is

$$\hat{e}_2 - e_2(E) = \frac{\gamma_1}{\gamma_1 + \gamma_2}(\hat{E} - E)$$

69

A Linear Example (Step 1)

- Recall from s.57 that

$$MAAC(E) = MAC_1(e_1) = MAC_2(e_2)$$

at the **MACE** solution

- Thus, we can find $MAAC(E)$ by
  - substituting e_1=$e_1(E)$ into $MAC_1(e_1)$; or
  - substituting e_2=$e_2(E)$ into $MAC_2(e_2)$;

70

A Linear Example (Step 1)

- Making this substitution either way yields

$$MAAC(E) = \varphi(\hat{E} - E)$$

where

$$\varphi = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} = \left(\sum_{i=1}^{2} \frac{1}{\gamma_i} \right)^{-1}$$

71

A Linear Example (Step 1)

- Recall that we can also construct the $MAAC(E)$ as the horizontal summation of $MAC_1(e_1)$ and $MAC_2(e_2)$.

- Let us do that for the linear example and confirm that we obtain the same solution as on the previous slide.

72

A Linear Example (Step 1)

- Recall the MAC for source 1:

$$MAC_1(e_1) = \gamma_1(\hat{e}_1 - e_1) \equiv m_1$$

- Take the inverse to obtain

$$e_1 = \hat{e}_1 - \frac{m_1}{\gamma_1}$$

73

A Linear Example (Step 1)

- Similarly, recall the MAC for source 2:

$$MAC_2(e_2) = \gamma_2(\hat{e}_2 - e_2) \equiv m_2$$

- Again, take the inverse to obtain

$$e_2 = \hat{e}_2 - \frac{m_2}{\gamma_2}$$

74

A Linear Example (Step 1)

- We now have each MAC expressed in a form where emissions is the subject of the equation.
- Why did we do this?
  - Because emissions is the variable measured on the horizontal axis.

75

A Linear Example (Step 1)

- Now add these inverses to obtain

$$E = e_1 + e_2 = \hat{e}_1 - \frac{m_1}{\gamma_1} + \hat{e}_2 - \frac{m_2}{\gamma_2}$$

- This is a horizontal summation precisely because emissions is measured on the horizontal axis.

76

A Linear Example (Step 1)

- Collect terms to obtain

$$E = \hat{E} - \left(\frac{m_1 \gamma_1 + m_2 \gamma_2}{\gamma_1 \gamma_2} \right)$$

77

A Linear Example (Step 1)

- Now impose the MACE condition:

$$m_1 = m_2 \equiv m$$

- Making that substitution yields

$$E = \hat{E} - m \left(\frac{\gamma_1 + \gamma_2}{\gamma_1 \gamma_2} \right)$$

78

A Linear Example (Step 1)

- Finally, take the inverse to obtain

$$MAAC(E) = m = \left(\frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2}\right)(\hat{E} - E)$$

- This is the same solution obtained earlier.

79

A Linear Example (Step 1)

- Note that in this example, *MAAC*(*E*) takes the same linear form as each of the individual MACs.

- This convenient symmetry of form does not necessarily hold in more general cases where the individual MACs are not linear.

80

Step 2(a): Optimal Aggregate Emissions

- We can now proceed to Step 2(a) of the policy problem:
  - we bring marginal damage and marginal aggregate abatement cost together to determine the optimal target with respect to aggregate emissions

81

Optimal Aggregate Emissions

- Optimal aggregate emissions are denoted E^*, and characterized by

$$MAAC(E^*) = MD(E^*)$$

- See Figure 5-13.

82

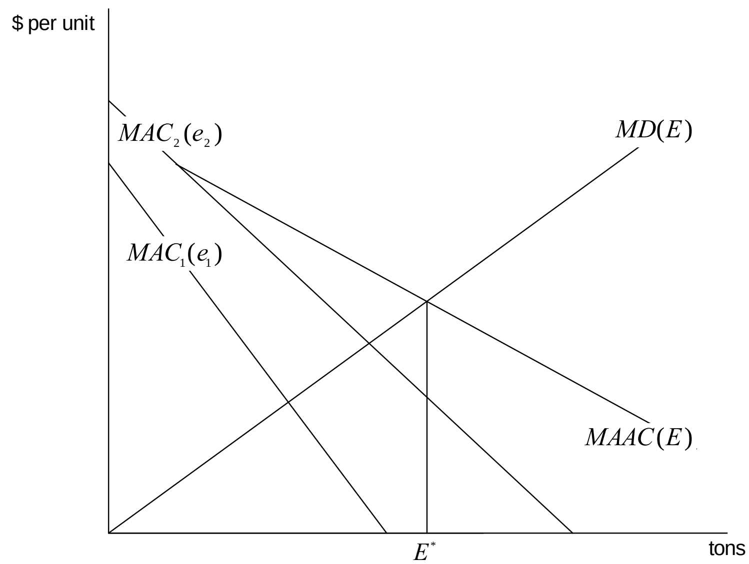


FIGURE 5-13

83

Optimal Aggregate Emissions

- The intuition behind this optimality condition is exactly the same as that in the case of a single source (recall Topic 3).

84

Optimal Aggregate Emissions

- In particular, if $E>E^*$ then reducing E to E^* yields a reduction in damage that exceeds the increase in aggregate abatement cost, and social net benefit rises; see Figures 5-14 through 5-16.

85

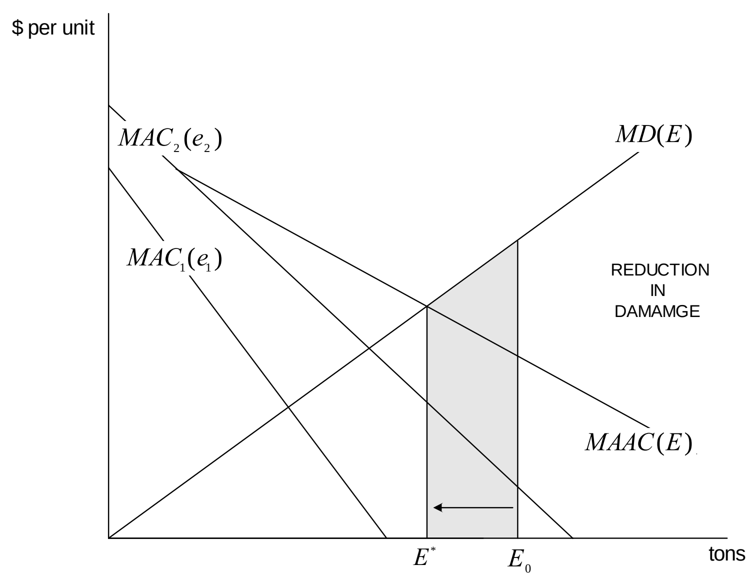


FIGURE 5-14

86

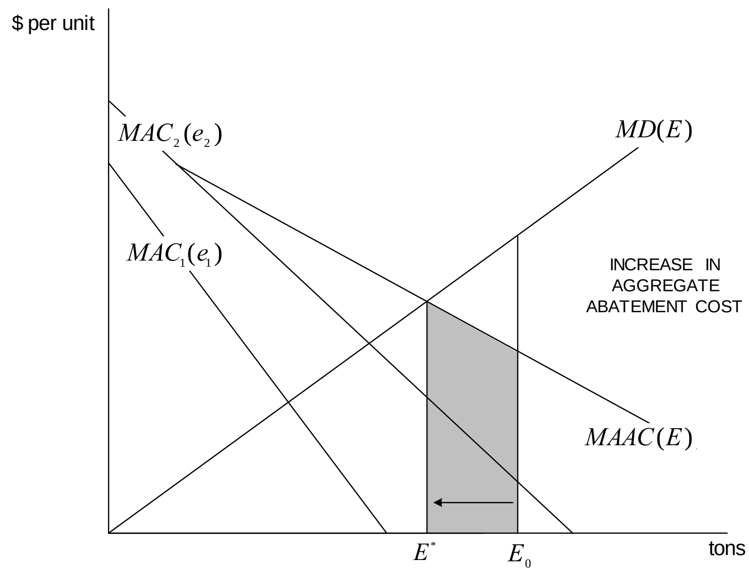


FIGURE 5-15 87

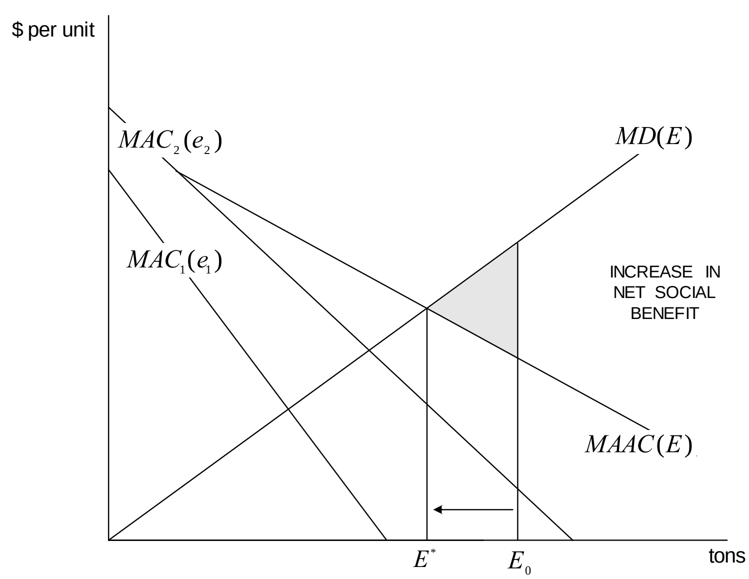


FIGURE 5-16 88

Optimal Aggregate Emissions

- Conversely, if $E<E^*$ then raising E to E^* yields a reduction in aggregate abatement cost that exceeds the increase in damage, and social net benefit rises; see Figures 5-17 through 5-19.

89

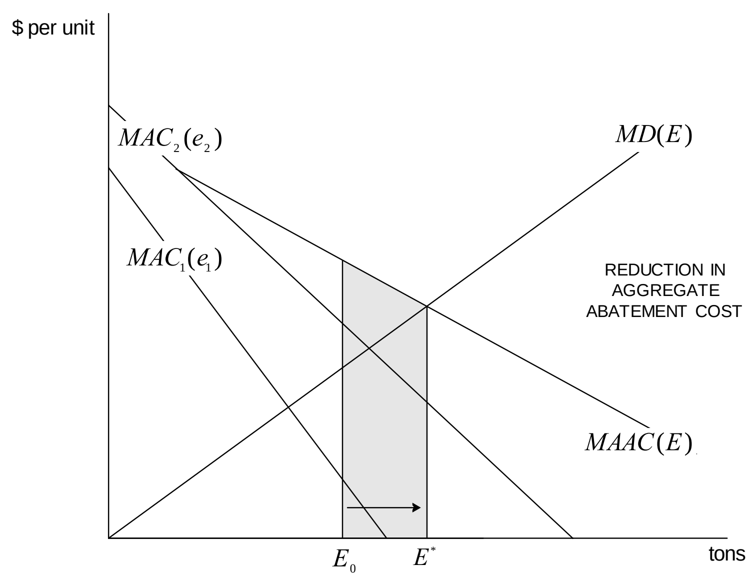


FIGURE 5-17

90

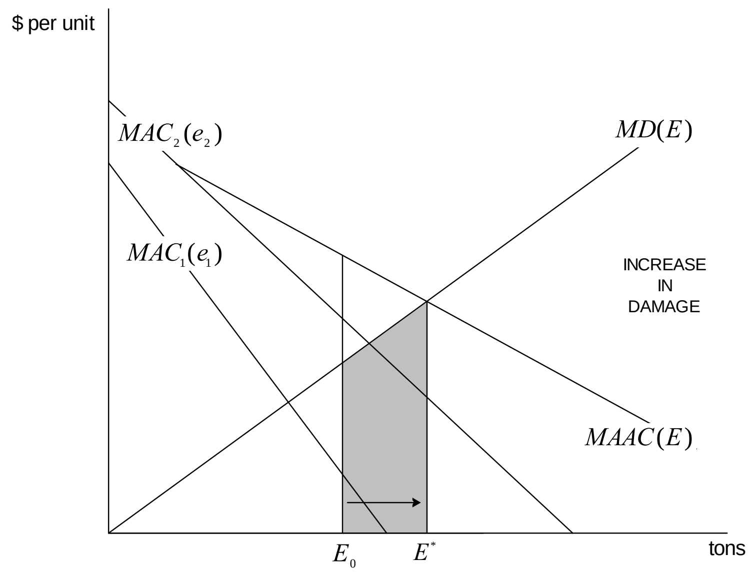


FIGURE 5-18

91

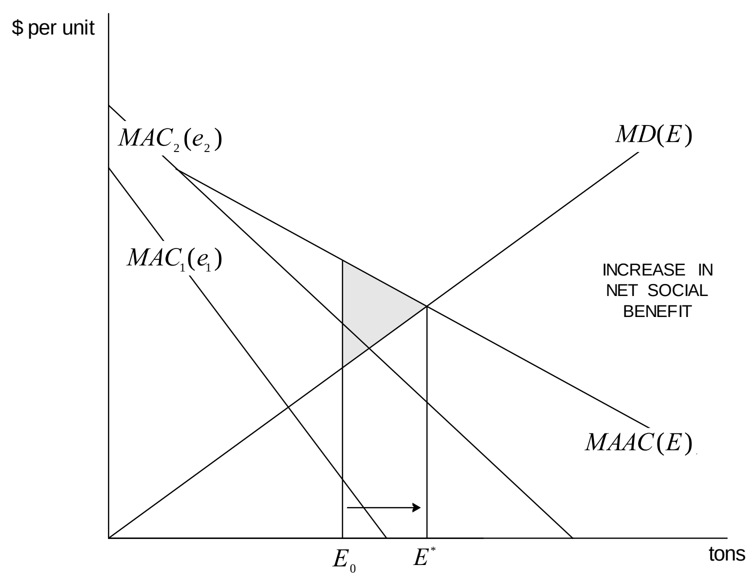


FIGURE 5-19

92

Step 2(b): The Optimal Allocation

- We can now proceed to the final step of the policy problem
  - we use the allocation rule from 1(a) to allocate the optimal aggregate emissions target between the two sources

93

The Optimal Allocation

- Recall from s.70 that

$$MAAC(E) = MAC_1(e_1) = MAC_2(e_2)$$

at the **MACE** solution, and we know from s.82 that optimal aggregate emissions solves

$$MAAC(E^*) = MD(E^*)$$

94

The Optimal Allocation

- Thus, we can find the **optimal allocations** for sources 1 and 2, denoted e_1^* and e_2^* respectively, as the solutions to

$$MAC_1(e_1^*) = MD(E^*)$$

and

$$MAC_2(e_2^*) = MD(E^*)$$

respectively; see Figure 5.20.

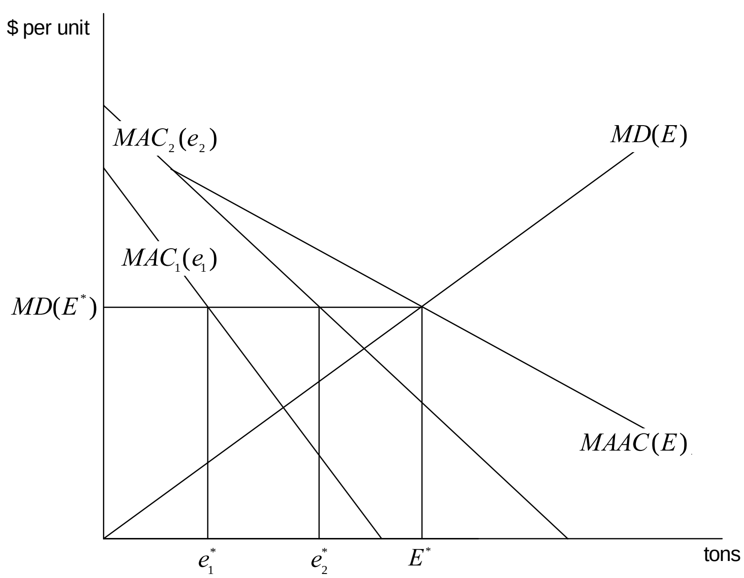


FIGURE 5-20

The Optimal Allocation

- Let us confirm the optimality of the overall solution (as depicted in Figure 5.20) with a simple graphical experiment.
- Starting at the optimal solution, suppose source 1 is allowed to emit one additional unit (which causes E to rise by one unit).
- The effect on its abatement cost and on damage are illustrated in Figure 5.21.

97

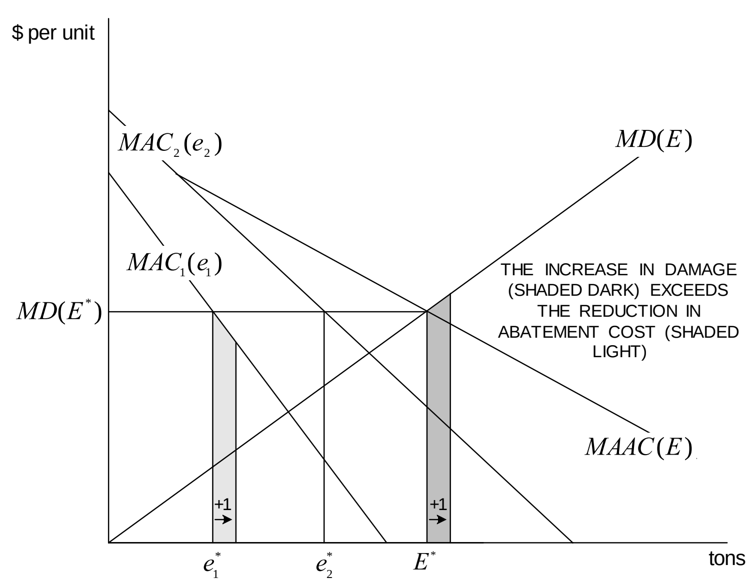


FIGURE 5-21

98

The Optimal Allocation

- It is clear from Figure 5-21 that an increase in emissions by source 1 creates an increase in damage larger than the reduction in abatement cost; net social benefit falls.

99

The Optimal Allocation

- Similarly, a reduction of one unit in the emissions allowed for source 1 causes net benefit to fall, because the increase in abatement cost exceeds the reduction in damage; see Figure 5-22.

100

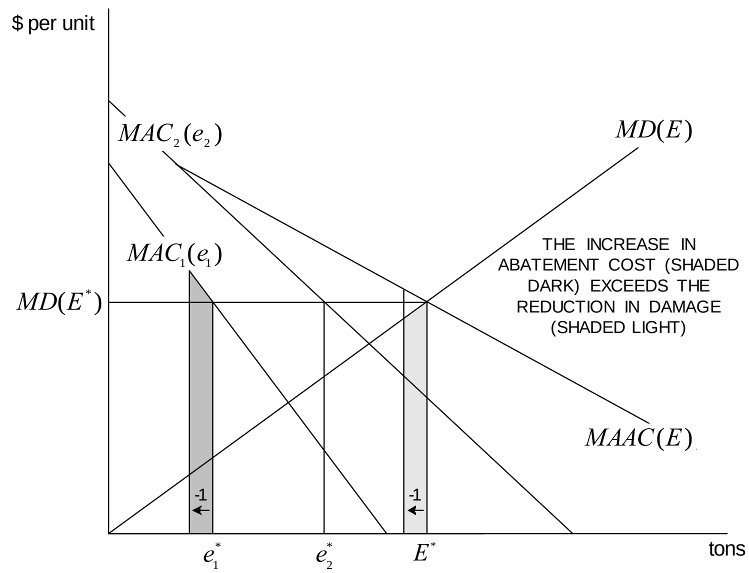


FIGURE 5-22

101

The Optimal Allocation

- A similar pair of experiments around the optimal emissions allocation for source 2 yields the same results: any deviation from e_2^* causes net social benefit to fall.

102

The Optimal Allocation

- To summarize, the **optimal solution** is e_1^* and e_2^* such that

$$e_1^* + e_2^* = E^*$$

and

$$MAC_1(e_1^*) = MAC_2(e_2^*) = MD(E^*)$$

103

A Linear Example (Step 2)

- Recall the linear example from s.58, where

$$MAC_1(e_1) = \gamma_1(\hat{e}_1 - e_1)$$

$$MAC_2(e_2) = \gamma_2(\hat{e}_2 - e_2)$$

104

A Linear Example (Step 2)

- In Step 1 of the policy problem for this example, we derived the MAAC (see s.71):

$$MAAC(E) = \varphi(\hat{E} - E)$$

where $\varphi = \dfrac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2}$ and $\hat{E} = \hat{e}_1 + \hat{e}_2$

105

A Linear Example (Step 2)

- Now suppose that MD is given by

$$MD(E) = \delta E$$

- Then the optimal solution for *E* solves

$$\delta E^* = \varphi(\hat{E} - E^*)$$

106

A Linear Example (Step 2)

- Solving this optimality condition for E^* yields

$$E^* = \left(\frac{\varphi}{\delta + \varphi}\right)\hat{E}$$

- We can now find e_1^* and e_2^* by making this substitution for E in $e_1^*(E)$ and $e_2^*(E)$ from Step 1(a).

107

5.4 A NUMERICAL EXAMPLE

108

A Numerical Example

- Topic 5 Review includes a solved example using the following linear functions:

$$MAC_1(e_1) = 240 - e_1$$

$$MAC_2(e_2) = 480 - 3e_2$$

$$MD(E) = \frac{9E}{4}$$

109

A Numerical Example

- In this example,

$$\gamma_1 = 1 \qquad \hat{e}_1 = 240$$

$$\gamma_2 = 3 \qquad \hat{e}_2 = 160$$

$$\delta = \frac{9}{4}$$

- See Figure 5-23.

110

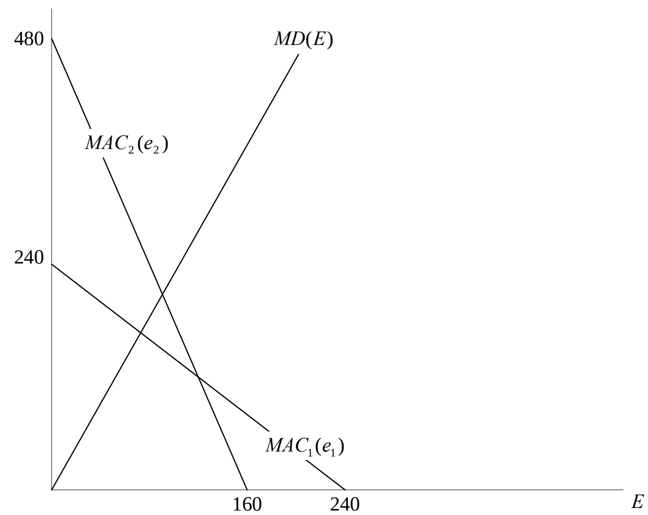


FIGURE 5-23

111

A Numerical Example

- The example is solved methodically in Topic 5 Review following the four steps that have yielded our characterization of the optimum.

- Here we will instead use direct substitution into our derivations from the linear example.

112

A Numerical Example

- Using the derivations for the linear example from s.105:

$$\varphi = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} = \frac{3}{4} \qquad \hat{E} = \hat{e}_1 + \hat{e}_2 = 400$$

113

A Numerical Example

- Thus, the marginal aggregate abatement cost function is

$$MAAC(E) = \frac{3}{4}(400 - E) = 300 - \frac{3E}{4}$$

- See Figure 5-24.

114

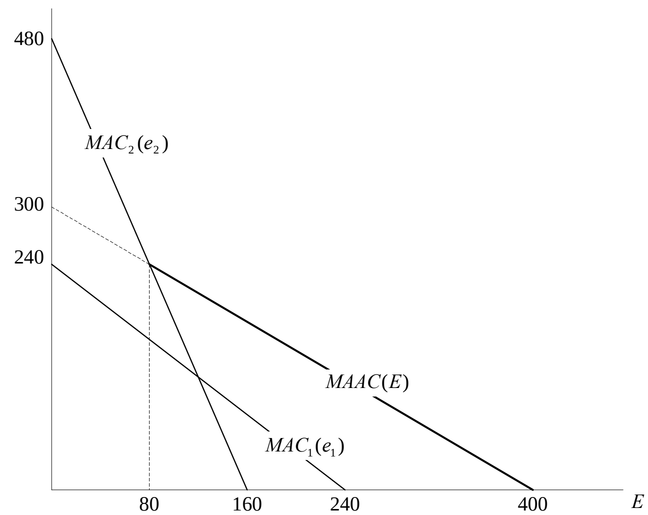


FIGURE 5-24

115

A Numerical Example

- Then from s.107, the optimal level of aggregate emissions is

$$E^* = \left(\frac{\varphi}{\delta+\varphi}\right)\hat{E} = \left(\frac{\frac{3}{4}}{\frac{9}{4}+\frac{3}{4}}\right)400 = 100$$

- See Figure 5-25.

116

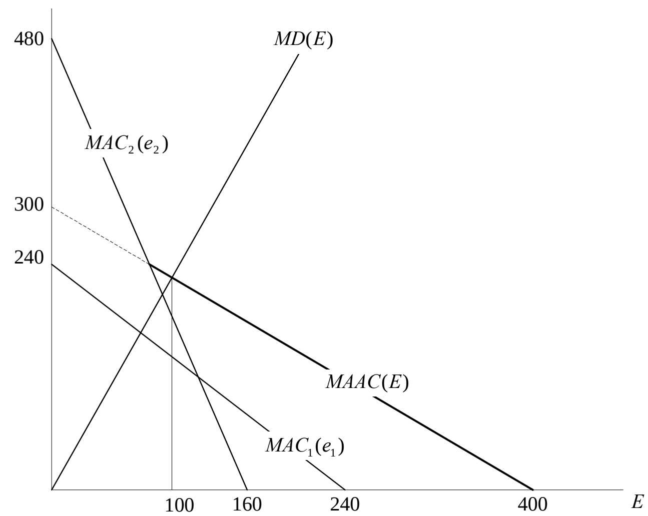


FIGURE 5-25

117

A Numerical Example

- Marginal damage at the optimum is

$$MD(E^*) = \frac{9}{4}(100) = 225$$

- See Figure 5-26.

118

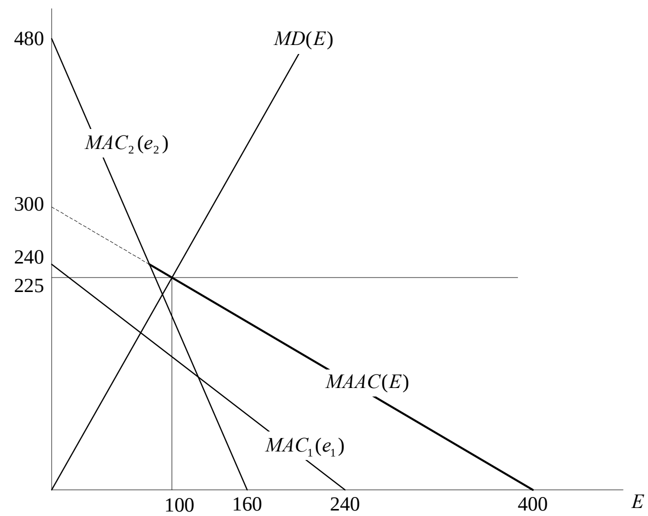


FIGURE 5-26

119

A Numerical Example

- Finally, we can solve for the optimal individual emissions:

$$MAC_1(e_1^*) = MD(E^*) \quad \Rightarrow \quad 240 - e_1^* = 225 \quad \Rightarrow \quad e_1^* = 15$$

$$MAC_2(e_2^*) = MD(E^*) \quad \Rightarrow \quad 480 - 3e_2^* = 225 \quad \Rightarrow \quad e_2^* = 85$$

- See Figure 5-27.

120

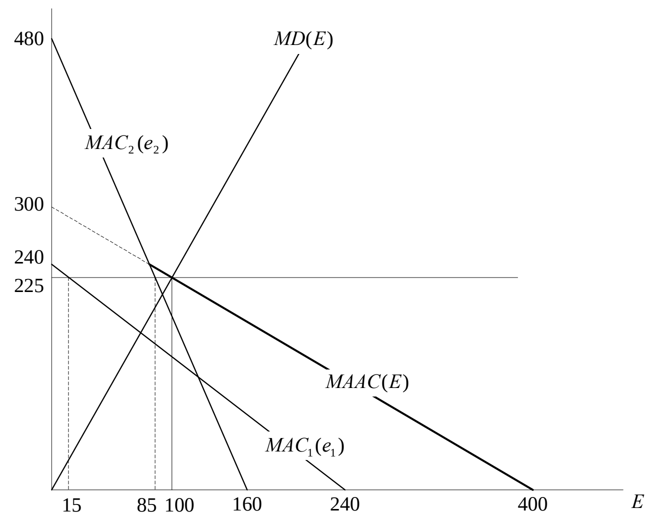


FIGURE 5-27

121

5.5 GENERALIZATION TO *n* SOURCES*

* Advanced Topic

122

Generalization to n Sources

- The logic behind the optimal solution for two sources (summarized in s.103) extends straightforwardly to n sources.

123

Generalization to n Sources

- Provided the pollutant is uniformly mixed with respect to all n sources, then we can still measure damage in terms of aggregate emissions,

$$E = \sum_{i=1}^{n} e_i$$

where e_i is emissions from source i.

124

Generalization to *n* Sources

- The **MACE** result still applies, in a generalized form: the aggregate abatement cost of achieving some target E_0 is minimized at the allocation $\{e_{i0}\}$ such that

$$\sum_{i=1}^{n} e_{i0} = E_0$$

$$MAC_i(e_{i0}) = MAC_j(e_{j0}) \quad \forall i, j$$

125

Generalization to *n* Sources

- The optimality condition for aggregate emissions is also a straightforward generalization of the result from s.82:

$$MAAC(E^*) = MD(E^*)$$

where

$$MAAC(E) = MAC_i(e_i) = MAC_j(e_j) \quad \forall i, j$$

126

Generalization to *n* Sources

- Finally, the optimal allocation rule for each source still applies:

$$MAC_i(e_i^*) = MD(E^*) \quad \forall i$$

127

Generalization to *n* Sources

- To summarize, the **optimal solution** when there are *n* sources of a uniformly mixed pollutant is $\{e_i^*\}$ such that

$$\sum_{i=1}^{n} e_i^* = E^*$$

and

$$MAC_i(e_i^*) = MD(E^*) \quad \forall i$$

128

The Linear Example for *n* Sources

- The MAC for source i is

$$MAC_i(e_i) = \gamma_i(\hat{e}_i - e_i)$$

and MD is

$$MD(E) = \delta E$$

129

The Linear Example for *n* Sources

- We know that the optimal solution is

$$\gamma_i(\hat{e}_i - e_i) = \delta E \qquad \forall i$$

- So for source *i* we can solve this to find

$$e_i = \hat{e}_i - \frac{\delta}{\gamma_i} E$$

130

The Linear Example for *n* Sources

- This must be true for all *i*, so we can <u>sum</u> across *i* on both sides of the equation to obtain

$$\sum_{i=1}^{n} e_i = \sum_{i=1}^{n} \hat{e}_i - \delta E \sum_{i=1}^{n} \left(\frac{1}{\gamma_i} \right)$$

131

The Linear Example for *n* Sources

- This can be written as

$$E = \hat{E} - \delta E \sum_{i=1}^{n} \left(\frac{1}{\gamma_i} \right)$$

which can then be solved for the optimal *E*.

132

The Linear Example for *n* Sources

- To obtain that solution in a convenient form, it is helpful to define

$$\varphi \equiv \left(\sum_{i=1}^{n} \frac{1}{\gamma_i} \right)^{-1}$$

- This is a generalization of our expression for φ from s.71 in the n=2 case.

133

The Linear Example for *n* Sources

- The optimality condition for E can now be written as

$$E = \hat{E} - \frac{\delta}{\varphi} E$$

134

The Linear Example for *n* Sources

- This solves for

$$E^{*} = \left(\frac{\varphi}{\delta + \varphi} \right) \hat{E}$$

- Thus, the solution has exactly the same form as that for the n=2 case (see s.107) but with a generalized definition of φ.

135

The Linear Example for *n* Sources

- We can interpret this optimal quantity of aggregate emissions in terms of the percentage reduction requirement for the set of sources as a whole:

$$R^{*} \equiv \frac{\hat{E} - E^{*}}{\hat{E}} = \frac{\delta}{\delta + \varphi}$$

136

The Linear Example for *n* Sources

- Note that R^* is
  - increasing in δ (the slope of the MD schedule); and
  - decreasing in φ (the slope of the MAAC schedule)

  reflecting the benefit and cost of abatement respectively.

137

The Linear Example for *n* Sources

- Now consider the allocation of E^* to the individual sources.
- From s.130 we know that at the optimal allocation for source *i*,

$$e_i = \hat{e}_i - \frac{\delta}{\gamma_i} E$$

138

The Linear Example for *n* Sources

- Substituting E^* for E then yields

$$e_i^* = \hat{e}_i - \frac{1}{\gamma_i}\left(\frac{\delta\varphi}{\delta+\varphi}\right)\hat{E}$$

139

The Linear Example for *n* Sources

- Express this allocation for source *i* in terms of the percentage reduction required:

$$r_i^* \equiv \frac{\hat{e}_i - e_i^*}{\hat{e}_i} = \frac{1}{\gamma_i \sigma_i}\left(\frac{\delta\varphi}{\delta+\varphi}\right)$$

where $\sigma_i = \hat{e}_i / \hat{E}$ is the **emissions share** for source *i* in the unregulated outcome.

140

The Linear Example for *n* Sources

- Note that for any pair of sources *j* and *k*,

$$\frac{r_j^*}{r_k^*} = \frac{\gamma_k \sigma_k}{\gamma_j \sigma_j}$$

141

The Linear Example for *n* Sources

- This tells us that relative abatement requirements (in terms of % reductions) at the optimum are inversely proportional to the relative MAC slopes, and inversely proportional to relative emissions shares in the unregulated outcome.

142

The Linear Example for *n* Sources

- Thus, the % reduction required for a large polluter (as measured by its unregulated emissions share) is *smaller* than the % reduction required for a small polluter, all other things equal.

143

The Linear Example for *n* Sources

- This might seem odd to a non-economist, but it reflects the fact that abatement cost is related to absolute abatement, and a given % reduction in emissions translates into a larger quantity of absolute abatement for a large polluter than for a small polluter.

144

The Linear Example for *n* Sources

- More generally, our results tell us that the optimal solution when there are many sources generally <u>cannot</u> be implemented with simple rules like
  – equal emissions allocations
  – equal absolute reduction requirements; or
  – equal % reduction requirements.

145

The Linear Example for *n* Sources

- So how can the social optimum be implemented?
- The ideal solution is to put a price on emissions.
- We begin to explore this approach in the next topic.

END

146

TOPIC 5 REVIEW QUESTIONS

These review questions begin with a solved example. It is recommended that you work through this example before commencing the questions.

A SOLVED EXAMPLE

Suppose MAC for source 1 is given by

$$MAC_1(e_1) = 240 - e_1$$

and marginal abatement cost for source 2 is given by

$$MAC_2(e_2) = 480 - 3e_2$$

In addition, suppose MD is given by

$$MD(E) = \frac{9E}{4}$$

where $E = e_1 + e_2$. See Figure R5-1.

Our **policy problem** is to determine the socially optimal level of aggregate emissions (denoted E^*), and the optimal allocation of E^* between the two sources.

<u>Step 1</u>

a) for any given aggregate emissions target, we find the allocation between the two sources that minimizes aggregate abatement cost.

b) we use the result from 1(a) to construct a marginal aggregate-abatement cost function in terms of the aggregate emissions target.

<u>Step 2</u>

a) we bring marginal damage and marginal aggregate abatement cost together to determine the *optimal* target in terms of aggregate emissions.

b) we use the allocation rule from 1(a) to allocate the optimal aggregate emissions target between the two sources.

Step 1(a): Least-Cost Allocation Rules

The **MACE** solution:

(1) $MAC_1(e_1) = MAC_2(e_2)$ <u>and</u> $e_1 + e_2 = E$

where E is the target level of aggregate emissions. Substituting for $MAC_1(e_1)$ and $MAC_2(e_2)$ yields

(2) $240 - e_1 = 480 - 3e_2$

Set $e_1 = E - e_2$ and substitute into (2):

(3) $240 - (E - e_2) = 480 - 3e_2$

and solve for e_2:

(4) $e_2(E) = 60 + \frac{E}{4}$

This is the <u>allocation rule</u> for source 2. Substitute (4) into $e_1 = E - e_2$ to obtain

(5) $e_1(E) = \frac{3E}{4} - 60$

This is the <u>allocation rule</u> for source 1.

Solutions (4) and (5) are valid if and only if $e_1(E) \geq 0$ and $e_2(E) \geq 0$; emissions cannot be negative. This is called an <u>interior solution</u>.

If $E < 80$ then $e_1(E) < 0$, and the interior solutions are not valid. In that case, we have a <u>corner solution</u>, in which *either* $e_1 = 0$ and $e_2 = E$, *or* $e_1 = E$ and $e_2 = 0$. We will return to the corner solution later. For the moment, we will focus on the interior solution (where $E \geq 80$).

Step 1(b): Construction of *MAAC(E)*

MAAC(E) tells us the marginal cost of reducing aggregate emissions. By virtue of the fact that $MAC_1(e_1) = MAC_2(e_2)$ at the least-cost solution, it follows that a marginal reduction in emissions is equally costly for each source, so the marginal cost of reducing aggregate emissions is equal to $MAC_1(e_1(E))$ and it is also equal to $MAC_2(e_2(E))$. That is,

(6) $MAAC(E) = MAC_1(e_1(E)) = MAC_2(e_2(E))$

Thus, we can construct *MAAC(E)* by substituting $e_1(E)$ into $MAC_1(e_1)$, or by substituting $e_2(E)$ into $MAC_2(e_2)$; we will get the same result either way.

Substituting $e_1(E)$ into $MAC_1(e_1)$ yields

(7) $MAAC(E) = 240 - \left(\frac{3E}{4} - 60\right) = 300 - \frac{3E}{4}$

Substituting $e_2(E)$ into $MAC_2(e_2)$ yields

(8) $MAAC(E) = 480 - 3\left(60 + \frac{E}{4}\right) = 300 - \frac{3E}{4}$

Graphically, $MAAC(E)$ is the horizontal summation of $MAC_1(e_1)$ and $MAC_2(e_2)$; see Figure R5-2. The vertical height measures the cost of a marginal reduction in emissions. That must be the same for each source as it is for the group of sources as a whole, because (6) must hold in the (interior) least-cost solution.

To see how we can obtain $MAAC(E)$ directly as the horizontal summation of $MAC_1(e_1)$ and $MAC_2(e_2)$, we first need to express each of these schedules with emissions as the subject. In particular, we can take the inverse of $MAC_1(e_1)$ to obtain

(9) $e_1 = 240 - m_1$

where m_1 denotes marginal abatement cost for source 1. Similarly, we can take the inverse of $MAC_2(e_2)$ to obtain

(10) $e_2 = 160 - \frac{m_2}{3}$

where m_2 denotes marginal abatement cost for source 2. Now set $m_1 = m_2 = m$ (because the marginal abatement costs must be equated) and then add (9) and (10) to obtain

(11) $E = 400 - m - \frac{m}{3} = 400 - \frac{4m}{3}$

Now take the inverse of (11) to make *m* the subject:

(12) $m = 300 - \frac{3E}{4}$

This is the $MAAC(E)$; see (8) above.

Note from Figure R5-2 that the solution for $MAAC(E)$ in (8) and (9) is valid only for $E \geq 80$; hence the dashed portion of the schedule.

Step 2(a): Determination of Optimal Aggregate Emissions

Optimal aggregate emissions are characterized by

(13) $MAAC(E) = MD(E)$

Substituting for $MAAC(E)$ and $MD(E)$ yields

(14) $300 - \frac{3E}{4} = \frac{9E}{4}$

Solve for E to obtain

(15) $E^* = 100$

See Figure R5-3. This solution is valid because $E^* \geq 80$. Had we found that $E^* < 80$ then the solution would not be valid; the intersection of $MAAC(E)$ and $MD(E)$ would be in the dashed portion of the schedule in Figure R5-3, and we know that the interior solution that underlies our construction of $MAAC(E)$ does not apply in that region.

STEP 2(b): Determination of the Optimal Allocation of Emissions Across Sources

We now use the least-cost allocation rules derived in Step 1(a) to allocation E^* across the two sources. Substitute $E = E^*$ into (5) to obtain

(16) $e_1^* = e_1(E^*) = \frac{3E^*}{4} - 60 = 15$

Substitute $E = E^*$ into (4) to obtain

(17) $e_2^* = e_2(E^*) = 60 + \frac{E^*}{4} = 85$

See Figure R5-4.

Implementation with a Pigouvian Tax (A Preview of Topic 6 Review)

The Pigouvian tax rule:

(18) $\tau^* = MD(E^*)$

In this case, we have

(19) $\tau^* = \frac{9E^*}{4} = 225$

Let us confirm that this tax rate does in fact implement the optimal solution. Source *i* responds to the tax by setting

(29) $MAC_i(e_i) = \tau$

Thus, for source 1 we have

(21) $240 - e_1 = \tau$

which solves for a response function:

(22) $e_1(\tau) = 240 - \tau$

Similarly, for source 2 we have

(23) $480 - 3e_2 = \tau$

which solves for a response function:

(24) $e_2(\tau) = 160 - \frac{\tau}{3}$

Setting $\tau = \tau^*$ in (22) and (24) yields

(25) $e_1(\tau^*) = 15$

and

(26) $e_2(\tau^*) = 85$

respectively. See Figure R5-5. Thus, the Pigouvian tax does induce the optimal solution.

The Corner Solutions

Recall that there are two possible corner solutions to the least-cost allocation problem, henceforth labeled CSA and CSB:

(CSA) $e_1 = 0$ and $e_2 = E$

(CSB) $e_1 = E$ and $e_2 = 0$

One of these solutions must apply if $E < 80$. The least-cost solution is the one that achieves the aggregate target at the lowest aggregate abatement cost, so we must construct the aggregate abatement cost for CSA and CSB and choose between them accordingly.

In CSA, we have $e_1 = 0$ and $e_2 = E$, so aggregate abatement cost is the sum of the areas in Figures R5-6 and R5-7. These are respectively,

(27) $AC_{1A} = 28800$

are

(28) $AC_{2A} = \dfrac{(160-E)(480-3E)}{2}$

Thus, aggregate abatement cost under CSA is

(29) $AC_A = 28800 + \dfrac{(160-E)(480-3E)}{2}$

In CSB, we have $e_1 = E$ and $e_2 = 0$, so aggregate abatement cost is the sum of the areas in Figures R5-8 and R5-9. These are respectively,

(30) $AC_{1B} = \dfrac{(240-E)(240-E)}{2}$

and

(31) $AC_{2B} = 38400$

Thus, aggregate abatement cost under CSA is

(32) $AC_B = \dfrac{(240-E)(240-E)}{2} + 38400$

It is straightforward to show that $AC_A < AC_B$ for any $E < 80$. Thus, the optimal corner solution when $E < 80$ is CSA: $e_1 = 0$ and $e_2 = E$.

REVIEW QUESTIONS

1. A uniformly mixed pollutant is one that spreads out unevenly within the receptive region, leading to measured pollutant concentrations that are unequal across the region.

A. True.

B. False.

2. Greenhouse gas emissions are a non-uniformly mixed pollutant in the sense that the damage done by climate change is likely to differ across countries.

A. True.

B. False.

3. For policy purposes, a key distinction between a uniformly mixed pollutant and a non-uniformly mixed pollutant is that for the former, total damage can be measured in terms of aggregate emissions, independently of individual source-locations.

A. True.

B. False.

4. In a setting with multiple sources of a uniformly mixed pollutant, the policy problem is solved in two steps, where

A. step 1 involves the choice of the aggregate emissions target, and step 2 involves the design of an allocation rule for assigning that target across sources.

B. step 1 involves the design of an allocation rule for any given target, and step 2 involves the construction of a marginal aggregate abatement cost function.

C. step 1 involves the design of an allocation rule for any given target and the construction of a marginal aggregate abatement cost function, and step 2 involves the choice of the optimal target and the allocation of that target across sources.

D. None of the above.

5. The marginal abatement cost equalization (**MACE**) rule tells us that an aggregate emissions target for a uniformly mixed pollutant should be allocated across sources such

A. marginal damage is equal to marginal aggregate abatement cost.

B. marginal aggregate abatement cost is minimized.

C. percentage abatement requirements are equated across sources.

D. None of the above.

6. Marginal aggregate abatement cost measures the minimum cost of reducing aggregate emissions by a marginal amount, and is constructed by taking the vertical summation of the marginal abatement cost schedules for the individual sources.

A. True.

B. False.

7. In general, the optimal policy for multiple sources of a uniformly mixed pollutant requires that each source reduce emissions by

A. the same amount.

B. the same percentage amount.

C. amounts that ensure that abatement costs are equalized across the sources.

D. None of the above.

Questions 8 – 17 relate to the following information. Suppose MAC for source 1 is given by

$$MAC_1(e_1) = 10 - e_1$$

and marginal abatement cost for source 2 is given by

$$MAC_2(e_2) = 12 - 3e_2$$

In addition, suppose MD is given by

$$MD(E) = E$$

where $E = e_1 + e_2$.

8. The unregulated emissions choices for sources 1 and 2 respectively are

A. $\hat{e}_1 = 4$ and $\hat{e}_2 = 10$

B. $\hat{e}_1 = 10$ and $\hat{e}_2 = 12$

C. $\hat{e}_1 = 1$ and $\hat{e}_2 = 3$

D. $\hat{e}_1 = 10$ and $\hat{e}_2 = 4$

9. The unregulated level of aggregate emissions is

A. $\hat{E} = 14$

B. $\hat{E} = 4$

C. $\hat{E} = 22$

D. None of the above.

10. The least-cost allocation of any aggregate-emissions target $E > 2/3$ requires an emissions allocation for source 1 equal to

A. $\frac{E}{2}$

B. $\frac{3E}{4} - \frac{1}{2}$

C. $\frac{2E}{3}$

D. 0

11. The least-cost allocation of any aggregate-emissions target $E < 2/3$ requires an emissions allocation for source 1 equal to

A. $\frac{E}{2}$

B. $\frac{3E}{4} - \frac{1}{2}$

C. $\frac{2E}{3}$

D. 0

Hint: can emissions for either source ever be *negative*?

12. The least-cost allocation of any aggregate-emissions target $E < 2/3$ requires an emissions allocation for source 2 equal to

A. $\frac{E}{4} + \frac{1}{2}$

B. $\frac{3E}{4}$

C. E

D. $\frac{E}{2}$

For **Questions 13 – 17**, confine consideration to the case where $E > 2/3$.

13. The $MAAC(E)$ schedule is

A. $22 - 4E$

B. $\frac{21}{2} - \frac{3E}{4}$

C. $2 - 2E$

D. $14 - \frac{3E}{4}$

14. $MAAC(\hat{E}) = 0$.

A. True.

B. False.

15. The socially optimal level of aggregate emissions is

A. 10

B. 8

C. 6

D. 2

16. The socially optimal level of emissions for source 1 is

A. 4

B. 9

C. 3

D. 2

17. The socially optimal level of emissions for source 2 is

A. 1

B. 9

C. 6

D. 2

Questions 18 – 26 relate to the following information. Suppose MAC for source 1 is given by

$$MAC_1(e_1) = 45 - 3e_1$$

and marginal abatement cost for source 2 is given by

$$MAC_2(e_2) = 70 - 2e_2$$

In addition, suppose MD is given by

$$MD(E) = \frac{4E}{5}$$

where $E = e_1 + e_2$.

18. The unregulated level of aggregate emissions is

A. $\hat{E} = 115$

B. $\hat{E} = 50$

C. $\hat{E} = 23$

D. None of the above.

19. The least-cost allocation of any aggregate-emissions target $E > 25/2$ requires an emissions allocation for source 1 equal to

A. $\frac{2E}{5} - 5$

B. $\frac{3E}{4} - \frac{1}{2}$

C. $\frac{3E}{5} + 5$

D. $\frac{E}{2}$

20. The least-cost allocation of any aggregate-emissions target $E > 25/2$ requires an emissions allocation for source 2 equal to

A. $\frac{2E}{5} - 5$

B. $\frac{3E}{4} - \frac{1}{2}$

C. $\frac{3E}{5} + 5$

D. $\frac{E}{2}$

21. The least-cost allocation of any aggregate-emissions target $E < 25/2$ requires an emissions allocation for source 2 equal to

A. $\frac{3E}{5} + 5$

B. $\frac{2E}{5} - 5$

C. E

D. 0

For **Questions 22 – 26**, confine consideration to the case where $E > 25/2$.

22. The *MAAC*(*E*) schedule is

A. $\frac{7E}{5}$

B. $\frac{25}{2} - 5E$

C. $115 - 5E$

D. $60 - \frac{6E}{5}$

23. The socially optimal level of aggregate emissions is

A. 10

B. 20

C. 30

D. 40

24. The socially optimal level of emissions for source 1 is

A. 23

B. 12

C. 7

D. 11

25. The socially optimal level of abatement for source 2 is

A. 23

B. 12

C. 7

D. 11

26. Aggregate abatement cost at the social optimum is

A. 210

B. 240

C. 90

D. 345

Questions 27 – 34 relate to the following information. Suppose MAC for source 1 is given by

$$MAC_1(e_1) = 385 - 5e_1$$

and marginal abatement cost for source 2 is given by

$$MAC_2(e_2) = 154 - 2e_2$$

In addition, suppose MD is given by

$$MD(E) = \frac{12E}{7(1+v)}$$

where $E = e_1 + e_2$ and v is the dollar investment in defensive action. This defensive action can be taken at only one of two levels: $v = 0$ or $v = 2$.

27. The least-cost allocation of any aggregate-emissions target $E > 231/5$ requires an emissions allocation for source 1 equal to

A. $\frac{2E}{5v} - 33$

B. $33 + \frac{2E}{7}$

C. $\frac{5E}{7} - 23$

D. $\frac{3E}{2(1+v)} + 23$

For **Questions 28 – 34**, confine consideration to the case where $E > 231/5$.

28. The *MAAC*(*E*) schedule is

A. $539 - 7E$

B. $539 - \frac{7E}{(1+v)}$

C. $220 - \frac{10E}{7}$

D. None of the above.

29. For any given level of defensive action *v*, the socially optimal level of aggregate emissions is

A. $\frac{770(1+v)}{11+5v}$

B. $\frac{110(1+v)}{11+77v}$

C. $\frac{70}{1+v}$

D. $70(1+v)$

30. If $v = 0$ then aggregate abatement cost at the social optimum is

A. 5040

B. 7080

C. 4320

D. 8990

31. If $v = 0$ then damage at the social optimum is

A. 6000

B. 3070

C. 4200

D. 5400

32. If $v = 2$ then total social cost at the social optimum is

A. 3220

B. 2740

C. 5200

D. 4840

33. The socially optimal level of defensive action is $v^* = 2$.

A. True.

B. False.

34. The net benefit of defensive action at $v = 2$ is

A. 3240

B. 4400

C. 2100

D. None of the above

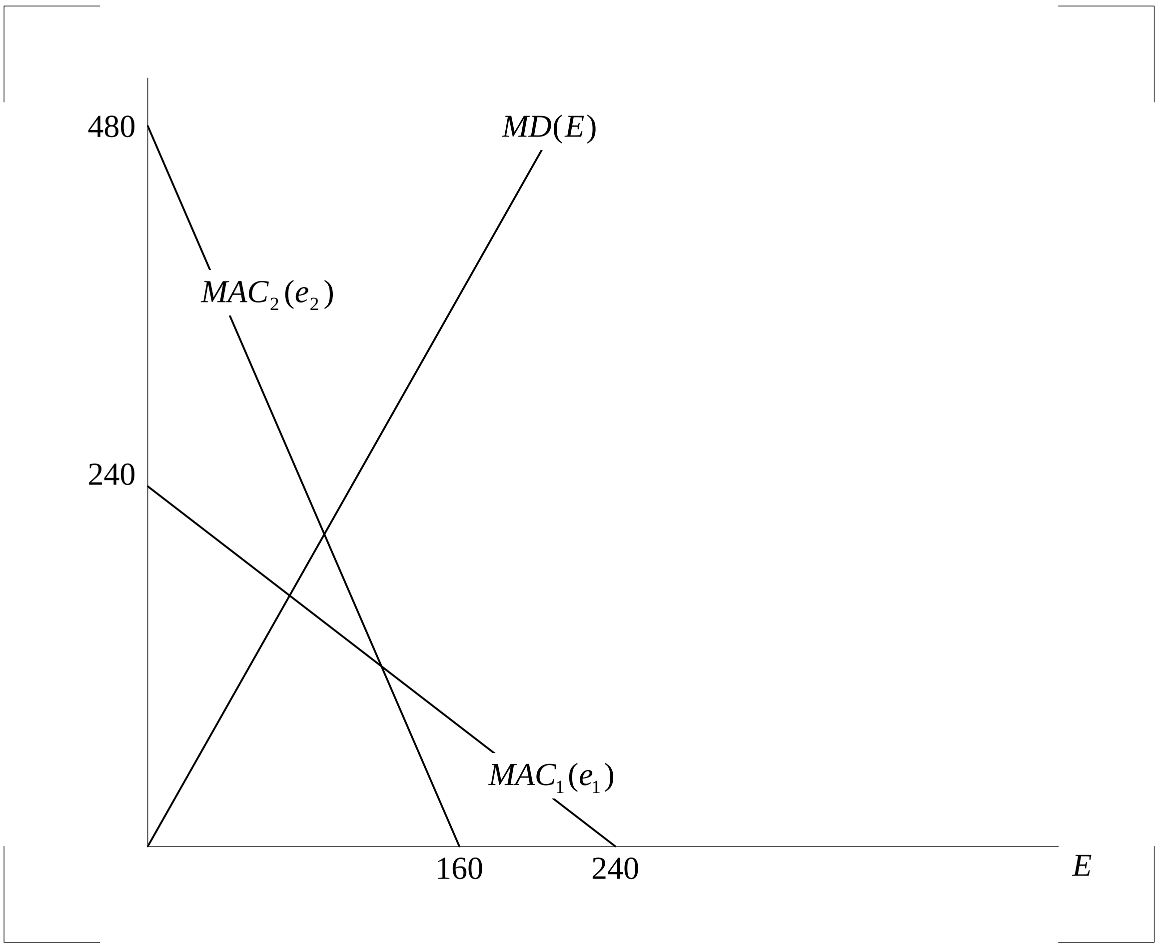


Figure R5-1

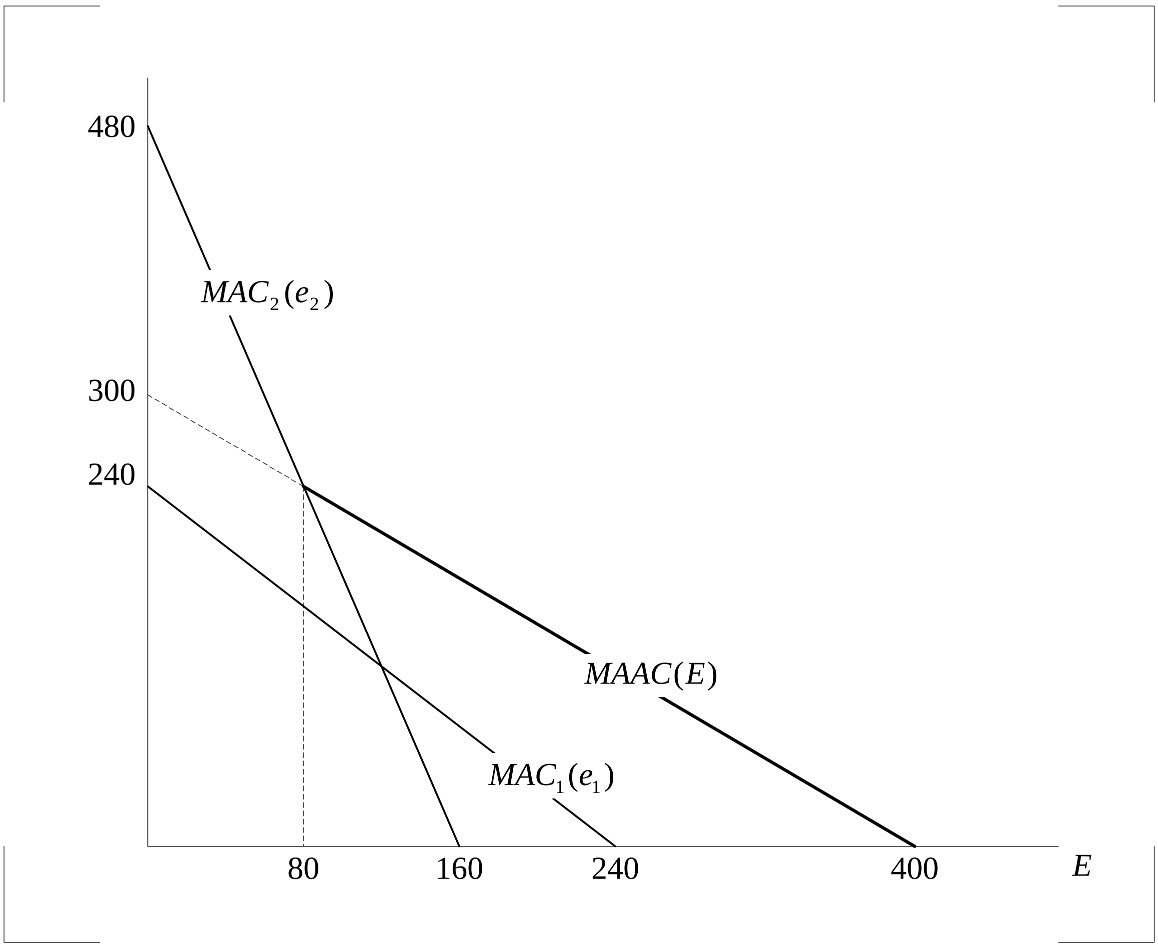


Figure R5-2

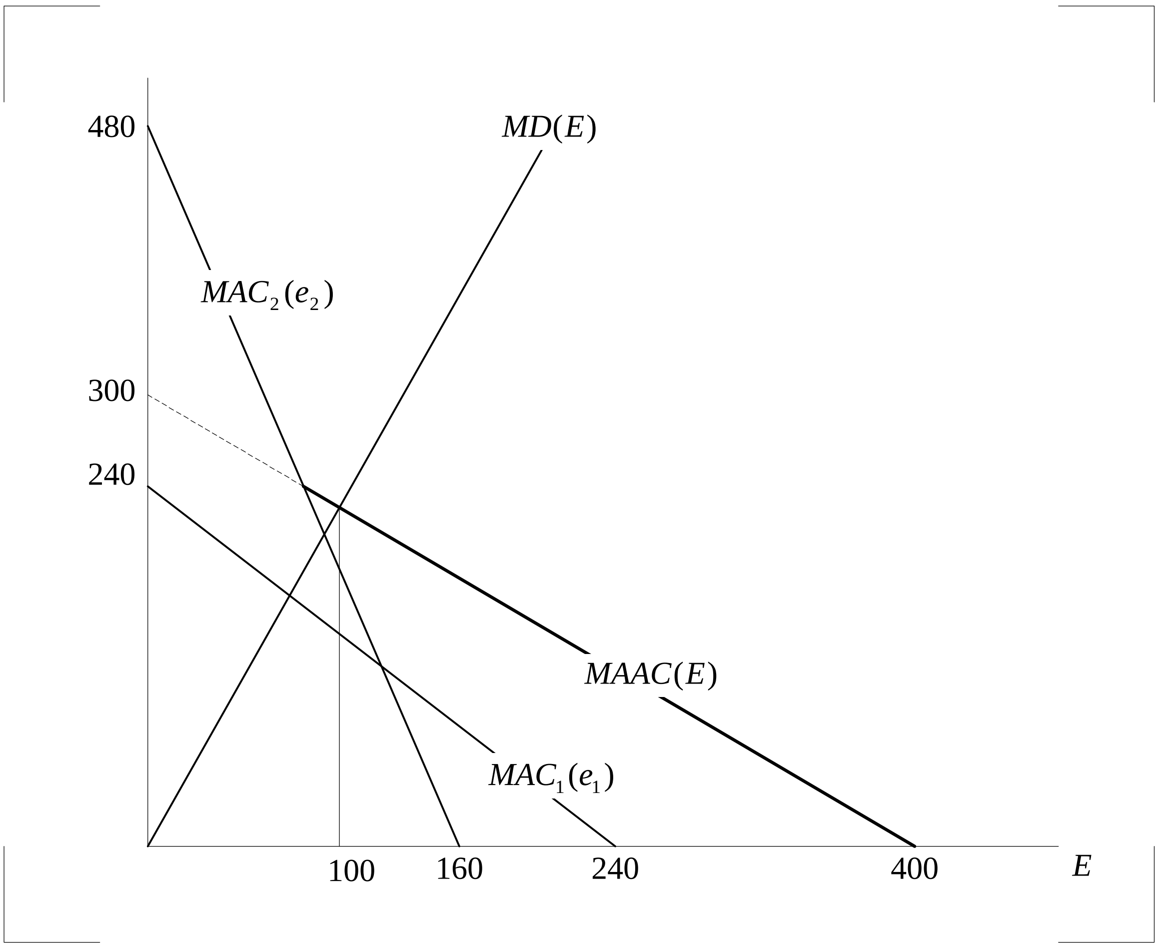


Figure R5-3

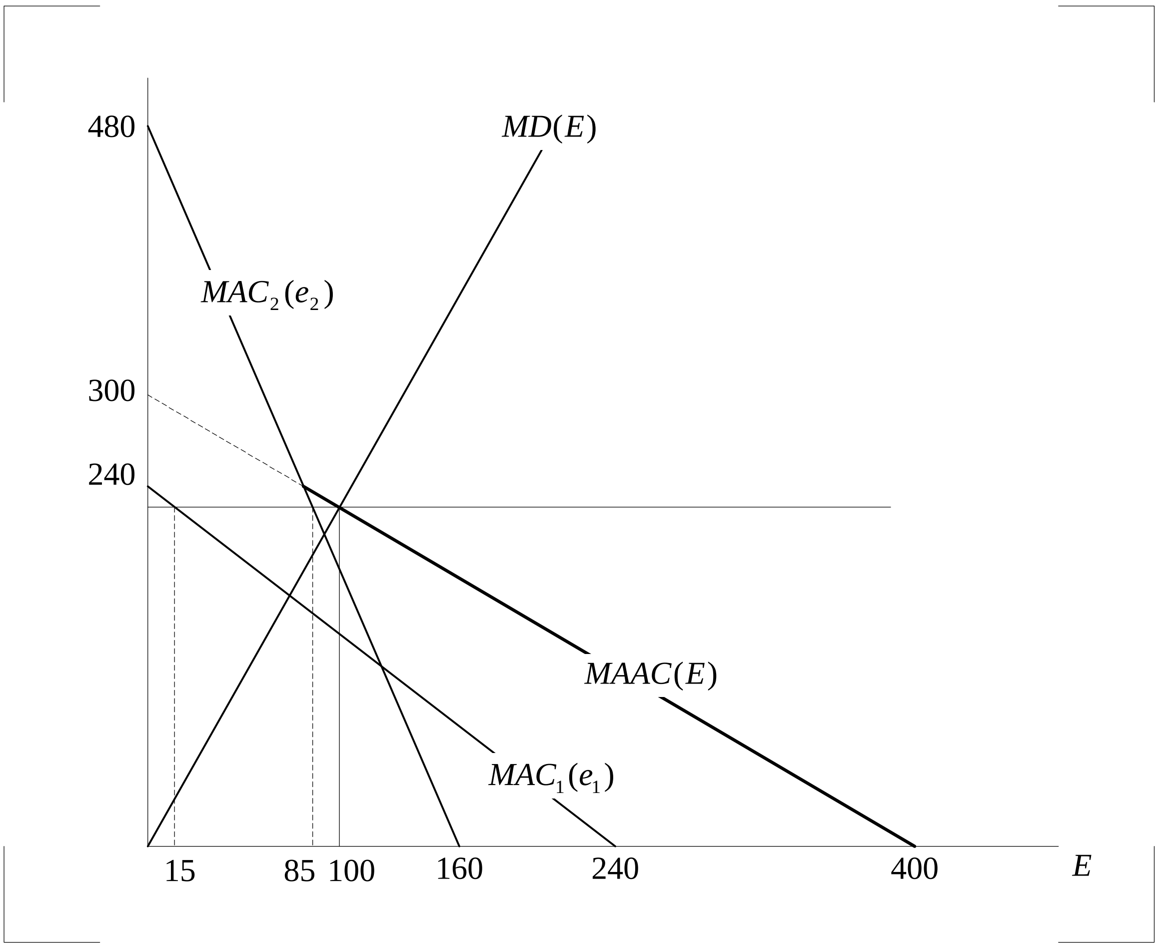


Figure R5-4

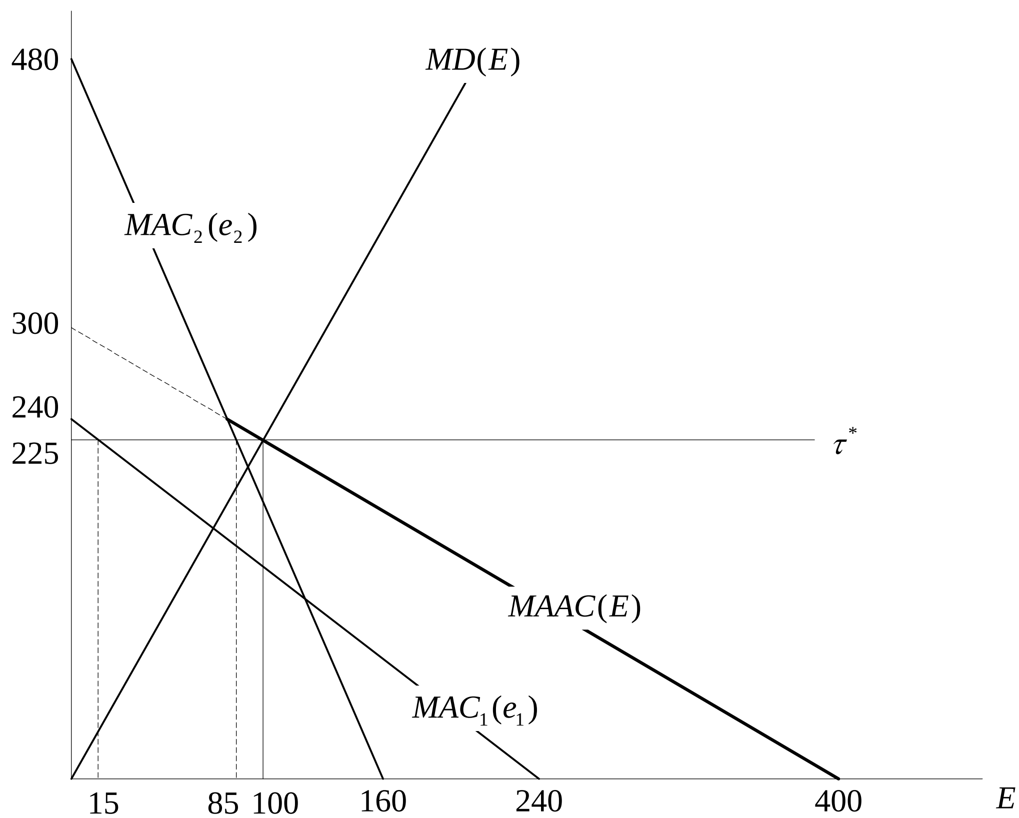

480
300
240
225
MD(E)
MAC2(e2)
τ*
MAAC(E)
MAC1(e1)
15
85
100
160
240
400
E

Figure R5-5

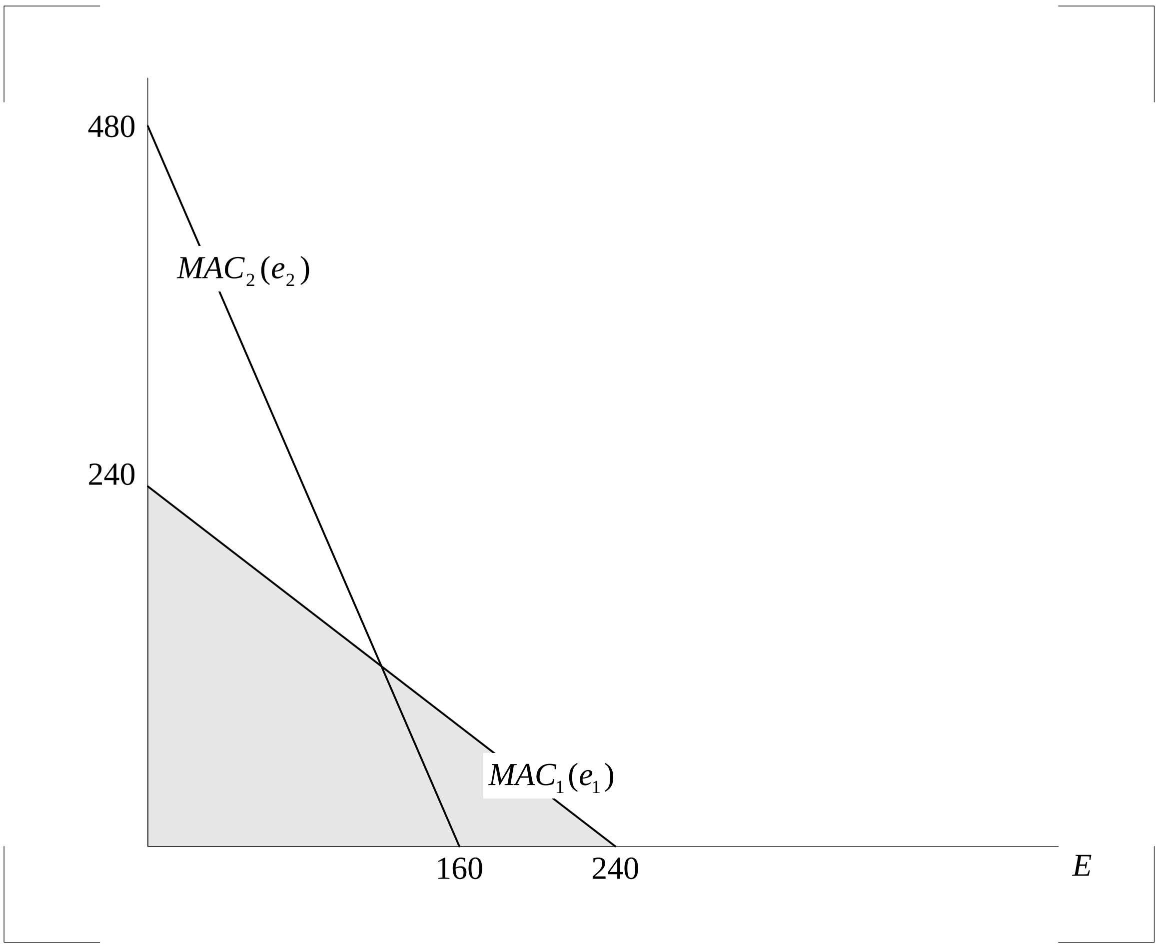


Figure R5-6

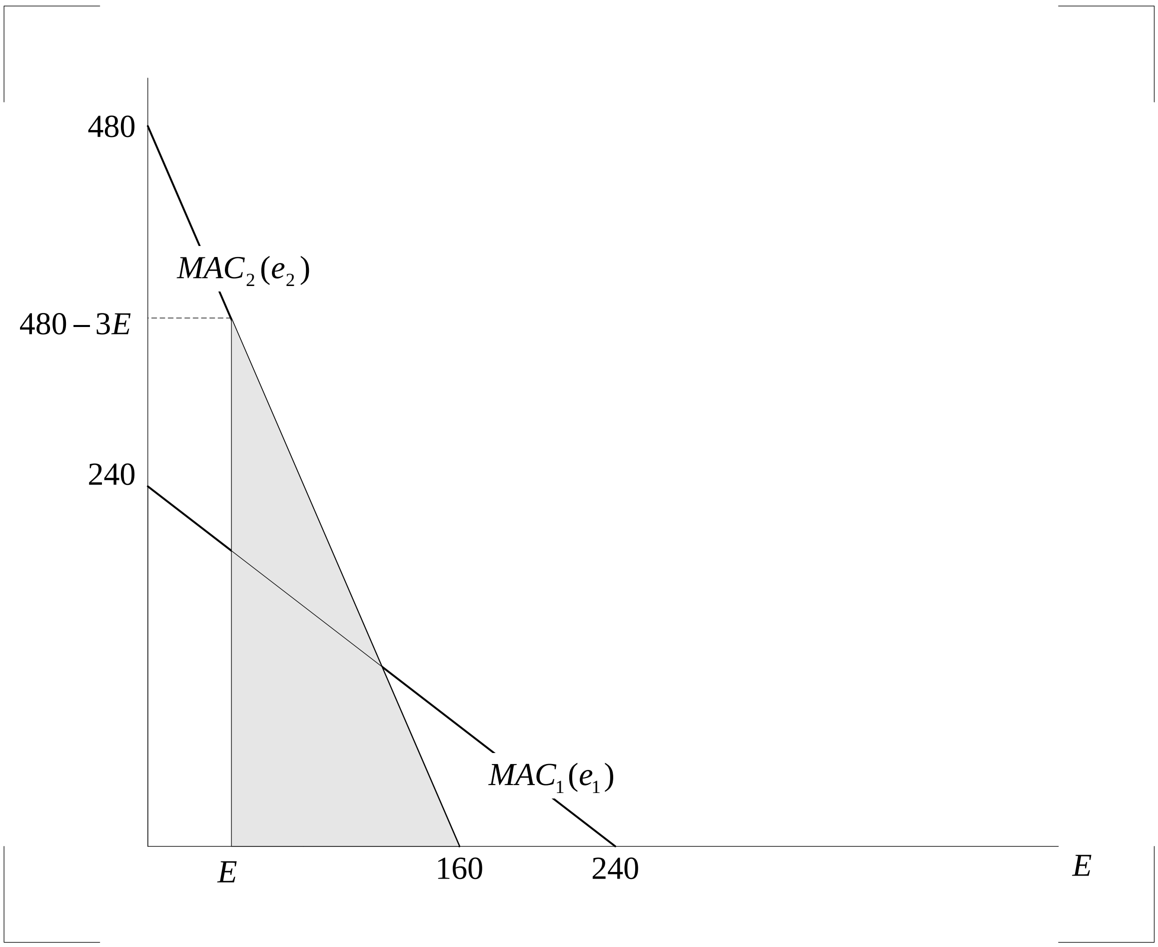


Figure R5-7

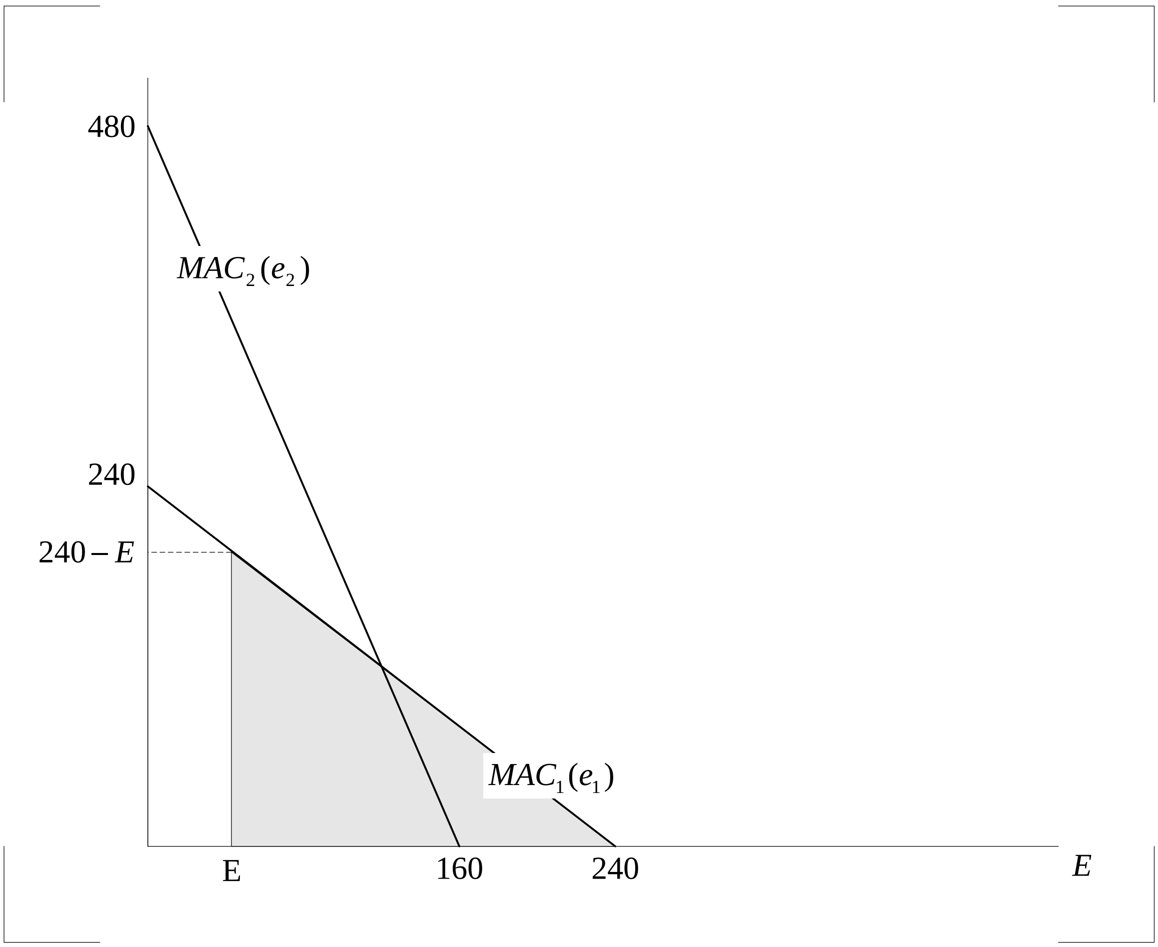


Figure R5-8

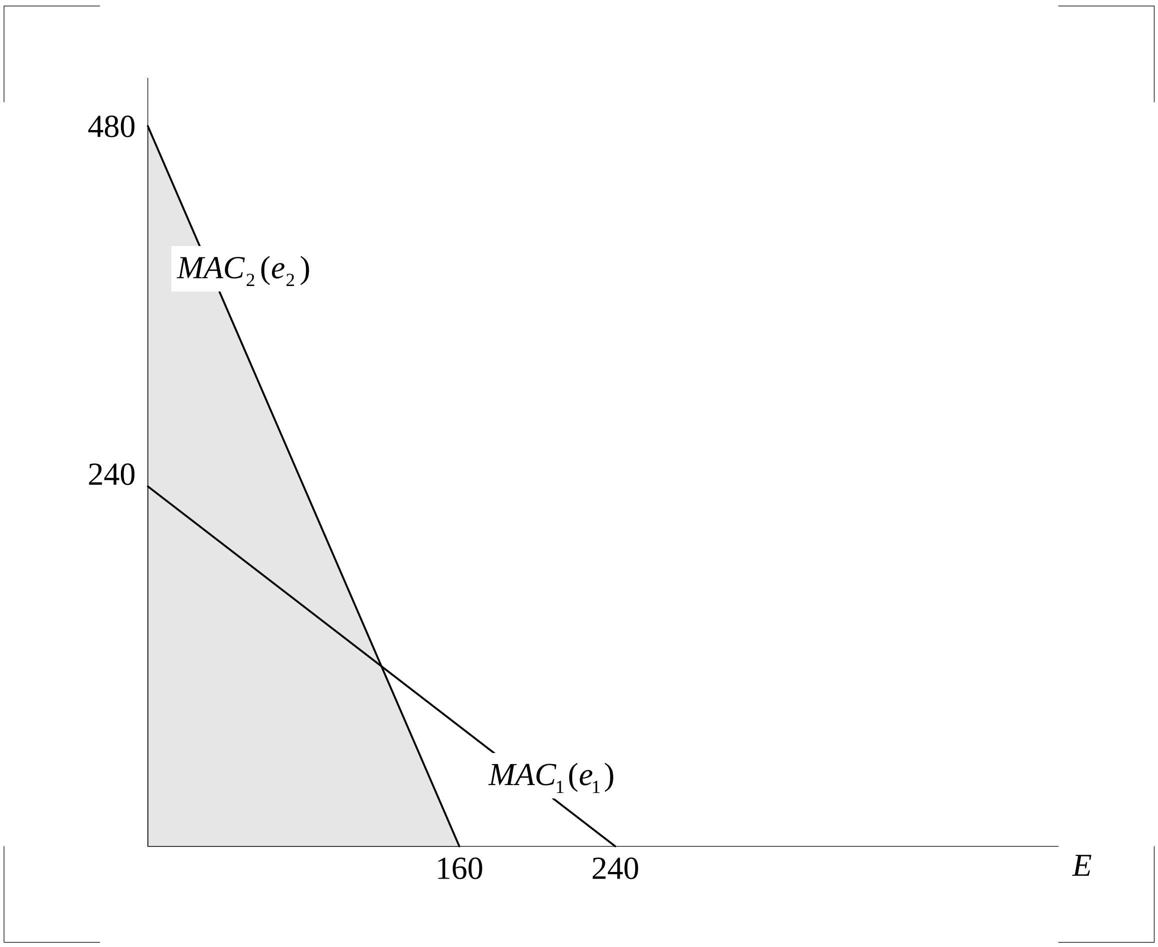


Figure R5-9

ANSWER KEY

1. B
2. B
3. A
4. C
5. D
6. B
7. D
8. D
9. A
10. B
11. D
12. C
13. B
14. A
15. C
16. A
17. D
18. B
19. A
20. C
21. C
22. D
23. C
24. C
25. B
26. B *
27. B
28. C
29. A
30. A
31. C
32. D
33. A
34. D **

* See Figure R5-10 below.

** See Figures R5-11 and R5-12 below.

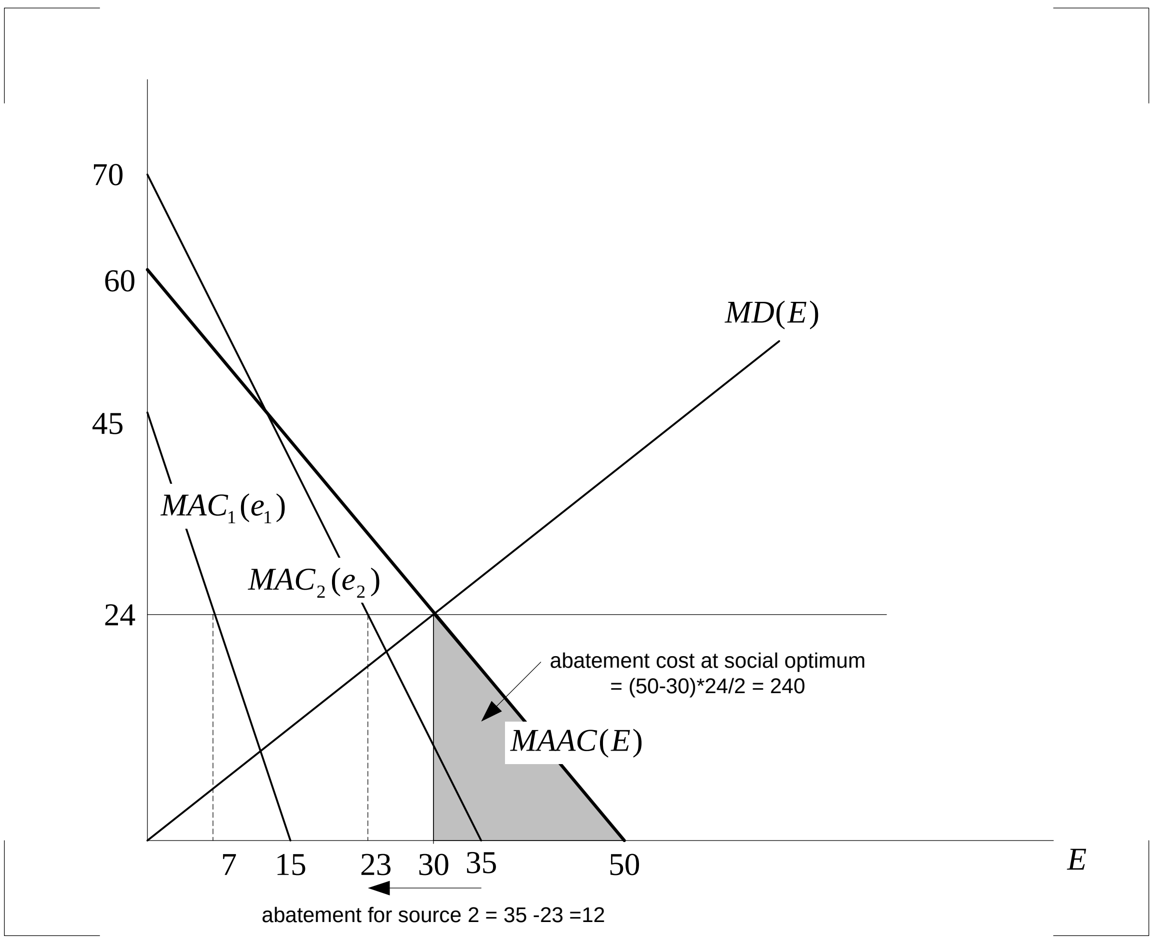

70
60
45
24
MD(E)
MAC1(e1)
MAC2(e2)
abatement cost at social optimum
= (50-30)*24/2 = 240
MAAC(E)
7
15
23
30
35
50
E
abatement for source 2 = 35 -23 =12

Figure R5-10

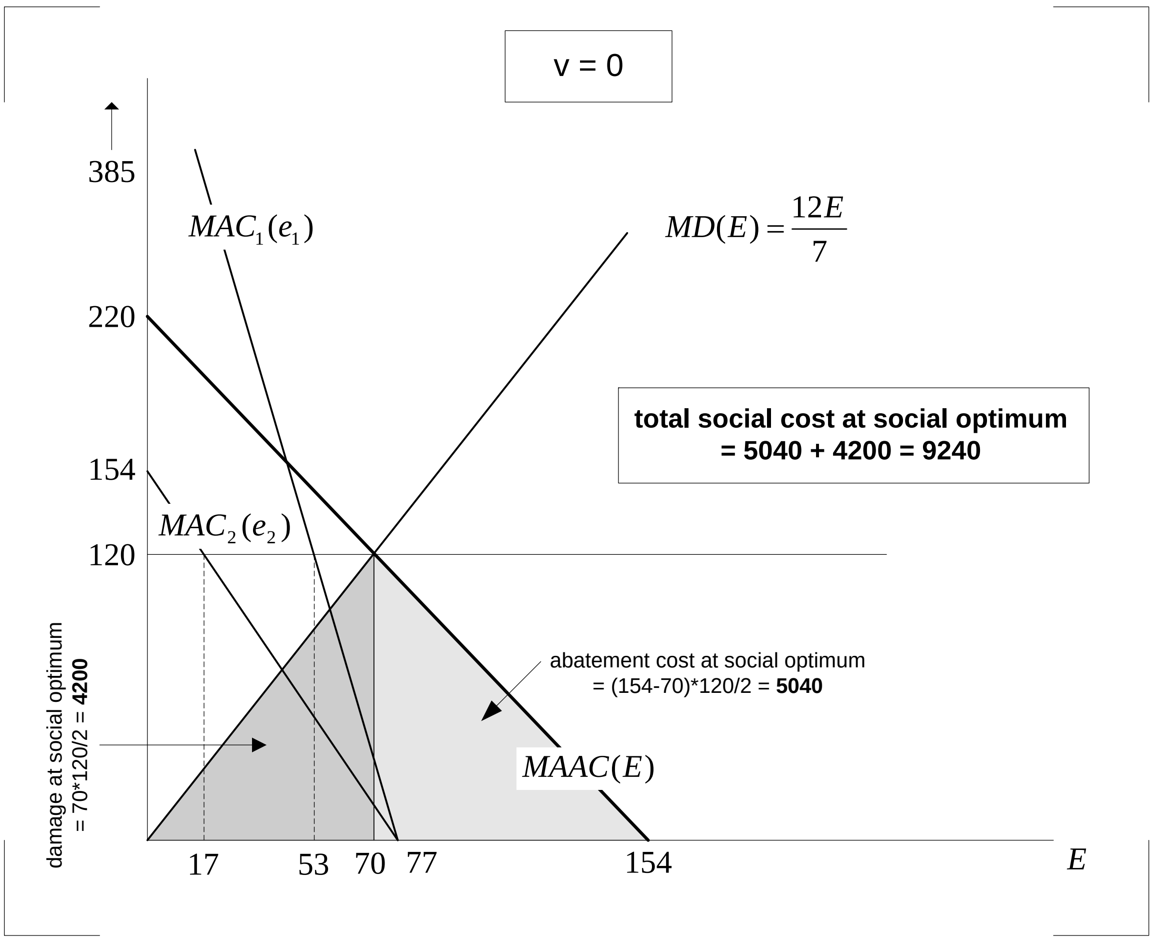


Figure R5-11

At $v = 0$, total social cost at the social optimum is 9240 (the sum of the shaded areas in Figure R5-11).

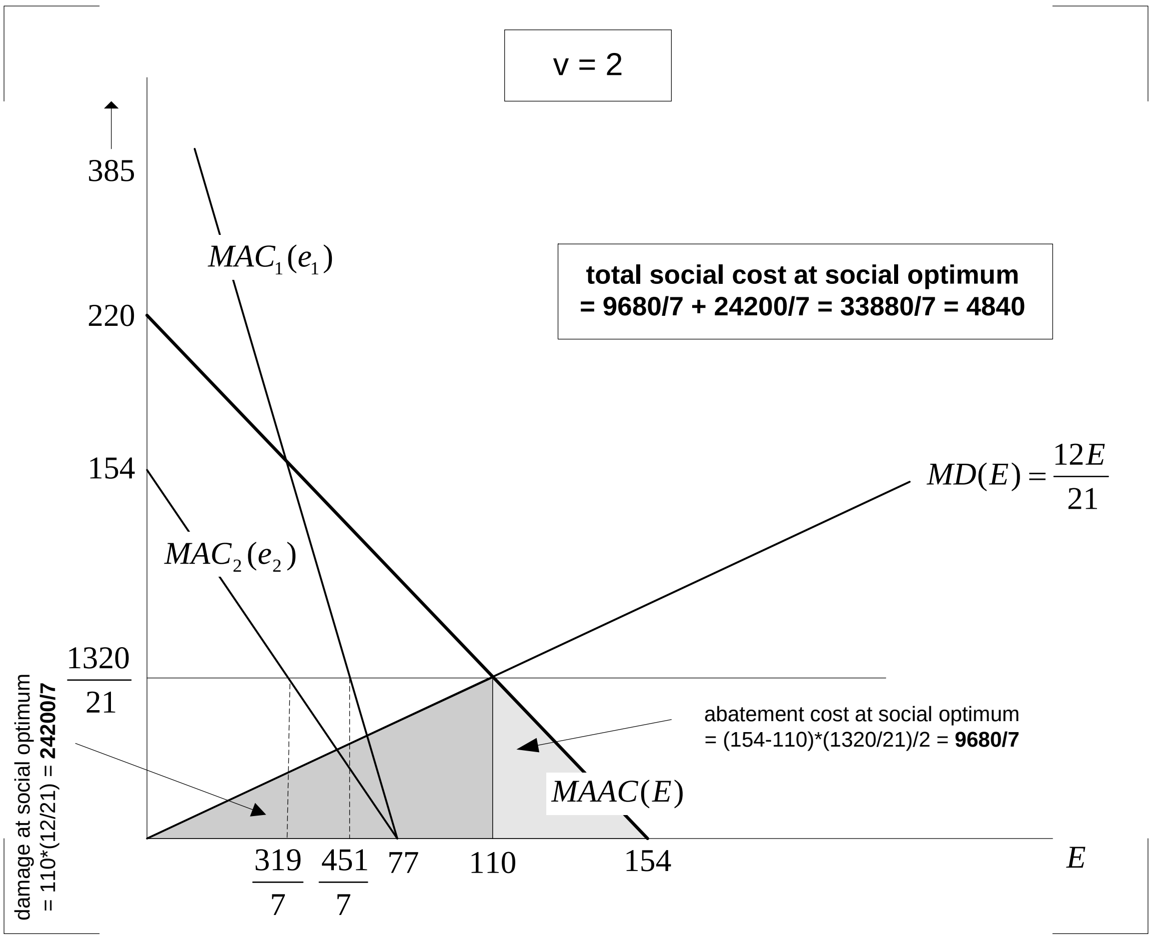


Figure R5-12

At $v = 2$, total social cost at the social optimum is 4840 (the sum of the shaded areas in Figure R5-12).

So from Figures R5-11 and R5-12, we see that the net benefit of setting $v = 2$ relative to $v = 0$ is (9240 – 4840) – 2 = 4398. This is positive, so $v = 2$ is better than $v = 0$, and since these are the only two available options, it follows that $v = 2$ is the optimal level of defensive action.

6. POLLUTION TAXES

1

OUTLINE

2

* Advanced Topic

3

6.1 INTRODUCTION

4

Introduction

- Recall from Topic 1.2 that policy-design is a two-part process:
  1. Specification of the policy goal
  2. Implementation of that goal via the application of policy instruments

5

Introduction

- Topics 3 – 5 dealt with the first part of this process, characterizing policy goals in terms of marginal damage and marginal abatement costs.
- We now turn to the second part of the process: **implementation** via the application of policy instruments.

6

Introduction

- We begin with the "textbook" policy instrument for the correction of externalities:
  - the **Pigouvian tax**

7

6.2 THE PIGOUVIAN TAX

8

The Pigouvian Tax

- The logic of placing a tax on pollution was first annunciated by Arthur Pigou, a British economist, in 1924.

9

The Pigouvian Tax

- That logic is compelling:
  - an externality arises from an action because the source agent does not account for the cost imposed on external agents
  - the purpose of the Pigouvian tax is to **internalize** that externality by imposing a tax on the action commensurate with the external cost

10

The Pigouvian Tax

- In the context of the pollution problem, the tax is set equal to marginal damage.

11

The Pigouvian Tax

- Let us now explore the details of how a Pigouvian tax on pollution works.
- We begin with a setting in which there is just one pollution source (such as the steel plant from Topic 3).

12

The Pigouvian Tax

- We will also assume that the MAC for the source is equal to true social MAC.
- This may not be true when there are other distortions in the market but we will abstract from that complication here; see Topic 6.8 for more detail.

13

A Tax on Pollution for a Single Source

- Recall from Topic 3 that the **private optimum** for the steel plant is $\hat{e}$, such that

$$MAC(\hat{e}) = 0$$

- The plant will choose this level of pollution in the **unregulated outcome**; see Figure 6.1

14

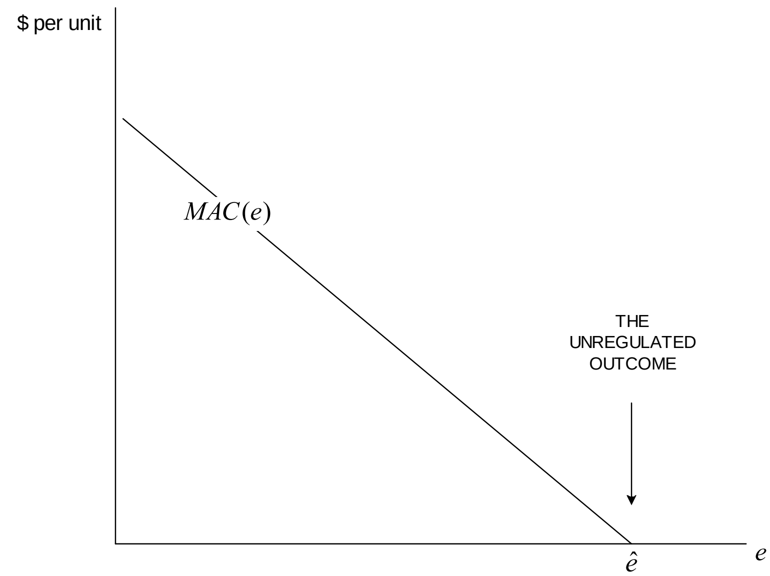


FIGURE 6-1

15

A Tax on Pollution for a Single Source

- Now suppose the regulatory authority (the "regulator") has the statutory power to levy a tax on pollution at τ dollars per ton.
- Thus, if the plant emits e tons of the pollutant then its **total tax payment** is

$$T = \tau e$$

16

A Tax on Pollution for a Single Source

- The plant will respond to this pollution tax by assessing whether it is cheaper to pay the tax on a given unit of pollution or to cut that unit of pollution and incur the associated abatement cost instead.

17

A Tax on Pollution for a Single Source

- This assessment on each unit of pollution will lead the plant to emit a quantity $e(\tau)$ such that

$$MAC(e(\tau)) = \tau$$

- See Figure 6-2.

18

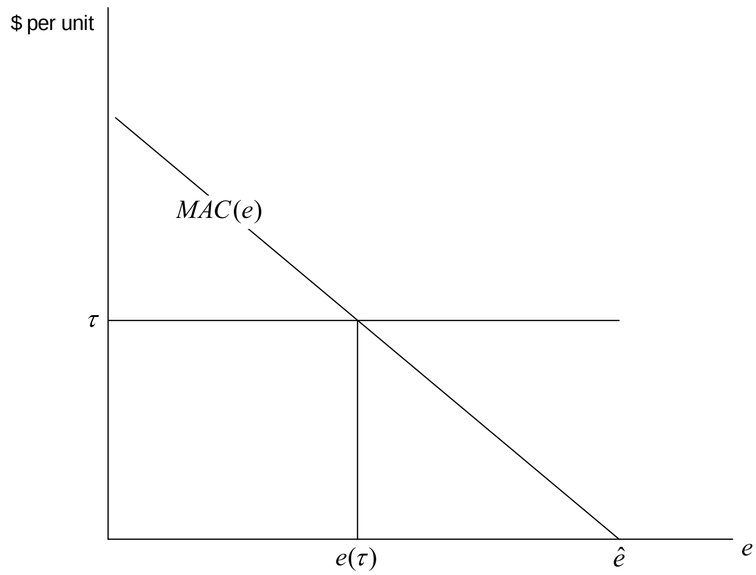


FIGURE 6-2

19

A Tax on Pollution for a Single Source

- To understand the logic of this response, suppose the plant initially chooses a higher quantity of emissions, $e_0 > e(\tau)$.
- At e_0, the plant could reduce its emissions to $e(\tau)$ and thereby reduce its total tax payment by more than the associated increase in abatement cost; see Figures 6-3 through 6-5.

20

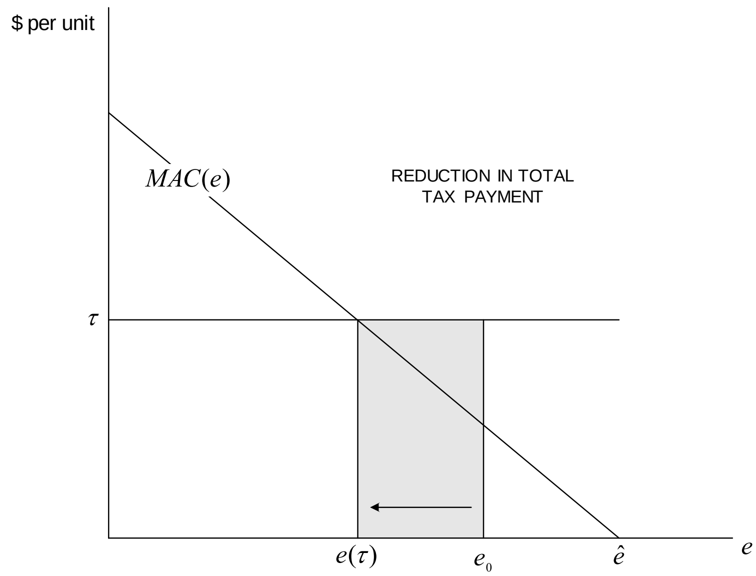


FIGURE 6-3 21

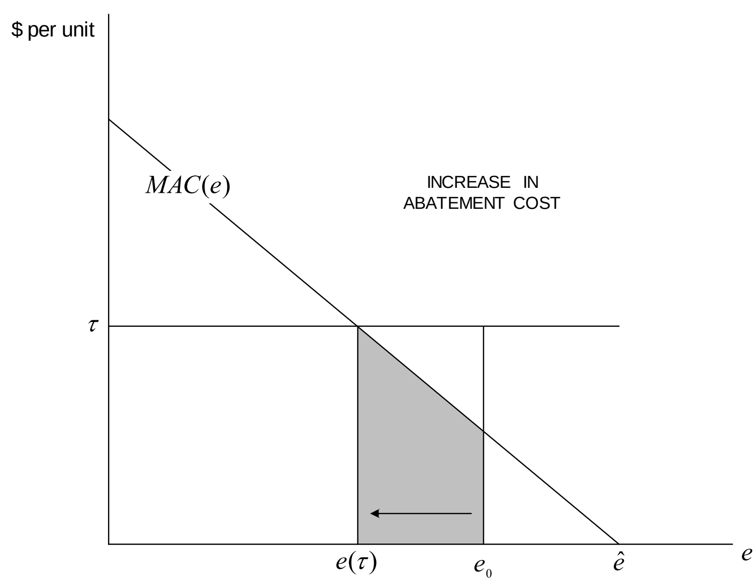


FIGURE 6-4 22

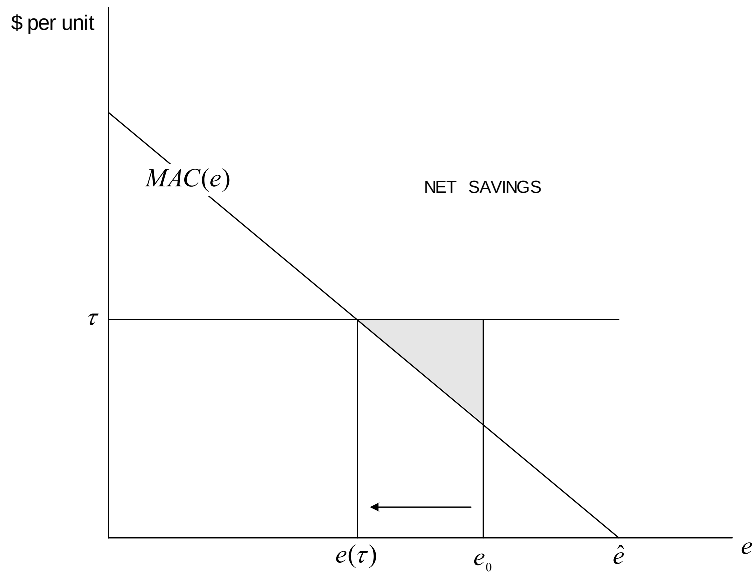


FIGURE 6-5

23

A Tax on Pollution for a Single Source

- Thus, if the plant is initially emitting e_0 it could profit by reducing emissions to $e(\tau)$.

24

A Tax on Pollution for a Single Source

- Similarly, if the plant initially chooses a lower quantity of emissions, $e_0 < e(\tau)$, then it could raise its emissions to $e(\tau)$ and thereby reduce its abatement cost by more than the associated increase in total tax payment; see Figures 6-6 through 6-8.

25

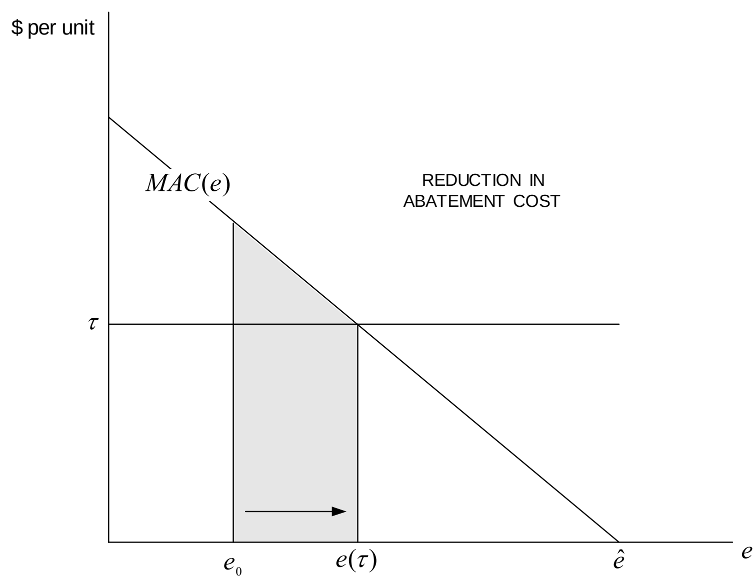


FIGURE 6-6

26

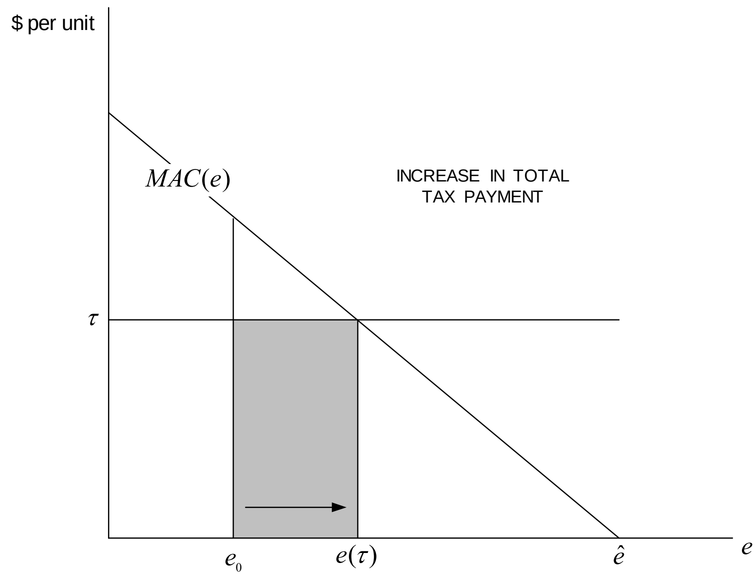


FIGURE 6-7 27

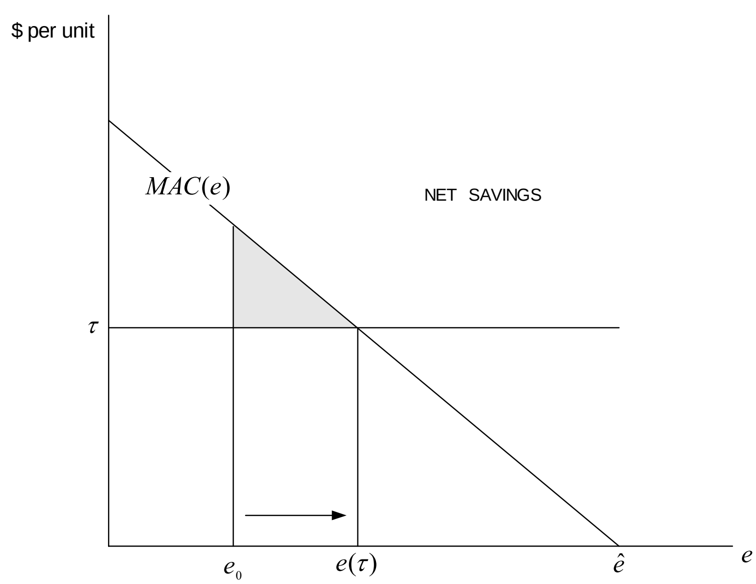


FIGURE 6-8 28

A Tax on Pollution for a Single Source

- We will refer to $e(\tau)$ as the **corrected private optimum** in the sense that this choice is privately optimal for the plant given that its incentives have been corrected by the tax.
- The Pigouvian tax is therefore also known as a corrective tax.

29

A Tax on Pollution for a Single Source

- Now that we know how the plant will respond in general to the tax, we can choose the tax rate to ensure that the corrected private optimum implements the policy goal.

30

A Tax on Pollution for a Single Source

- Recall from Topic 3 that the socially optimal emissions level is e^* such that

$$MAC(e^*) = MD(e^*)$$

- And we know that the plant chooses $e(\tau)$ such that

$$MAC(e(\tau)) = \tau$$

31

A Tax on Pollution for a Single Source

- Thus, $e(\tau) = e^*$ if and only if $\tau = \tau^*$, where

$$\tau^* = MD(e^*)$$

- That is, the optimal tax rate is set equal to MD evaluated at the social optimum.

- This is the **Pigouvian rule**. See Figure 6-9.

32

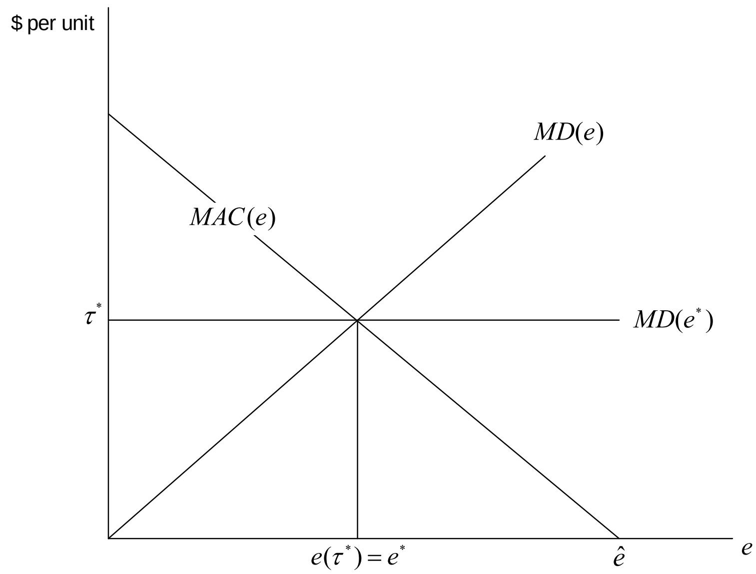


FIGURE 6-9

33

A Tax on Pollution for a Single Source

- Note that the tax paid on the marginal unit of emissions is just equal to the MD caused by that unit of emissions.
- It is in this sense that the Pigouvian tax internalizes the externality.

34

A Tax on Pollution for a Single Source

- The total tax paid by the plant at the corrected private optimum is

$$T^* = \tau^* e(\tau^*)$$

- See Figure 6-10.

35

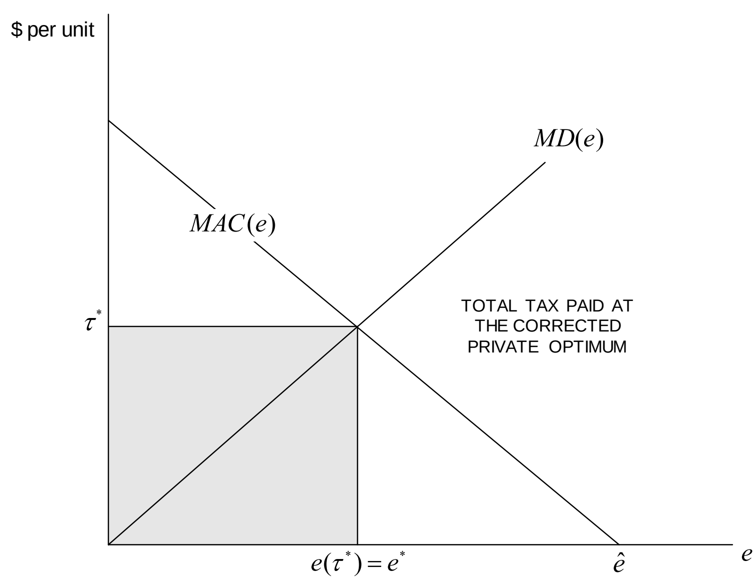


FIGURE 6-10

36

A Tax on Pollution for a Single Source

- In addition, the plant incurs the cost of abatement associated with reducing emissions from $\hat{e}$ to e^*; see Figure 6-11.

37

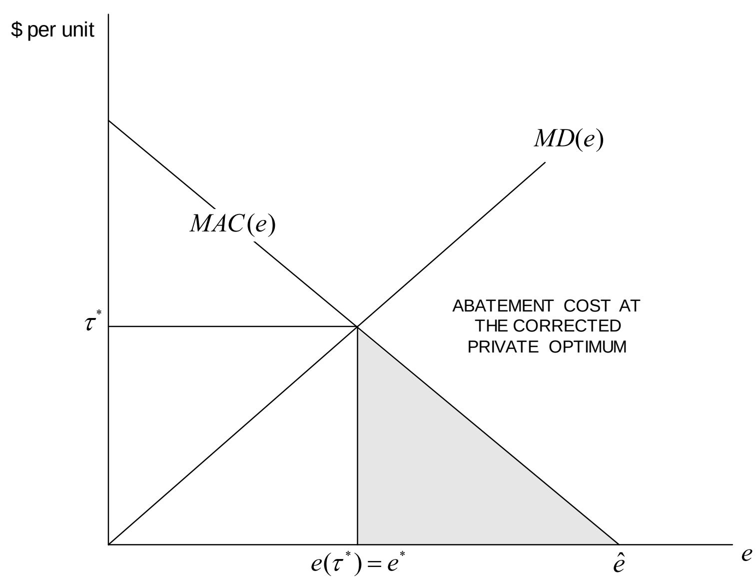


FIGURE 6-11

38

A Tax on Pollution for a Single Source

- The sum of these amounts is called the **compliance cost** of the tax policy for the plant; see Figure 6-12.

39

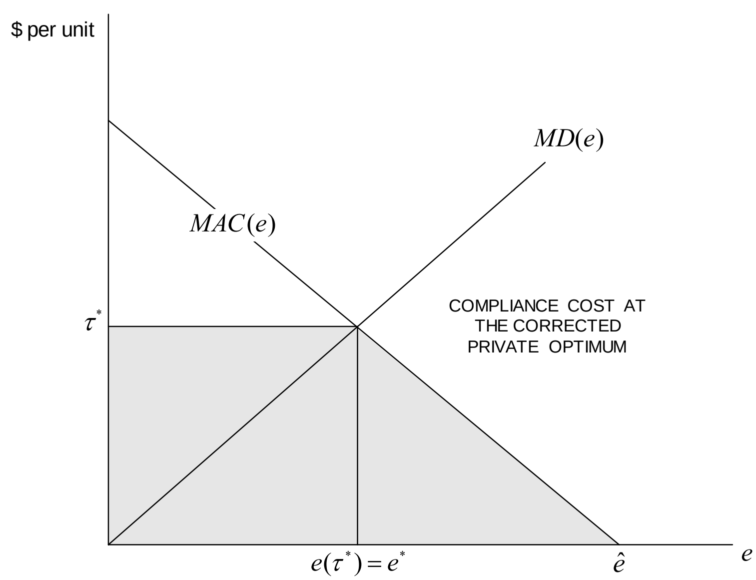


FIGURE 6-12

40

A Tax on Pollution for a Single Source

- The distinction between the compliance cost and the total tax paid is an important one.
- It is common among non-economists to think of the cost of the regulation for the plant as simply the total tax paid on emissions, but this understates the full cost.
- The full cost includes the cost of cutting emissions in response to the tax.

41

6.3 A LINEAR EXAMPLE

42

A Linear Example

- Suppose MAC for the plant is

$$MAC(e) = \gamma(\hat{e} - e)$$

- The plant will respond to a pollution tax τ by setting

$$\gamma(\hat{e} - e) = \tau$$

43

A Linear Example

- Solving this behavioral equation yields the corrected private optimum

$$e(\tau) = \hat{e} - \frac{\tau}{\gamma}$$

44

A Linear Example

- It is instructive to express this response to the tax in terms of abatement:

$$\hat{e} - e(\tau) = \frac{\tau}{\gamma}$$

- Thus, abatement is increasing in the size of the tax but decreasing in the slope of the MAC.

45

A Linear Example

- Thus, the impact of a tax of any given size depends critically on abatement cost.
- If γ is very high – implying that the plant finds it very costly to reduce emissions – then even a large tax will not induce a large reduction in emissions.
- We will return to this point in a moment.

46

A Linear Example

- The Pigouvian rule sets the tax rate equal to MD evaluated at the social optimum e^*, which we derived in Topic 3.5:

$$e^* = \left(\frac{\gamma}{\gamma+\delta}\right)\hat{e}$$

47

A Linear Example

- Thus,

$$\tau^* = \delta e^* = \delta\left(\frac{\gamma}{\gamma+\delta}\right)\hat{e}$$

48

A Linear Example

- Substituting τ^* for τ in $e(\tau)$ then yields

$$e(\tau^*) = \hat{e} - \frac{\delta\left(\dfrac{\gamma}{\gamma+\delta}\right)\hat{e}}{\gamma}$$

49

A Linear Example

- Simplifying this expression yields

$$e(\tau^*) = \left(\frac{\gamma}{\gamma+\delta}\right)\hat{e} = e^*$$

- Thus, the Pigouvian tax implements the social optimum as a corrected private optimum.

50

A Linear Example

- Recall that the tax will not cause a large reduction in emissions if γ is high.
- This does not mean the tax has failed to do its job.
- The purpose of the Pigouvian tax is not to reduce pollution *per se*; it is to reduce pollution to its socially optimal quantity.

51

A Linear Example

- If γ is very high then the socially optimal quantity of pollution is also high (for any δ), and so it is fully appropriate for the tax not to cause a substantial reduction in emissions.

52

A Linear Example

- These properties of the optimal solution are illustrated in Figure 6-13, where the Pigouvian tax and the optimal quantity of abatement are plotted, measured on the LHS and RHS vertical axes respectively, against γ on the horizontal axis.

53

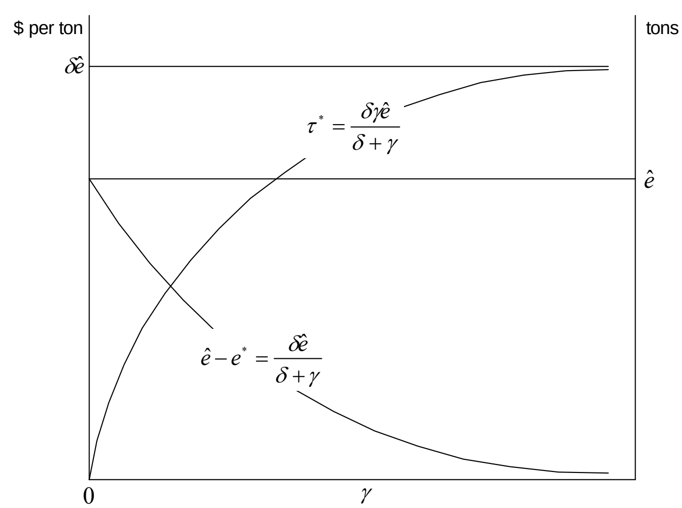


FIGURE 6-13

54

A Linear Example

- Note that $\tau^* = 0$ and $e^* = \hat{e}$ if $\gamma = 0$ (abatement is costless).

- At the opposite extreme, $\tau^* \rightarrow \delta\hat{e}$ and abatement $\rightarrow 0$ as $\gamma \rightarrow \infty$ (abatement is infinitely costly).

55

6.4 THE PIGOUVIAN TAX WITH MULTIPLE SOURCES

56

The Pigouvian Tax with Multiple Sources

- The logic of the Pigouvian tax extends straightforwardly to a case with multiple sources, at least in the case of a uniformly-mixed pollutant.

57

The Pigouvian Tax with Multiple Sources

- In that case, recall that damage is a function only of aggregate emissions, so the characteristics of any individual source are irrelevant to the damage its emissions cause; all that matters for damage is the quantity of emissions from that source.

58

The Pigouvian Tax with Multiple Sources

- Since the purpose of the Pigouvian tax is to internalize that damage, the tax rate should be the same for all sources.

59

The Pigouvian Tax with Multiple Sources

- For example, a ton of carbon emitted by a small firm or a poor household causes exactly the same damage as a ton of carbon emitted by a large firm or a rich household, and critically, the purpose of the carbon tax is to internalize that damage.
- Thus, they should all face the same tax rate.

60

The Pigouvian Tax with Multiple Sources

- What should that tax rate be?
- First consider a setting with two sources.
- We know from the logic of Topic 6.2 that any source faced with a tax τ on emissions will choose $e_i(\tau)$ such that

$$MAC_i(e_i(\tau)) = \tau$$

61

The Pigouvian Tax with Multiple Sources

- Moreover, we know from Topic 5 that the social optimum when there are two sources is e_1^* and e_2^* such that

$$e_1^* + e_2^* = E^*$$

and

$$MAC_1(e_1^*) = MAC_2(e_2^*) = MD(E^*)$$

62

The Pigouvian Tax with Multiple Sources

- Thus, the tax will implement the social optimum if and only if $\tau=\tau^*$, where

$$\tau^* = MD(E^*)$$

- That is, the tax is set equal to marginal damage evaluated at the social optimum; see Figure 6.14.

63

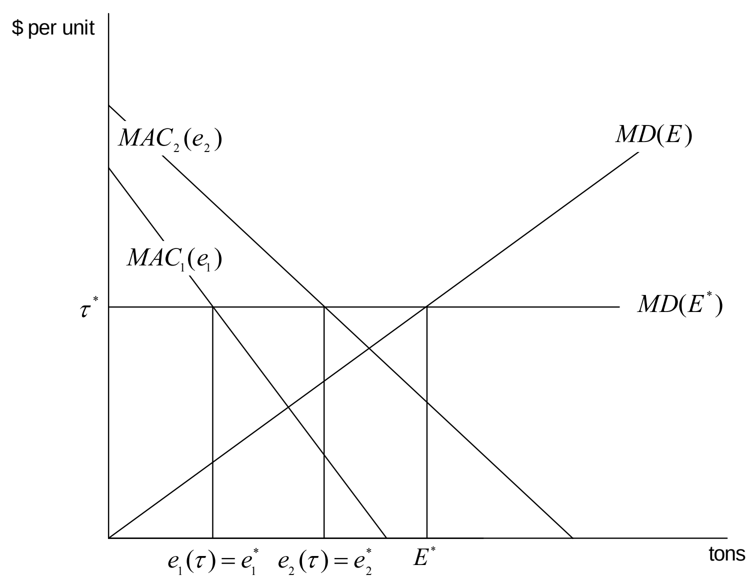


FIGURE 6-14

64

The Pigouvian Tax with Multiple Sources

- This result generalizes to a setting with any number of sources, provided the pollutant is uniformly mixed.
- In all such settings, the **Pigouvian tax** is

$$\tau^* = MD(E^*)$$

65

The Pigouvian Tax with Multiple Sources

- It is important to stress the power of the Pigouvian tax in achieving the social optimum here.
- First, the tax implements the **MACE** solution from Topic 5 because each source faces the same tax rate.

66

The Pigouvian Tax with Multiple Sources

- In particular, source *i* responds to the tax by choosing $e_i(\tau)$ such that

$$MAC_i(e_i(\tau^*)) = \tau^*$$

- Thus, for any two sources *j* and *k* we have

$$MAC_j(e_j(\tau^*)) = \tau^* = MAC_k(e_k(\tau^*))$$

67

The Pigouvian Tax with Multiple Sources

- That is, MACs are equated across sources, because each source faces the same tax rate.

- Thus, we know that the aggregate abatement induced by the tax is achieved at the lowest possible cost.

68

The Pigouvian Tax with Multiple Sources

- Second, the tax rate is equal to $MD(E^{*})$, so

$$MAC_j(e_j(\tau^*)) = MAC_k(e_k(\tau^*)) = MD(E^*) \quad \forall j,k$$

- Thus, the quantity of aggregate emissions (and hence, aggregate abatement) induced by the tax is socially optimal.

69

6.5 THE LINEAR EXAMPLE WITH MULTIPLE SOURCES

70

The Linear Example

- We have seen in Topic 5.3 that a setting with n sources is a generalization of the two-source case.

71

The Linear Example

- In particular, optimal emissions are

$$E^* = \left(\frac{\varphi}{\delta + \varphi} \right) \hat{E}$$

where

$$\varphi \equiv \left(\sum_{i=1}^{n} \frac{1}{\gamma_i} \right)^{-1}$$

72

The Linear Example

- The Pigouvian tax required to implement this outcome is $\tau^{*}=MD(E^{*})$. Thus,

$$\tau^{*} = \delta\left(\frac{\varphi}{\delta+\varphi}\right)\hat{E}$$

73

The Linear Example

- In comparison, recall the optimal tax when there is only one source:

$$\tau^{*} = \delta\left(\frac{\gamma}{\delta+\gamma}\right)\hat{e}$$

- This is just a special case of the more general setting.

74

6.6 AN ALTERNATIVE PRESENTATION*

* Advanced Topic

An Alternative Presentation

- We will sometimes find it useful to think of the optimal tax rate as equating the marginal benefit and marginal cost of the tax policy.

An Alternative Presentation

- The marginal benefit of the tax is the reduction in damage due to the induced reduction in emissions.

- The marginal cost of the tax is the increase in abatement cost due to the induced reduction in emissions.

77

An Alternative Presentation

- Let $E(\tau)$ denote aggregate emissions in response to the tax, and define the **marginal abatement response** (MAR) as the amount by which emissions fall when τ is increased by a small amount.
- Let $MAR(\tau)$ denote this marginal abatement response.

78

An Alternative Presentation

- Then the marginal benefit of the tax is

$$MB(\tau) = MAR(\tau)MD(E(\tau))$$

- Interpretation:
  – an increase in τ causes a reduction in E (as measured by MAR), and this reduction in E then causes a reduction in D (as measured by MD).

79

An Alternative Presentation

- The marginal cost of the tax is

$$MC(\tau) = MAR(\tau)MAAC(E(\tau))$$

- Interpretation:
  – an increase in τ causes a reduction in E (as measured by MAR), and this reduction in E then causes an increase in abatement cost (as measured by MAAC).

80

An Alternative Presentation

- We know that each source responds to the tax by equating its own MAC to the tax rate, and this means that aggregate emissions in response to the tax, denoted $E(\tau)$, must satisfy

$$MAAC(E(\tau)) = \tau$$

81

An Alternative Presentation

- Thus, we can write the marginal cost of the tax as

$$MC(\tau) = MAR(\tau)\tau$$

- The optimal tax is τ^*, where

$$MB(\tau^*) = MC(\tau^*)$$

82

An Alternative Presentation

- Making the substitutions, and noting that *MAR*(τ) can be divided out of both sides, this **optimal tax rule** becomes

$$MD(E(\tau)) = \tau$$

83

An Alternative Presentation

- In the linear example we know that source *i* responds to the tax by setting

$$e_i(\tau) = \hat{e}_i - \frac{\tau}{\gamma_i}$$

84

An Alternative Presentation

- Thus, aggregate emissions in response to the tax are

$$E(\tau) = \sum_{i=1}^{n} e_i(\tau) = \hat{E} - \tau \sum_{i=1}^{n} \frac{1}{\gamma_i}$$

85

An Alternative Presentation

- If the MD schedule is $MD(E)=\delta E$, then we can express this MD in terms of τ:

$$MD(\tau) = \delta E(\tau) = \delta\left(\hat{E} - \frac{\tau}{\varphi}\right)$$

where

$$\varphi \equiv \left(\sum_{i=1}^{n} \frac{1}{\gamma_i}\right)^{-1}$$

86

An Alternative Presentation

- Hence, the optimal tax rule is

$$\delta\left(\hat{E}-\frac{\tau}{\varphi}\right)=\tau$$

- See Figure 6-15.

87

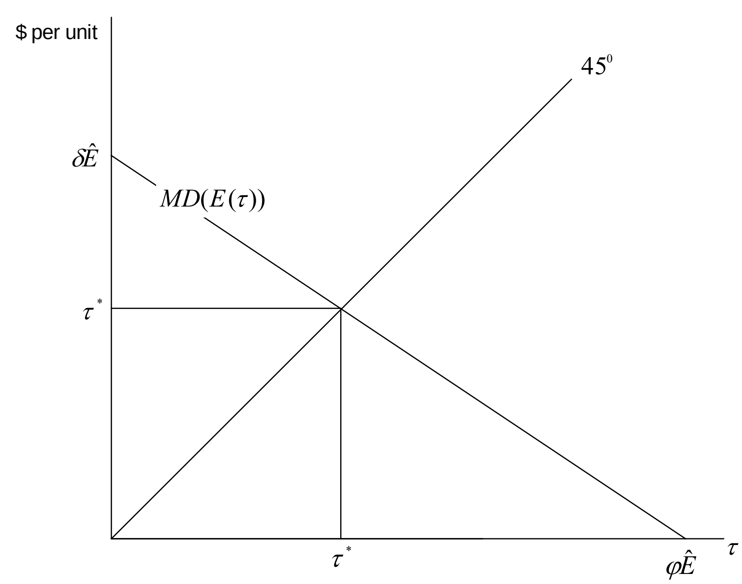


FIGURE 6-15

88

An Alternative Presentation

- Solving for the optimal τ yields

$$\tau^* = \left(\frac{\delta\varphi}{\delta + \varphi} \right) \hat{E}$$

- This of course is exactly the same solution we derived earlier.

89

6.7 PROPERTY RIGHTS AND THE POLLUTER-PAYS PRINCIPLE

90

Property Rights and the Polluter-Pays Principle

- The primary purpose of the Pigouvian tax is to correct incentives: it internalizes the external cost of pollution and thereby causes the polluting source to choose the social optimum.

Property Rights and the Polluter-Pays Principle

- A distinct but inseparable property of the Pigouvian rule is its embodiment of the so-called “polluter-pays principle”.

- Recall from Topic 2.8 the discussion of property rights in the context of a polluting plant and a farm exposed to its emissions.

Property Rights and the Polluter-Pays Principle

- We argued that in principle, a contract could be signed between the plant and the farmer if property rights are well-defined.

- In particular, if the farmer has the right not to be harmed then the plant would have to pay the farmer for being allowed to emit.

93

Property Rights and the Polluter-Pays Principle

- Conversely, if the plant has the right to emit then the farmer must pay the plant for any abatement it undertakes.
- The Pigouvian tax policy effectively assigns property rights to the state:
  - polluters must pay the state for being allowed to emit.

94

Property Rights and the Polluter-Pays Principle

- Indeed, we can think of the tax policy as a contract between the state and the polluters, in which polluters buy emission rights from the state at a "price" of τ dollars per unit.

95

The Tax as an Emissions Trading Price

- To pursue this analogy further, think of $MD(E)$ as the supply of emissions rights, reflecting the willingness-to-accept (WTA) of the state (representing its citizens) to give up those rights.

96

The Tax as an Emissions Trading Price

- Similarly, think of $MAAC(E)$ as the demand for emissions rights, reflecting the willingness-to-pay (WTP) for those rights by the polluters (as measured by their abatement costs avoided).

97

The Tax as an Emissions Trading Price

- The equilibrium price – where supply and demand are equal – is the Pigouvian tax rate.

98

The Tax as an Emissions Trading Price

- We can also construct useful analogies to "producer surplus" and "consumer surplus" in this market for emissions rights.

99

The Tax as an Emissions Trading Price

- The "producer" is the supplier of emissions rights (the state), and the producer surplus in this context is a **resource rent**.
- This reflects the idea – discussed in Section 1.2 – that the capacity of the environment to accept waste is a resource.
- Here the state is selling the right to use some of that capacity.

100

The Tax as an Emissions Trading Price

- This resource rent to the state is illustrated in Figure 6-16.

101

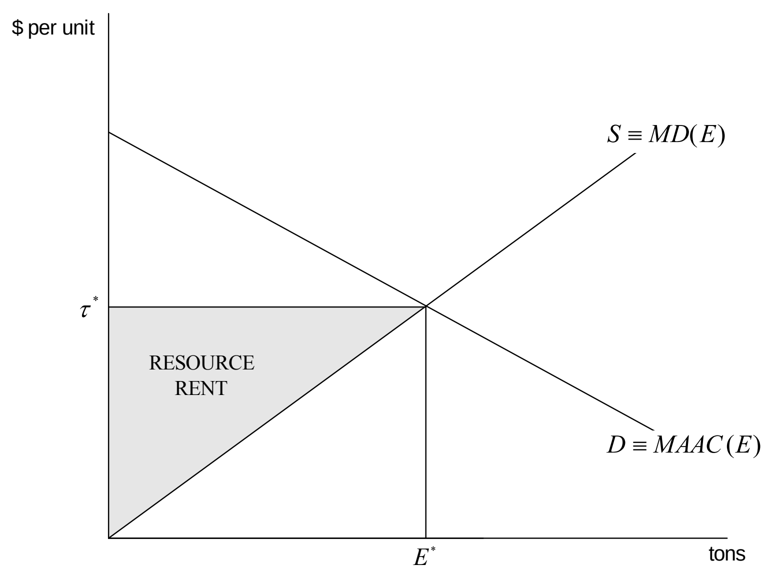


FIGURE 6-16

102

The Tax as an Emissions Trading Price

- Note that this producer surplus measure is constructed in the usual way:
  - it measures the difference between the payment actually received for the E^* units sold (the total tax payment by polluters), and the minimum amount the state would have been willing to accept to supply those units, as measured by the damage caused by E^* (the area under the MD schedule).

103

The Tax as an Emissions Trading Price

- The key point here is worth re-emphasizing:
  - the total tax payment made by polluters exceeds the damage caused by those emissions, and the difference is the resource rent collected by the state.
- Thus, the total tax payment comprises two parts, as depicted in Figure 6-17.

104

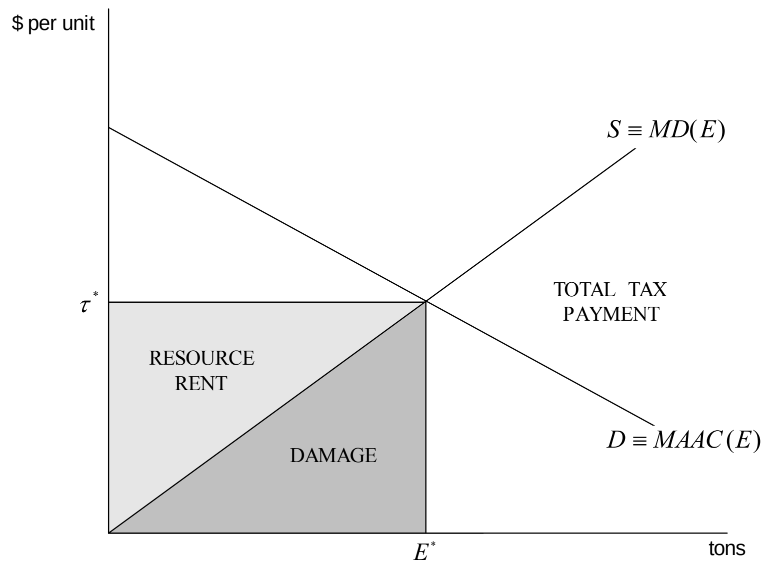


FIGURE 6-17

105

The Tax as an Emissions Trading Price

- It is sometimes argued that this “excessive” tax payment – relative to the actual damage caused – is a problem with the Pigouvian tax, or that polluters are being charged more than is “fair”.

106

The Tax as an Emissions Trading Price

- However, suppliers receive producer surplus in any trade setting in which supply is upward-sloping, and there is no reason to think that trade in emission rights should be treated as exceptional.
- Producers do not give away their producer surplus, and it is not clear why the state should give away its resource rent.

107

The Tax as an Emissions Trading Price

- Just as the state receives producer surplus from the trade in emissions rights, the buyers of those rights receive consumer surplus.
- This consumer surplus to polluters is illustrated in Figure 6-18.

108

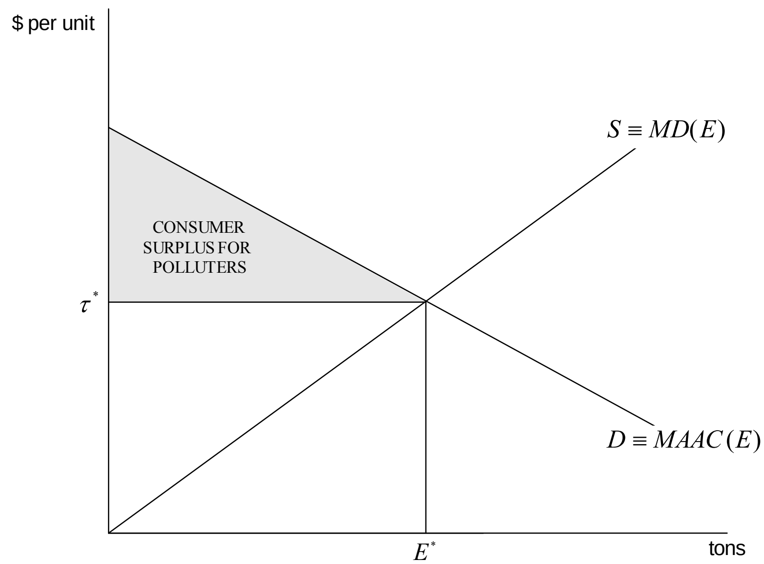


FIGURE 6-18

109

The Tax as an Emissions Trading Price

- This consumer surplus measure is constructed in the usual way:
  – it is the difference between the amount the buyers of emission rights would have been willing to pay, as measured by the abatement cost avoided (the area under *MAAC*(*E*) from 0 to E^*), and the amount they actually paid (the total tax payment)

110

The Tax as an Emissions Trading Price

- Note that the state could in principle exploit its monopoly position as sole supplier of emission rights, and set a price higher than τ, thereby extracting some of the consumer surplus from polluters.

111

The Tax as an Emissions Trading Price

- However, the proper role of the state is to maximize <u>total</u> surplus – not is own resource rent – because the consumer surplus to polluters is eventually dispersed to the citizens who own polluting firms or reside in polluting households.

112

Property Rights and Politics

- The entanglement of incentive correction and the assignment of property rights in pollution tax policy points to why the politics of environmental policy can be so contentious.

113

Property Rights and Politics

- For example, a polluting plant might not object to pollution pricing if it owns the emissions rights, but might understandably lobby against a Pigouvian tax policy that implicitly assigns those rights to the state.

114

Property Rights and Politics

- In general, it is critical to understand both the incentive effects and the distributional implications of environmental policy options, and we will continue to emphasize this throughout the course.

115

6.8 ADJUSTMENTS TO THE PIGOUVIAN RULE*

* Advanced Topic

116

Adjustments to the Pigouvian Rule

- The Pigouvian rule – where the tax rate is set equal to $MD(E^*)$ – is derived under the assumption that the MAC for the source is equal to true social MAC.
- In some settings, this may not be true, and an adjustment to the tax rate may be needed to reflect this.

117

Monopoly

- As an example, consider a setting with a single source, and where that source is a monopoly in its product market.

118

Monopoly

- Suppose emissions-intensity is fixed and equal to one, so cutting output is the only way to reduce emissions.
- This means that output and emissions are effectively synonymous here, so we can express abatement cost and damage in terms of output, x.

119

Monopoly

- The profit-maximizing strategy for the monopolist is to produce where marginal cost and marginal revenue are equated.
- This monopoly solution is denoted x^M in Figure 6-19, where $p(x)$ denotes market demand, and $MR(x)$ denotes marginal revenue.

120

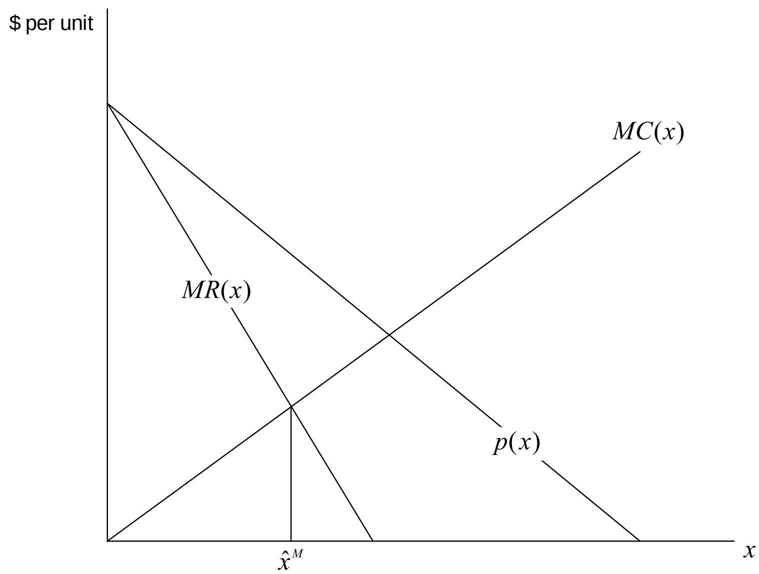


FIGURE 6-19

121

Monopoly

- The marginal <u>private</u> net benefit from production for the monopolist is

$$MNPB(x) = MR(x) - MC(x)$$

- This is the marginal <u>private</u> abatement cost for the monopolist in this fixed-intensity setting.

122

Monopoly

- In contrast, the marginal <u>social</u> net benefit from production – leaving aside the damage from pollution – is

$$MNSB(x) = p(x) - MC(x)$$

- This is the marginal <u>social</u> abatement cost in this fixed-intensity setting.

123

Monopoly

- What is the source of this difference?
- The demand schedule measures the WTP of consumers, and hence reflects the marginal social benefit of production.
- However, the monopolist cannot extract that WTP unless it can use a perfectly discriminating pricing strategy, setting a different price for each unit sold.

124

Monopoly

- If it is constrained to set a single price (due to the possibility of *ex post* arbitrage among consumers) then it sells only to those consumers with a relatively high WTP.

125

Monopoly

- This means that there are consumers who are willing to pay the marginal cost of production, but it is not profitable for the monopolist to sell to those consumers since this would require that it reduce its price for all consumers.
- Thus, output that would have positive net social benefit is nonetheless not produced.

126

Monopoly

- Figure 6-20 depicts *MNPB*(x) alongside *MNSB*(x) – rescaled relative to Figure 6-19 – together with the marginal damage function (expressed in terms of x).
- The social optimum is x^*, where

$$MNSB(x^*) = MD(x^*)$$

127

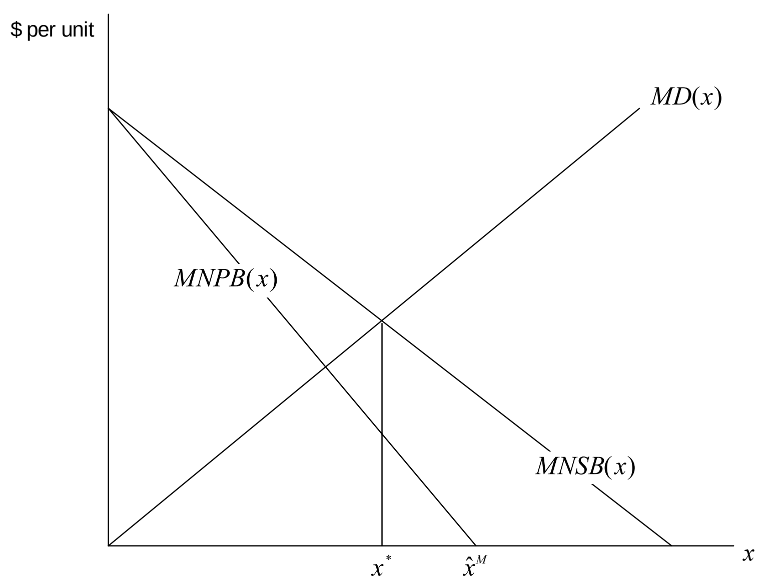


FIGURE 6-20

128

Monopoly

- Now suppose we attempt to implement this social optimum with a tax on emissions (a tax on output in this fixed-intensity setting), and the tax is set according to the Pigouvian rule:

$$\tau^* = MD(x^*)$$

129

Monopoly

- The response of the monopolist is to equate *MNPB*(*x*) to the tax rate, and therefore choose $x(\tau^*)$, as illustrated in Figure 6-21.

130

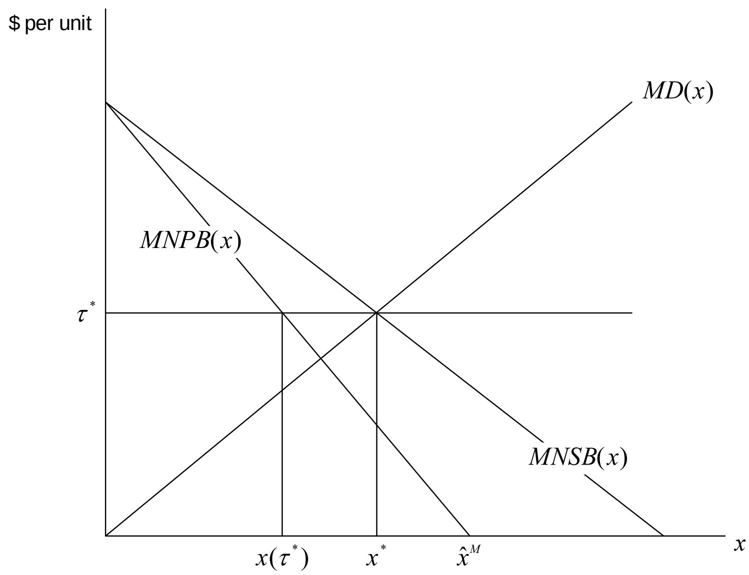


FIGURE 6-21

131

Monopoly

- It is clear from Figure 6-21 that the Pigouvian tax rule does <u>not</u> implement the social optimum here.
- In response to the tax, the monopolist chooses an output <u>lower</u> than the social optimum.
- Why?

132

Monopoly

- The monopolist undervalues output from a social perspective – leading to the usual under-production distortion in monopoly – and therefore over-responds to the tax.

133

Monopoly

- Implementation of the social optimum in this setting requires a tax rate lower than the Pigouvian tax rate, denoted τ^{**}, such that $x(\tau^{**})=x^*$, where

$$MNPB(x(\tau^{**})) = \tau^{**}$$

- See Figure 6-22.

134

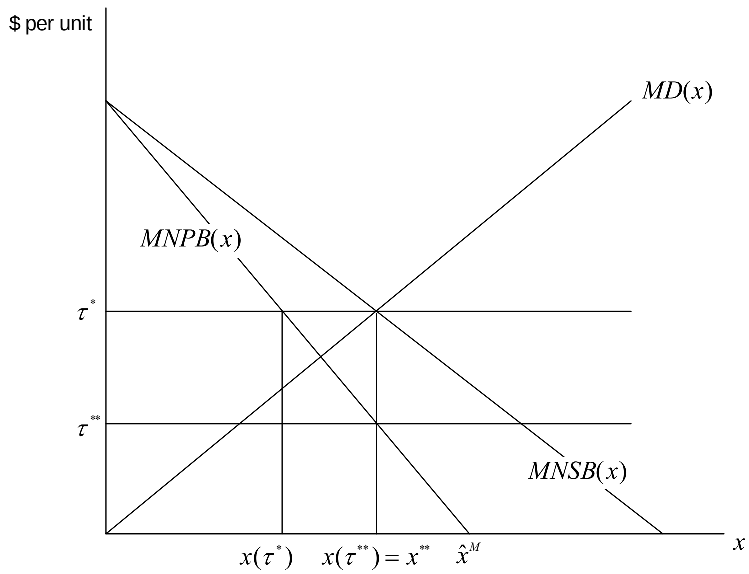


FIGURE 6-22

135

Monopoly

- In this particular setting – with fixed emissions-intensity – the adjusted pollution tax can implement the social optimum.
- In more general settings, this is typically not possible.

136

Monopoly

- As a general rule, achieving the social optimum in the presence of multiple distortions requires as many policy instruments as there are distortions.
- As an analogy, imagine trying to hit two targets with a single arrow; it is only possible if those targets happen to be perfectly aligned.

137

Monopoly

- In settings where emissions-intensity is not fixed – where emissions and output are not rigidly aligned – the pollution tax cannot achieve the social optimum, and the adjustment must strike a balance between exacerbating the monopoly under-production problem on one hand, and internalizing the pollution externality on the other .

138

Other Distortions

- The monopoly distortion is just one example of instances where marginal private abatement cost (MPAC) and marginal social abatement cost (MSAC) can differ.
- Less extreme cases of imperfect competition can yield a similar distortion.

139

Other Distortions

- In other instances, intensity-reduction measures undertaken by a source may have their own associated unpriced externalities.
- For example, a tax on airborne emissions might induce a plant to capture those emissions using a fluid-based filtration system, leading to the discharge of polluted effluent.

140

Other Distortions

- If that effluent is not also properly taxed, then the private cost of cutting emissions will understate the true social cost of cutting those emissions.

141

Other Distortions

- Factor market distortions can also be important, as for example where labor-market frictions prevent wages from adjusting when a pollution tax causes the contraction of an entire industry (as with the impact of a carbon tax on the coal industry).

142

Other Distortions

- The array of potential distortions in an economy means that making optimal adjustments to the Pigouvian rule can be extremely complicated.

- In general, making such adjustments is <u>not</u> the best policy.

143

Other Distortions

- The best policy is to address those other distortions directly, with policies targeted specifically at them.
- For example, competition policy should be used to address a monopoly distortion; labor market policy should be used to address labor market frictions; etc.

144

Other Distortions

- Addressing other distortions with targeted policies then allows the pollution tax to specifically target the pollution externality, as it is designed to do.
- Making adjustments to the Pigouvian rule should be viewed as a “second-best” approach for instances where properly targeted alternative policies are unavailable.

END

TOPIC 6 REVIEW QUESTIONS

It is recommended that you study again the worked example from Topic 5 Review Questions prior to answering these questions (especially for Questions 13 – 31). It contains a guide to answering these questions.

1. The primary goal of a Pigouvian tax on pollution is to
A. raise revenue.
B. reduce emissions.
C. internalize the external cost of that pollution.
D. transfer property rights from polluters to the state.

2. A polluting source responds to a pollution tax by reducing emissions until its abatement cost is just equal to the amount of tax paid.
A. True.
B. False.

3. The Pigouvian rule sets the tax rate equal to marginal damage evaluated at the private optimum.
A. True.
B. False.

Questions 4 to 6 relate to the following information. There is a single pollutant source with marginal abatement cost given by

$$MAC(e) = 100 - 20e$$

The marginal damage schedule is

$$MD(e) = 5e$$

4. If this source faces a tax rate $\tau = 40$ then its emissions are

A. 1

B. 2

C. 3

D. 4

5. The Pigouvian tax rate is

A. 5

B. 20

C. 25

D. 10

6. If this source faces the Pigouvian tax rate then its compliance cost is

A. 35

B. 125

C. 90

D. 65

Questions 7 to 9 relate to the following information. There is a single pollutant source with marginal abatement cost given by

$$MAC(e) = 90 - 15e$$

The marginal damage schedule is

$$MD(e) = \delta e$$

7. If this source faces a tax rate τ then its emissions are

A. $e(\tau) = 6 - \dfrac{\tau}{15}$

B. $e(\tau) = 90 - 15\tau$

C. $e(\tau) = \dfrac{90 - 15\tau}{\tau}$

D. None of the above.

8. The Pigouvian tax rate is

A. δ

B. $\dfrac{90\delta}{15 + \delta}$

C. $100 + \delta$

D. $\dfrac{90}{15 + \delta}$

9. Compliance cost for this source under the Pigouvian tax rate is

A. $\dfrac{90\delta}{(15 + \delta)^2}$

B. $(100 + \delta)\delta$

C. $\dfrac{270\delta(\delta + 30)}{(15 + \delta)^2}$

D. None of the above

10. In a setting with multiple sources of a uniformly mixed pollutant, the Pigouvian tax rate

A. is the same for all sources.

B. differs across sources according their relative marginal abatement costs.

C. differs across sources according to their relative contribution to aggregate emissions.

D. differs across sources in a manner that ensures that abatement costs are equated across those sources.

11. In a setting with multiple sources of a uniformly mixed pollutant, the Pigouvian rule sets the tax rate for each source at marginal damage evaluated at the socially optimal emissions level for that source.

A. True.

B. False.

12. The Pigouvian tax rule implements the **MACE** rule.

A. True.

B. False.

Questions 13 to 18 relate to the following information. There are two sources of a uniformly mixed pollutant. Marginal abatement cost for source 1 is given by

$$MAC_1(e_1) = 20 - e_1$$

and marginal abatement cost for source 2 is given by

$$MAC_2(e_2) = 24 - 3e_2$$

Marginal damage is given by

$$MD(E) = E$$

where $E = e_1 + e_2$.

13. For $E > 4/3$, the $MAAC(E)$ schedule is

A. $44 - 4E$

B. $4 - 4E$

C. $2 - 2E$

D. $21 - \frac{3E}{4}$

14. The Pigouvian tax rate is

A. 12

B. 8 for source 1 and 4 for source 2

C. 4 for source 1 and 8 for source 2

D. None of the above

15. Abatement cost for source 1 under the Pigouvian tax is

A. 168

B. 72

C. 24

D. 96

16. Compliance cost for source 2 under the Pigouvian tax is

A. 168

B. 72

C. 24

D. 96

17. The resource rent collected by the state under the Pigouvian tax rate is

A. 168

B. 72

C. 24

D. 96

18. The relationship between your answers to Qs. 15 – 17 is a coincidence.

A. True.

B. False.

19. In general, the source that makes the largest percentage emissions cut in response to the Pigouvian tax incurs the highest compliance cost.

A. True.

B. False.

20. In general, the source that incurs the highest abatement cost in response to the Pigouvian tax also incurs the highest compliance cost.

A. True.

B. False.

Questions 21 – 30 relate to the following information. There are two sources of a uniformly mixed pollutant. Marginal abatement cost for source 1 is given by

$$MAC_1(e_1) = 90 - 3e_1$$

and marginal abatement cost for source 2 is given by

$$MAC_2(e_2) = 70 - 2e_2$$

Marginal damage is given by

$$MD(E) = 4E$$

where $E = e_1 + e_2$.

21. For $E > 20/3$, the *MAAC*(*E*) schedule is

A. $\frac{7E}{5}$

B. $\frac{25}{2} - 5E$

C. $115 - 5E$

D. $78 - \frac{6E}{5}$

22. The Pigouvian tax rate is

A. 20

B. 40

C. 60

D. 80

23. Relative to its unregulated emissions level, source 1 cuts emissions (in response to the Pigouvian tax) by

A. 41%

B. 67%

C. 77%

D. 86%

24. Relative to its unregulated emissions level, source 2 cuts emissions (in response to the Pigouvian tax) by

A. 41%

B. 67%

C. 77%

D. 86%

25. Relative to the unregulated emissions level, the Pigouvian tax causes aggregate emissions to fall by

A. 41%

B. 67%

C. 77%

D. 86%

26. Abatement cost for source 1 under the Pigouvian tax is

A. 600

B. 760

C. 480

D. 240

27. Abatement cost for source 2 under the Pigouvian tax is

A. 480

B. 900

C. 1300

D. 600

28. Compliance cost for source 1 under the Pigouvian tax is

A. 1700

B. 760

C. 1200

D. 1360

29. Compliance cost for source 2 under the Pigouvian tax is

A. 1700

B. 760

C. 1200

D. 1360

30. The resource rent collected by the state under the Pigouvian tax rate is

A. 1200

B. 1300

C. 670

D. 450

ANSWER KEY

1. C
2. B
3. B
4. C
5. B
6. C
7. A
8. B
9. C
10. A
11. B
12. A
13. D
14. A
15. B
16. B
17. B
18. A
19. B
20. B
21. D
22. C
23. B*
24. D*
25. C*
26. A
27. B
28. C
29. C
30. D

* See Calculations below.

Q23

The percentage change in emissions is

$$\left(\frac{\hat{e}_1 - e_1(\tau)}{\hat{e}_1}\right)100 = \left(\frac{30-10}{30}\right)100 = 66.\dot{6} \approx 67$$

Q24

The percentage change in emissions is

$$\left(\frac{\hat{e}_2 - e_2(\tau)}{\hat{e}_2}\right)100 = \left(\frac{35-5}{35}\right)100 = 85.7 \approx 86$$

Q25

The percentage change in emissions is

$$\left(\frac{\hat{E} - E(\tau)}{\hat{E}}\right)100 = \left(\frac{(30+35)-(10+5)}{30+35}\right)100 = 76.9 \approx 77$$

7. EMISSIONS TRADING

1

OUTLINE

2

* Advanced Topic

3

7.1 INTRODUCTION

4

Introduction

- A tax policy puts a price on emissions, and then allows sources to choose emissions in response to that price.

- An alternative regulatory approach is to fix the quantity of aggregate emissions directly, and allocation each source a share of that aggregate.

5

Introduction

- We will see in this topic that if these shares are tradeable then sources will trade them among themselves in a way that achieves a cost-minimizing implementation of the aggregate emissions quota.

6

Introduction

- We will also see that this trade-based approach is equivalent to the tax-based approach if the regulator knows the MACs of the individual sources.

7

Introduction

- In contrast, if the regulator does not know the individual MACs, then the two policy approaches will typically yield different outcomes, and one approach can be preferable to the other, depending on the circumstances.

8

Introduction

- In Topics 9 and 10 we will examine this question of comparative policy-performance when abatement costs are unknown.

- Our goal in this topic is simply to see how emissions trading works.

9

7.2 THE BASICS OF EMISSIONS TRADING

10

The Basics of Emissions Trading

- Suppose the regulator chooses an aggregate emissions target $\tilde{E}$, measured in tons.
- It then creates a total of $\tilde{E}$ **emission permits** (or licenses), each of which entitles the holder to emit one ton of emissions.
- In this sense, the regulator "caps" aggregate emissions.

11

The Basics of Emissions Trading

- We can think of this capped level of aggregate emissions as the **aggregate supply of permits** on the permit market, and it is fixed by construction.

12

The Basics of Emissions Trading

- We will later return to the question of how the initial allocation of permits across sources is decided, but for the most part it has no bearing on the key results that follow.

13

The Basics of Emissions Trading

- Once the initial allocations are made, permits can be traded without restriction among permit holders.

- Hence, an emissions-trading program is sometimes referred to as a **"cap and trade"** program.

14

The Basics of Emissions Trading

- If there are a sufficiently large number of sources, and the initial allocation is sufficiently dispersed across sources, then the market for permits will be perfectly competitive.

15

The Basics of Emissions Trading

- That is, each source can buy or sell as many permits as it wishes at the market price, and that price is the same for all buyers and sellers.

- Let p denote this market price for permits.

16

The Basics of Emissions Trading

- This price means that each source faces an opportunity cost of emitting:
  - a unit of emissions requires a permit, and that permit must either be purchased at price p, or retained out of the initial allocation at a foregone sale price p.
- Either way, the opportunity cost of emitting a ton of emissions is p.

17

The Basics of Emissions Trading

- In this regard, the permit price acts like a tax, and our previous analysis of the response to a tax applies equally here.
- The key difference here is that the price is determined by trading among sources rather than being set by the regulator, as the tax would be.

18

The Basics of Emissions Trading

- Let us recap the logic of how a source will respond to the permit price.
- Each source assesses whether it is cheaper to pay the permit price on a given unit of pollution or to cut that unit of pollution and incur the associated abatement cost instead.

19

The Basics of Emissions Trading

- This assessment on each unit of pollution will lead source *i* to emit a quantity $e_i(p)$ such that

$$MAC_i(e_i(p)) = p$$

- See Figure 7-1.

20

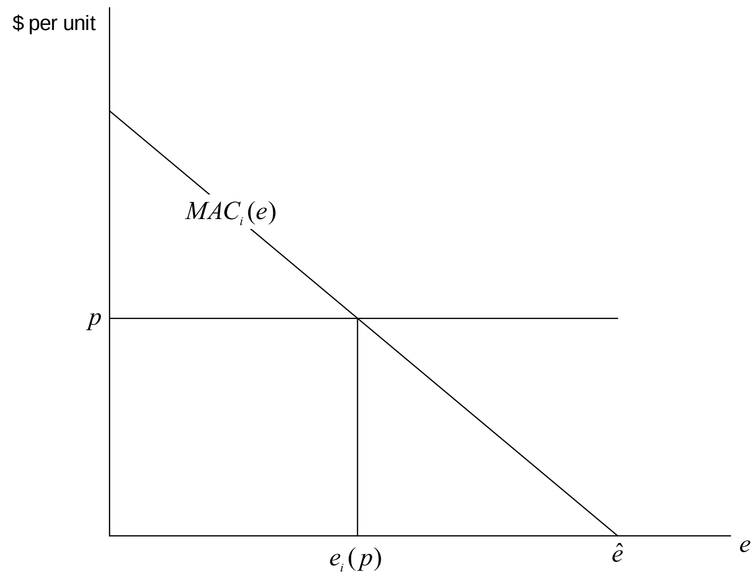


FIGURE 7-1

21

The Basics of Emissions Trading

- Note that this emissions choice in response to the permit price is independent of how many permits the source is allocated initially.
- The price of a permit is the opportunity cost of emitting regardless of whether a permit must be purchased, or the sale of a permit forgone.

22

The Basics of Emissions Trading

- The critical feature of permit trading is that all sources face the same price for permits.
- This means that for any two sources *j* and *k,* we have

$$MAC_j(e_j(p)) = p = MAC_k(e_k(p))$$

23

The Basics of Emissions Trading

- This is the most important property of the emissions trading program:
  - MACs are equated across sources, and so the aggregate emissions target is achieved at the lowest possible cost.

24

The Equilibrium Price

- Since source *i* chooses e_i such that $MAC_i(e_i(p))=p$, we can interpret the MAC schedule for that source as its **permit-demand schedule**.
- It tells us how many permits the source wishes to hold at any given permit price; recall Figure 7-1.

25

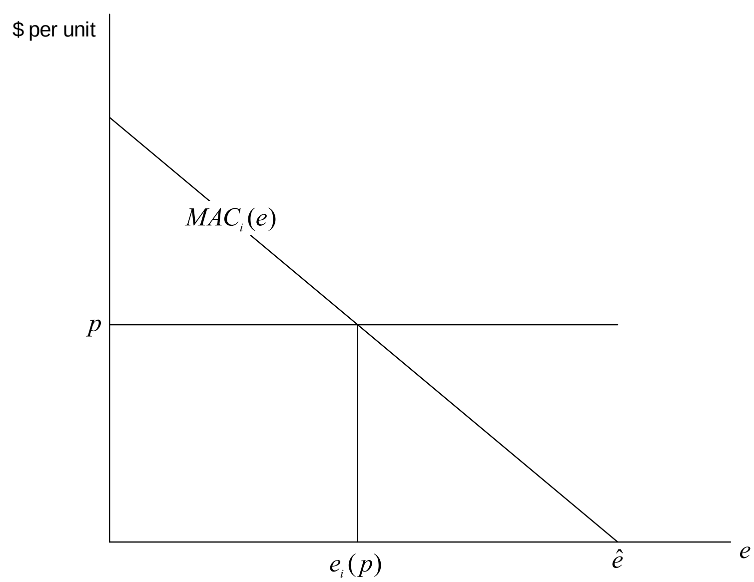


FIGURE 7-1 (repeat)

26

The Equilibrium Price

- We can then construct the **aggregate demand** for permits across *n* sources, denoted $E(p)$, as the summation of the individual demands:

$$E(p) = \sum_{i=1}^{n} e_i(p)$$

27

The Equilibrium Price

- Graphically, we can represent this aggregate demand as the $MAAC(E)$ schedule across the sources.
- Figure 7-2 depicts this horizontal summation for the case of n=2, highlighting the fact that $E(p)=e_1(p)+e_2(p)$ at any given price.

28

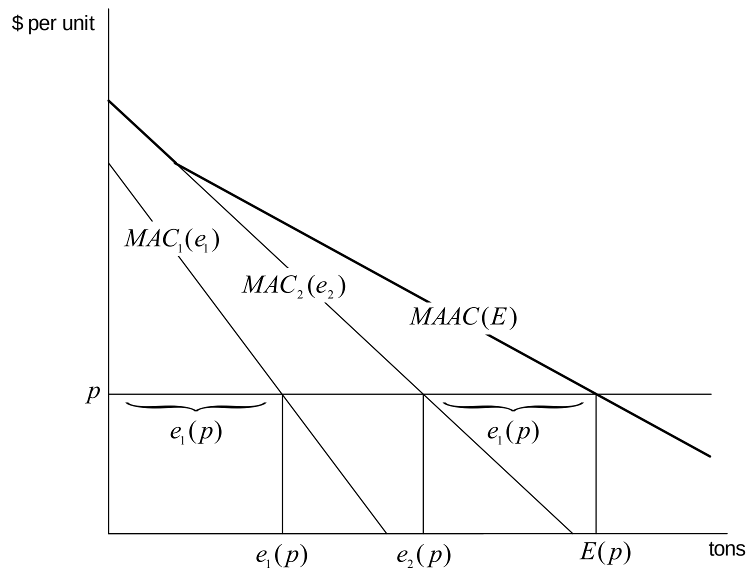


FIGURE 7-2

29

The Equilibrium Price

- The market is in equilibrium when the aggregate demand for permits is equal to the aggregate supply.
- Thus, the **equilibrium price** of permits is $\tilde{p}$, where

$$E(\tilde{p}) = \tilde{E}$$

30

The Equilibrium Price

- Since we know that

$$MAAC(E(p)) = p$$

it follows that

$$\tilde{p} = MAAC(\tilde{E})$$

- See Figure 7-3 for the case of n=2.

31

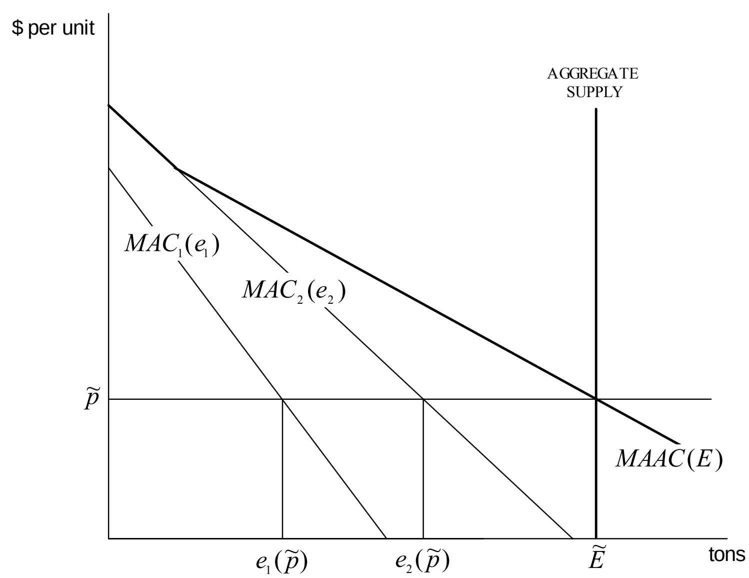


FIGURE 7-3

32

The Equilibrium Price

- Figure 7-3 summarizes the key features of the emissions trading equilibrium:
  - aggregate emissions are equal to the policy target;
  - MACs are equated across sources

33

Buyers and Sellers

- The aggregate demand for permits must be equal to the aggregate supply at the equilibrium price, but at that price, there will be buyers and sellers of permits, depending on the initial allocations.

34

Buyers and Sellers

- In particular, suppose source i has initial allocation $a_i > e_i(\tilde{p})$, then source i is a seller of permits; see Figure 7-4.

35

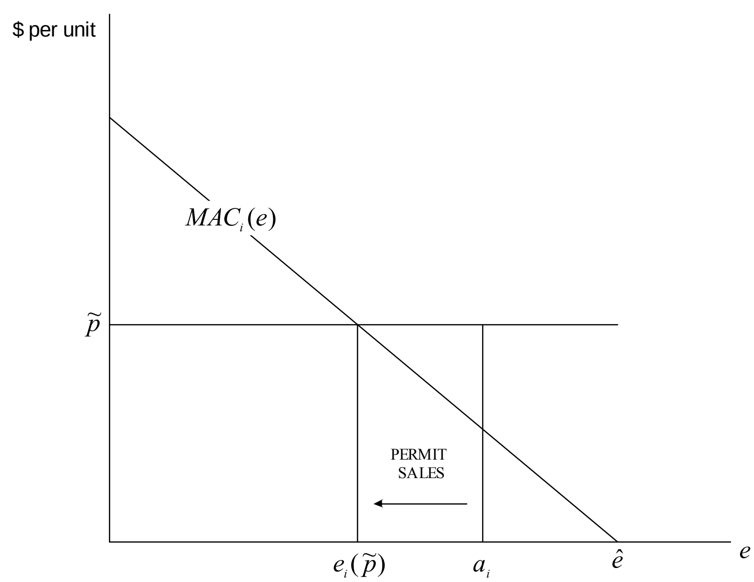


FIGURE 7-4

36

Buyers and Sellers

- Conversely, if $a_i < e_i(\tilde{p})$ then source i is a buyer of permits; see Figure 7-5.

37

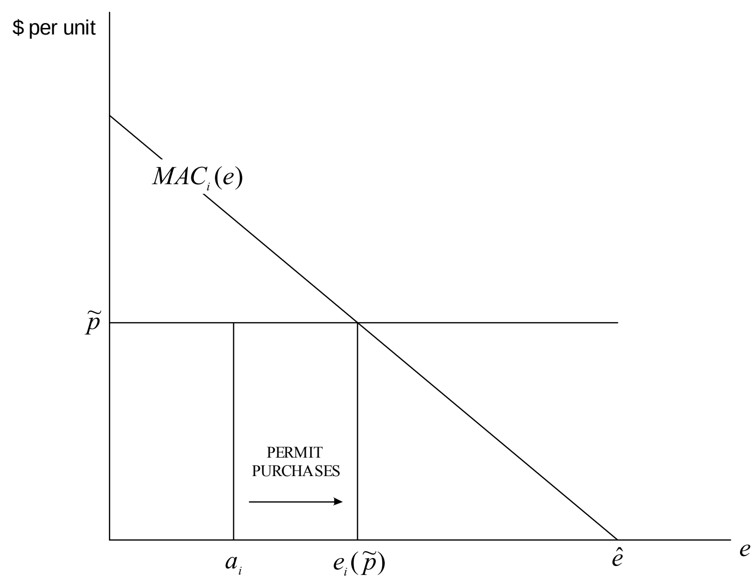


FIGURE 7-5

38

7.3 THE LINEAR CASE

39

The Linear Case

- Suppose there are n sources, and the MAC for source i is given by

$$MAC_i(e_i) = \gamma_i(\hat{e}_i - e_i)$$

40

The Linear Case

- Faced with permit price p, source i sets its emissions where

$$\gamma_i(\hat{e}_i - e_i) = p$$

41

The Linear Case

- Solving this behavioral response yields the permit-demand schedule for source i

$$e_i(p) = \hat{e}_i - \frac{p}{\gamma_i}$$

42

The Linear Case

- Aggregating across n sources yields the aggregate demand for permits:

$$E(p) = \hat{E} - p\sum_{i=1}^{n}\left(\frac{1}{\gamma_i}\right)$$

43

The Linear Case

- If the regulator issues a total of $\tilde{E}$ permits, then the equilibrium condition is

$$\hat{E} - p\sum_{i=1}^{n}\left(\frac{1}{\gamma_i}\right) = \tilde{E}$$

- See Figure 7-6.

44

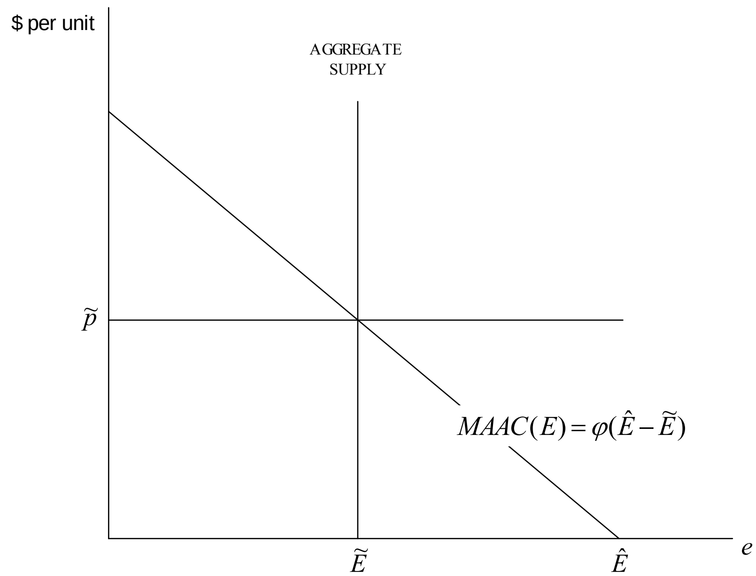


FIGURE 7-6

45

The Linear Case

- Solving for p yields

$$\tilde{p} = \varphi(\hat{E} - \tilde{E})$$

where

$$\varphi = \left(\sum_{i=1}^{n} \frac{1}{\gamma_i} \right)^{-1}$$

46

The Linear Case

- Two key properties of $\tilde{p}$:
  - it is decreasing in $\tilde{E}$: the more permits issued, the lower is the equilibrium price
  - it is increasing in φ (where φ measures the slope of the MAAC schedule): the more costly is abatement, the higher is the equilibrium price of permits

47

7.4 THE PERMIT PRICE IN RELATION TO THE OPTIMAL TAX

48

The Permit Price in Relation to an Optimal Tax

- Suppose that MAAC is known.

- Then we know that the optimal quantity of aggregate emissions is E^*, such that

$$MAAC(E^*) = MD(E^*)$$

49

The Permit Price in Relation to an Optimal Tax

- We also know that the tax rate needed to implement that optimal quantity is

$$\tau^* = MD(E^*)$$

- See Figure 7-7.

50

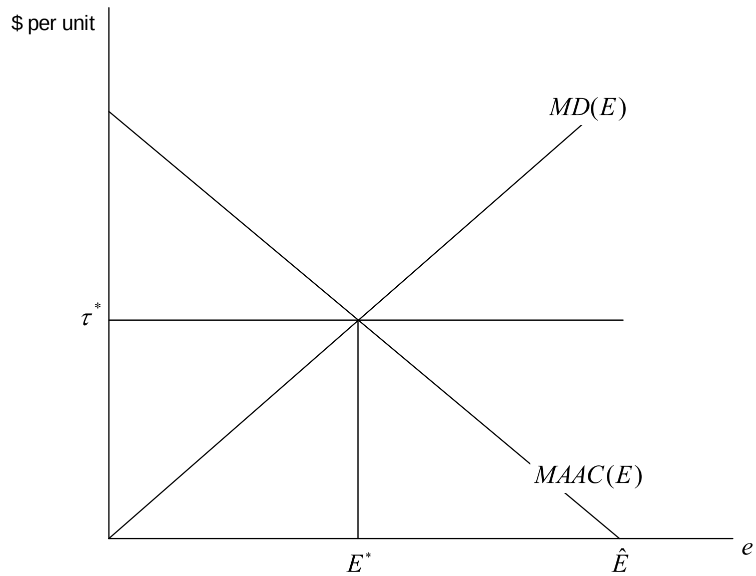


FIGURE 7-7

51

The Permit Price in Relation to an Optimal Tax

- Now suppose the regulator uses an emissions trading program instead of a tax.
- It will issue E^* permits and allow those permits to trade.

52

The Permit Price in Relation to an Optimal Tax

- We know that the equilibrium price will be

$$p^* = MAAC(E^*)$$

- See Figure 7-8.

53

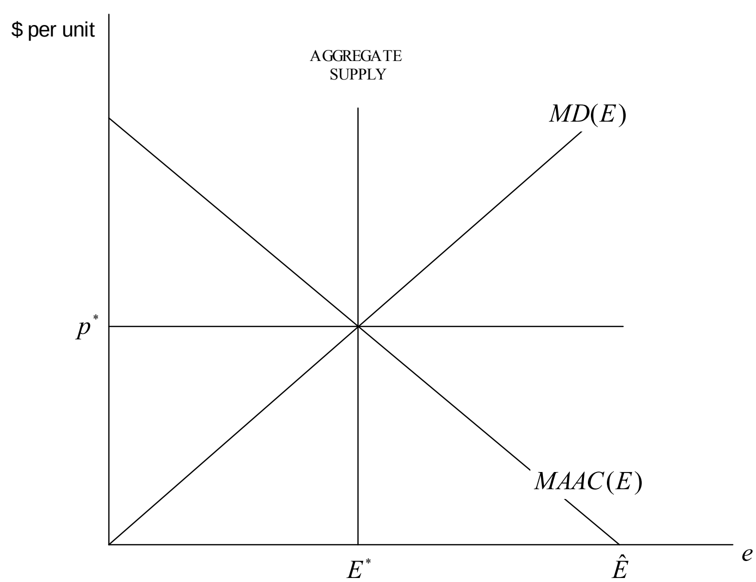


FIGURE 7-8

54

The Permit Price in Relation to an Optimal Tax

- It follows that

$$p^* = MD(E^*) = \tau^*$$

- That is, the equilibrium price is equal to the optimal tax when the emissions quantity is chosen optimally.

55

The Permit Price in Relation to an Optimal Tax

- This tells us that a pollution tax and an emissions trading program are equivalent policies when MAAC is known, at least in terms of equilibrium prices and quantities.
- However, they can differ in terms of their distributional properties, and we consider that issue next.

56

7.5 PROPERTY RIGHTS AND THE INITIAL ALLOCATION OF PERMITS

57

Property Rights and the Initial Allocation of Permits

- Recall from Section 7.2 that we put aside the question of how permits are initially allocated across sources.
- The initial allocation has no bearing on the results we have derived so far, provided that the allocation is sufficiently dispersed across sources to ensure that no source has market power in the permit market.

58

Property Rights and the Initial Allocation of Permits

- The key issue with respect to the initial allocation relates to the implied assignment of property rights.
- Recall from Topic 6 that we can think of the capacity of the environment to accept a flow of waste as a resource.

59

Property Rights and the Initial Allocation of Permits

- The use of a pollution tax implicitly assigns ownership of that resource to the state, where total tax revenue comprises two parts:
  - compensation to the owner for the cost of the resource used (environmental damage); plus
  - a resource rent
- See Figure 7-9.

60

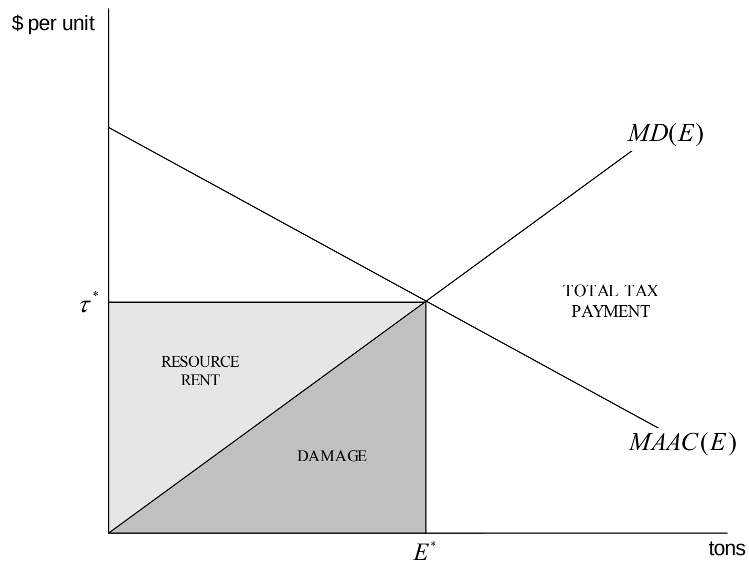


FIGURE 7-9

61

Property Rights and the Initial Allocation of Permits

- Under an emissions trading policy, the implied assignment of ownership over the resource depends critically on how the initial allocation of permits is made.
- There are two broad approaches:
  - permits must be purchased from the state
  - permits are allocated without charge (*gratis*)

62

Permits Purchased from the State

- In a setting where the MAAC is known, the state can in principle sell all permits at a price equal to the equilibrium market price, since it can predict that price with certainty.
- The sale would then generate exactly the same revenue that an optimal tax would generate, with the same implied assignment of property rights to the state.

63

Permits Purchased from the State

- If the MAAC is not known then the eventual market trading price cannot be predicted with certainty.
- Moreover, if polluting sources do not know the MAAC then they too are unable to predict the eventual trading price, and they will be uncertain about what a permit will eventually be worth.

64

Permits Purchased from the State

- In such a setting, permits would typically be sold by **auction**, where sources bid for permits based on their own expectation of what the permits will eventually be worth in the market.
- The same approach is used for an initial public offering of corporate shares, and for the sale of government and corporate bonds.

65

Permits Purchased from the State

- There are many types of auction in practice, and there is a rich body of theory on auction-design.
- We will not delve into that theory here since it requires a knowledge of game theory and mathematics beyond what is assumed for this course.

66

Permits Purchased from the State

- For our purposes, it is sufficient to note that it is generally not possible for the state to extract the entire eventual trading value of the permits issued.

67

Gratis Allocation

- Under a *gratis* allocation scheme, permits are initially allocated to sources free-of-charge, based on some predetermined allocation rule.

68

Gratis Allocation

- A typical rule would tie the allocation to existing or historical emissions.
- For example, suppose we express the total permit issue as some fraction β of existing (unregulated) aggregate emissions:

$$\widetilde{E} = \beta \hat{E}$$

69

Gratis Allocation

- Then the allocation to source *i* might be based on its own existing emissions, using the same fractional rule:

$$a_i = \beta \hat{e}_i$$

- This approach to the allocation is typically called "**grandfathering**".

70

Gratis Allocation

- A grandfathering rule for allocating permits effectively splits property rights over the resource between the state and the existing polluters.
- In particular, it implies that source i has ownership over l_i emissions rights; it can then trade these freely in the permit market.

71

A Middle-Ground Approach

- If all permits are issued *gratis* (whether or not by grandfathering), the state receives no revenue from that issue, and there is no resource rent captured.
- A middle-ground approach would allocate some fraction of the total issue on a *gratis* basis, and then auction the rest.

72

A Middle-Ground Approach

- For example, if the total issue is

$$\tilde{E} = \beta\hat{E}$$

then the regulator might set aside a fraction α of that issue for auction, and give source i a grandfathered *gratis* allocation equal to

$$a_i = (1-\alpha)\beta\hat{e}_i$$

73

A Middle-Ground Approach

- Ideally, α would be chosen optimally based on the competing objectives of the regulator, including fiscal objectives and political objectives relating to property rights.

74

7.6 OFFSET TRADING

75

Offset Trading

- An **offset** is a measurable and verifiable reduction in emissions that can be bought and sold across sources.
- It is sometimes called a "tradable emissions credit".

76

Offset Trading when Emissions are Capped

- In a setting where aggregate emissions are capped, trading in offsets is equivalent to trading in permits.

77

Offset Trading when Emissions are Capped

- For example, suppose the regulator has capped aggregate emissions for a particular set of sources but has not instituted a formal emissions trading program.
- In such a setting, suppose source *i* wishes to expand its production, and as a consequence, increase its emissions by Δe_i.

78

Offset Trading when Emissions are Capped

- Doing so would violate the aggregate cap, and so permission would not be granted by the regulator.
- However, suppose source *i* can reach an agreement with another source within the capped set, source *j*, whereby source *j* reduces its emissions by Δe_i, thereby leaving aggregate emissions unchanged.

79

Offset Trading when Emissions are Capped

- In exchange for this offsetting reduction by source *j*, source *i* makes a payment to source *j*.
- Because aggregate emissions are still capped, source *j* is unable to "undo" the offset by increasing its emissions after the trade.

80

Offset Trading when Emissions are Capped

- More generally, any source *j* within the capped set could unilaterally reduce its emissions by Δe_i, and register this reduction with the regulator as an "offset" (or emissions credit).
- Source *j* could then post this offset for sale on an offset market.

81

Offset Trading when Emissions are Capped

- Trading in such offsets would establish a market price, and the equilibrium price per ton of the pollutant would be exactly the same as the price of a permit, had a permit trading scheme been in operation.

82

Offset Trading when Emissions are Capped

- Thus, provided the regulator is willing to allow offset trading, emissions trading will arise endogenously in any setting where aggregate emissions are capped.

83

"Offsets" without an Emissions Cap

- In a setting where aggregate emissions are not capped, the meaning of an "offset" becomes unclear.

- To see this, consider the following scenario.

84

“Offsets” without an Emissions Cap

- Suppose an owner of forested land does not currently find it profitable to log her land, and so leaves the trees standing.

- Those standing trees are a store of carbon, and if they are still growing, they continue to capture carbon from the atmosphere.

85

“Offsets” without an Emissions Cap

- Now suppose that the land-owner becomes aware of a market for carbon offsets, through which entities who create emissions can purchase offsets, and thereby claim – perhaps for marketing or public-relations purposes – to have offset their own emissions.

86

“Offsets” without an Emissions Cap

- The land-owner then claims that she had in fact intended to log her forest, and thereby release the stored carbon, but now pledges not to do so.
- That pledge is enough to create an “offset”.
- The “offset” is then purchased by a source of new emissions who then claims to have “neutralized” those emissions via the offset.

87

“Offsets” without an Emissions Cap

- By this accounting, aggregate emissions have not risen.
- However, in the true **base case** – the state that would have been realized in the absence of the offset market – the release of carbon from the trees would not have occurred anyway.

88

“Offsets” without an Emissions Cap

- Relative to that base case, aggregate emissions have risen by the amount of the “offset”.

89

“Offsets” without an Emissions Cap

- The source of the problem here is that the base case cannot be verified: an outside observer cannot know that the trees would not have been logged in the absence of the offset market because the costs and benefits of doing so are known only to the landowner.

90

“Offsets” without an Emissions Cap

- In general, verifying a **counter-factual** – that is, ascertaining what would have happened but did not happen, due to some intervention – is extremely difficult.

91

“Offsets” without an Emissions Cap

- This problem does not arise if aggregate emissions are capped.
- In that setting, our land-owner would need a permit to release the carbon in her trees, and she would have acquired that permit only if she had indeed intended to log the trees.

92

“Offsets” without an Emissions Cap

- If she then sells that permit instead, her emissions are truly lower than in the base case, and aggregate emissions remain unchanged when the permit is used by a new source.

93

7.7 EMISSIONS TRADING WITH NON-UNIFORMLY MIXED POLLUTANTS*

* Advanced Topic

94

Emissions Trading with Non-Uniformly Mixed Pollutants

- We have so far focused exclusively on the design of policies for uniformly-mixed pollutants.
- Recall that for these pollutants, damage is a function of aggregate emissions only; the specific location of individual sources does not matter.

95

Emissions Trading with Non-Uniformly Mixed Pollutants

- If pollutants are non-uniformly mixed then the location of sources does matter, and a permit trade from one source to another may not leave damage unchanged even though aggregate emissions remain unchanged.

96

Emissions Trading with Non-Uniformly Mixed Pollutants

- Emissions trading can still be useful in such settings, though it is necessarily more complicated.
- We will not undertake a comprehensive treatment here; instead we will develop some key concepts, mostly in the context of a simple example.

97

Emissions Trading with Non-Uniformly Mixed Pollutants

- The first step is to divide the regulated region into geographical zones, each of which has a monitoring station (a “receptor point”) that measures the **ambient concentration** of the regulated pollutant (typically measured in parts per million in the surrounding air or water).

98

Emissions Trading with Non-Uniformly Mixed Pollutants

- Damage is typically a function of these ambient concentrations, so they usually form the basis of the policy targets.
- This is also true of a uniformly mixed pollutant but because it is uniformly mixed, the pollutant concentration is the same all across the region, and so depends only on aggregate emissions in the region.

99

Emissions Trading with Non-Uniformly Mixed Pollutants

- In contrast, for non-uniformly mixed pollutants, the ambient concentration in any given zone depends on the location of sources in relation to that zone, and to geographical characteristics like wind patterns and topography, or currents in the water.

100

Emissions Trading with Non-Uniformly Mixed Pollutants

- To illustrate how emissions trading works in this situation, we will focus on a simple example in which there are just two sources in the region.

101

Emissions Trading with Non-Uniformly Mixed Pollutants

- Imagine a setting in which two sources of the regulated pollutant are located roughly along an east-west line.
- The prevailing wind is from the west, and to the east of the sources is a ring of hills that traps much of the pollution.
- The regulator divides the region into two zones, as illustrated in Figure 7-10.

102

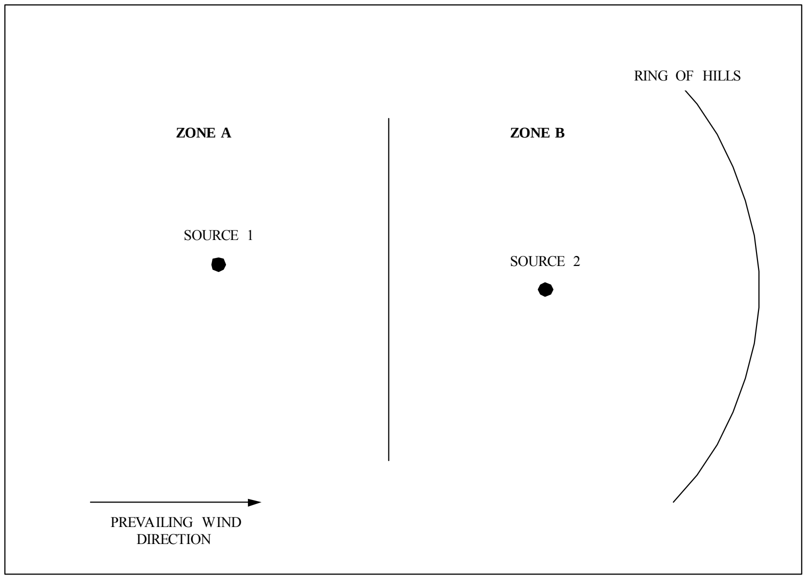


FIGURE 7-10

103

Emissions Trading with Non-Uniformly Mixed Pollutants

- Based on data collected for the region, the regulator estimates a set of **transfer coefficients** $\{\lambda\}$, where λ_{ik} relates emissions from source *i* to ambient concentrations at the receptor point in zone *k*.
- These transfer coefficients measure the extent to which each source contributes to the pollutant concentration in each zone.

104

Emissions Trading with Non-Uniformly Mixed Pollutants

- For example, suppose the coefficients for source 1 are λ_{1A}=0.6 and λ_{1B}=0.4.
- This tells us that for every ton of the pollutant emitted by source 1 on an average day, 60% stays in zone A and 40% travels to zone B.
- Suppose the corresponding coefficients for source 2 are λ_{2A}=0.1 and λ_{2B}=0.9.

105

Emissions Trading with Non-Uniformly Mixed Pollutants

- The stark asymmetry between these coefficients reflects the non-uniformly mixed nature of the pollutant, and the prevailing wind direction.
- For a uniformly mixed pollutant, all the coefficients would be equal to one half.

106

Emissions Trading with Non-Uniformly Mixed Pollutants

- The transfer coefficients determine the **pollutant exposure** for each zone, as a function of emissions from each source.

107

Emissions Trading with Non-Uniformly Mixed Pollutants

- Suppose current emissions are 100 tons from source 1, and 50 tons from source 2.
- Then the pollutant exposures are

$$X_A = (0.6)100 + (0.1)50 = 65$$
$$X_B = (0.4)100 + (0.9)50 = 85$$

for zones A and B respectively.

108

Emissions Trading with Non-Uniformly Mixed Pollutants

- Note that $X_A + X_B = 150$; that is, all emissions end up in either zone A or zone B (because the hills prevent further travel).

109

Emissions Trading with Non-Uniformly Mixed Pollutants

- Note also that zone B has the greatest pollutant exposure even though source 2 emits only half the emissions of source 1.

- This reflects the geographic characteristics of the region.

110

Emissions Trading with Non-Uniformly Mixed Pollutants

- In general, the pollutant exposure in zone *k* from *n* sources within the region is

$$X_k(e) = \sum_{i=1}^{n} \lambda_{ik} e_i$$

where e_i is emissions from source *i*, and λ_{ik} is the transfer coefficient between source *i* and zone *k*.

111

Emissions Trading with Non-Uniformly Mixed Pollutants

- Now suppose that in our two-source example, the regulator introduces an upper limit on ambient concentrations in any zone that translates into a maximum pollutant exposure of 50 tons.

112

Emissions Trading with Non-Uniformly Mixed Pollutants

- The combinations of e_1 and e_2 that satisfy these zone-requirements with strict equality are given by

$$X_A(e_1, e_2) = (0.6)\,e_1 + (0.1)\,e_2 = 50$$
$$X_B(e_1, e_2) = (0.4)\,e_1 + (0.9)\,e_2 = 50$$

for zones A and B respectively.

113

Emissions Trading with Non-Uniformly Mixed Pollutants

- These equations are **iso-exposure schedules**; they identify combinations of e_1 and e_2 that yield a given exposure in the zone concerned.

- They are illustrated in Figure 7-11.

114

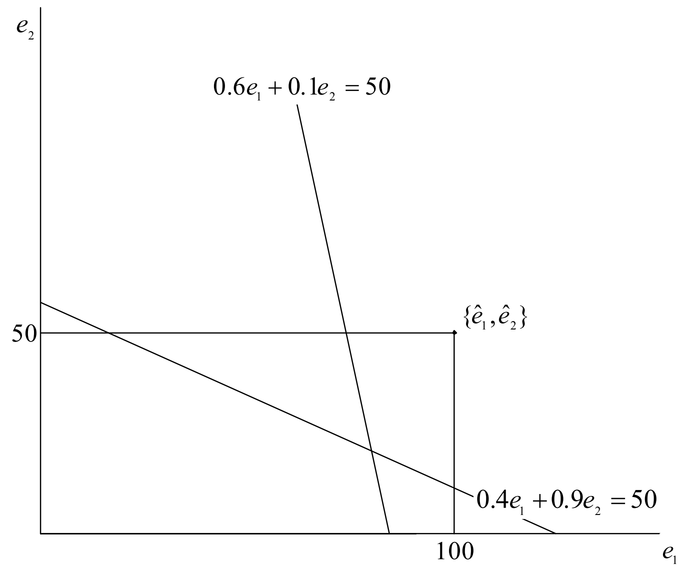


FIGURE 7-11

115

Emissions Trading with Non-Uniformly Mixed Pollutants

- The regulatory requirements are met for any combination of e_1 and e_2 on or below <u>both</u> of these schedules.
- Thus, the requirements are met for $\{e_1, e_2\}$ pairs within the shaded region of Figure 7-12, including the boundary of that region.

116

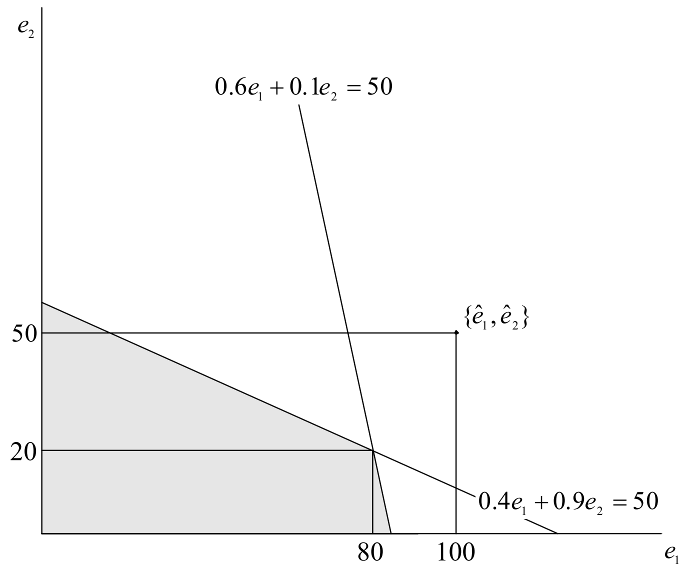


FIGURE 7-12

117

Emissions Trading with Non-Uniformly Mixed Pollutants

- Note from Figure 7-12 that there is just one combination of e_1 and e_2 that satisfies the new requirements with strict equality:

  $\tilde{e}_1$=80 and $\tilde{e}_2$=20
- Suppose the regulator imposes these limits on the two sources.

118

Emissions Trading with Non-Uniformly Mixed Pollutants

- Now consider the possibility of emissions trading: could the two sources trade to a different pair of emissions, at which total abatement costs are lower than at $\{\tilde{e}_1, \tilde{e}_2\}$ but neither zone limit is exceeded?

119

Emissions Trading with Non-Uniformly Mixed Pollutants

- Recall that trade in a uniformly mixed pollutant is one-for-one:
  – a permit or offset for one ton purchased from a selling source allows the buying source to increase its emissions by exactly one ton.
- Thus, trading is always worthwhile until MACs are exactly equal across sources.

120

Emissions Trading with Non-Uniformly Mixed Pollutants

- The same is <u>not</u> true for trade in a non-uniformly mixed pollutant.

- Trade between sources must typically occur at an **exchange rate** that is not one-for-one if the trade is to keep pollution exposures within the allowed limits.

121

Emissions Trading with Non-Uniformly Mixed Pollutants

- Moreover, the required exchange rate depends on the direction of trade.
- In the context of our example,
  - for each ton purchased from source 1, source 2 can increase emissions by only 4/9 tons; and
  - for each ton purchased from source 2, source 1 can increase emissions by only 1/6 tons.
- See Figure 7-13.

122

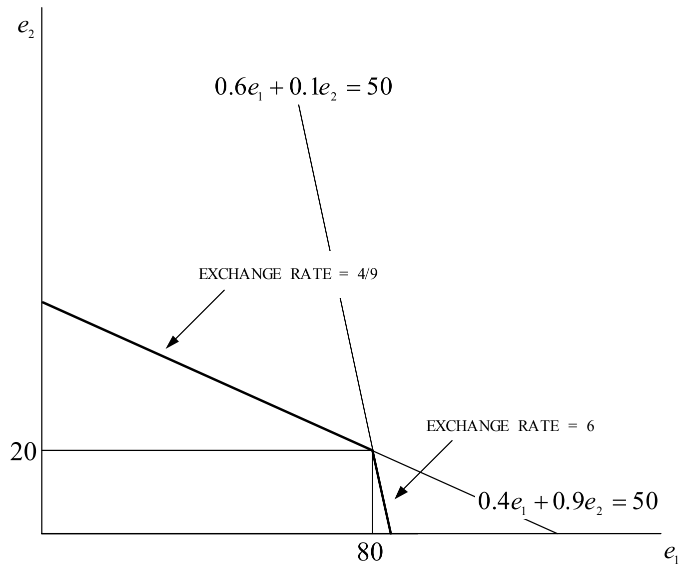


FIGURE 7-13

123

Emissions Trading with Non-Uniformly Mixed Pollutants

- Critically, this means that a trade from source 2 to source 1 (at exchange rate 6) is worthwhile if and only if

$$\frac{MAC_1(\tilde{e}_1)}{MAC_2(\tilde{e}_2)} > 6$$

124

Emissions Trading with Non-Uniformly Mixed Pollutants

- That is, the cost savings for source 1 from having an additional unit of emissions must be at least 6 times the additional cost incurred by source 2 from giving up that unit in order to make trade at an exchange rate of 6 to 1 worthwhile for both sources.

125

Emissions Trading with Non-Uniformly Mixed Pollutants

- Conversely, a trade from source 1 to source 2 (at exchange rate 4/9) is worthwhile if and only if

$$\frac{MAC_2(\tilde{e}_2)}{MAC_1(\tilde{e}_1)} > \frac{9}{4}$$

126

Emissions Trading with Non-Uniformly Mixed Pollutants

- Thus, <u>no</u> trade will occur if

$$\frac{1}{6} < \frac{MAC_2(\tilde{e}_2)}{MAC_1(\tilde{e}_1)} < \frac{9}{4}$$

127

Emissions Trading with Non-Uniformly Mixed Pollutants

- For example, suppose MACs are linear:

$$MAC_1(e_1) = \gamma_1(100 - e_1)$$

$$MAC_2(e_2) = \gamma_2(50 - e_2)$$

128

Emissions Trading with Non-Uniformly Mixed Pollutants

- Then at $\tilde{e}_1$=80 and $\tilde{e}_2$=20,

$$MAC_1(\tilde{e}_1) = 20\gamma_1$$

$$MAC_2(\tilde{e}_2) = 30\gamma_2$$

129

Emissions Trading with Non-Uniformly Mixed Pollutants

- Thus, <u>no</u> trade will occur if

$$\frac{1}{9} < \frac{\gamma_2}{\gamma_1} < \frac{3}{2}$$

130

Emissions Trading with Non-Uniformly Mixed Pollutants

- Trade will occur if and only if γ_2/γ_1 lies outside these bounds.
- In particular, if $\gamma_1 > 9\gamma_2$ then source 1 will purchase emissions from source 2 (at exchange rate 6), as illustrated in Figure 7-14.

131

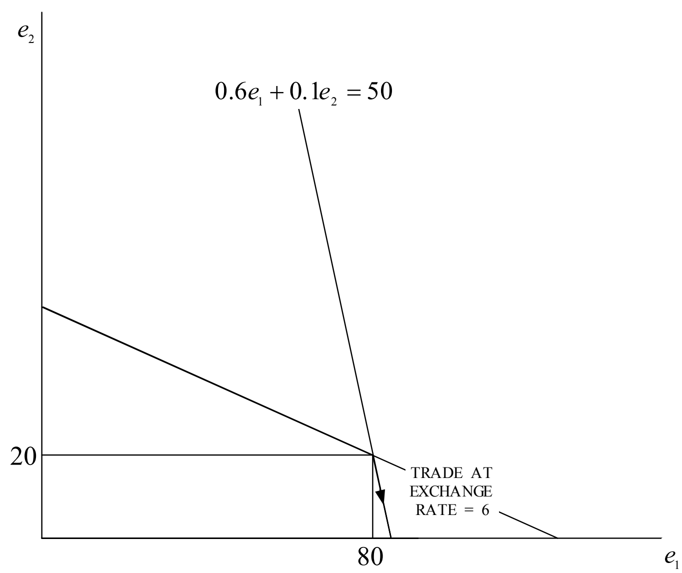


FIGURE 7-14

132

Emissions Trading with Non-Uniformly Mixed Pollutants

- Conversely, if $\gamma_2 > 3\gamma_1/2$ then source 2 will purchase emissions from source 1 (at exchange rate 4/9), as illustrated in Figure 7-15.

133

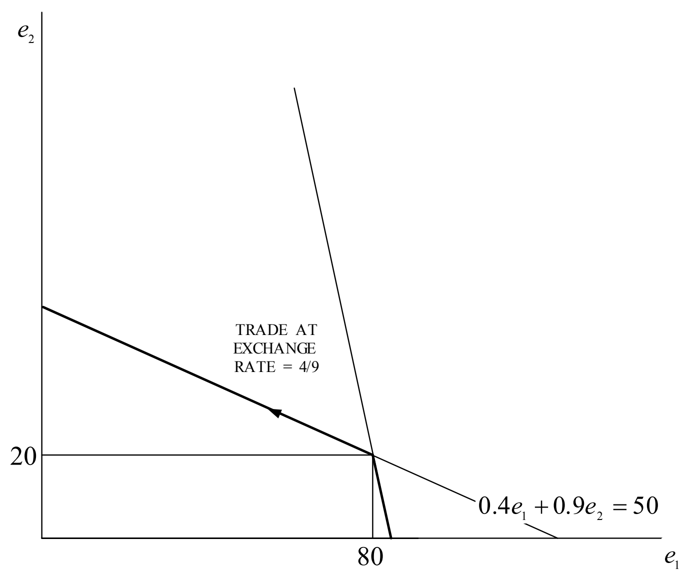


FIGURE 7-15

134

Emissions Trading with Non-Uniformly Mixed Pollutants

- It is important to note that if trade does occur then the pollutant exposure after trade must be <u>lower</u> than the limit in one of the zones.

- See Figure 7-16.

135

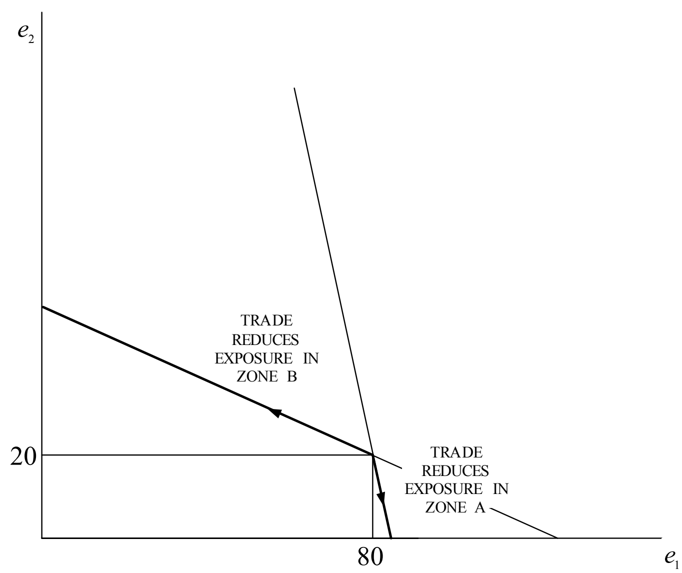


FIGURE 7-16

136

Emissions Trading with Non-Uniformly Mixed Pollutants

- This raises an important question:
  - should the regulator allow a trade-off between exposures in the two zones – by allowing trade to raise exposure in one zone if it reduces exposure in the other zone by enough – and thereby facilitate more trade, and hence, reduce aggregate abatement costs?

137

Emissions Trading with Non-Uniformly Mixed Pollutants

- The answer depends on the objective function of the regulator, particularly in terms of how damage is defined.

138

Emissions Trading with Non-Uniformly Mixed Pollutants

- Suppose the regulator uses a **linear tradeoff rule**, and a quadratic damage function, where damage for the region as a whole is defined as

$$D(X_A, X_B) = \delta(\omega_A X_A + \omega_B X_B)^2$$

where ω_A and ω_B are "exposure weights".

139

Emissions Trading with Non-Uniformly Mixed Pollutants

- It can be shown – using a little calculus – that the regulator would then allow trade at a fixed exchange rate such that a one ton change in source 2 emissions must be offset by a change of R tons in source 1 emissions, where

$$R = \frac{\omega_A \lambda_{1A} + \omega_B \lambda_{1B}}{\omega_A \lambda_{2A} + \omega_B \lambda_{2B}}$$

140

Emissions Trading with Non-Uniformly Mixed Pollutants

- Figure 7-17 illustrates an example, where sources 1 and 2 can trade in either direction at exchange rate *R*.

141

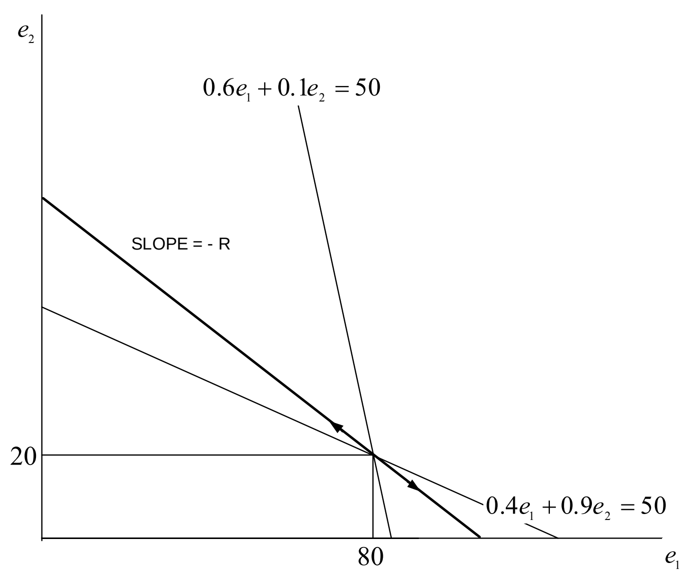


FIGURE 7-17

142

Emissions Trading with Non-Uniformly Mixed Pollutants

- In the special case where $\omega_A = \omega_B$ (equal weight on exposure in both zones), and all emissions stay within the regulated region, this exchange rate is – 1.

- Thus, this special case is equivalent to one with a uniformly mixed pollutant.

END

TOPIC 7 REVIEW QUESTIONS

Questions 1 to 16 relate to the following information. There are two sources of a uniformly mixed pollutant with marginal abatement costs given by

$$MAC_1(e_1) = 200 - e_1$$

and

$$MAC_2(e_2) = 240 - 3e_2$$

respectively. The marginal damage schedule is

$$MD(E) = \frac{3E}{4}$$

where $E = e_1 + e_2$.

1. Marginal aggregate abatement cost is

A. $210 - \frac{3E}{4}$

B. $440 - 4E$

C. $40 - 2E$

D. $440 + \frac{3E}{4}$

2. The socially-optimal aggregate emissions level is

A. 180

B. 140

C. 160

D. 120

3. Socially-optimal emissions for sources 1 and 2 are, respectively

A. 95 and 45

B. 80 and 60

C. 75 and 15

D. 90 and 30

4. The Pigouvian tax rate in this setting is

A. 120

B. 100

C. 105

D. 215

Suppose the regulator introduces a cap-and-trade program with an aggregate emissions cap equal to a fraction α of the unregulated emissions level. Each source is allocated that same fraction of its own unregulated emissions, *gratis*. Suppose emissions are traded under competitive conditions.

5. If the trading price of permits is *p*, then demand for permits by sources 1 and 2 is, respectively

A. $e_1(p) = 200 - p$ and $e_2(p) = 80 - \frac{p}{3}$

B. $e_1(p) = p - 200$ and $e_2(p) = \frac{p}{3} - 80$

C. $e_1(p) = 200 + p$ and $e_2(p) = 80 + \frac{p}{3}$

D. $e_1(p) = 200 - \frac{p}{3}$ and $e_2(p) = 80 - p$

6. Aggregate demand for permits is

A. $E(p) = \frac{4p}{3} - 280$

B. $E(p) = 280 + \frac{4p}{3}$

C. $E(p) = 120 - \frac{4p}{3}$

D. $E(p) = 280 - \frac{4p}{3}$

Suppose $\alpha = 4/5$.

7. The aggregate supply of permits is

A. 184

B. 145

C. 196

D. 224

8. The equilibrium price of permits is

A. 96

B. 42

C. 56

D. 104

9. The equilibrium demand for permits by sources 1 and 2 is, respectively

A. 158 and 66

B. 132 and 64

C. 78 and 134

D. None of the above

10. In equilibrium, source 1 sells 2 permits to source 2.

A. True.

B. False.

Now suppose $\alpha = 1/2$.

11. The aggregate supply of permits is

A. 220

B. 140

C. 180

D. 200

12. This aggregate emissions cap is equal to the socially-optimal aggregate emissions level.

A. True.

B. False.

13. The equilibrium price of permits is

A. 120

B. 100

C. 105

D. 215

14. The equilibrium demand for permits by sources 1 and 2 is, respectively

A. 95 and 45

B. 80 and 60

C. 75 and 15

D. 90 and 30

15. In equilibrium, source 1 buys 5 permits from source 2.

A. True.

B. False.

16. The equilibrium price of permits is equal to the Pigouvian tax rate in this setting.

A. True.

B. False.

Questions 17 to 27 relate to the following information. There are two sources of a uniformly mixed pollutant with marginal abatement costs given by

$$MAC_1(e_1) = 50 - \frac{e_1}{2}$$

and

$$MAC_2(e_2) = 100 - 2e_2$$

respectively. The marginal damage schedule is

$$MD(E) = 2E$$

where $E = e_1 + e_2$.

17. Marginal aggregate abatement cost is

A. $150 - \frac{3E}{4}$

B. $50 - 2E$

C. $50 - \frac{2E}{3}$

D. $60 - \frac{2E}{5}$

18. The socially-optimal aggregate emissions level is

A. 35

B. 40

C. 25

D. 15

Suppose the regulator introduces a cap-and-trade program with an aggregate emissions cap equal to a fraction α of the unregulated emissions level. Each source is allocated that same fraction of its own unregulated emissions, *gratis*. Suppose emissions are traded under competitive conditions.

19. If the trading price of permits is *p*, then demand for permits by sources 1 and 2 is, respectively

A. $e_1(p) = 100 - 2p$ and $e_2(p) = 50 - 2p$

B. $e_1(p) = 200 - p$ and $e_2(p) = 50 - \frac{p}{2}$

C. $e_1(p) = 100 - 2p$ and $e_2(p) = 50 - \frac{p}{2}$

D. None of the above

20. Aggregate demand for permits is

A. $E(p) = 150 - \frac{2p}{5}$

B. $E(p) = 150 + \frac{5p}{2}$

C. $E(p) = 150 - \frac{5p}{2}$

D. $E(p) = 150 + \frac{2p}{5}$

Suppose $\alpha = 1/2$.

21. The aggregate supply of permits is

A. 75

B. 125

C. 65

D. 100

22. The equilibrium price of permits is

A. 35

B. 25

C. 20

D. 30

23. The equilibrium demand for permits by sources 1 and 2 is, respectively

A. 40 and 30

B. 40 and 35

C. 55 and 35

D. 55 and 30

24. In equilibrium, source 1 sells 10 permits to source 2.

A. True.

B. False.

Now suppose α is chosen to implement the socially-optimal level of aggregate emissions.

25. The equilibrium price of permits is

A. 50

B. 75

C. 60

D. 45

26. In equilibrium, source 1 sells its entire allocation to source 2.

A. True.

B. False.

27. The equilibrium price of permits is equal to the Pigouvian tax rate in this setting.

A. True.

B. False.

ANSWER KEY

1. A

2. B

3. A

4. C

5. A

6. D

7. D

8. B

9. A

10. A

The initial allocation for each sources is determined by the size of alpha, which is set at 4/5 for Q7 – Q10 (see the line above Q7). Thus, the allocation to source 1 is a1 = (4/5)*ehat1 = (4/5)200 = 160. Similarly, for source 2, a2 = (4/5)80 = 64. The aggregate allocation is A = 160 + 64 = 224. Setting this aggregate allocation equal to aggregate demand from Q6D gives us the equilibrium price, p=42. Substituting that price into the demands from sources 1 and 2 (from 5A), gives us the equilibrium emissions for each source: 158 and 66 for sources 1 and 2 respectively. So, source 2 wants to emit 66 but was only allocated 64, so it buys 2 permits from source 1 (which was allocated 160 but only wants to emit 158, and so is willing to sell exactly 2 permits).

11. B

12. A

13. C

14. A

15. B

16. A

17. D

18. C

19. C
20. C
21. A
22. D
23. B
24. A
25. A
26. A
27. A

8. POLLUTION TAXES WHEN DAMAGE IS UNCERTAIN

OUTLINE

2

8.1 INTRODUCTION

Introduction

- Our analysis of pollution taxes in Topic 6 assumed that the regulator knows the true $MD(E)$ and $MAAC(E)$, as needed to set the optimal pollution tax rate.
- In practice, the regulator may not have complete information about either.

4

Introduction

- Information on *MD*(*E*) can be gathered through the scientific study of physical impacts, and via economic valuation techniques which place dollar values on those physical impacts.
- However, in practice there is always some residual uncertainty about the relationship between emissions and damage.

5

Introduction

- In this topic we examine the implications of this uncertainty about the damage function for the design of a pollution tax.
- In Topic 9 we consider the implications of uncertainty about abatement costs.

6

8.2 BELIEFS AND EXPECTED LOSS

Beliefs and Expected Loss

- To keep matters as simple as possible, let us focus on the linear case where marginal damage is given by

$$MD(E) = \delta E$$

and where δ could be one of two values, δ_L and $\delta_H > \delta_L$, as illustrated in Figure 8-1.

8

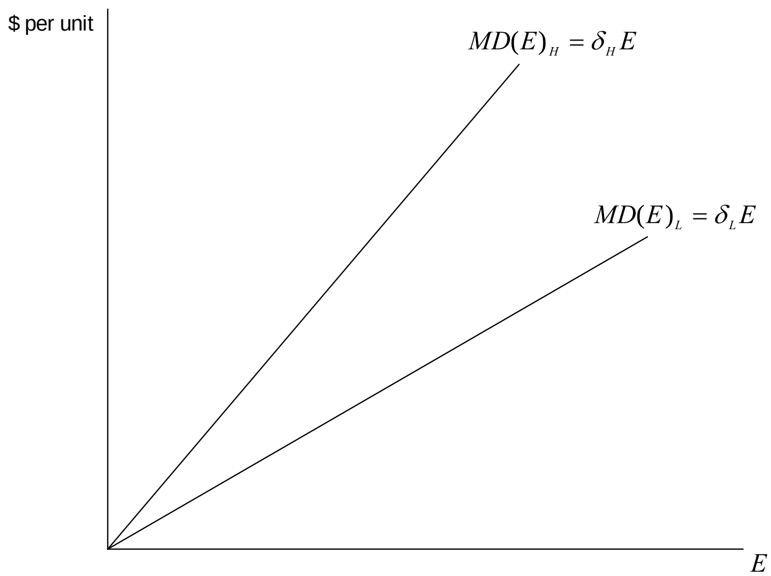


FIGURE 8-1

9

Beliefs and Expected Loss

- Suppose that marginal aggregate abatement cost is given by

$$MAAC(E) = \varphi(\hat{E} - E)$$

and that both $\hat{E}$ and φ are known.

10

Beliefs and Expected Loss

- Under **full information**, where the regulator knows whether the true δ is δ_L or δ_H, the tax would be set at its usual Pigouvian value:

$$\tau_i^* = \frac{\delta_i \varphi \hat{E}}{\varphi + \delta_i}$$

where i = L or H; see Figure 8-2.

11

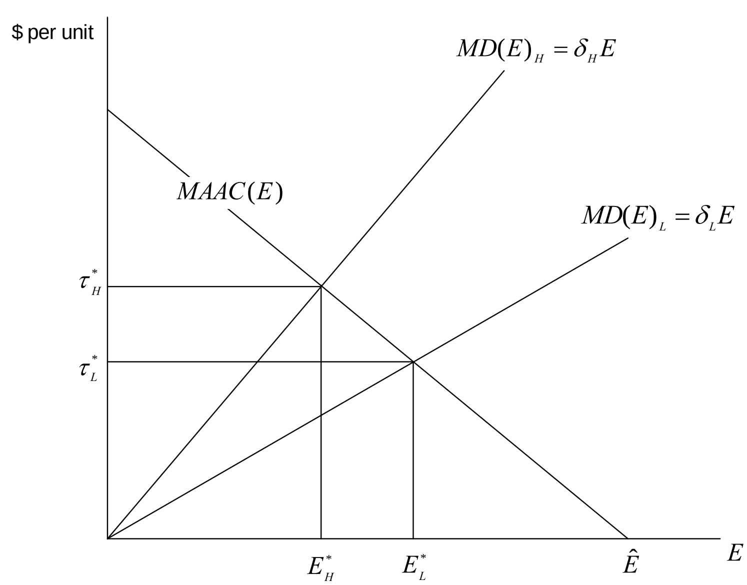


FIGURE 8-2

12

Beliefs and Expected Loss

- Under full information, these taxes would implement the socially optimal quantity of emissions, as illustrated in Figure 8-2.

13

Beliefs and Expected Loss

- Now suppose that the regulator does not know the true value of δ; it could be either δ_L or δ_H.
- If the regulator sets the tax rate at τ^*_L but $\delta=\delta_H$, then emissions will be too high, and there will be an associated loss of social surplus, denoted $LSS_H(\tau^*_L)$, equal to the shaded area in Figure 8-3.

14

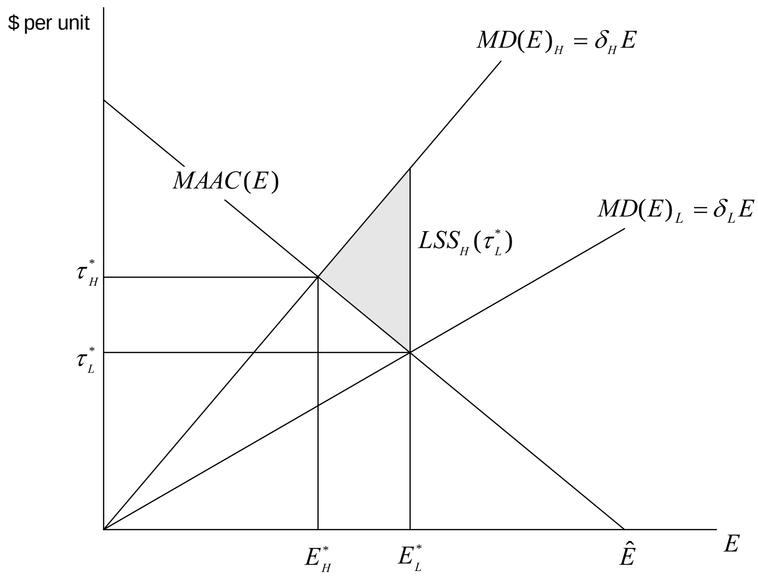


FIGURE 8-3

15

Beliefs and Expected Loss

- Conversely, if the regulator sets the tax rate at τ^*_H but $\delta=\delta_L$, then emissions will be too low, and there will be an associated loss of social surplus, denoted $LSS_L(\tau^*_H)$, equal to the shaded area in Figure 8-4.

16

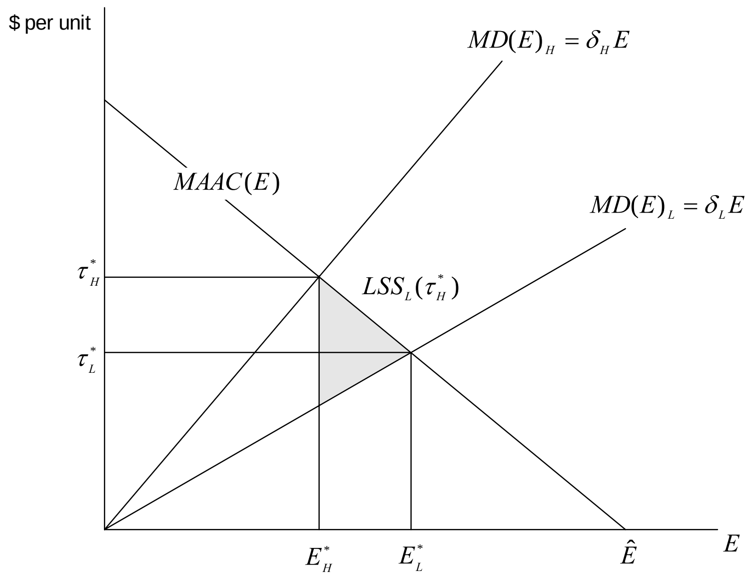


FIGURE 8-4

17

Beliefs and Expected Loss

- What is the optimal tax rate here?
- The optimal rate strikes a balance between these two extremes, based on the **beliefs** of the regulator, as described by the probabilities it assigns to the L and H values of δ.

18

Beliefs and Expected Loss

- Suppose the regulator assigns probability ρ to δ_L, and probability (1-ρ) to δ_H.
- It is important to be clear that these are not objective probabilities in the sense that the true value of δ is decided by a coin toss or some other random trial.

19

Beliefs and Expected Loss

- These are subjective probabilities in the sense that they summarize the beliefs of the regulator as to the relative likelihoods of the two possible values of δ.
- New information – via scientific study, for example – could lead to a revision of these beliefs, based on the principles of **Bayesian learning**.

20

Beliefs and Expected Loss

- We will not explore Bayesian learning here, but it is important to remember that beliefs in practice are continually revised in response to new information.

- Here we will simply take ρ as given.

21

Beliefs and Expected Loss

- Suppose the regulator sets the tax rate at τ, somewhere between τ^*_L and τ^*_H.
- The aggregate response to this tax rate by the regulated sources is $E(\tau)$, defined by

$$MAAC(E(\tau)) = \tau$$

and illustrated in Figure 8-5.

22

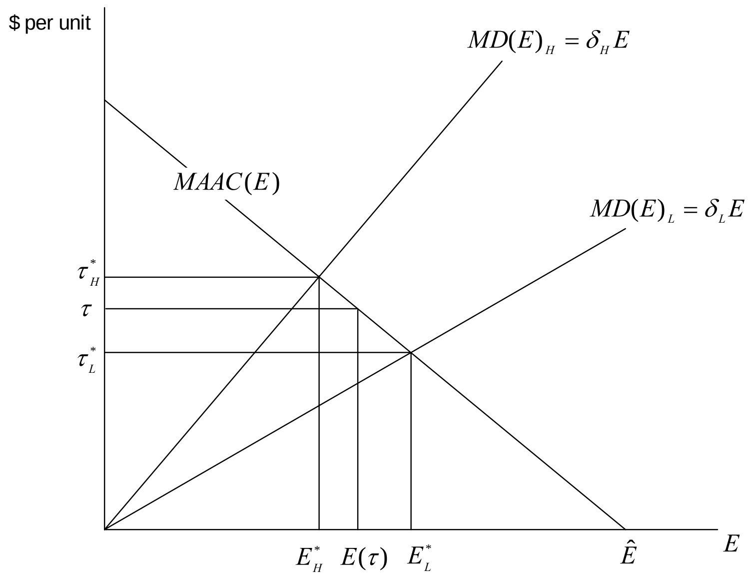


FIGURE 8-5

23

Beliefs and Expected Loss

- We know from Topic 6 that in the linear case, $E(\tau)$ is given by

$$E(\tau) = \hat{E} - \frac{\tau}{\varphi}$$

24

Beliefs and Expected Loss

- If $\delta=\delta_L$, then τ will induce an emissions level that is too low, and if $\delta=\delta_H$, then τ will induce an emissions level that is too high.
- The associated social surplus losses are $LSS_L(\tau)$ and $LSS_H(\tau)$, respectively, as illustrated in Figure 8-6.

25

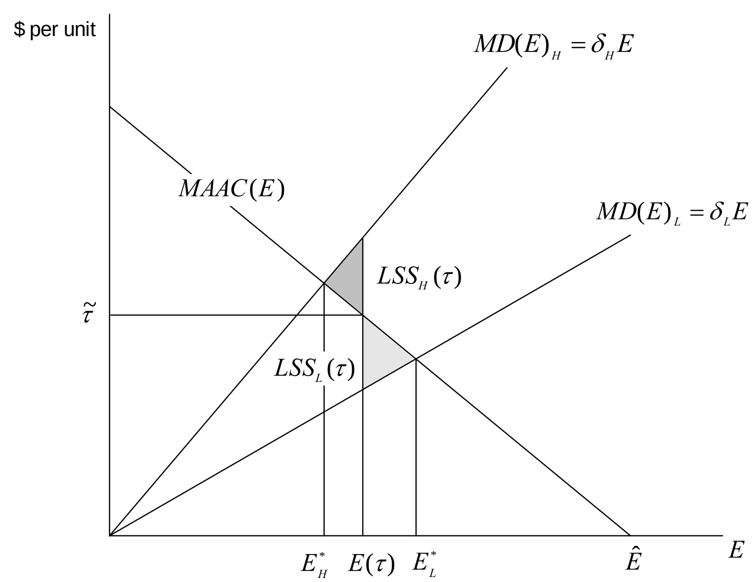


FIGURE 8-6

26

Beliefs and Expected Loss

- From the perspective of the regulator, $LSS_L(\tau)$ and $LSS_H(\tau)$ occur with probability ρ and (1-ρ), respectively.

27

Beliefs and Expected Loss

- Let us now define the **expected loss of social surplus** as

$$\mathbf{E}[LSS(\tau)] = \rho LSS_L(\tau) + (1-\rho) LSS_H(\tau)$$

28

Beliefs and Expected Loss

- The goal of the regulator is to minimize this expected loss through its choice of τ.
- We need calculus to solve this loss-minimization problem directly, but we can approach the problem from a different angle to find a solution, using expected marginal damage.

29

8.3 EXPECTED MARGINAL DAMAGE AND OPTIMAL EMISSIONS

Expected Marginal Damage and Optimal Emissions

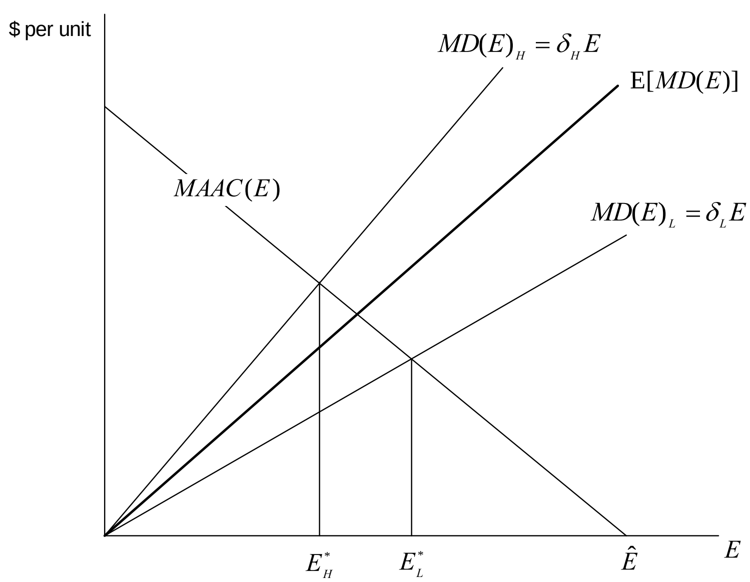
- **Expected marginal damage** is defined as

$$\mathrm{E}[MD(E)] = \rho MD(E)_L + (1-\rho)MD(E)_H$$

where $MD(E)_L=\delta_L E$ and $MD(E)_H=\delta_H E$ in the linear case.

- E[$MD(E)$] for that linear case is illustrated in Figure 8.7, drawn for ρ=1/2.

31



FIGUFRE 8-7

32

Expected Marginal Damage and Optimal Emissions

- In the linear case, we have

$$\mathrm{E}[MD(E)] = \rho\delta_L E + (1-\rho)\delta_H E = \bar{\delta}E$$

where

$$\bar{\delta} = \rho\delta_L + (1-\rho)\delta_H$$

is the **expected value** of δ.

33

Expected Marginal Damage and Optimal Emissions

- We can now characterize the optimal quantity of emissions as $\tilde{E}$, defined by

$$MAAC(\tilde{E}) = \mathrm{E}[MD(\tilde{E})]$$

- See Figure 8.8.

34

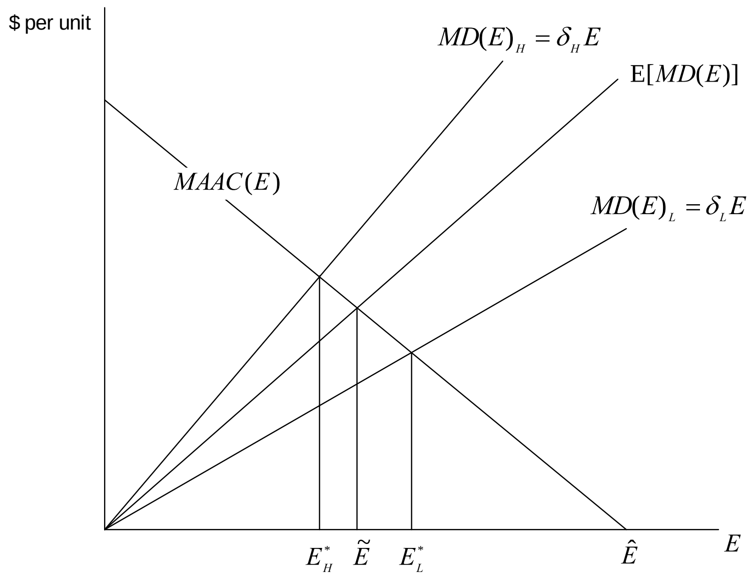


FIGURE 8-8

35

Expected Marginal Damage and Optimal Emissions

- In the linear case, we have

$$\varphi(\hat{E} - \tilde{E}) = \bar{\delta}\tilde{E}$$

which solves for

$$\tilde{E} = \frac{\varphi\hat{E}}{\varphi + \bar{\delta}}$$

36

Expected Marginal Damage and Optimal Emissions

- Thus, optimal emissions under uncertain damage has the same form as optimal emissions under full information except that δ^{bar} takes the place of δ.
- Moreover, this result extends to a setting in which there are more than two possible values of δ.

37

Expected Marginal Damage and Optimal Emissions

- For example, suppose there are n possible values of δ, where $\delta = \delta_i$ has probability ρ_i.
- Then

$$\bar{\delta} = \sum_{i=1}^{n} \rho_i \delta_i$$

and the optimal quantity of emissions still takes the same form.

38

Expected Marginal Damage and Optimal Emissions

- In general, if $MD(E)$ is <u>linear in the uncertain parameter</u>, then the optimal quantity of emissions takes the same form as the full-information quantity, where the uncertain parameter is replaced by its expected value.

39

The Asymmetry of Loss

- It is important to note that the optimal quantity of emissions is <u>not</u> simply equal to the probability-weighted average of the quantities that would be optimal for each value of δ under full information.

40

The Asymmetry of Loss

- Specifically, we can show that

$$\tilde{E} < \mathbf{E}[E^*] \equiv \rho E_L^* + (1-\rho)E_H^*$$

where

$$E_i^* = \frac{\varphi \hat{E}}{\varphi + \delta_i}$$

is the full-information E for i=L and H.

41

The Asymmetry of Loss

- Optimal emissions are <u>lower</u> than the probability-weighted average of the full-information quantities because the cost of emissions being too high is greater than the cost of emissions being too low.

42

The Asymmetry of Loss

- To see this, consider Figure 8-9, which illustrates the social surplus losses from the full-information quantities when they are mismatched with the true MD schedule.
- (Note that these are the same areas that appeared on Figures 8.3 and 8.4).

43

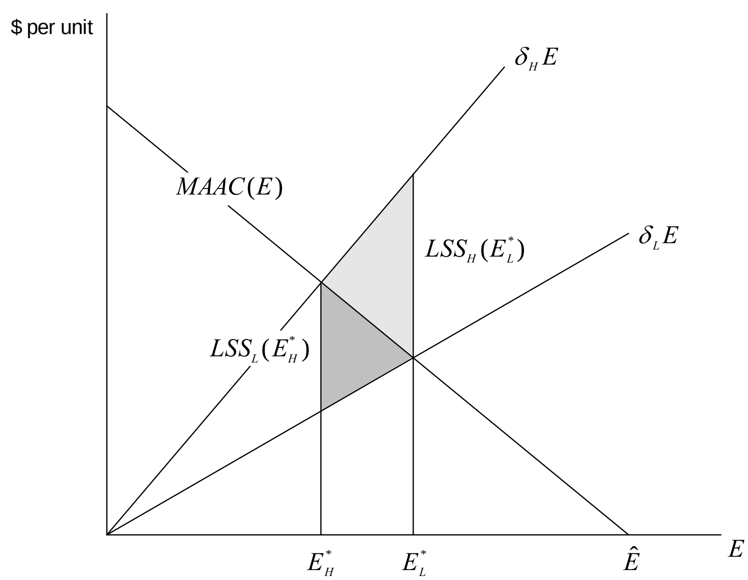


FIGURE 8-9

44

The Asymmetry of Loss

- It is clear from Figure 8-9 that the loss from "over-emitting" (the lighter shaded area) is greater than the loss from "under-emitting" (the darker shaded area).
- Why? Because the two MD schedules diverge: the gap between the schedules grows as *E* rises.

45

The Asymmetry of Loss

- In the linear case it can be shown that the ratio of these two shaded areas has a very simple form:

$$\frac{LSS_H(E_L^*)}{LSS_L(E_H^*)} = \frac{\varphi + \delta_H}{\varphi + \delta_L}$$

46

The Asymmetry of Loss

- Thus, the bigger the difference between δ_L and δ_H , the more quickly the two MD schedules diverge, and the higher is the relative loss from over-emitting.

47

The Asymmetry of Loss

- The <u>optimal</u> quantity reflects this asymmetry, and so $\tilde{E}$ is closer to E_H^* than a simple probability-weighted averaging of E_L^* and E_H^* would dictate.

- This relationship between $\tilde{E}$ and the expected E^* is illustrated in Figure 8-10 which plots both against 1-ρ.

48

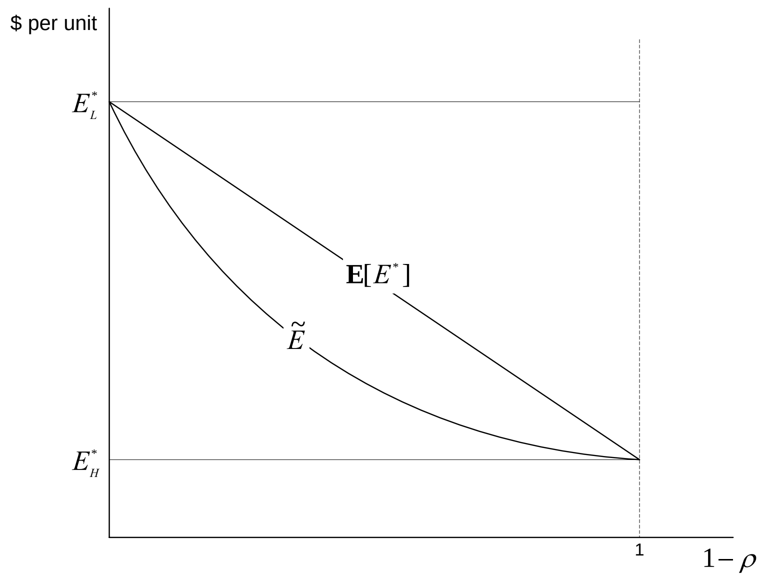


FIGURE 8-10

49

8.4 IMPLEMENTATION WITH A TAX

Implementation with a Tax

- Now let us return to the optimal tax.
- The tax rate needed to implement $\tilde{E}$ is

$$\tilde{\tau} = MAAC(\tilde{E})$$

- See Figure 8-11.

51

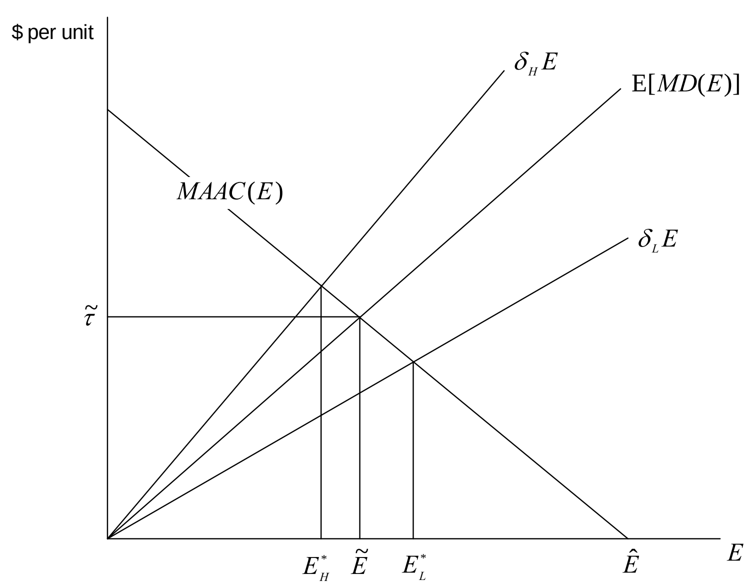


FIGURE 8-11

52

Implementation with a Tax

- In the linear case, this solves for

$$\tilde{\tau} = \frac{\bar{\delta}\varphi\hat{E}}{\varphi + \bar{\delta}}$$

53

Implementation with a Tax

- Note that this optimal tax rate has the same form as the standard Pigouvian tax rate under full information except that $\bar{\delta}$ takes the place of δ (recall s.11).
- Note too that the tax rate is equal to expected marginal damage, evaluated at $\tilde{E}$, as per the Pigouvian rule.

54

Implementation with a Tax

- Recall from that optimal emissions are lower than the expected value of the full-information quantities (due the asymmetry of loss); see Figure 8-10.

55

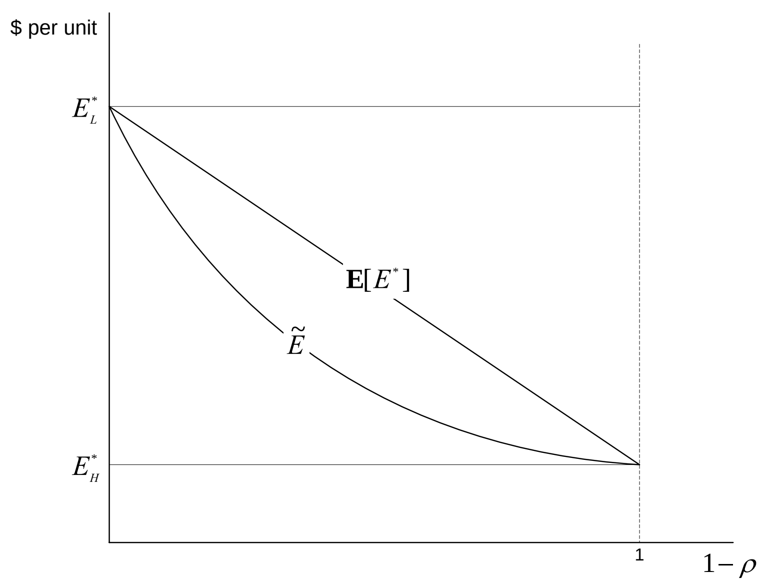


FIGURE 8-10 (Repeat)

56

Implementation with a Tax

- It follows that the optimal tax is <u>higher</u> than the expected value of the full-information taxes.

57

Implementation with a Tax

- Specifically, we can show that

$$\tilde{\tau} > \mathbf{E}[\tau^*] \equiv \rho\tau_L^* + (1-\rho)\tau_H^*$$

where

$$\tau_i^* = \frac{\delta_i \varphi \hat{E}}{\varphi + \delta_i}$$

is the full-information tax for *i*=L and H.

58

Implementation with a Tax

- This relationship between $\tilde{\tau}$ and the expected τ^* is illustrated in Figure 8-12 which plots both against ρ.

59

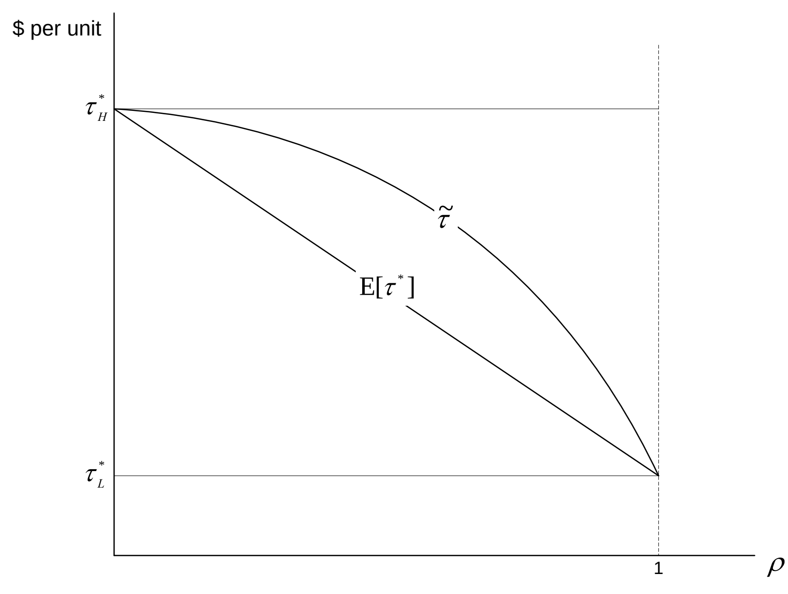


FIGURE 8-12

60

Implementation with a Tax

- Note that Figure 8-12 has ρ measured on the axis, whereas Figure 8-10 has 1- ρ measured on the axis.
- The difference is because in both cases, the optimal policy must be equal to the full-information policy when either ρ=0 or ρ=1.

61

8.5 THE VALUE OF INFORMATION

The Value of Information

- Recall from Section 8.1 that we initially framed the policy problem in terms of minimizing the expected loss of social surplus.
- In particular, the optimal tax is chosen to minimize the probability-weighted sum of the two areas in Figure 8.6, as reproduced in Figure 8.13.

63

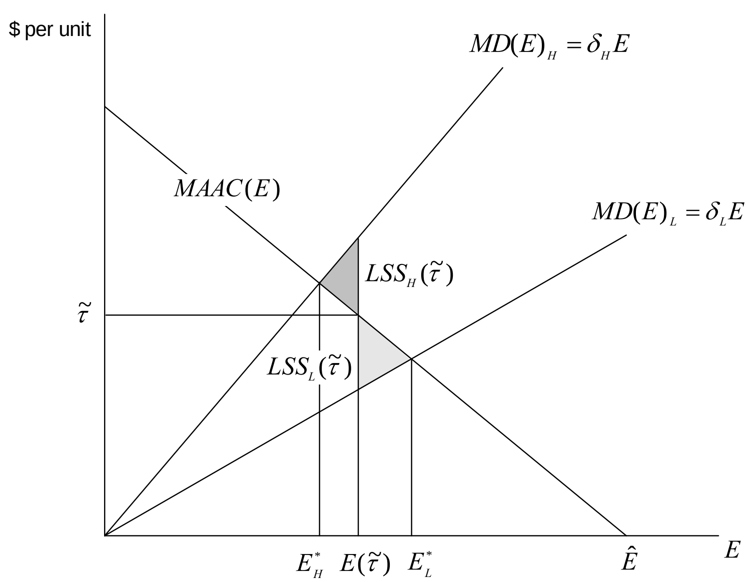


FIGURE 8-13

64

The Value of Information

- The optimal policy minimizes this expected loss but it cannot eliminate the loss since the true value of δ remains unknown.
- This raises an interesting question:
  - What is the value of information about δ in terms of the expected loss avoided under the optimal policy?

65

The Value of Information

- We will not work through the mathematics here, but it can be shown that the expected loss under the optimal policy is

$$\mathrm{E}[LSS] = \frac{1}{\psi}\left(\frac{E_L^* E_H^*}{2(\phi + \bar{\delta})}\right)$$

where ψ measures the **precision** of beliefs.

66

The Value of Information

- In particular,

$$\psi = \frac{1}{(\delta_H - \delta_L)^2 \rho(1-\rho)}$$

- In general, the precision of a probability density is the reciprocal of its variance.

67

The Value of Information

- Note that if ρ=0 or ρ =1 then there is no uncertainty; beliefs are perfectly precise and the expected loss is zero.
- In all other cases, beliefs have some imprecision, and that imprecision is increasing in the extent to which δ_L and δ_H differ, as measured by $(\delta_H - \delta_L)^2$.

68

The Value of Information

- Plotting ψ against ρ shows us that beliefs are least precise when ρ =1/2; see Figure 8.14.

69

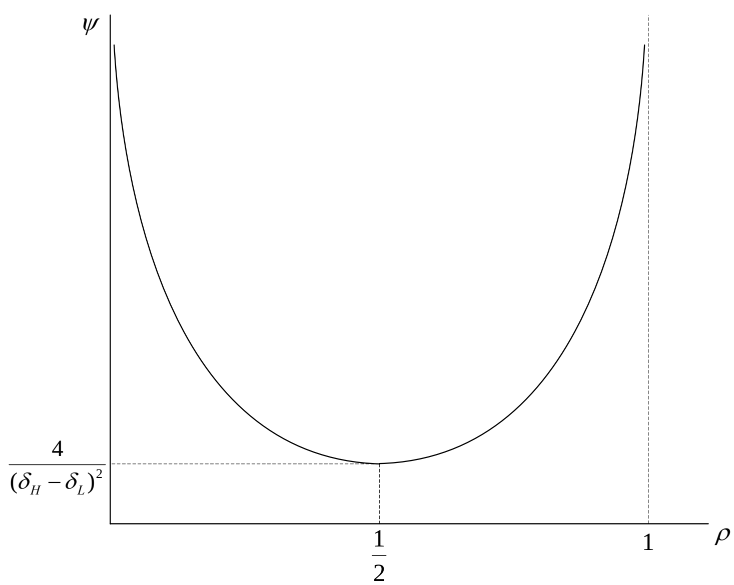


FIGURE 8-14

70

The Value of Information

- This has an intuitive interpretation:
  - beliefs are least precise when both values of δ are equally likely, and the difference between them is very large; that is, the regulator cannot pin down a value for δ with much confidence at all.

71

The Value of Information

- New information, through scientific research for example, allows the regulator to improve the precision of its beliefs, and this reduces the expected loss from the optimal policy.
- That reduction in expected loss is a measure of the value of that information.

72

Preview: An Unpredictable Response to the Tax

- Implementation of optimal emissions using the tax is straightforward here despite the uncertainty around MD, because the response to that tax is not uncertain.
- Once the optimal quantity of emissions has been chosen – based on expected marginal damage – the regulator can achieve that quantity with certainty using the tax.

73

An Unpredictable Response to the Tax

- In sharp contrast, if MAAC is uncertain then the response to the tax itself is uncertain, and we have a much more complicated policy problem to solve.
- We examine that problem in the next topic.

END

74

TOPIC 8 REVIEW QUESTIONS

These review questions begin with a solved example. It is recommended that you work through this example before commencing the questions.

A SOLVED EXAMPLE

There are multiple sources of a uniformly mixed pollutant, with marginal aggregate abatement cost given by

$$MAAC(E) = 840 - 3E$$

The marginal damage schedule is

$$MD(E) = \delta E$$

where the true value of δ is unknown by the regulator. The regulator's beliefs about δ are as follows: $\delta = \delta_L = 1$ with probability $\rho = 1/2$; and $\delta = \delta_H = 3$ with probability $1 - \rho = 1/2$.

1. Calculate the full-information Pigouvian tax rates in the low and high damage states, denoted τ_L^* and τ_H^* respectively.

(a) Solution for τ_L^*

First find E_L^*, characterized by, $MAAC(E) = MD(E)_L$:

(1) $840 - 3E = E$

Solving equation (1) yields

(2) $E_L^* = 210$

The associated tax rate is then calculated as $\tau_L^* = MD(E_L^*)_L$:

(3) $\tau_L^* = \delta_L E_L^* = 210$

(b) Solution for τ_H^*

First find E_H^*, characterized by, $MAAC(E) = MD(E)_H$:

(4) $840 - 3E = 3E$

Solving equation (4) yields

(5) $E_H^* = 140$

The associated tax rate is then calculated as $\tau_H^* = MD(E_H^*)_H$:

(6) $\tau_H^* = \delta_L E_H^* = 420$

2. Suppose the regulator sets the tax rate at $\tau = \tau_L^*$. Calculate the resulting loss of social surplus in the high-damage state, denoted $LSS_H(\tau_L^*)$.

Emissions will be too high in this situation. We need to calculate the shaded area in Figure 8-3 from the slides, reproduced here as Figure R8-1 (which is not drawn to scale). This area is

(7) $$LSS_H(\tau_L^*) = \int_{E_H^*}^{E_L^*} MD(E)_H \, dE - \int_{E_H^*}^{E_L^*} MAAC(E) dE$$

To calculate this area without using calculus, we first need to find $MD(E_L^*)_H$. This is equal to

(8) $MD(E_L^*)_H = \delta_H E_L^* = 630$

as illustrated in Figure R8-1. We can then calculate the area of the shaded triangle as

(9) $$LSS_H(\tau_L^*) = \frac{[MD(E_L^*)_H - \tau_L^*][E_L^* - E_H^*]}{2}$$

$$= \frac{[630 - 210][210 - 140]}{2}$$

$$= 14700$$

3. Suppose the regulator sets the tax rate at $\tau = \tau_L^*$. Calculate the resulting loss of social surplus in the low-damage state, denoted $LSS_L(\tau_L^*)$.

By definition, this tax rate is optimal for the damage state so there is no loss of social surplus. Thus, $LSS_L(\tau_L^*) = 0$.

4. Suppose the regulator sets the tax rate at $\tau = \tau_L^*$. Calculate the resulting expected loss of social surplus, denoted $\mathbf{E}[LSS(\tau_L^*)]$.

This is calculated as the probability-weighted sum of the loss values we calculated in parts 3. and 2. above (the former being zero). That is,

(10) $$\mathbf{E}[LSS(\tau_L^*)] = \rho LSS_L(\tau_L^*) + (1-\rho)LSS_H(\tau_L^*) = (1/2)0 + (1/2)14700 = 7350$$

Note that ρ has been defined as the probability of the L state, so it is the weight assigned to the loss in the L state (which is zero). The residual weight is assigned to the loss in H state (which is 14700).

5. Suppose the regulator sets the tax rate at $\tau = \tau_H^*$. Calculate the resulting loss of social surplus in the low-damage state, denoted $LSS_L(\tau_H^*)$.

Emissions will be too low in this situation. We need to calculate the shaded area in Figure 8-4 from the slides, reproduced here as Figure R8-2 (which is not drawn to scale). This area is

(11) $$LSS_L(\tau_H^*) = \int_{E_H^*}^{E_L^*} MAAC(E)dE - \int_{E_H^*}^{E_L^*} MD(E)_L \, dE$$

To calculate this area without using calculus, we first need to find $MD(E_H^*)_L$. This is equal to

(12) $$MD(E_H^*)_L = \delta_L E_H^* = 140$$

as illustrated in Figure R8.2. We can then calculate the area of the shaded triangle as

(13) $$LSS_L(\tau_H^*) = \frac{[\tau_H^* - MD(E_H^*)_L][E_L^* - E_H^*]}{2}$$

$$= \frac{[420-140][210-140]}{2}$$

$$= 9800$$

6. Suppose the regulator sets the tax rate at $\tau = \tau_H^*$. Calculate the resulting loss of social surplus in the high-damage state, denoted $LSS_H(\tau_H^*)$.

By definition, this tax rate is optimal for the damage state so there is no loss of social surplus. Thus, $LSS_H(\tau_H^*) = 0$.

7. Suppose the regulator sets the tax rate at $\tau = \tau_H^*$. Calculate the resulting expected loss of social surplus, denoted $\mathbf{E}[LSS(\tau_H^*)]$.

This is calculated as the probability-weighted sum of the loss values we calculated in parts 5. and 6. above (the latter being zero). That is,

(14) $\mathbf{E}[LSS(\tau_H^*)] = \rho LSS_L(\tau_H^*) + (1-\rho)LSS_H(\tau_H^*) = (1/2)9800 + (1/2)0 = 4900$

Note that ρ has been defined as the probability of the L state, so it is the weight assigned to the loss in the L state (which is 9800). The residual weight is assigned to the loss in H state (which is zero).

8. If the regulator could only choose between τ_L^* and τ_H^*, which tax rate would it choose?

In this example, we know from comparing (14) with (10) that the expected loss is smallest under τ_H^*, so the regulator would choose that rate. However, we know that the regulator can almost always do better (and never worse) by choosing a tax rate somewhere between τ_L^* and τ_H^*, and we now turn to calculating that optimal rate. We start by deriving the expected marginal damage schedule.

9. Derive the expected marginal damage function.

The expected marginal damage function is

(15) $\mathbf{E}[MD(E)] = \rho MD(E)_L + (1-\rho)MD(E)_H$

$$= \rho\delta_L E + (1-\rho)\delta_H E$$

$$= (\rho\delta_L + (1-\rho)\delta_H)E$$
$$= ((1/2)1 + (1/2)3)E$$
$$= 2E$$

10. Calculate the optimal level of aggregate emissions in this setting, denoted $\tilde{E}$.

We need to find the solution to $\mathbf{E}[MD(E)] = MAAC(E)$. In this case, we need to solve

(16) $2E = 840 - 3E$

which solves for $\tilde{E} = 168$. See Figure R8-3 (which reproduces Figure 8-8 from the slides in the context of this example).

11. Calculate the optimal tax rate in this setting, denoted $\tilde{\tau}$.

The optimal tax rate is the rate need to implement $\tilde{E}$. That is, we need to find a tax rate such that the response to that tax rate yields the optimal level of emissions. This is easy (because we know the marginal aggregate abatement cost schedule). We just need to choose τ to ensure that

(17) $\tau = MAAC(\tilde{E})$

In this example,

(18) $\tilde{\tau} = 840 - 3(168) = 336$

See Figure R8-4 (which reproduces Figure 8-10 from the slides in the context of this example).

12. Calculate the probability-weighted average of the full-information tax rates, denoted $\bar{\tau}^*$.

This is simply equal to the probability-weighted average of the tax rates we calculated in parts 1(a) and 1(b) above:

(19) $\bar{\tau}^* = \rho\tau_L^* + (1-\rho)\tau_H^* = (1/2)210 + (1/2)420 = 315$

Note that is lower than the optimal tax rate we calculated in part 11. Why? The divergence of the low MD and high MD schedules means that the loss of surplus from over-emitting (by under-taxing) is greater than the loss of surplus from under-emitting (by over-taxing), so the optimal tax rate is set higher than a simple probability-weighted average of the full-information rates. (See slide 8-47).

13. Calculate the loss of social surplus if the regulator sets the tax rate at $\tau = \tilde{\tau}$ and the true damage state is L. Let $LSS_L(\tilde{\tau})$ denote this loss.

We need to calculate the lower shaded area in Figure 8-11 from the slides, reproduced here as Figure R8-5 (which is not drawn to scale). This area is

(20) $$LSS_L(\tilde{\tau}) = \int_{\tilde{E}}^{E_L^*} MAAC(E)dE - \int_{\tilde{E}}^{E_L^*} MD(E)_L dE$$

To calculate this area without using calculus, we first need to find $MD(\tilde{E})_L$. This is equal to

(21) $$MD(\tilde{E})_L = \delta_L \tilde{E} = 168$$

as illustrated in Figure R8.5. We can then calculate the area of the shaded triangle as

(22) $$LSS_L(\tilde{\tau}) = \frac{[\tilde{\tau} - MD(\tilde{E})_L][E_L^* - \tilde{E}]}{2}$$

$$= \frac{[336 - 168][210 - 168]}{2}$$

$$= 3528$$

14. Calculate the loss of social surplus if the regulator sets the tax rate at $\tau = \tilde{\tau}$ and the true damage state is H. Let $LSS_H(\tilde{\tau})$ denote this loss.

We need to calculate the upper shaded area in Figure 8-11 from the slides, reproduced here as Figure R8-6 (which is not drawn to scale). This area is

(23) $$LSS_H(\tilde{\tau}) = \int_{E_H^*}^{\tilde{E}} MD(E)_H dE - \int_{E_H^*}^{\tilde{E}} MAAC(E)dE$$

To calculate this area without using calculus, we first need to find $MD(\tilde{E})_H$. This is equal to

(24) $$MD(\tilde{E})_H = \delta_H \tilde{E} = 504$$

as illustrated in Figure R8.6. We can then calculate the area of the shaded triangle as

(25) $$LSS_H(\tilde{\tau}) = \frac{[MD(\tilde{E})_H - \tilde{\tau}][\tilde{E} - E_H^*]}{2}$$

$$= \frac{[504 - 336][168 - 140]}{2}$$

$$= 2352$$

15. Calculate the expected loss of social surplus if the regulator sets the tax rate at $\tau = \tilde{\tau}$, denoted $\mathbf{E}[LSS(\tilde{\tau})]$.

This is calculated as the probability-weighted sum of the loss values we calculated in parts 13. and 14. above. That is,

(26) $$\mathbf{E}[LSS(\tilde{\tau})] = \rho LSS_L(\tilde{\tau}) + (1-\rho)LSS_H(\tilde{\tau}) = (1/2)3528 + (1/2)2352 = 2940$$

Note that this is less than the expected loss of surplus under either τ_L^* or τ_H^* from parts 4. and 7. above; the optimal tax is clearly better than either of these extremes. Next we will see that it is also better than a simple weighted average of the two extremes.

16. Calculate the expected loss of social surplus if the regulator sets the tax rate at $\tau = \bar{\tau}^*$, denoted $\mathbf{E}[LSS(\bar{\tau}^*)]$.

Recall from part 12. that $\bar{\tau}^* = 315$. To calculate the expected loss under this tax rate we need to calculate the loss under L damage scenario, the loss under the H damage scenario, and then take the probability-weighted average of those two losses. Those two losses are measured by the lower and upper shaded areas respectively in Figure R8-7. To calculate these areas, we first need to calculate the aggregate emissions response to the tax rate, denoted $E(\bar{\tau}^*)$ in the figure. We find this by equating $MAAC(E)$ to the tax rate, and solving for E. This yields $E(\bar{\tau}^*) = 175$. It is then straightforward to calculate the two areas, using the same reasoning we have used above. The calculated values are

(27) $$LSS_L(\bar{\tau}^*) = \frac{[315 - 175][210 - 175]}{2} = 2450$$

and

(28) $$LSS_H(\bar{\tau}^*) = \frac{[525 - 315][175 - 140]}{2} = 3675$$

The expected loss of surplus under this tax rate is the probability-weighted average of these losses:

(29) $\mathbf{E}[LSS(\overline{\tau}^*)] = \rho LSS_L(\overline{\tau}^*) + (1-\rho)LSS_H(\overline{\tau}^*) = (1/2)2450 + (1/2)3675 = 3062.5$

In comparison, recall that the expected loss under the optimal tax rate is only 2940.

Questions 1 to 19 relate to the following information. There are multiple sources of a uniformly mixed pollutant, with marginal aggregate abatement cost given by

$$MAAC(E) = 200 - 2E$$

The marginal damage schedule is

$$MD(E) = \delta E$$

where the true value of δ is unknown by the regulator. The regulator's beliefs about δ are as follows: $\delta = 2$ with probability $\rho = 1/3$; and $\delta = 8$ with probability $1-\rho = 2/3$.

1. The full-information Pigouvian tax rates in the low and high damage states, denoted τ_L^* and τ_H^* respectively, are

A. $\tau_L^* = 100/3$ and $\tau_H^* = 320/3$

B. $\tau_L^* = 100$ and $\tau_H^* = 160$

C. $\tau_L^* = 160$ and $\tau_H^* = 100$

D. None of the above.

2. If the regulator sets the tax rate at $\tau = \tau_L^*$ and $\delta = 2$ then the resulting loss of social surplus is zero.

A. True.

B. False.

3. If the regulator sets the tax rate at $\tau = \tau_L^*$ and $\delta = 8$ then the resulting loss of social surplus, denoted $LSS_H(\tau_L^*)$, is

A. 1800

B. 2300

C. 4500

D. 0

4. If the regulator sets the tax rate at $\tau = \tau_L^*$ then the expected loss of social surplus is

A. 3000

B. 4500

C. 5200

D. 1500

5. If the regulator sets the tax rate at $\tau = \tau_H^*$ then the expected loss of social surplus is

A. 1800

B. 3000

C. 5200

D. 600

6. If the regulator must for some reason choose either τ_L^* or τ_H^* then it will choose τ_H^*.

A. True.

B. False.

7. Expected marginal damage is

A. $\mathbf{E}[MD(E)] = 2E$

B. $\mathbf{E}[MD(E)] = 8E$

C. $\mathbf{E}[MD(E)] = 6E$

D. $\mathbf{E}[MD(E)] = 5E$

8. The optimal level of aggregate emissions – the level that minimizes the expected loss of social surplus – is where $\mathbf{E}[MD(E)] = MAAC(E)$.

A. True.

B. False.

9. The optimal level of aggregate emissions in this setting is

A. 20

B. 25

C. 30

D. 50

10. The optimal tax rate in this setting, denoted $\tilde{\tau}$, is

A. 100

B. 140

C. 150

D. 160

11. The probability-weighted average of the full-information tax rates, denoted $\bar{\tau}^*$, is

A. 100

B. 140

C. 150

D. 160

12. The divergence of the low damage and high damage MD schedules accounts for the difference between your answers to Qs. 10 and 11 above.

A. True.

B. False.

13. If the regulator sets the tax rate at $\tau = \tilde{\tau}$ and $\delta = 2$ then the resulting loss of social surplus, denoted $LSS_L(\tilde{\tau})$, is

A. 1250

B. 1500

C. 1800

D. 1950

14. If the regulator sets the tax rate at $\tau = \tilde{\tau}$ and $\delta = 8$ then the resulting loss of social surplus, denoted $LSS_H(\tilde{\tau})$, is

A. 125

B. 150

C. 180

D. 195

15. If the regulator sets the tax rate at $\tau = \tilde{\tau}$ then the expected loss of social surplus is

A. 350

B. 500

C. 600

D. 850

Compare this with your answers to Qs. 4 and 5 above.

16. If the regulator sets the tax rate at $\tau = \tilde{\tau}$ then the tax rate is equal to expected marginal damage at the level of aggregate emissions induced by the tax.

A. True.

B. False.

17. If the regulator sets the tax rate at $\tau = \bar{\tau}^*$ (from Q.11 above) then aggregate emissions will be

A. 20

B. 25

C. 30

D. 50

18. If the regulator sets the tax rate at $\tau = \bar{\tau}^*$ then the tax rate is less than expected marginal damage at the level of aggregate emissions induced by the tax.

A. True.

B. False.

19. If the regulator sets the tax rate at $\tau = \bar{\tau}^*$ then the expected loss of social surplus is

A. 350

B. 500

C. 600

D. 850

Compare this with your answer to Q.15 above.

Questions 20 to 28 relate to the following information. There are multiple sources of a uniformly mixed pollutant, with marginal aggregate abatement cost given by

$$MAAC(E) = 6000 - 15E$$

The marginal damage schedule is

$$MD(E) = \delta E$$

where the true value of δ is unknown by the regulator. The regulator's beliefs about δ are as follows: $\delta = 5$ with probability $\rho = 3/5$; and $\delta = 15$ with probability $1 - \rho = 2/5$.

20. The full-information optimal aggregate emissions levels in the low and high damage states, denoted E_L^* and E_H^* respectively, are

A. $E_L^* = 3000$ and $E_L^* = 1500$

B. $E_L^* = 300$ and $E_H^* = 200$

C. $E_L^* = 1500$ and $E_H^* = 3000$

D. $E_L^* = 200$ and $E_H^* = 300$

21. The full-information Pigouvian tax rates in the low and high damage states, denoted τ_L^* and τ_H^* respectively, are

A. $\tau_L^* = 3000$ and $\tau_L^* = 1500$

B. $\tau_L^* = 300$ and $\tau_H^* = 200$

C. $\tau_L^* = 1500$ and $\tau_H^* = 3000$

D. $\tau_L^* = 200$ and $\tau_H^* = 300$

22. If the regulator sets the tax rate at $\tau = \tau_L^*$ then the expected loss of social surplus is

A. 60000

B. 150000

C. 100000

D. 75000

23. If the regulator sets the tax rate at $\tau = \tau_H^*$ then the expected loss of social surplus is

A. 60000

B. 150000

C. 100000

D. 75000

24. The optimal level of aggregate emissions in this setting is

A. 200

B. 250

C. 300

D. 350

25. The optimal tax rate in this setting, denoted $\tilde{\tau}$, is

A. 1500

B. 2250

C. 3000

D. 4250

26. The probability-weighted average of the full-information tax rates, denoted $\bar{\tau}^*$, is

A. 2100

B. 2700

C. 2250

D. 1750

27. If the regulator sets the tax rate at $\tau = \tilde{\tau}$ then the expected loss of social surplus is

A. 15000

B. 22500

C. 27500

D. 30000

28. If the regulator sets the tax rate at $\tau = \bar{\tau}^*$ (from Q.26) then the expected loss of social surplus is

A. 29500

B. 31200

C. 33500

D. 37000

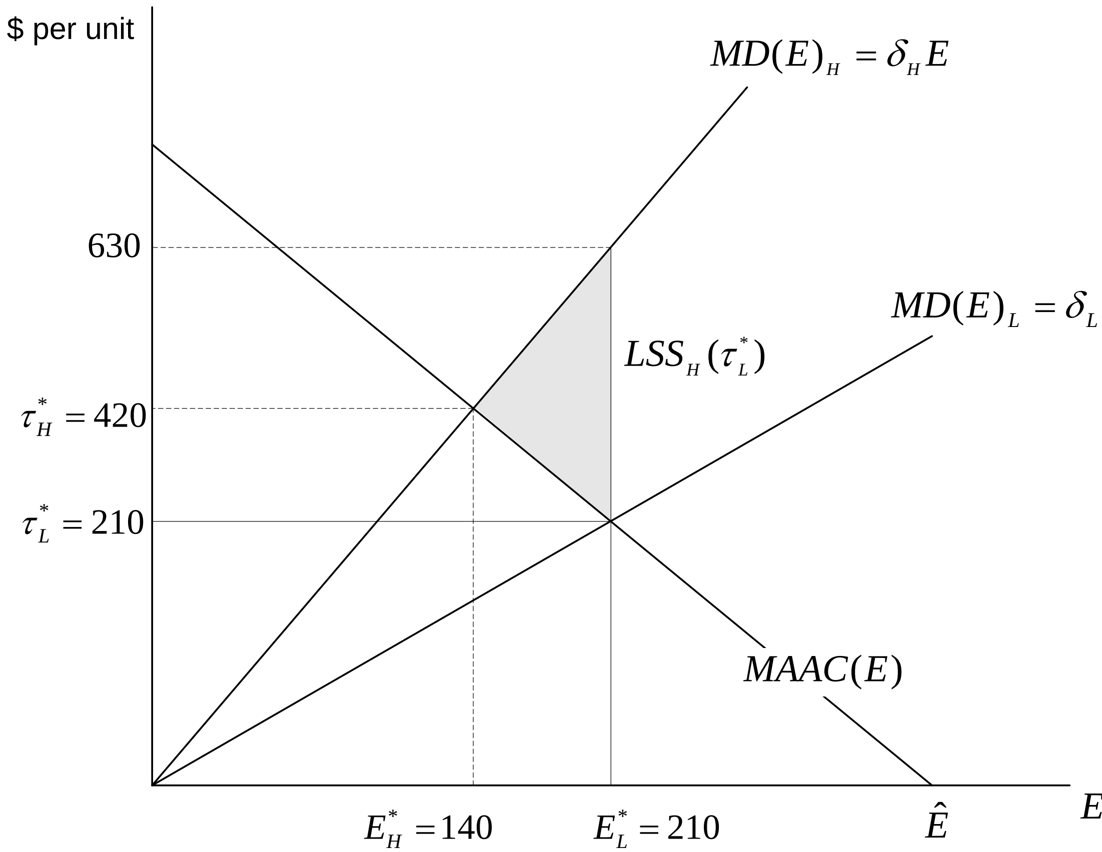

$ per unit
$MD(E)_H = \delta_H E$
630
$MD(E)_L = \delta_L E$
$LSS_H(\tau_L^*)$
$\tau_H^* = 420$
$\tau_L^* = 210$
$MAAC(E)$
$E_H^* = 140$
$E_L^* = 210$
$\hat{E}$
E

Figure R8-1

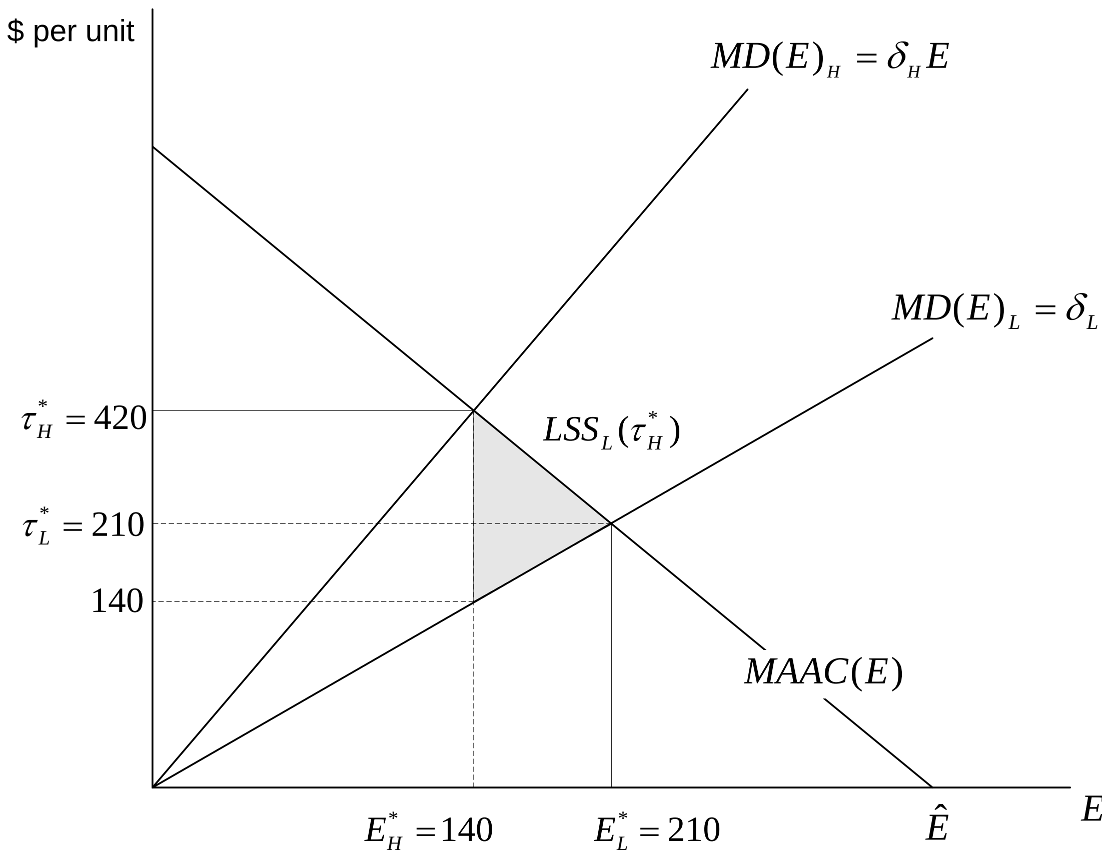

$ per unit
$MD(E)_H = \delta_H E$
$MD(E)_L = \delta_L E$
$\tau_H^* = 420$
$LSS_L(\tau_H^*)$
$\tau_L^* = 210$
140
$MAAC(E)$
E
$E_H^* = 140$
$E_L^* = 210$
$\hat{E}$

Figure R8-2

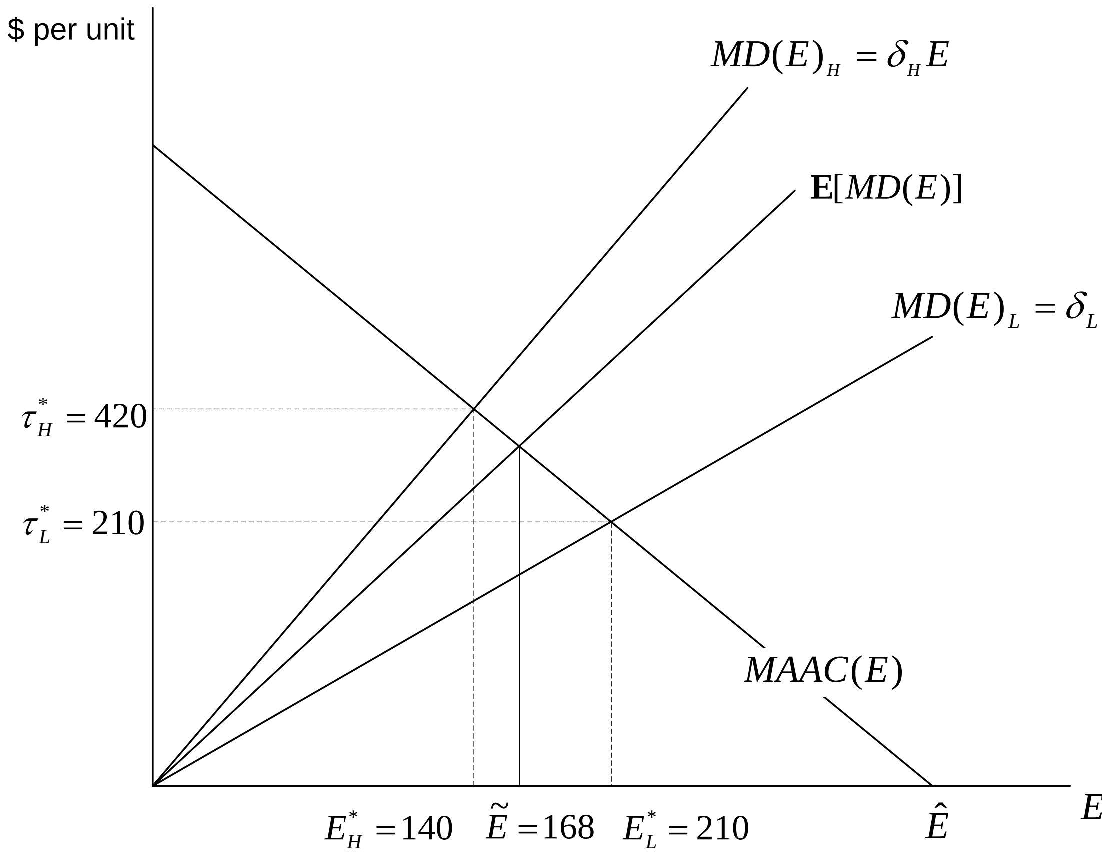

$ per unit
$MD(E)_H = \delta_H E$
$\mathbf{E}[MD(E)]$
$MD(E)_L = \delta_L E$
$\tau_H^* = 420$
$\tau_L^* = 210$
$MAAC(E)$
$E_H^* = 140$
$\tilde{E} = 168$
$E_L^* = 210$
$\hat{E}$
E

Figure R8-3

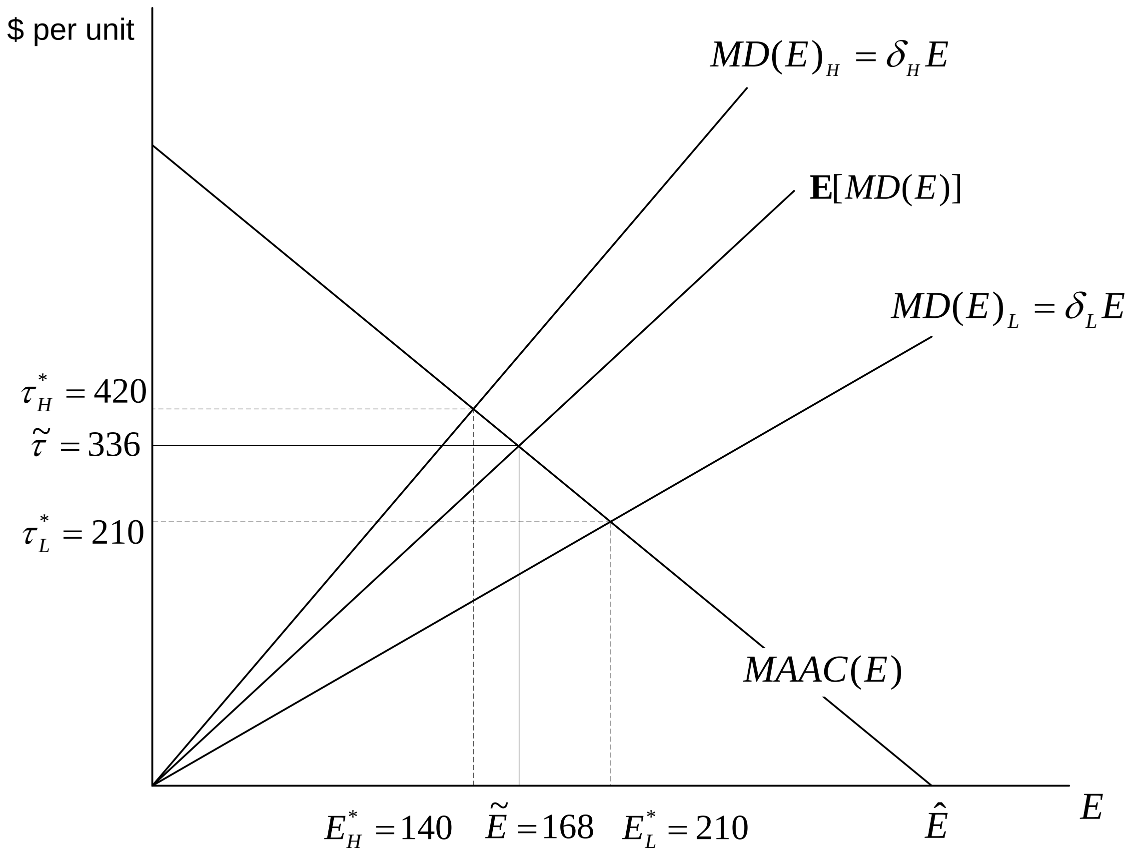

$ per unit
$MD(E)_H = \delta_H E$
$\mathbf{E}[MD(E)]$
$MD(E)_L = \delta_L E$
$\tau_H^* = 420$
$\tilde{\tau} = 336$
$\tau_L^* = 210$
$MAAC(E)$
$E_H^* = 140$
$\tilde{E} = 168$
$E_L^* = 210$
$\hat{E}$
E

Figure R8-4

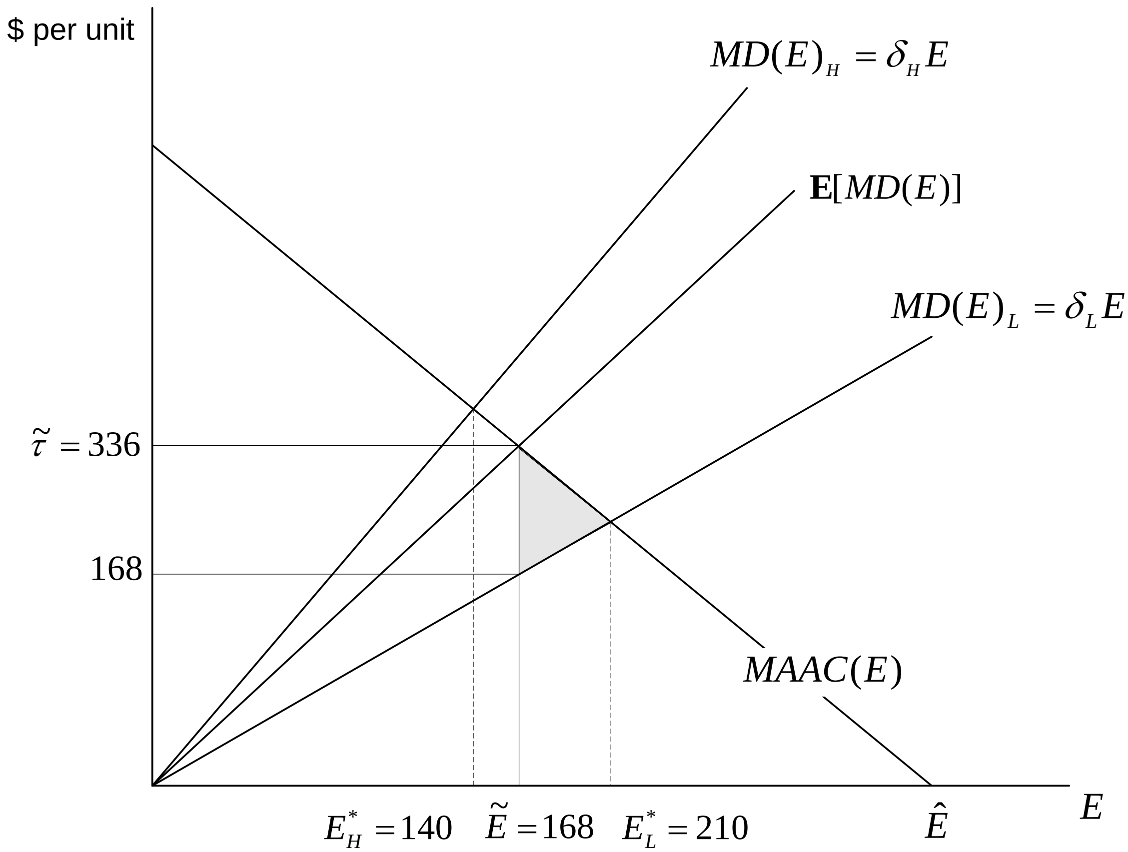

$ per unit
$MD(E)_H = \delta_H E$
$\mathbf{E}[MD(E)]$
$MD(E)_L = \delta_L E$
$\tilde{\tau} = 336$
168
$MAAC(E)$
$E_H^* = 140$
$\tilde{E} = 168$
$E_L^* = 210$
$\hat{E}$
E

Figure R8-5

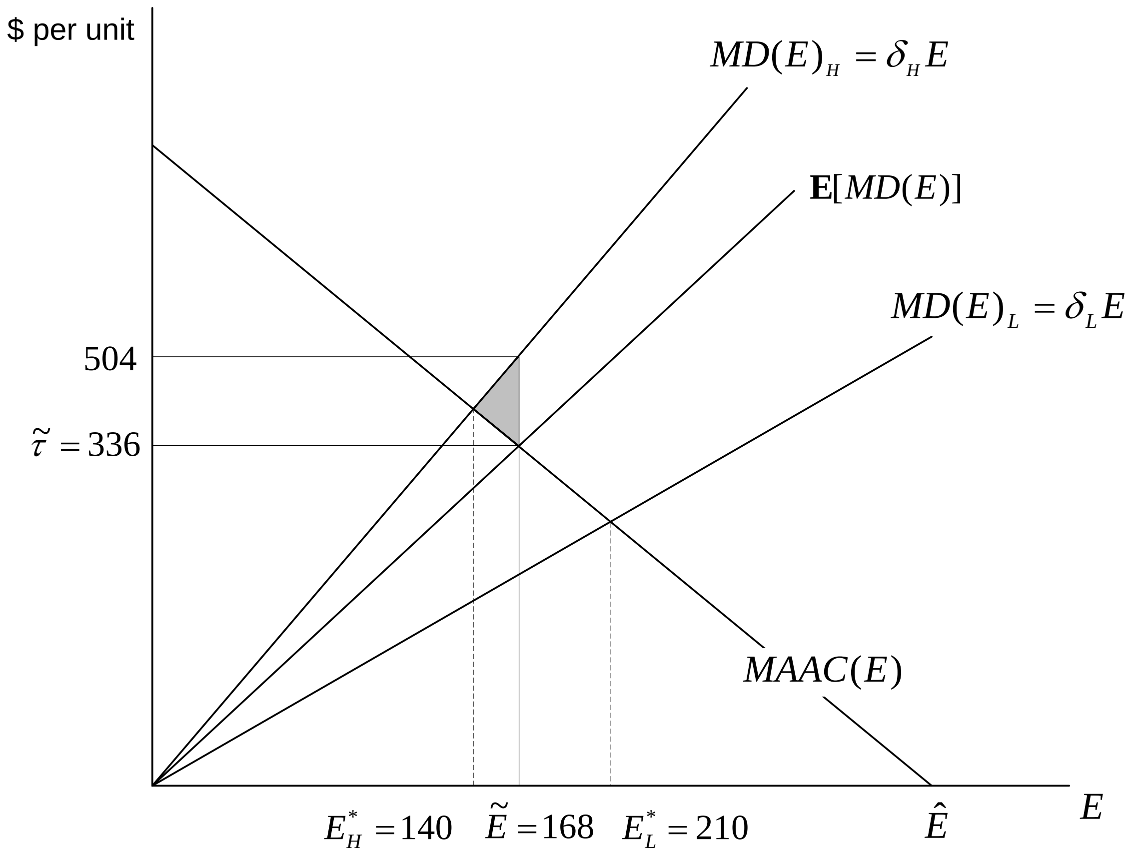

$ per unit
$MD(E)_H = \delta_H E$
$\mathbf{E}[MD(E)]$
$MD(E)_L = \delta_L E$
504
$\tilde{\tau} = 336$
$MAAC(E)$
$E_H^* = 140$
$\tilde{E} = 168$
$E_L^* = 210$
$\hat{E}$
E

Figure R8-6

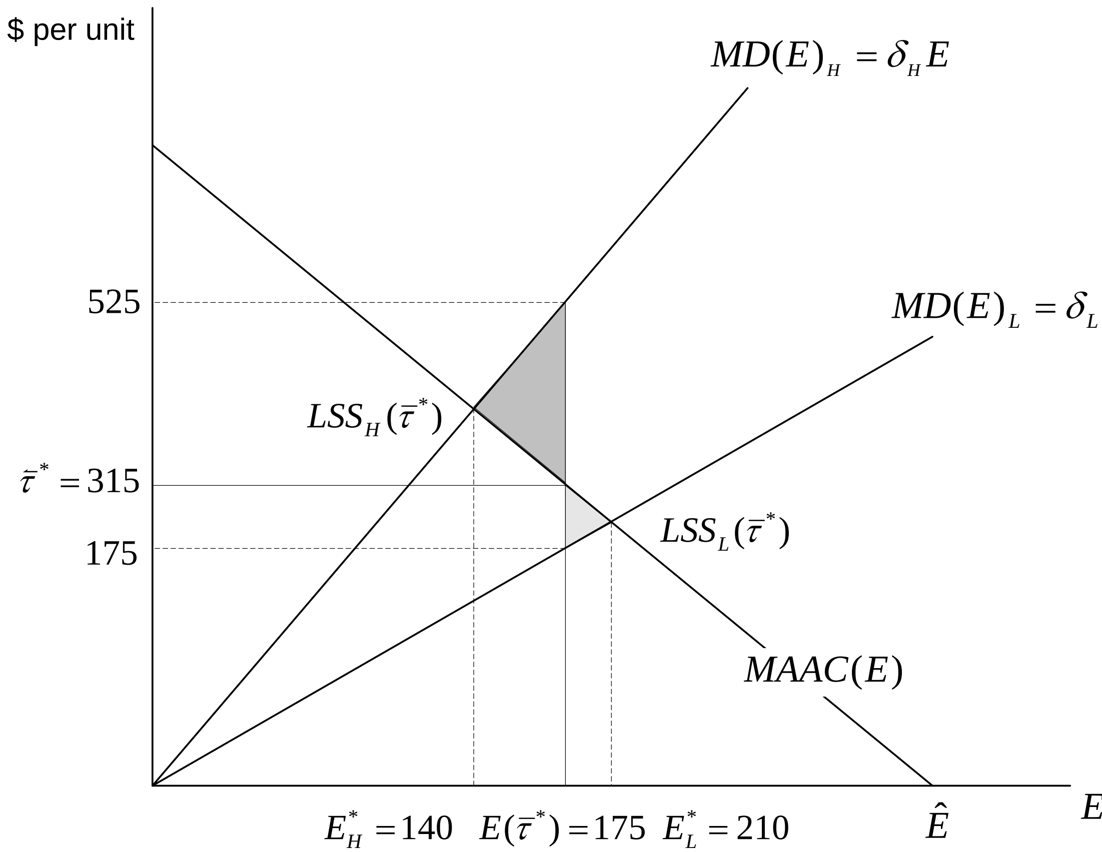

$ per unit
$MD(E)_H = \delta_H E$
$MD(E)_L = \delta_L E$
525
$LSS_H(\bar{\tau}^*)$
$\bar{\tau}^* = 315$
175
$LSS_L(\bar{\tau}^*)$
$MAAC(E)$
$E_H^* = 140$
$E(\bar{\tau}^*) = 175$
$E_L^* = 210$
$\hat{E}$
E

Figure R8-7

ANSWER KEY

1. B
2. A
3. C
4. A
5. D
6. A
7. C
8. A
9. B
10. C
11. B
12. A
13. A
14. A
15. B
16. A
17. C
18. A
19. C
20. B
21. C
22. A
23. A
24. B
25. B
26. A
27. D
28. B

9. POLLUTION TAXES WHEN ABATEMENT COST IS UNCERTAIN

1

OUTLINE

* Advanced Topic

2

9.1 INTRODUCTION

3

Introduction

- The previous topic examined the design of a pollution tax when damage is uncertain.
- In that setting, the key problem lies with the choice of optimal emissions.
- Once that choice is made, implementation via a tax is straightforward if the MAC schedules are known.

4

Introduction

- In this topic we extend consideration to a setting where MACs are not known by the regulator.
- This makes the tax policy design problem much more complicated because the regulator does not know how sources will respond to the tax.

5

Introduction

- The key problem for the regulator in this regard is that sources typically have better information about their abatement costs than the regulator does.
- That is, there is **asymmetric information** about abatement costs.

6

Introduction

- This introduces an important strategic element to the interaction between the regulator and the polluting sources.
- If the regulator simply asks sources to report their MAC schedules, then those sources have an incentive to provide false information if they believe it can influence the pollution tax rate to their advantage.

7

Introduction

- Let us first examine this problem in the simplest possible setting: where there is a single polluting source.

8

9.2 A SINGLE POLLUTING SOURCE

A Single Polluting Source

- Consider a setting in which a single source could have one of two possible MACs, denoted $MAC(e)_L$ and $MAC(e)_H$, as illustrated in Figure 9-1.

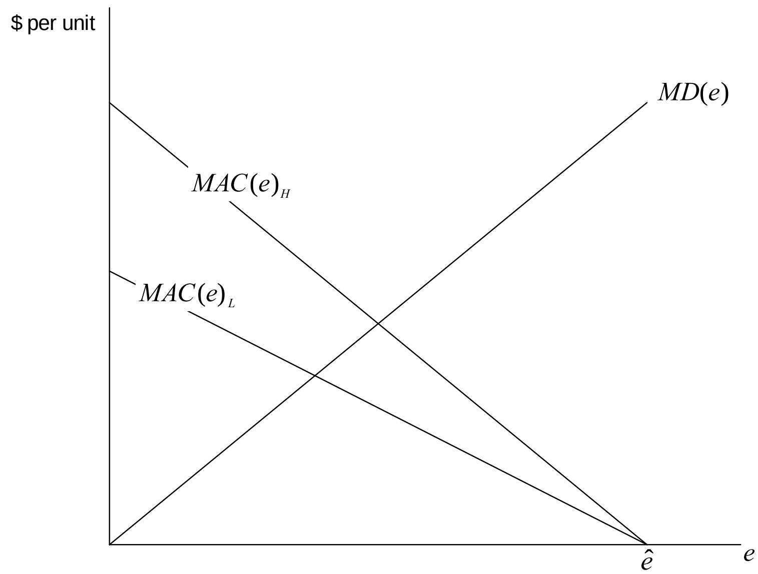


FIGURE 9-1

11

A Single Polluting Source

- We are assuming here that the two MACs have different slopes but the same horizontal intercept, at $\hat{e}$, which is known by the regulator.
- We will refer to this case as a setting with **slope uncertainty**.

12

A Single Polluting Source

- In contrast, **intercept uncertainty** refers to a setting where $\hat{e}$ is unknown.
- This case is less interesting, for two reasons.
  - First, $\hat{e}$ is the unregulated outcome, and in principle, should be observable.
  - Second, slope uncertainty presents a much greater policy problem than intercept uncertainty, for reasons that will become clear.

13

A Single Polluting Source

- If the source has $MAC(e)_L$, we will call it the "L type"; if it has $MAC(e)_H$, we will call it the "H type".
- If the regulator could distinguish between these two types, it would set the tax at either τ^*_L or τ^*_H accordingly, and the source would respond with the socially-optimal emissions level, as illustrated in Figure 9-2.

14

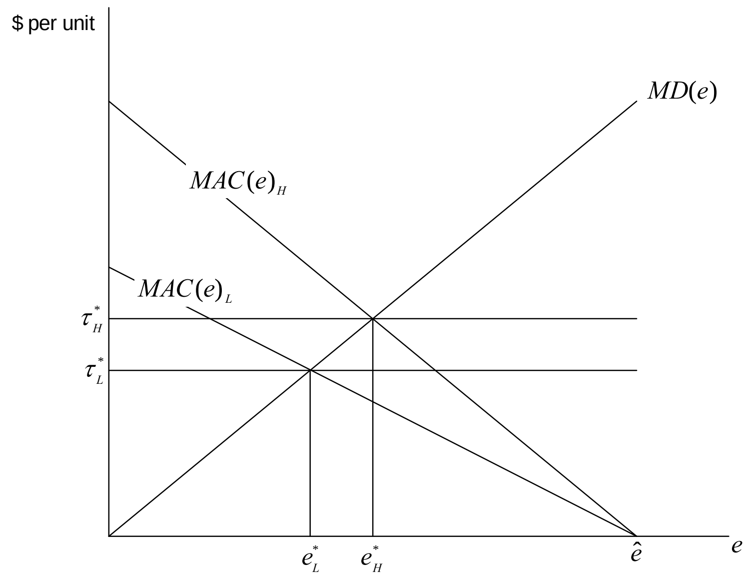


FIGURE 9-2

15

A Single Polluting Source

- However, if the regulator cannot distinguish between types, then it cannot determine the correct the Pigouvian tax.
- What can the regulator do in that situation?

16

Pooling and Separating Equilibria

- Suppose the regulator simply asks the source to report its type, and then sets the tax rate based on this report.
- Does the source have an incentive to report truthfully?

17

Pooling and Separating Equilibria

- To answer this question, we must compare the compliance cost for the source if it reports truthfully with its compliance cost if it reports falsely.

18

Pooling and Separating Equilibria

- To begin, let us consider that comparison for the H type.
- If it reports truthfully, then it will face tax rate τ^*_H , and choose emissions e^*_H .
- Its compliance cost will be the shaded area in Figure 9-3.

19

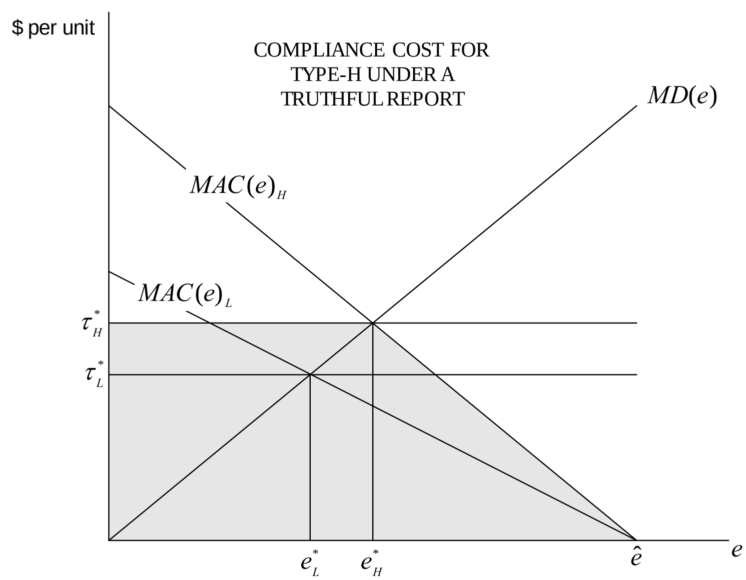


FIGURE 9-3

20

Pooling and Separating Equilibria

- If instead the H type makes a false report, and claims that it is L type, then it will face tax rate τ^*_L .
- <u>If</u> it could then set emissions in response to this tax rate as it would like, it would choose $e_H(\tau^*_L)$ as illustrated in Figure 9-4, and incur a compliance cost equal to the shaded area in that figure.

21

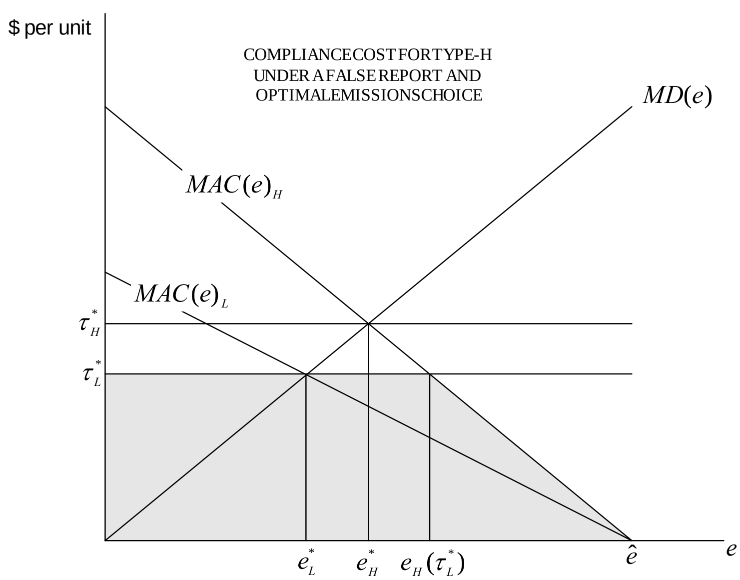


FIGURE 9-4

22

Pooling and Separating Equilibria

- Comparing the compliance costs in Figures 9-3 and 9-4, it is clear that making a false report is the best choice under those conditions.

23

Pooling and Separating Equilibria

- <u>However</u>, if the source does choose $e_H(\tau^*_L)$ in response to τ^*_L then the regulator will immediately infer that it is in fact the H type, since the L type would never choose $e_H(\tau^*_L)$; it would choose e^*_L .

24

Pooling and Separating Equilibria

- Thus, for the false report to be credible, the H type must choose e^*_L; that is, it must mimic the L type by responding to the tax as the L type would.
- The associated compliance cost is equal to the shaded area in Figure 9-5.

25

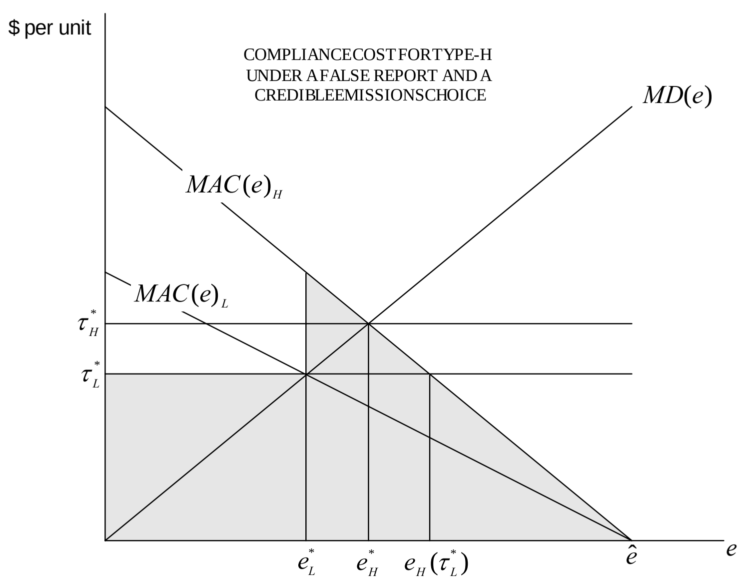


FIGURE 9-5

26

Pooling and Separating Equilibria

- By comparing the shaded areas in Figure 9-5 and 9-3, we can construct the cost saving from reporting falsely.
- This cost saving is depicted as the difference between the light-shaded and dark-shaded areas in Figure 9-6.

27

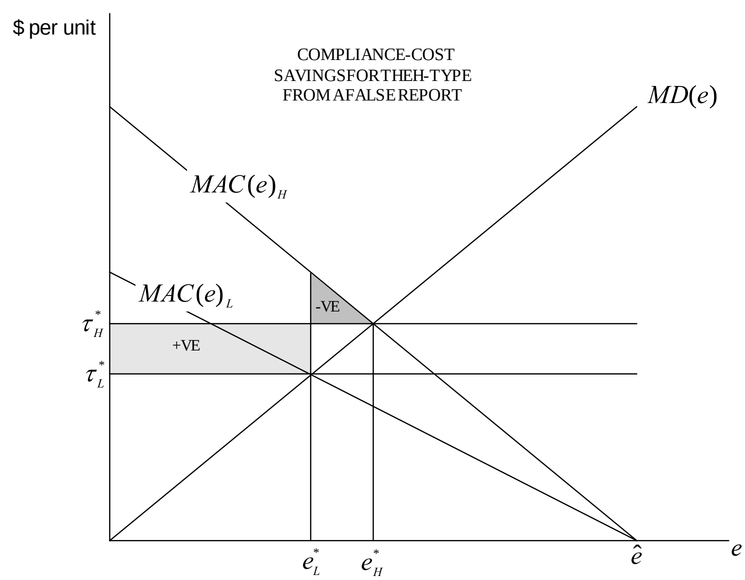


FIGURE 9-6

28

Pooling and Separating Equilibria

- Based on Figure 9-6, it appears that the net savings from a false report are positive for the H type.
- However, we cannot be sure that this would be true under a different set of MACs.

29

Pooling and Separating Equilibria

- For example, in the case illustrated in Figure 9-7, savings for the H type from a false report are negative.

30

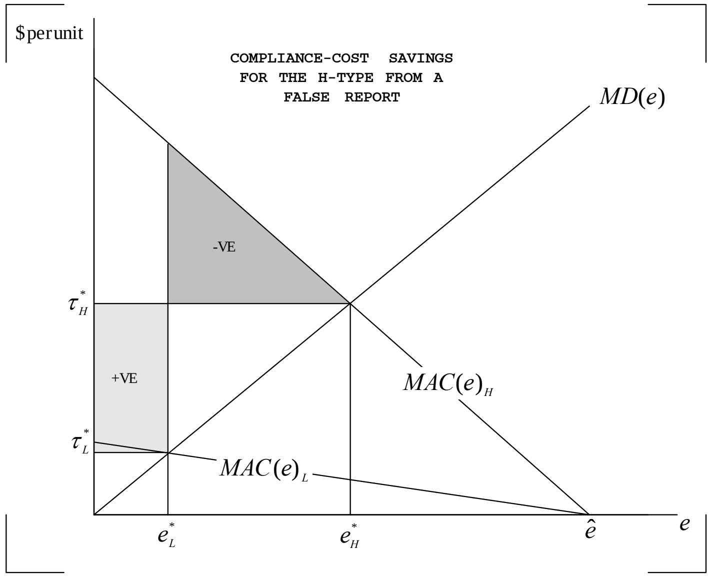


FIGURE 9-7

31

Pooling and Separating Equilibria

- To examine this question more closely, let us consider the linear model on which the figures are based.

32

Pooling and Separating Equilibria

- Suppose MD is given by

$$MD(e) = \delta e$$

and MAC is given by

$$MAC(e)_i = \gamma_i(\hat{e} - e)$$

where i=L or H.

33

Pooling and Separating Equilibria

- Under full-information, socially optimal emissions are

$$e_i^* = \frac{\gamma_i \hat{e}}{\gamma_i + \delta}$$

where i=L or H.

34

Pooling and Separating Equilibria

- The corresponding full-information Pigouvian taxes are

$$\tau_i^* = \frac{\delta\gamma_i \hat{e}}{\gamma_i + \delta}$$

where $i=L$ or H.

35

Pooling and Separating Equilibria

- From Figure 9-6 (or Figure 9-7) we can calculate the compliance-cost savings for the H type (CCS_H) from a false report as

$$CCS_H = (\tau_H^* - \tau_L^*)e_L^* - \frac{\gamma_H (e_H^* - e_L^*)^2}{2}$$

36

Pooling and Separating Equilibria

- Making the substitutions for τ^*_i and e^*_i, yields the following requirement for CCS_H>0:

$$\gamma_L > \frac{\gamma_H^2}{3\gamma_H + 2\delta}$$

37

Pooling and Separating Equilibria

- This tells us that false reporting by the H type is worthwhile if and only if the difference between γ_L and γ_H is not too large.
- This threshold relationship between γ_L and γ_H is illustrated in Figure 9-8 for a fixed value of δ.

38

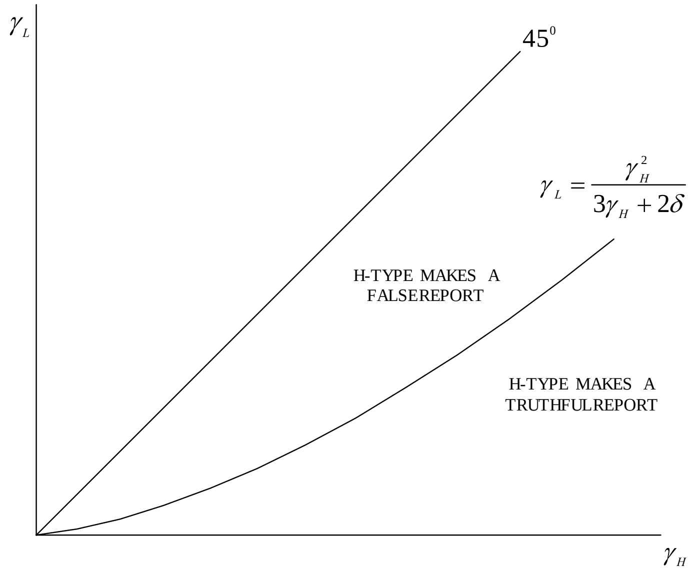


FIGURE 9-8

39

Pooling and Separating Equilibria

- If γ_L is <u>below</u> this threshold then the H type will report truthfully.
- Interpretation:
  - If γ_L is much lower than γ_H then e^*_L is much lower than e^*_H, and this makes mimicking the L type very costly for the H type; it is better-off if it reports truthfully.

40

Pooling and Separating Equilibria

- In general, mimicking is worthwhile only if the subject of that mimicking is not too different from the mimic itself, otherwise too much effort – in our context, too much cost – must be incurred to make the mimicking credible.

41

Pooling and Separating Equilibria

- Now let us consider the incentives for truthful reporting by the L type.

- If it reports truthfully, then it will face tax rate τ^*_L, and choose emissions e^*_L.
- Its compliance cost will be the shaded area in Figure 9-9.

42

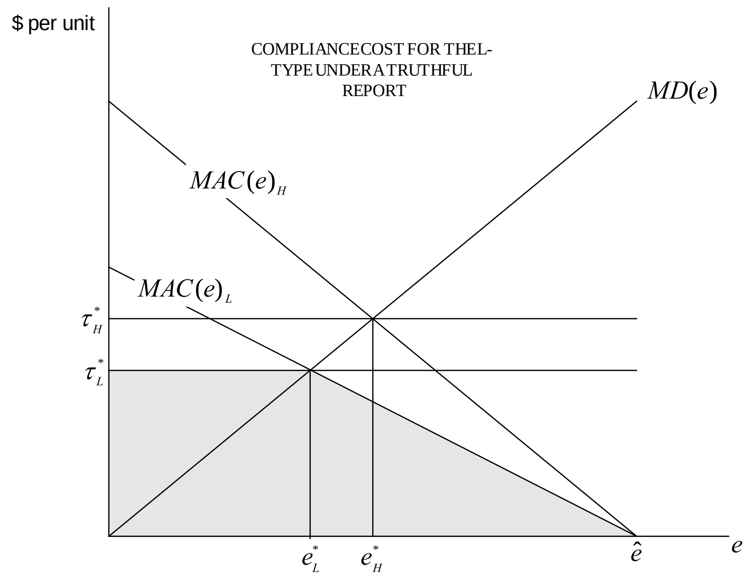


FIGURE 9-9

43

Pooling and Separating Equilibria

- Conversely, if the L type reports falsely, it will face tax rate τ^*_H, and its only <u>credible</u> response is to choose e^*_H.
- The associated compliance cost is equal to the shaded area in Figure 9-10.

44

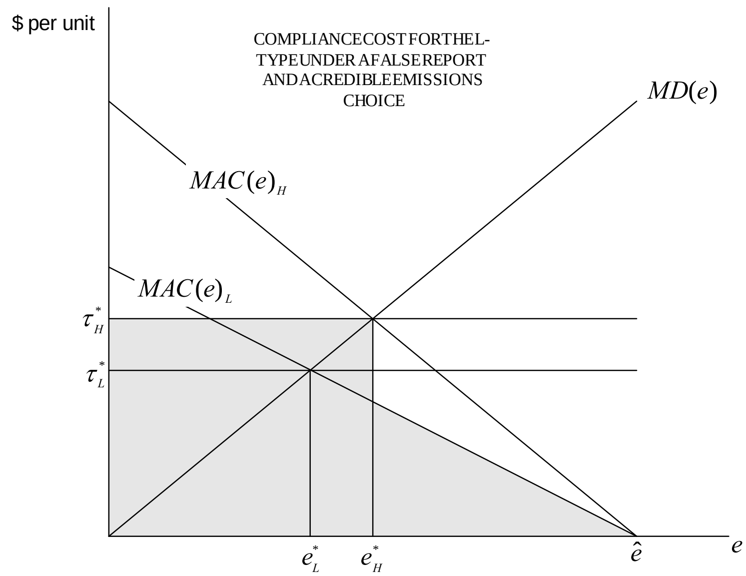


FIGURE 9-10

45

Pooling and Separating Equilibria

- It is clear from Figures 9-9 and 9-10 that the L type has no incentive to make a false report; doing so always leads to a higher compliance cost.

46

Pooling and Separating Equilibria

- In summary, the L type will always report truthfully, and the H type will report truthfully if and only if γ_L lies on or below the threshold in Figure 9-8.

47

Pooling and Separating Equilibria

- This means there are two possible outcomes here, depending on parameter values:
  - a **pooling equilibrium,** where both types make the same report (one falsely reporting L, the other truthfully reporting L); and
  - a **separating equilibrium,** where each type makes a different report (both truthful).
- See Figure 9-11.

48

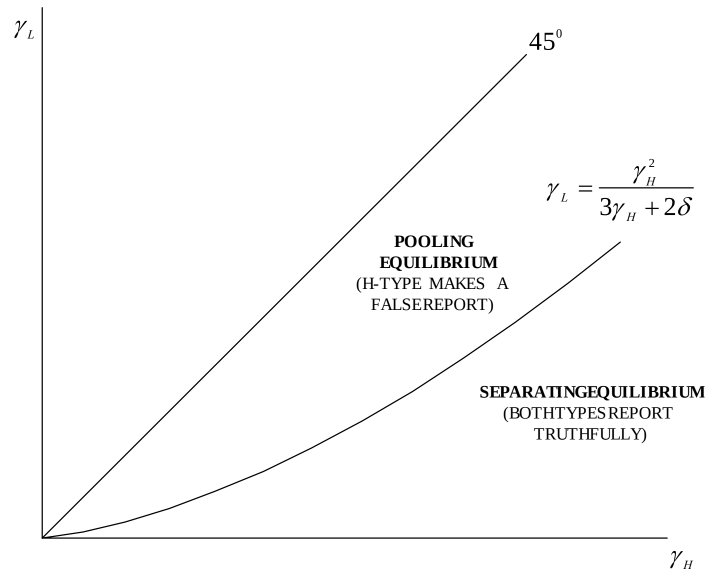


FIGURE 9-11

49

Pooling and Separating Equilibria

- In the separating equilibrium, the Pigouvian tax implements the social optimum: the emissions choice by the source equates MD with the true MAC, regardless of type.
- In the pooling equilibrium, the social optimum is implemented only if the source is the L type.

50

Pooling and Separating Equilibria

- If the source is the H type then the Pigouvian tax does not implement the social optimum in the pooling equilibrium, and there is an associated loss of social surplus, equal to the shaded area in Figure 9-12.

51

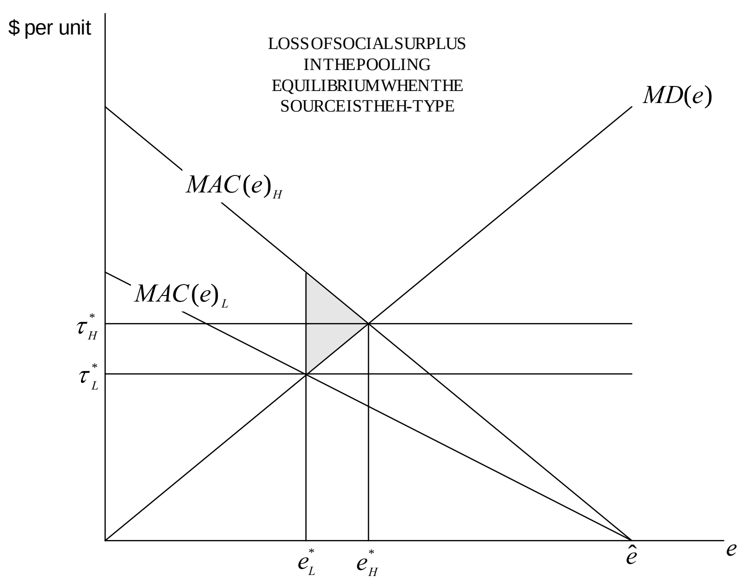


FIGURE 9-12

52

Pooling and Separating Equilibria

- Note that the H type abates too much in the pooling equilibrium relative to the social optimum.
- It does so despite the elevated abatement cost associated with that choice because the tax savings from mimicking the L type are sufficiently high to make the excessive abatement worthwhile.

53

9.3 SECOND-BEST POLICY FOR A SINGLE POLLUTING SOURCE*

* Advanced Topic

54

Second-Best Policy for a Single Polluting Source

- Is there a tax policy that can implement the social optimum under the pooling equilibrium conditions?
- As one might expect, there is no policy that can achieve socially-optimal emissions and at the same time extract the full resource rent from the source under all conditions.

55

Second-Best Policy for a Single Polluting Source

- However, there are two alternative policy approaches that can achieve socially-optimal emissions if the regulator forgoes some of the resource rent.
- We will refer to these as
  - an increasing marginal rate policy
  - an incentive-compatible fixed-rate policy
- Consider each in turn.

56

The Increasing Marginal Rate Policy

- The standard Pigouvian rule prescribes a fixed tax rate; each unit of emissions is charged the same tax rate.
- Suppose instead the regulator charges a different tax rate on each unit of emissions according to its marginal damage.

57

The Increasing Marginal Rate Policy

- Such a tax rule sets the **marginal tax rate** according to the MD schedule:

$$\tau(e) = MD(e)$$

- Since the MD schedule is upward-sloping, this means that the marginal tax rate rises as emissions rise.

58

The Increasing Marginal Rate Policy

- This type of tax rule may be familiar from income tax systems, which often use increasing marginal tax rates (typically rising in discrete jumps as income rises).
- In that context, increasing marginal tax rates are usually motivated by distributional goals not informational ones.

59

The Increasing Marginal Rate Policy

- Faced with a tax schedule given by

$$\tau(e) = MD(e)$$

a source of type *i* will respond by emitting up to the point where $MAC(e)_i = \tau(e)$, as illustrated in Figure 9-13.

60

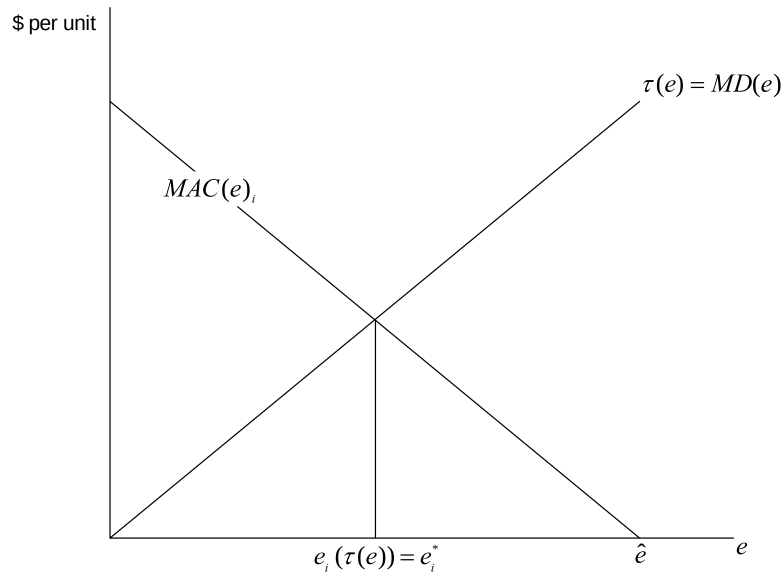


FIGURE 9-13

61

The Increasing Marginal Rate Policy

- Since the tax schedule matches the MD schedule exactly, this response to the tax schedule implements the socially optimal emissions level.
- This is true regardless of whether the source is the L type or the H type (or any other type, in a case with more than two types).

62

The Increasing Marginal Rate Policy

- Thus, the increasing marginal rate (IMR) policy always induces socially-optimal emissions in a setting with a single source.

63

The Increasing Marginal Rate Policy

- Note that under this IMR policy, the standard Pigouvian tax rate is charged only on the marginal unit of emissions; all other units are charged their own specific tax rate, each one lower than the Pigouvian rate.

64

The Increasing Marginal Rate Policy

- This means that the revenue generated by the IMR policy is exactly equal to the damage done, since the total tax payment is simply the area under the tax schedule; see Figure 9-14.

65

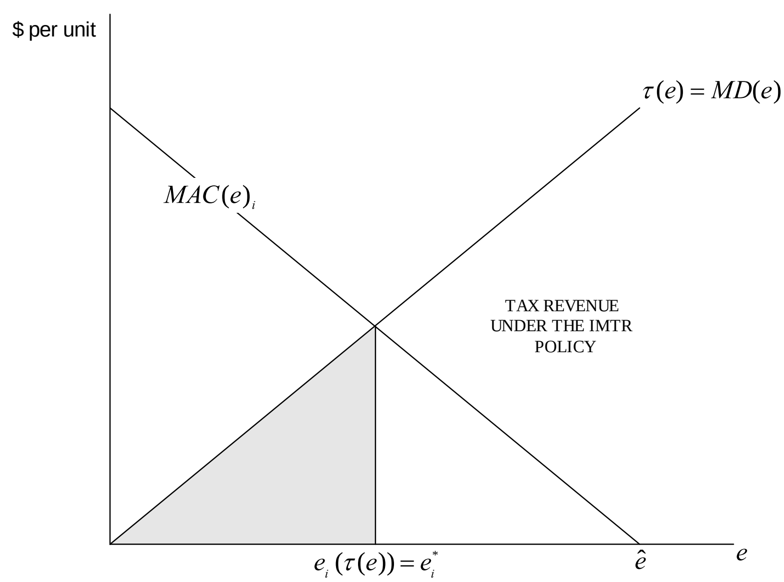


FIGURE 9-14

66

The Increasing Marginal Rate Policy

- Thus, the regulator forgoes the entire resource rent under the IMR policy.

67

Incentive-Compatible Fixed Rates

- An incentive-compatible fixed-rate (ICFR) policy charges a single tax rate for all units of emissions, in the spirit of the Pigouvian tax, but sets those rates to ensure that the L type always chooses e^*_L, and the H type always chooses e^*_H.

68

Incentive-Compatible Fixed Rates

- Specifically, the source is offered a choice between two "tax contracts", L and H:
  - the L contract specifies that the source emit e^*_L, and pay a tax rate τ_L on each unit;
  - the H contract specifies that the source emit e^*_H, and pay a tax rate τ_H on each unit.

69

Incentive-Compatible Fixed Rates

- The key to the policy is to choose τ_L and τ_H to ensure that the tax contracts are **incentive compatible**:
  - compliance cost for the L type must be lower under the L contract than under the H contract; and
  - compliance cost for the H type must lower under the H contract than under the L contract.

70

Incentive-Compatible Fixed Rates

- Thus, the ICFR policy is designed to induce a separating equilibrium under all conditions.

71

Incentive-Compatible Fixed Rates

- The ICFR policy is based on a key result in economic theory called the **revelation principle**.
- Roughly-speaking, it states that the best contract in a setting with asymmetric information is one in which all agents reveal themselves truthfully; that is, one that induces a separating equilibrium.

72

Incentive-Compatible Fixed Rates

- Let us examine the design of the ICFR policy in the context of the linear example.

- The first step is to derive incentive-compatibility conditions for each type.

73

Incentive-Compatible Fixed Rates

- For the L type, compliance cost under the L contract is

$$CC_{LL} = e_L^* \tau_L + \frac{\gamma_L (\hat{e} - e_L^*)}{2}$$

74

Incentive-Compatible Fixed Rates

- In comparison, compliance cost for the L type under the H contract is

$$CC_{LH} = e_H^* \tau_H + \frac{\gamma_L (\hat{e} - e_H^*)}{2}$$

75

Incentive-Compatible Fixed Rates

- The **incentive compatibility condition** for the L type (**IC$_L$**) is

$$CC_{LL} \leq CC_{LH}$$

76

Incentive-Compatible Fixed Rates

- Making the appropriate substitutions for CC_{LL} and CC_{LH}, and rearranging the expression, the $\mathbf{IC_L}$ condition can be written as a linear relationship in τ_H and τ_L :

$$\tau_H \geq a\tau_L + b$$

77

Incentive-Compatible Fixed Rates

where

$$a = \frac{\gamma_L(\gamma_H + \delta)}{\gamma_H(\gamma_L + \delta)}$$

and

$$b = \frac{\delta^2(\gamma_H - \gamma_L)(\gamma_H + \gamma_L + 2\delta)}{2(\gamma_H + \delta)(\gamma_L + \delta)^2}$$

78

Incentive-Compatible Fixed Rates

- For the moment, we are not too concerned about these specific expressions for *a* and *b*, except to note that $b>0$ since $\gamma_H>\gamma_L$.

79

Incentive-Compatible Fixed Rates

- The $\mathbf{IC_L}$ condition is illustrated in Figure 9-15, where the shaded region depicts combinations of τ_L and τ_H where

$$CC_{LL} \leq CC_{LH}$$

80

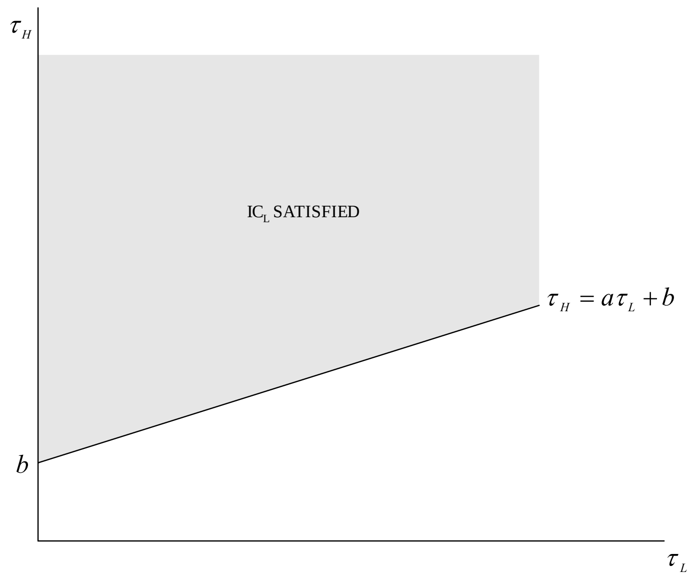


FIGURE 9-15

81

Incentive-Compatible Fixed Rates

- <u>Any</u> combination of τ_L and τ_H in the shaded region of Figure 9-15 will induce the L type to choose the L contract rather than the H contract.

82

Incentive-Compatible Fixed Rates

- Note that the **IC_L** threshold is upward-sloping:
  - a higher value of τ_L makes the H contract relatively more attractive, so a higher value of τ_H is needed to offset that effect.

83

Incentive-Compatible Fixed Rates

- Now let us derive the incentive-compatibility condition for the H type.

- Compliance cost for the H type under the H contract is

$$CC_{HH} = e_H^* \tau_H + \frac{\gamma_H (\hat{e} - e_H^*)}{2}$$

84

Incentive-Compatible Fixed Rates

- In comparison, compliance cost for the H type under the L contract is

$$CC_{HL} = e_L^* \tau_L + \frac{\gamma_H (\hat{e} - e_L^*)}{2}$$

85

Incentive-Compatible Fixed Rates

- The incentive compatibility condition for the H type (**IC$_H$**) is

$$CC_{HH} \leq CC_{HL}$$

86

Incentive-Compatible Fixed Rates

- Making the appropriate substitutions for CC_{HH} and CC_{HL}, and rearranging the expression, the $\mathbf{IC_H}$ condition can be written as a linear relationship in τ_H and τ_L :

$$\tau_H \leq a\tau_L + b\frac{\gamma_H}{\gamma_L}$$

where a and b are as given on s.76.

87

Incentive-Compatible Fixed Rates

- The $\mathbf{IC_H}$ condition is illustrated in Figure 9-16, where the shaded region depicts combinations of τ_L and τ_H where

$$CC_{HH} \leq CC_{HL}$$

88

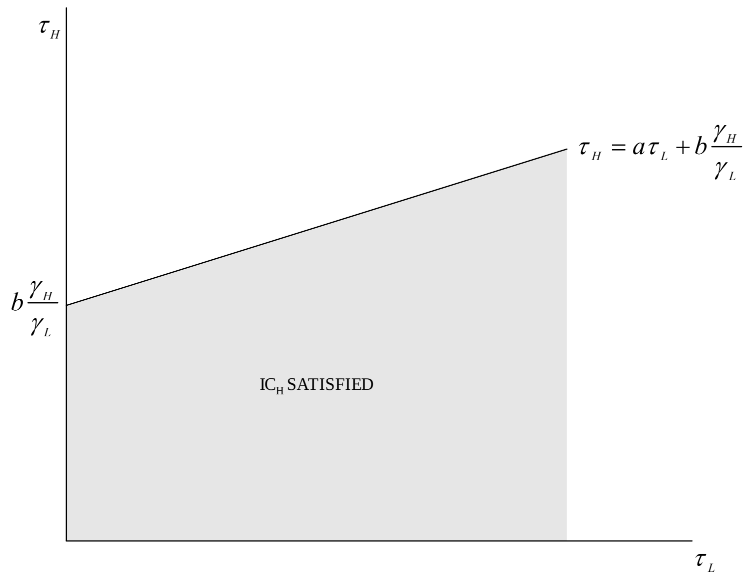


FIGURE 9-16

89

Incentive-Compatible Fixed Rates

- Any combination of τ_L and τ_H below the threshold in Figure 9-16 will induce the H type to choose the H contract rather than the L contract.

90

Incentive-Compatible Fixed Rates

- Note that this threshold is upward-sloping:
  - a higher value of τ_H makes the L contract relatively more attractive, so a higher value of τ_L is needed to offset that effect.

91

Incentive-Compatible Fixed Rates

- Now let us combine Figures 9-16 and 9-15 to identify the combinations of τ_L and τ_H under which both IC conditions are satisfied; see Figure 9-17.

92

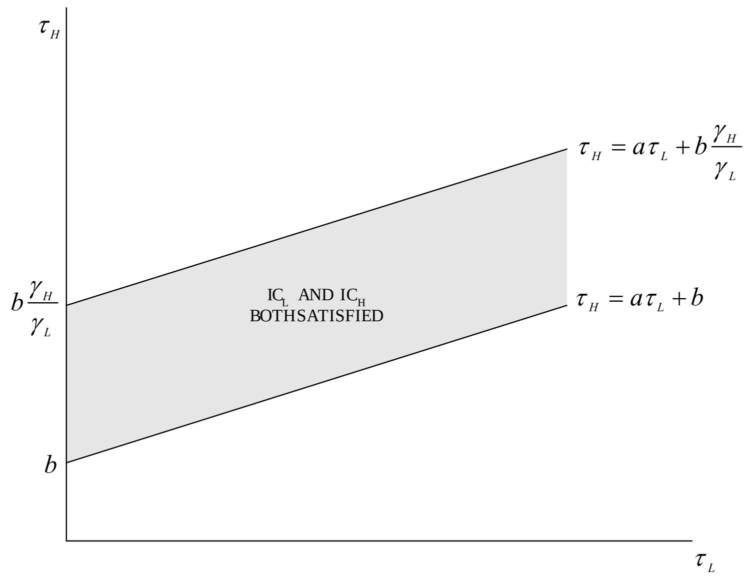


FIGURE 9-17

93

Incentive-Compatible Fixed Rates

- Any combination of τ_L and τ_H in the shaded region of Figure 9-17, when paired with e_L^* and e_H^* respectively in a tax contract, is an incentive compatible policy:
  - the policy will induce the L type to choose the L contract, and the H type to choose the H contract.

94

Incentive-Compatible Fixed Rates

- How do the Pigouvian taxes (τ_L^* and τ_H^*) relate to the incentive-compatible policy?
- Recall from Section 9.1 that the Pigouvian tax rule induces a separating equilibrium if and only if

$$\gamma_L \leq \frac{\gamma_H^2}{3\gamma_H + 2\delta}$$

95

Incentive-Compatible Fixed Rates

- If this condition is satisfied, then the Pigouvian rule is an incentive-compatible policy; see Figure 9-18.

96

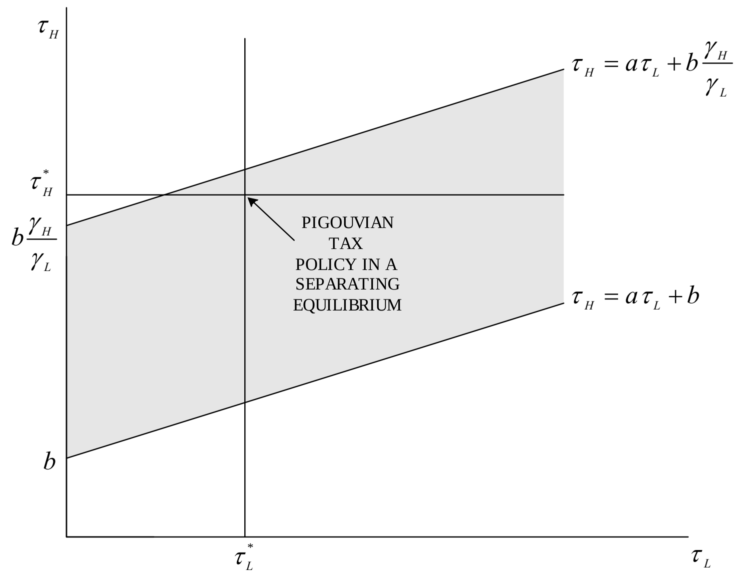


FIGURE 9-18

97

Incentive-Compatible Fixed Rates

- Conversely, if

$$\gamma_L > \frac{\gamma_H^2}{3\gamma_H + 2\delta}$$

then the Pigouvian tax rule induces a pooling equilibrium; it is not incentive compatible.
- See Figure 9-19.

98

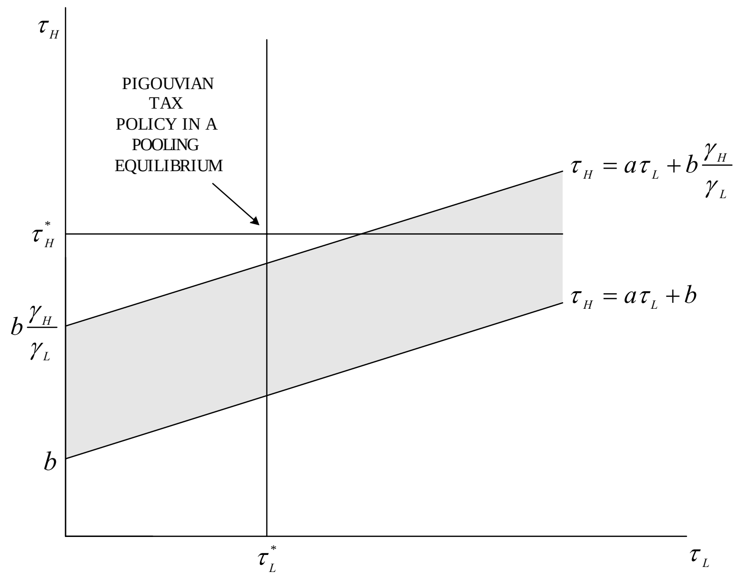


FIGURE 9-19

99

Incentive-Compatible Fixed Rates

- In the case where the Pigouvian tax policy is not incentive compatible (as in Figure 9-19) which ICFR policy should the regulator choose?
- Recall that all ICFR policies induce socially optimal emissions, so they differ only in terms of revenue raised (and hence, compliance costs).

100

Incentive-Compatible Fixed Rates

- Suppose the goal of the regulator is to find an ICFR policy that is **revenue-equivalent** to a Pigouvian tax policy under truthful reporting.

- What does “revenue-equivalent” mean in this setting?

101

Incentive-Compatible Fixed Rates

- Recall that the regulator does not know whether the source is the L type or the H type until the source reveals itself through the tax contract it chooses.
- Thus, when constructing the policy, the regulator cannot know how much revenue it will <u>actually</u> generate.

102

Incentive-Compatible Fixed Rates

- However, the regulator can calculate the **expected revenue** from a policy:
  - the amount of revenue it will generate “on average”, given the relative likelihoods of the source being the L type or the H type.

103

Incentive-Compatible Fixed Rates

- Let π denote the subjective probability that the source is type L, and (1- π) denote the subjective probability that it is type H.
- Then **expected revenue** from an ICFR policy with tax rates τ_L and τ_H is

$$\mu(\tau_L, \tau_H) = \pi\tau_L e_L^* + (1-\pi)\tau_H e_H^*$$

104

Incentive-Compatible Fixed Rates

- In comparison, expected revenue from the Pigouvian rule under truthful reporting would be

$$\mu^* = \pi\tau_L^* e_L^* + (1-\pi)\tau_H^* e_H^*$$

105

Incentive-Compatible Fixed Rates

- Setting $\mu(\tau_L,\tau_H)=\mu^*$ and solving for τ_H yields a linear relationship between τ_H and τ_L that ensures revenue-equivalence with the Pigouvian rule under truthful reporting:

$$\tau_H(\tau_L;\mu^*) = -\left(\frac{\pi e_L^*}{(1-\pi)e_H^*}\right)\tau_L + \frac{\mu^*}{(1-\pi)e_H^*}$$

106

Incentive-Compatible Fixed Rates

- Graphically, this relationship between τ_H and τ_L is an **iso-revenue schedule** passing through the Pigouvian tax policy with slope

$$-\frac{\pi e_L^*}{(1-\pi)e_H^*}$$

It is depicted in Figure 9-20.

107

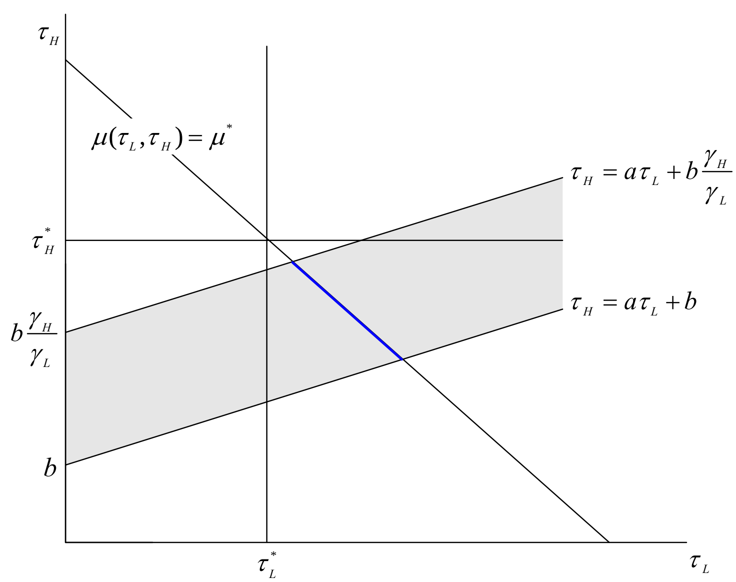


FIGURE 9-20

108

Incentive-Compatible Fixed Rates

- The highlighted segment of the iso-revenue schedule passing through the shaded region of Figure 9-20 identifies the set of ICFR policies that yield the same expected revenue as the Pigouvian tax policy under truthful reporting.

109

Incentive-Compatible Fixed Rates

- Note that there are a continuum of ICFR policies that satisfy this criterion.

- The one closest to the Pigouvian tax policy is $\{\tau_L^{IC}, \tau_H^{IC}\}$, depicted in Figure 9-21 as the “near Pigouvian” ICFR policy.

110

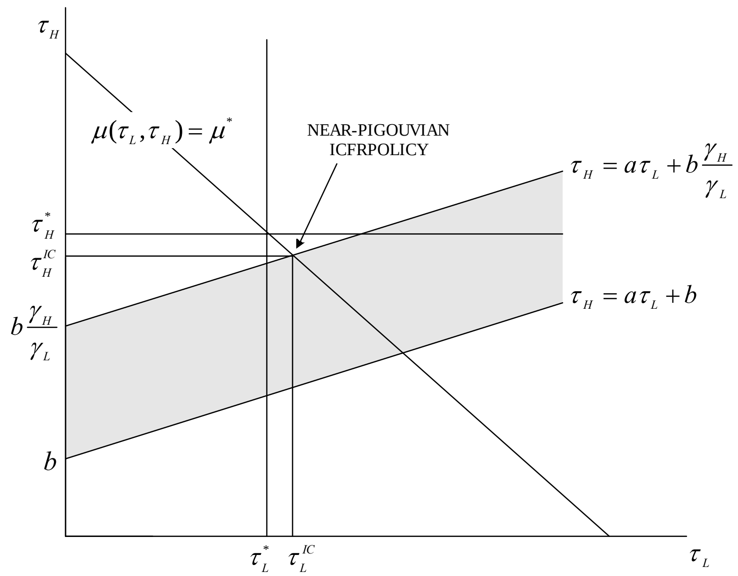


FIGURE 9-21

111

Incentive-Compatible Fixed Rates

- The $\{\tau_L^{IC}, \tau_H^{IC}\}$ policy sets a tax rate for the H type that is just low enough to induce a separating equilibrium, and a tax rate for the L type that ensures revenue-equivalence (in expected value terms) with a Pigouvian tax policy under truthful reporting.

112

Incentive-Compatible Fixed Rates

- Note that this revenue-equivalent policy over-taxes the L source (relative to the Pigouvian rule) and under-taxes the H source, so actual revenue is always either higher or lower than it would be under complete information.

113

Incentive-Compatible Fixed Rates

- How does this revenue-equivalent ICFR policy compare with the IMR policy we examined earlier?
- We know that the IMR policy induces socially optimal emissions, as does an ICFR policy, so there must exist an ICFR policy that is equivalent to the IMR policy.
- Let us find that policy.

114

Incentive-Compatible Fixed Rates

- Recall that tax revenue under the IMR policy is exactly equal to damage.
- In the linear case, damage at the optimum is

$$D(e_i^*) = \frac{\delta(e_i^*)^2}{2}$$

for i=L or H

115

Incentive-Compatible Fixed Rates

- An ICFR policy yields exactly this revenue if tax rates are set equal to one-half their Pigouvian values.

116

Incentive-Compatible Fixed Rates

- Thus, the ICFR policy that matches the IMR policy is one that sets

$$\tau_L = \frac{\tau_L^*}{2} \qquad \text{and} \qquad \tau_H = \frac{\tau_H^*}{2}$$

- See Figure 9-22.

117

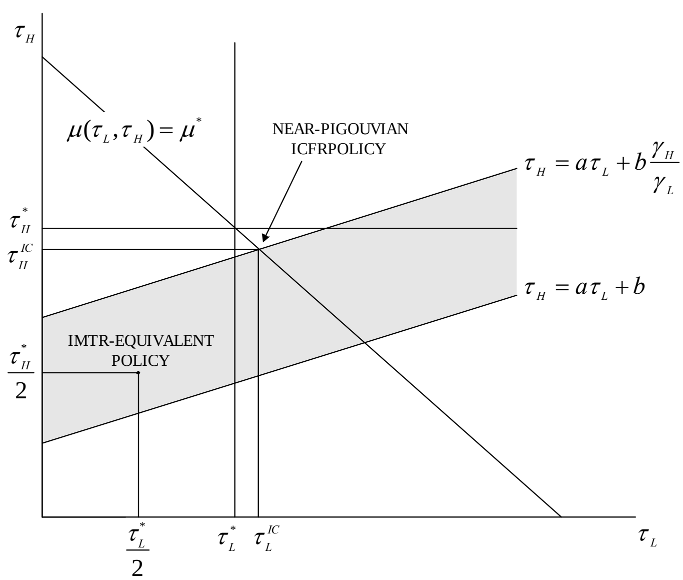


FIGURE 9-22

118

9.4 MULTIPLE SOURCES*

* Advanced Topic

119

Multiple Sources

- The policy-design problem is much more complicated when there are multiple sources (even when the pollutant is uniformly-mixed).

120

Multiple Sources

- With ICFR policies and IMR policies, we now face a conflict between incentive compatibility on one hand and the minimization of aggregate abatement cost on the other, and it is generally not possible to implement the social optimum.
- Why?

121

Multiple Sources

- Recall from Topic 6 that implementation of the social optimum requires that all sources face the same tax rate.

122

Multiple Sources

- This ensures that marginal abatement costs are equated across sources, which in turn ensures that the socially optimal level of aggregate emissions is achieved at least-cost (the **MACE** solution).
- However, assigning the same tax rate to all sources is not compatible with an ICFR policy.

123

Multiple Sources

- For example, consider a setting in which there are two sources, and from the perspective of the regulator, each one could be either an L type or an H type.
- In that case there are three possible states of the world as viewed by the regulator:
  - both sources are H type; both sources are L type; there is one source of each type.

124

Multiple Sources

- This in turn means that there are three possible values for the optimal quantity of aggregate emissions, and the regulator needs to know the true state of the world in order to determine the correct value.

125

Multiple Sources

- In principle, we can construct an ICFR policy that will reveal this information via the separating-equilibrium responses to that policy, but the associated tax rates must differ according to type.

126

Multiple Sources

- This is clearly incompatible with the least-cost requirement that all sources face the same tax rate, regardless of type.

127

Multiple Sources

- In general, if different tax rates must be assigned to different types in order to circumvent the information asymmetry, then marginal abatement costs cannot be equated across sources, and the social optimum cannot be implemented.

128

Multiple Sources

- Similarly, an IMR policy cannot induce the social optimum in this setting.
- In particular, if the sources are of different type then <u>any</u> IMR schedule will induce individual responses where MACs are <u>not</u> equated; see Figure 9-23.

129

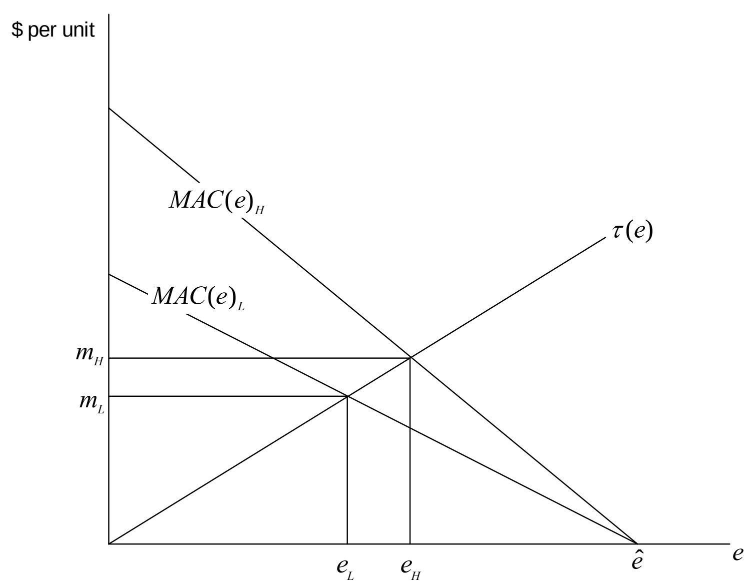


FIGURE 9-23

130

Multiple Sources

- Theoretical work in policy design has developed some creative and sophisticated mechanisms that can in principle achieve better outcomes than ICFR policies or IMR policies in multiple-source settings.

131

Multiple Sources

- However, these schemes typically require the implementation of complex source-specific pricing whereby each regulated entity faces a pricing scheme tailored to fit its own individual characteristics.
- This requirement is not easily reconciled with the practical realities of real-world regulation.

132

Multiple Sources

- In practice, the regulator must operate in a world where uncertainty about the MAAC cannot be fully resolved.
- How should a standard Pigouvian tax be chosen in such a setting?
- We investigate this question next.

133

9.5 CHOOSING A TAX RATE UNDER UNRESOLVED UNCERTAINTY

134

Choosing a Tax Rate Under Unresolved Uncertainty

- We will illustrate the essential elements of the tax design problem using our simple linear model.

135

Choosing a Tax Rate Under Unresolved Uncertainty

- Recall that MD is given by

$$MD(E) = \delta E$$

and MAC for source *i* is given by

$$MAC_i(e_i) = \gamma_i(\hat{e}_i - e_i)$$

136

Choosing a Tax Rate Under Unresolved Uncertainty

- We know from Ch.6 that source *i* will respond to a tax τ by setting emissions at

$$e_i(\tau) = \hat{e}_i - \frac{\tau}{\gamma_i}$$

137

Choosing a Tax Rate Under Unresolved Uncertainty

- Aggregate emissions in response to the tax are

$$E(\tau) = \sum_{i=1}^{n} e_i(\tau) = \hat{E} - \frac{\tau}{\varphi}$$

where

$$\varphi = \left(\sum_{i=1}^{n} \frac{1}{\gamma_i} \right)^{-1}$$

138

Choosing a Tax Rate Under Unresolved Uncertainty

- If the regulator does not know γ_i and $\hat{e}_i$ for every source, then both $\hat{E}$ and φ are unknown.

- The regulator must therefore construct **beliefs** about these parameters based on its best estimates of the individual parameters for the individual sources.

139

Choosing a Tax Rate Under Unresolved Uncertainty

- The statistical formalities of this process are beyond the scope of this course.
- Here we will assume the simplest possible representation of these beliefs, whereby $\hat{E}$ is assumed to be known, and φ is believed to be φ_L with probability π, and φ_H with probability (1-π).

140

Choosing a Tax Rate Under Unresolved Uncertainty

- Thus, we are assuming that the uncertainty takes the same form that we used for our analysis of a single source in Section 9.1 but we are now interpreting that form of uncertainty in an aggregate sense.
- In particular, the MAAC could be one of two possibilities, as illustrated in Figure 9-24.

141

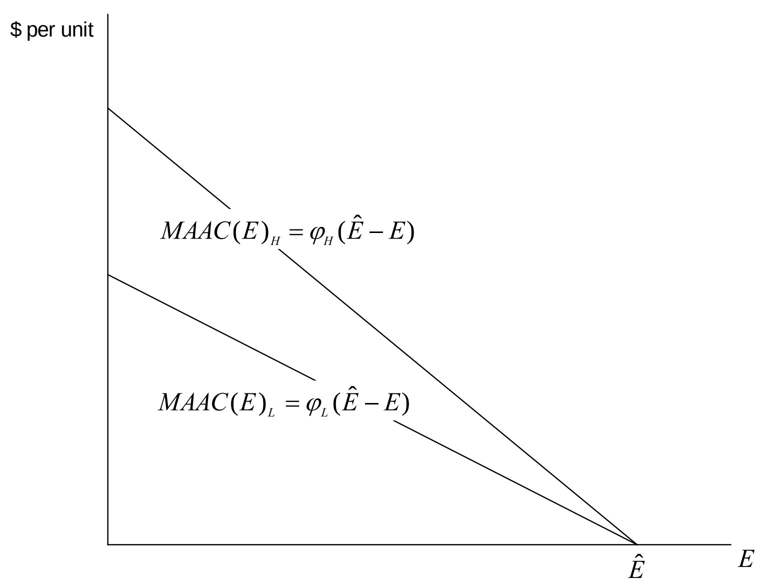


FIGURE 9-24

142

Choosing a Tax Rate Under Unresolved Uncertainty

- This in turn means that socially optimal aggregate emissions could be one of two possible values,

$$E_L^* = \frac{\varphi_L \hat{E}}{(\varphi_L + \delta)} \qquad \text{or} \qquad E_H^* = \frac{\varphi_H \hat{E}}{(\varphi_H + \delta)}$$

- See Figure 9-25.

143

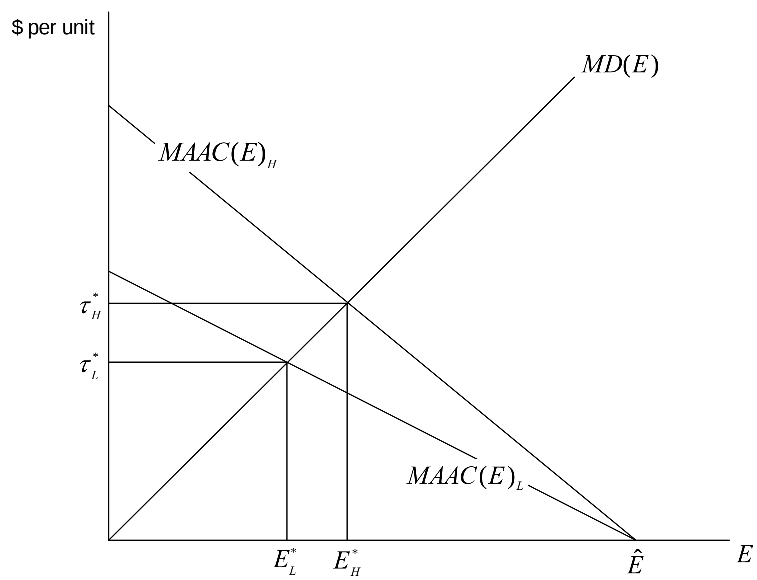


FIGURE 9-25

144

Choosing a Tax Rate Under Unresolved Uncertainty

- The full-information Pigouvian taxes in this setting are τ_L^* and τ_H^* , as depicted in Figure 9-25.

145

Choosing a Tax Rate Under Unresolved Uncertainty

- If the regulator sets a tax rate τ then the aggregate response will be

  $$E(\tau)_L = \hat{E} - \frac{\tau}{\varphi_L} \quad \text{or} \quad E(\tau)_H = \hat{E} - \frac{\tau}{\varphi_H}$$

  according to whether the true MAAC is L or H respectively.
- See Figure 9-26.

146

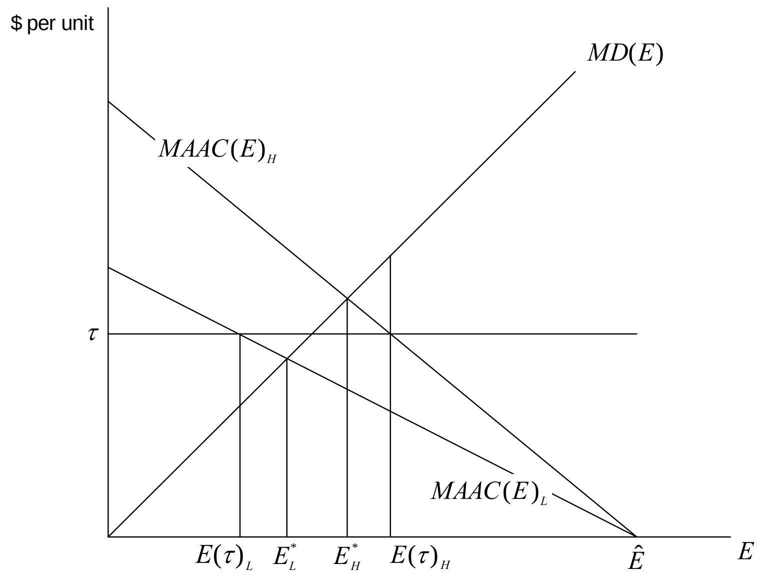


FIGURE 9-26

147

Choosing a Tax Rate Under Unresolved Uncertainty

- Suppose the regulator sets the tax at τ_L^*.
- If L is the true state of the world, then the tax will implement the social optimum, but if H is the true state of the world, then the tax will induce a level of emissions that is too high, with an associated loss of social surplus equal to the area $LSS_H(\tau_L^*)$ in Figure 9-27.

148

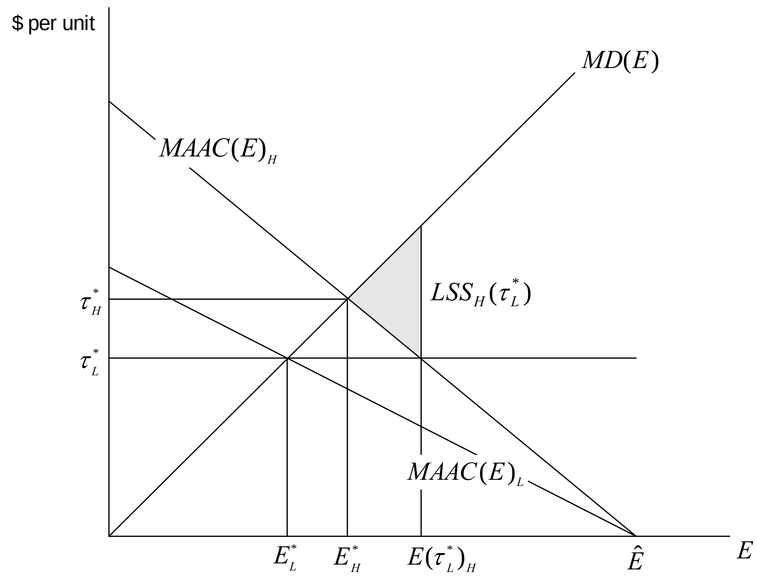


FIGURE 9-27

149

Choosing a Tax Rate Under Unresolved Uncertainty

- Conversely, if the regulator sets the tax at τ_H^* and H is the true state of the world, then it will implement the social optimum.
- However, if L is the true state of the world, then the tax will induce a level of emissions that is too low, with an associated loss of social surplus equal to the area $LSS_L(\tau_H^*)$ in Figure 9-28.

150

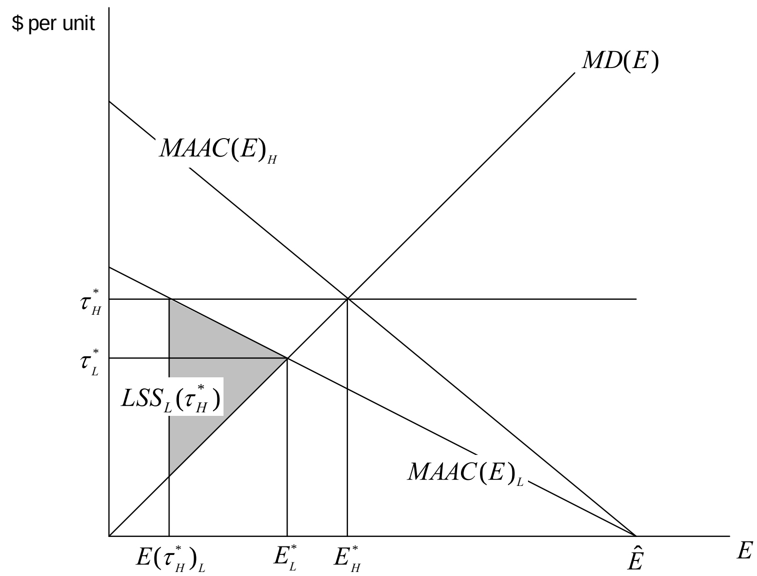


FIGURE 9-28

151

Choosing a Tax Rate Under Unresolved Uncertainty

- The goal of the regulator is to strike a balance between these two extreme outcomes, by setting a tax rate somewhere between τ_L^* and τ_H^*.
- In particular, suppose the regulator sets the tax at rate τ, as in Figure 9-29.

152

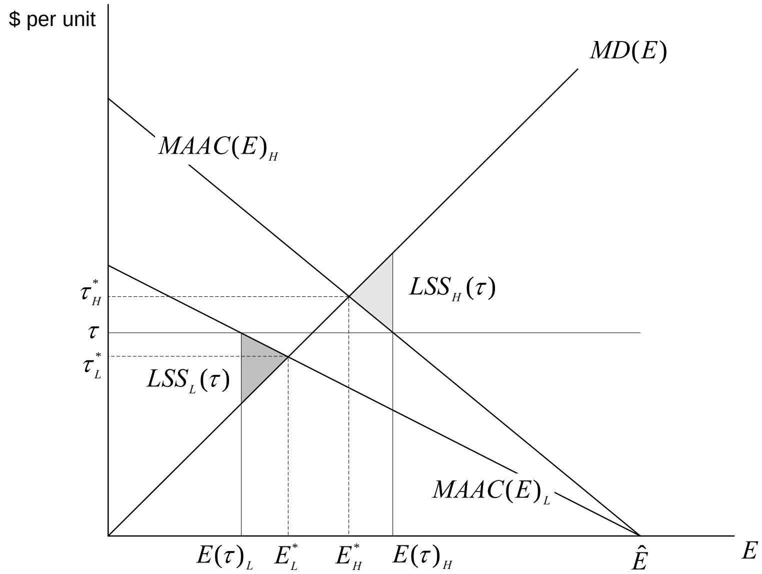


FIGURE 9-29

153

Choosing a Tax Rate Under Unresolved Uncertainty

- If L is the true state of the world then τ will induce too much abatement, while if H is the true state of the world then τ will induce too little abatement.
- The loss of social surplus associated with these errors is $LSS_L(\tau)$ and $LSS_H(\tau)$ respectively, as depicted in Figure 9-29.

154

Choosing a Tax Rate Under Unresolved Uncertainty

- The goal of the regulator is to minimize the **expected loss** from these errors, defined as the probability-weighted sum of the associated surplus losses:

$$\mathbf{E}[LSS(\tau)] = \pi LSS_L(\tau) + (1-\pi)LSS_H(\tau)$$

155

Choosing a Tax Rate Under Unresolved Uncertainty

- Thus, if π is small (there is a high probability that the true state is H) then the regulator should set a relatively high tax rate since this reduces the size of the most likely loss, $LSS_H(\tau)$; see Figure 9-30.

156

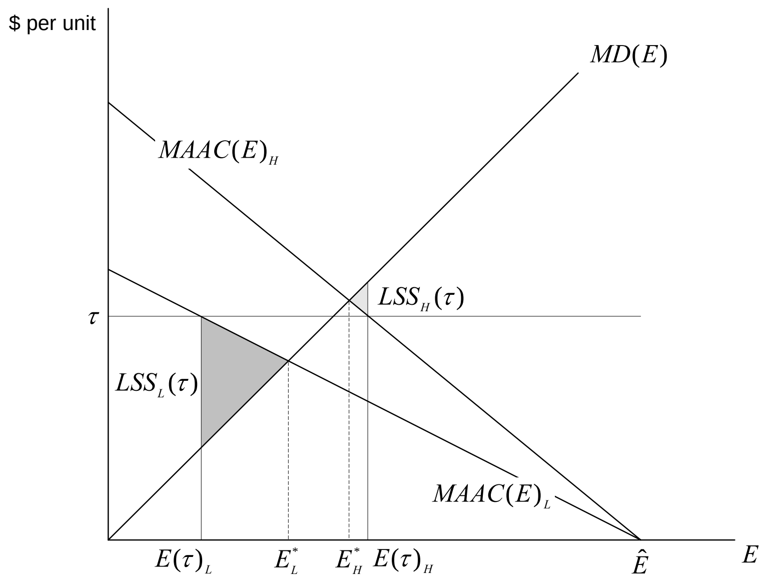


FIGURE 9-30

157

Choosing a Tax Rate Under Unresolved Uncertainty

- Conversely, if π is large (there is a high probability that the true state is L) then the regulator should set a relatively low tax rate since this reduces the size of $LSS_L\ (\tau)$; see Figure 9-31.

158

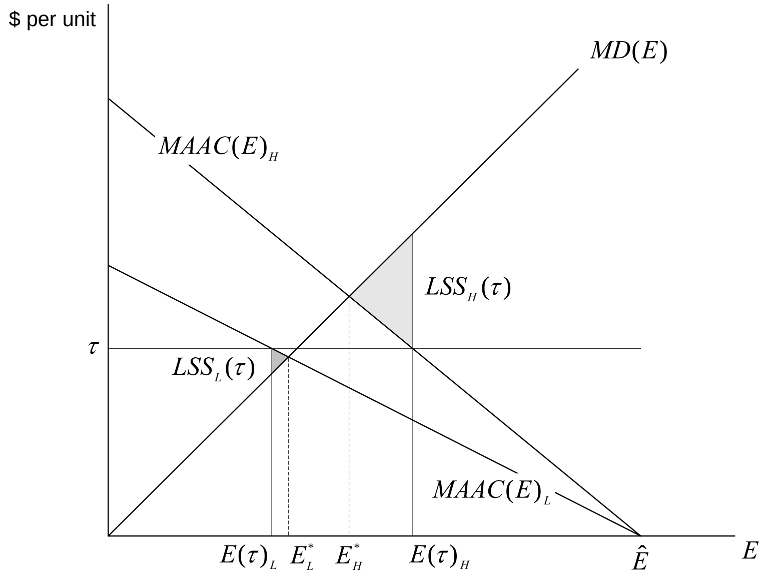


FIGURE 9-31

159

Choosing a Tax Rate Under Unresolved Uncertainty

- Finding the optimal tax rate, denoted $\tilde{\tau}$, requires the use of calculus even in the simple linear case, so we will not derive it here (it is reported in Appendix A9).
- However, it is instructive to examine its relationship to π in a graph; see Figure 9-32.

160

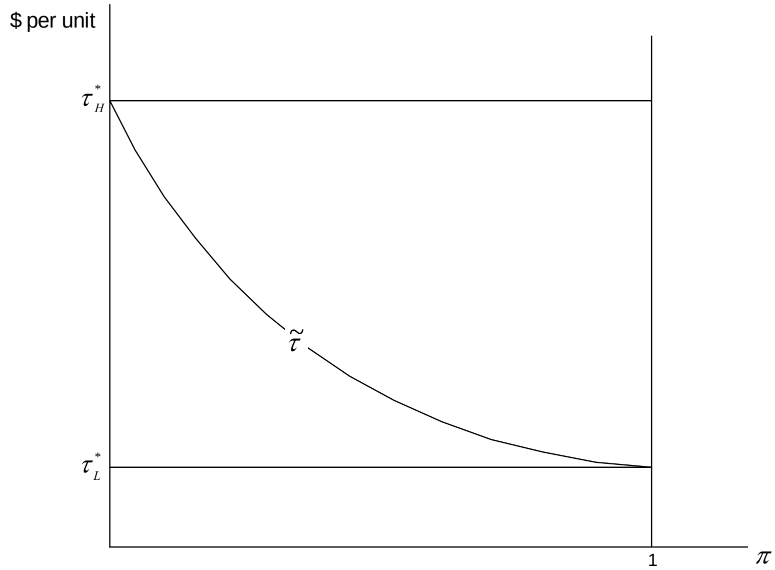


FIGURE 9-32

161

Choosing a Tax Rate Under Unresolved Uncertainty

- At π=0, we are effectively in a setting with no uncertainty where the true state is H, and so $\tilde{\tau} = \tau_H{}^*$.
- As π rises there is an increasingly higher probability that MAAC is relatively low, and so a lower tax rate is required to ensure that sources do not over-abate in response to the tax.

162

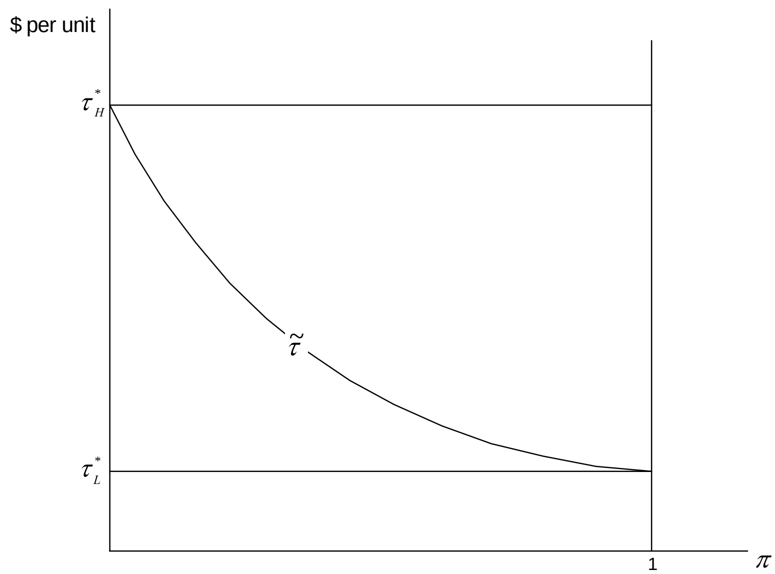


FIGURE 9-32 (repeat)

163

Choosing a Tax Rate Under Unresolved Uncertainty

- At π=1, we are effectively in a setting with no uncertainty where the true state is L, and so $\tilde{\tau} = \tau_L{}^*$.

164

Choosing a Tax Rate Under Unresolved Uncertainty

- Now consider the relationship between $\tilde{\tau}$ and $\mathbf{E}[\tau^*]$, where

$$\mathbf{E}[\tau^*] = \pi\tau_L^* + (1-\pi)\tau_H^*$$

is the probability-weighted average of the full-information tax rates; see Figure 9-33.

165

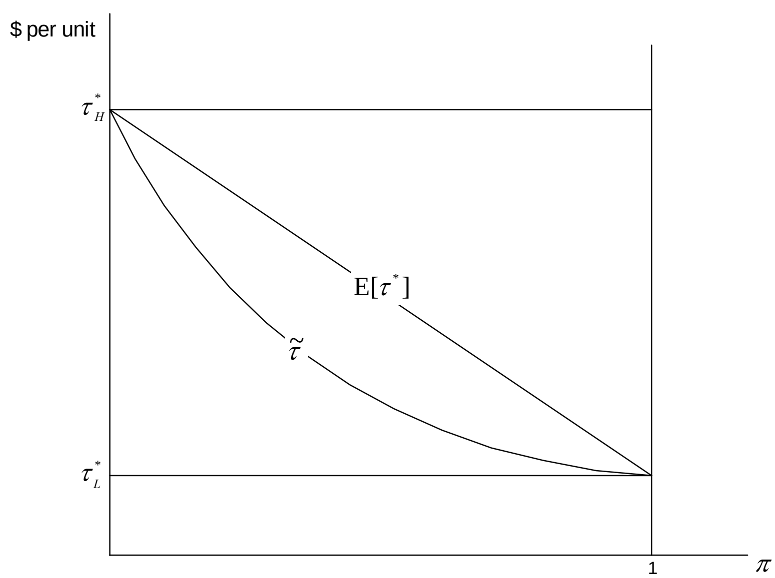


FIGURE 9-33

166

Choosing a Tax Rate Under Unresolved Uncertainty

- Note that **E**[τ^*] is a simple linear function in Figure 9-33, with slope

$$\frac{-\pi}{1-\pi}$$

- Why is $\tilde{\tau}$ < **E**[τ^*], except at the extremes where there is no effective uncertainty?

167

Choosing a Tax Rate Under Unresolved Uncertainty

- To understand why, consider Figure 9-34, which combines Figures 9-27 and 9-28 to illustrate the social surplus losses from the full-information taxes when they are mismatched with the true MAAC schedules.

168

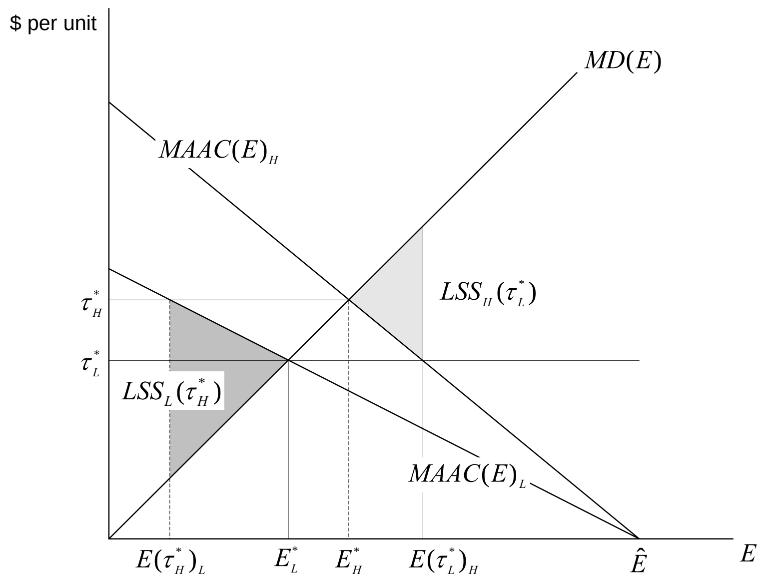


FIGURE 9-34

169

Choosing a Tax Rate Under Unresolved Uncertainty

- It is clear from Figure 9-34 that the loss from “over-taxing” (the darker shaded area) is greater than the loss from “under-taxing” (the lighter shaded area).
- Why? Because the two MAC schedules <u>diverge</u>: the gap between the schedules grows as abatement rises.

170

Choosing a Tax Rate Under Unresolved Uncertainty

- The optimal tax reflects this asymmetry, and so $\tilde{\tau}$ is closer to $\tau_L{}^*$ than a simple probability-weighted averaging of $\tau_L{}^*$ and $\tau_H{}^*$ would dictate.

171

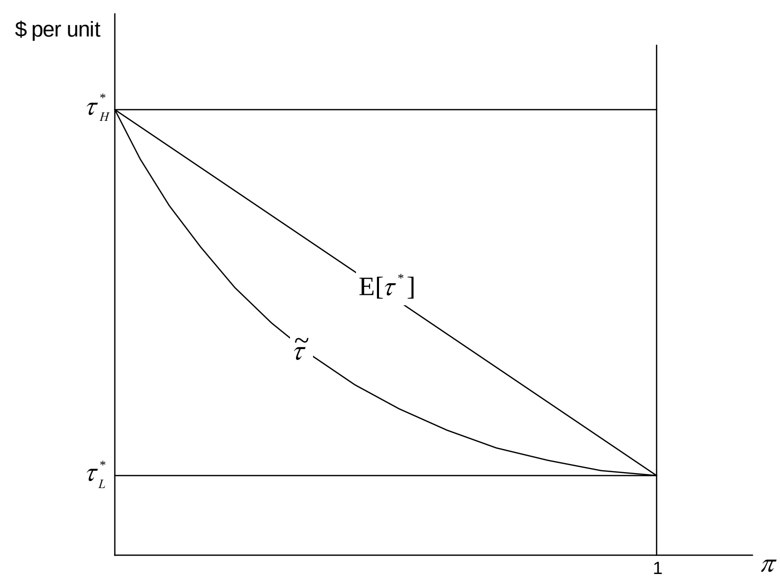


FIGURE 9-33 (repeat)

172

Choosing a Tax Rate Under Unresolved Uncertainty

- Recall from Topic 8 that a similar argument relating to uncertain MD meant that in that setting, $\tilde{\tau} > \mathbf{E}[\tau^*]$.
- See Figure 8-12 from that topic.

173

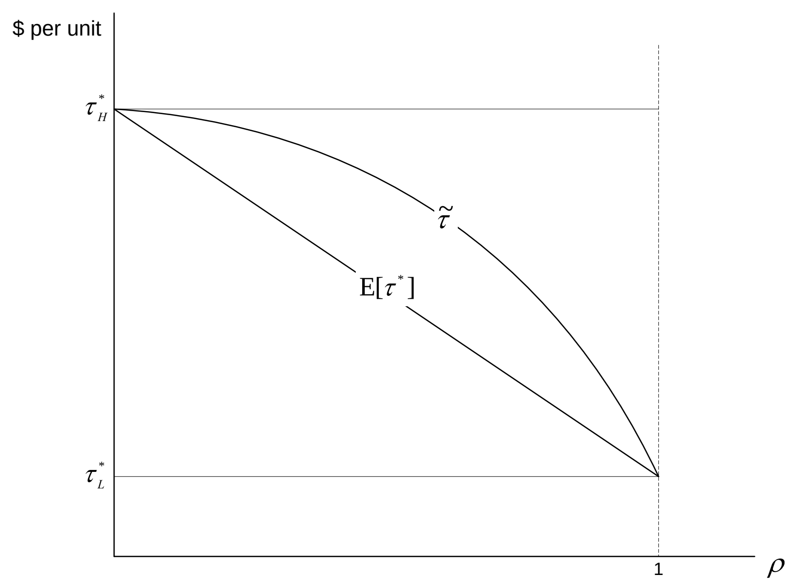


FIGURE 8-12 (Repeat) 174

APPENDIX A9: THE OPTIMAL TAX RATE

175

The Optimal Tax Rate

- The optimal tax rate under unresolved uncertainty in the linear case:

$$\tilde{\tau} = \frac{\delta \hat{E} \Phi}{\delta\left(\pi \frac{\varphi_H}{\varphi_L} + (1-\pi)\frac{\varphi_L}{\varphi_H} \right) + \Phi}$$

- where

$$\Phi = \pi \varphi_H + (1-\pi)\varphi_L$$

176

The Optimal Tax Rate

- Note that Φ is not equal to φ^{bar} except in the special case where π=1/2.

END 177

TOPIC 9 REVIEW QUESTIONS

These review questions begin with a solved example. It is recommended that you work through this example before commencing the questions.

A SOLVED EXAMPLE

The example relates to the following information. There are multiple sources of a uniformly mixed pollutant with marginal aggregate abatement cost given by

$$MAAC(e) = \phi(2160 - E)$$

where ϕ could be one of two values: $\phi_L = 1$ or $\phi_H = 2$. The regulator cannot distinguish between these two possibilities, henceforth denoted the "L-cost scenario" and the "H-cost scenario" respectively. Its beliefs are that $\phi = \phi_L$ with probability 3/10.

The marginal damage schedule is

$$MD(e) = \delta E$$

where $\delta = 1$.

1. Calculate the full-information Pigouvian tax rates, denoted τ_L^* and τ_H^* for the L-cost scenario and H-cost scenario respectively.

(a) <u>Solution for τ_L^*</u>

First find E_L^*, characterized by, $MAAC(E)_L = MD(E)$:

(1) $\quad 2160 - E = E$

Solving equation (1) yields

(2) $\quad E_L^* = 1080$

The associated tax rate is then calculated as $\tau_L^* = MD(E_L^*)$:

(3) $\quad \tau_L^* = \delta E_L^* = 1080$

(b) <u>Solution for τ_H^*</u>

First find E_H^*, characterized by, $MAAC(E)_H = MD(E)$:

(4) $4320 - 2E = E$

Solving equation (4) yields

(5) $E_H^* = 1440$

The associated tax rate is then calculated as $\tau_H^* = MD(E_H^*)$:

(6) $\tau_H^* = \delta E_H^* = 1440$

2. Suppose the regulator sets $\tau = \tau_L^*$. Calculate aggregate emissions in the H-cost scenario.

In general, the aggregate response to a tax rate τ is such that

(7) $MAAC(E) = \tau$

In this example, we have

(8) $\phi(2160 - E) = \tau$

which solves for

(9) $E(\tau) = 2160 - \dfrac{\tau}{\phi}$

where $\phi = 1$ in the L-cost scenario, and $\phi = 2$ in the H-cost scenario. Thus, if $\tau = \tau_L^*$ then aggregate emissions in the H-cost scenario are

(10) $E(\tau_L^*)_H = 2160 - \dfrac{\tau_L^*}{2}$

Making the substitution for τ_L^* from (3) yields

(11) $E(\tau_L^*)_H = 1620$

3. Suppose the regulator sets $\tau = \tau_H^*$. Calculate aggregate emissions in the L-cost scenario.

Using (9) with $\phi = 1$ and $\tau = \tau_H^*$, we have

(12) $E(\tau_H^*)_L = 720$

4. If the regulator sets $\tau = \tau_L^*$ and $\phi = \phi_H$, what is the loss of social surplus?

We need to calculate the shaded area in Figure 9.27, reproduced here as Figure R9-1. This area is

(13) $$LSS(\tau_L^*)_H = \int_{E_H^*}^{E(\tau_L^*)_H} MD(E)dE - \int_{E_H^*}^{E(\tau_L^*)_H} MAAC(E)_H \, dE$$

To calculate this area without using calculus, we first need to find $MD(E(\tau_L^*)_H)$. This is equal to

(14) $MD(E(\tau_L^*)_H) = \delta E(\tau_L^*)_H = 1620$

since $\delta = 1$ in this example. We can then calculate the area of the shaded triangle as

(15) $$LSS(\tau_L^*)_H = \frac{[MD(E(\tau_L^*)_H) - \tau_L^*][E(\tau_L^*)_H - E_H^*]}{2}$$

$$= \frac{[1620 - 1080][1620 - 1440]}{2}$$

$$= 48600$$

5. If the regulator sets $\tau = \tau_H^*$ and $\phi = \phi_L$, what is the loss of social surplus?

We need to calculate the shaded area in Figure 9.28, reproduced here as Figure R9-2. This area is

(16) $$LSS(\tau_H^*)_L = \int_{E(\tau_H^*)_L}^{E_L^*} MAAC(E)_L \, dE - \int_{E(\tau_H^*)_L}^{E_L^*} MD(E)dE$$

To calculate this area without using calculus, we first need to find $MD(E(\tau_H^*)_L)$. This is equal to

(17) $MD(E(\tau_H^*)_L) = \delta E(\tau_H^*)_L = 720$

since $\delta = 1$ in this example. We can then calculate the area of the shaded triangle as

(18) $$LSS(\tau_H^*)_L = \frac{[\tau_H^* - MD(E(\tau_H^*)_L)][E_L^* - E(\tau_H^*)_L]}{2}$$
$$= \frac{[1440-720][1080-720]}{2}$$
$$= 129600$$

6. Calculate the expected loss of social surplus when $\tau = \tau_L^*$

The L-cost scenario occurs with probability 3/10, and under that scenario τ_L^* implements the social optimum, with no associated loss of social surplus. The H-cost scenario occurs with probability 7/10, and under that scenario τ_L^* creates a loss of social surplus equal to $LSS(\tau_L^*)_H$, as calculated in Result 4 above.

Thus, the expected loss of social surplus is

(19) $$\mathbf{E}[LSS(\tau_L^*)] = \frac{3}{10}0 + \frac{7}{10}LSS(\tau_L^*)_H = 0 + \frac{7}{10}48600 = 34020$$

7. Calculate the expected loss of social surplus when $\tau = \tau_H^*$

The L-cost scenario occurs with probability 3/10, and under that scenario τ_H^* creates a loss of social surplus equal to $LSS(\tau_H^*)_L$, as calculated in Result 5 above. The H-cost scenario occurs with probability 7/10, and under that scenario τ_H^* implements the social optimum, with no associated loss of social surplus. Thus, the expected loss of social surplus is

(20) $$\mathbf{E}[LSS(\tau_H^*)] = \frac{3}{10}LSS(\tau_H^*)_L + \frac{7}{10}0 = \frac{3}{10}129600 + 0 = 38880$$

8. Calculate the probability-weighted average of the full-information taxes, denoted $\mathbf{E}[\tau^*]$.

Using (3) and (6),

(21) $$\mathbf{E}[\tau^*] = \frac{3}{10}\tau_L^* + \frac{7}{10}\tau_H^* = \frac{3}{10}1080 + \frac{7}{10}1440 = 1332$$

9. Suppose the regulator sets $\tau = \mathbf{E}[\tau^*]$. Calculate emissions under the L-cost scenario.

We know from (9) above that

(22) $$E(\tau) = 2160 - \frac{\tau}{\phi}$$

Setting $\phi = 1$ and $\tau = \mathbf{E}[\tau^*]$ from (21) yields

(23) $$E(\mathbf{E}[\tau^*]))_L = 828$$

10. Suppose the regulator sets $\tau = \mathbf{E}[\tau^*]$. Calculate emissions under the H-cost scenario.

Substituting $\phi = 2$ and $\tau = \mathbf{E}[\tau^*]$ in (22) yields

(24) $$E(\mathbf{E}[\tau^*])_H = 1494$$

11. Suppose the regulator sets $\tau = \mathbf{E}[\tau^*]$. Calculate the expected loss of social surplus.

We need to calculate the probability-weighted average of the shaded areas in Figure 9-29 evaluated at $\tau = \mathbf{E}[\tau^*]$. The figure is reproduced here as Figure R9-3. First calculate $LSS(\mathbf{E}[\tau^*])_L$:

(25) $$LSS(\mathbf{E}[\tau^*])_L = \int_{E(\mathbf{E}[\tau^*])_L}^{E_L^*} MAAC(E)_L\, dE - \int_{E(\mathbf{E}[\tau^*])_L}^{E_L^*} MD(E) dE$$

To calculate this area without using calculus, we first need to find $MD(E(\mathbf{E}[\tau^*])_L)$; see Figure R8-3. This is equal to

(26) $MD(E(\mathbf{E}[\tau^*])_L) = \delta E(\mathbf{E}[\tau^*])_L = 828$

since $\delta = 1$ in this example. We can then calculate the area of the shaded triangle as

(27) $$LSS(\mathbf{E}[\tau^*])_L = \frac{[\mathbf{E}[\tau^*] - MD(E(\mathbf{E}[\tau^*])_L)][E_L^* - E(\mathbf{E}[\tau^*])_L]}{2}$$

$$= \frac{[1332 - 828][1080 - 828]}{2}$$

$$= 63504$$

Now calculate $LSS(\mathbf{E}[\tau^*])_H$, from Figure R8-3:

(28) $$LSS(\mathbf{E}[\tau^*])_H = \int_{E_H^*}^{E(\mathbf{E}[\tau^*])} MD(E)dE - \int_{E_H^*}^{E(\mathbf{E}[\tau^*])} MAAC(E)_H \, dE$$

To calculate this area without using calculus, we first need to find $MD(E(\mathbf{E}[\tau^*])_H)$; see Figure R8-3. This is equal to

(29) $MD(E(\mathbf{E}[\tau^*])_H) = \delta E(\mathbf{E}[\tau^*])_H = 1494$

since $\delta = 1$ in this example. We can then calculate the area of the shaded triangle as

(30) $$LSS(\mathbf{E}[\tau^*])_H = \frac{[MD(E(\mathbf{E}[\tau^*])_H) - \mathbf{E}[\tau^*]][E(\mathbf{E}[\tau^*])_H - E_H^*]}{2}$$

$$= \frac{[1494 - 1332][1494 - 1440]}{2}$$

$$= 4374$$

We can now calculate the probability-weighted average of $LSS(\mathbf{E}[\tau^*])_L$ and $LSS(\mathbf{E}[\tau^*])_H$ to find the expected loss of social surplus when $\tau = \mathbf{E}[\tau^*]$:

(31) $$\mathbf{E}[LSS(\mathbf{E}[\tau^*])] = \frac{3}{10} LSS(\mathbf{E}[\tau^*])_L + \frac{7}{10} LSS(\mathbf{E}[\tau^*])_H$$

$$= \frac{3}{10} 63504 + \frac{7}{10} 4374$$

$$= 22113$$

Compare this result to those from Results 6 & 7 above. The expected loss of social surplus from using $\tau = \mathbf{E}[\tau^*]$ is less than from using either τ_L^* or τ_H^* because the former policy takes into account the relative probabilities of the tow possible cost scenarios.

However, we can do even better than setting $\tau = \mathbf{E}[\tau^*]$ because this policy does not take into account the asymmetry of losses under the L-cost and H-cost scenarios that arises from the different slopes of $MAAC(E)_L$ and $MAAC(E)_H$. The next few questions relate to an optimal tax policy, which does take account of this asymmetry. In particular, the optimal tax, minimizes

(32) $$\mathbf{E}[LSS(\tau)] = \frac{3}{10}LSS(\tau)_L + \frac{7}{10}LSS(\tau)_H$$

where $LSS(\tau)_L$ denotes the loss of social surplus under the L-cost scenario, and $LSS(\tau)_H$ denotes the loss of social surplus under the H-cost scenario.

We need calculus to find the optimal tax, but it can be shown that the optimal tax in this example is $\tilde{\tau} = 1248$.

12. Suppose the regulator sets $\tau = \tilde{\tau}$. Calculate emissions under the L-cost scenario.
Substituting $\phi = 1$ and $\tau = 1248$ in (22) yields

(33) $$E(\tilde{\tau})_L = 912$$

13. Suppose the regulator sets $\tau = \tilde{\tau}$. Calculate emissions under the H-cost scenario.
Substituting $\phi = 2$ and $\tau = 1248$ in (22) yields

(34) $$E(\tilde{\tau})_H = 1536$$

14. Suppose the regulator sets $\tau = \tilde{\tau}$. Calculate the expected loss of social surplus.
We need to calculate the probability-weighted average of the shaded areas in Figure 9-29 evaluated at $\tau = \tilde{\tau}$. The figure is reproduced here as Figure R9-4. First calculate $LSS(\tilde{\tau})_L$:

(35) $$LSS(\tilde{\tau})_L = \int_{E(\tilde{\tau})_L}^{E_L^*} MAAC(E)_L\,dE - \int_{E(\tilde{\tau})_L}^{E_L^*} MD(E)\,dE$$

To calculate this area without using calculus, we first need to find $MD(E(\tilde{\tau})_L)$; see Figure R9-4. This is equal to

(36) $$MD(E(\tilde{\tau})_L) = \delta E(\tilde{\tau})_L = 912$$

since $\delta = 1$ in this example. We can then calculate the area of the shaded triangle as

(37) $$LSS(\tilde{\tau})_L = \frac{[\tilde{\tau} - MD(E(\tilde{\tau})_L)][E_L^* - E(\tilde{\tau})_L]}{2}$$

$$= \frac{[1248 - 912][1080 - 912]}{2}$$

$$= 28224$$

Now calculate $LSS(\tilde{\tau})_H$, from Figure R9-4:

(38) $$LSS(\tilde{\tau})_H = \int_{E_H^*}^{E(\tilde{\tau})} MD(E)dE - \int_{E_H^*}^{E(\tilde{\tau})} MAAC(E)_H \, dE$$

To calculate this area without using calculus, we first need to find $MD(E(\tilde{\tau})_H)$; see Figure R8-4. This is equal to

(39) $$MD(E(\tilde{\tau})_H) = \delta E(\tilde{\tau})_H = 1536$$

since $\delta = 1$ in this example. We can then calculate the area of the shaded triangle as

(40) $$LSS(\tilde{\tau})_H = \frac{[MD(E(\tilde{\tau})_H) - \tilde{\tau}][E(\tilde{\tau})_H - E_H^*]}{2}$$

$$= \frac{[1536 - 1248][1536 - 1440]}{2}$$

$$= 13824$$

We can now calculate the probability-weighted average of $LSS(\tilde{\tau})_L$ and $LSS(\tilde{\tau})_H$ to find the expected loss of social surplus when $\tau = \tilde{\tau}$:

(41) $$\mathbf{E}[LSS(\tilde{\tau})] = \frac{3}{10} LSS(\tilde{\tau})_L + \frac{7}{10} LSS(\tilde{\tau})_H$$

$$= \frac{3}{10} 28224 + \frac{7}{10} 13824$$

$$= 18144$$

Compare this result with that from Result 11 above. The expected loss of social surplus under the optimal tax is about 18% lower than the expected loss of social surplus when

$\tau = \mathbf{E}[\tau^*]$. This superior performance of the optimal tax reflects the fact that it takes into account the asymmetry of losses associated with under-taxing versus over-taxing.

REVIEW QUESTIONS

Questions 1 to 4 relate to the following information. There is a single pollutant source with marginal abatement cost given by

$$MAC(e) = \gamma(100 - e)$$

where γ could be one of two values: $\gamma_L = 5$ or $\gamma_H = 10$. The regulator cannot distinguish between these two types, henceforth denoted the "L type" and the "H type" respectively. The marginal damage schedule is

$$MD(e) = 15e$$

1. The full-information Pigouvian tax rates, denoted τ_L^* and τ_H^* for the L and H type respectively, are

A. $\tau_L^* = 200$ and $\tau_H^* = 400$

B. $\tau_L^* = 375$ and $\tau_H^* = 600$

C. $\tau_L^* = 525$ and $\tau_H^* = 250$

D. $\tau_L^* = 350$ and $\tau_H^* = 175$

Suppose the regulatory strategy is to ask the source to report its type, and then set the tax rate based on this report.

2. If the H type makes a truthful report then its compliance cost is

A. 42000

B. 18000

C. 33000

D. None of the above.

3. If the H type makes a false report then its compliance cost is

A. 33000

B. 46875

C. 42000

D. 37500

4. The regulatory strategy will induce a

A. pooling equilibrium

B. separating equilibrium

Questions 5 to 8 relate to the following information. There is a single pollutant source with marginal abatement cost given by

$$MAC(e) = \gamma(400 - e)$$

where γ could be one of two values: $\gamma_L = 1$ or $\gamma_H = 5$. The regulator cannot distinguish between these two types, henceforth denoted the "L type" and the "H type" respectively. The marginal damage schedule is

$$MD(e) = 3e$$

5. The full-information Pigouvian tax rates, denoted τ_L^* and τ_H^* for the L and H type respectively, are

A. $\tau_L^* = 175$ and $\tau_H^* = 125$

B. $\tau_L^* = 750$ and $\tau_H^* = 300$

C. $\tau_L^* = 300$ and $\tau_H^* = 750$

D. $\tau_L^* = 125$ and $\tau_H^* = 175$

Suppose the regulatory strategy is to ask the source to report its type, and then set the full-information tax rate based on this report.

6. If the H type makes a truthful report then its compliance cost is

A. 243750

B. 56250

C. 255000

D. 75000

7. If the H type makes a false report then its compliance cost is

A. 33200

B. 255000

C. 75000

D. 198750

8. The regulatory strategy will induce a

A. pooling equilibrium

B. separating equilibrium

9. There is a single pollutant source with marginal abatement cost given by

$$MAC(e) = \gamma(200 - e)$$

where γ could be one of two values: γ_L or $\gamma_H = 5$. The regulator cannot distinguish between these two types, henceforth denoted the "L type" and the "H type" respectively, and it does not know γ_L. The marginal damage schedule is

$$MD(e) = 30e$$

Suppose the regulatory strategy is to ask the source to report its type, and then set the tax rate based on this report. This strategy will induce a separating equilibrium if and only if

A. $\gamma_L > 1/3$

B. $\gamma_L > 1/4$

C. $\gamma_L < 1/3$

D. $\gamma_L < 1/4$

10. There is a single pollutant source with marginal aggregate abatement cost given by

$$MAC(e) = \gamma(200 - e)$$

where γ could be one of two values: $\gamma_L = 1$ or $\gamma_H = 5$. The regulator cannot distinguish between these two types, henceforth denoted the "L type" and the "H type" respectively. The marginal damage schedule is

$$MD(e) = \delta e$$

Suppose the regulatory strategy is to ask the source to report its type, and then set the full-information tax rate based on this report. This strategy will induce a pooling equilibrium if and only if

A. $\delta > 5$

B. $\delta < 5$

C. $\delta > 7$

D. $\delta < 7$

11. In relation to the regulatory strategy described in Qs.1 – 10 above, emissions in a pooling equilibrium are always too low.

A. True.

B. False.

12. In relation to the regulatory strategy described in Qs.1 – 10 above, a pooling equilibrium is most likely when the two types have similar costs.

A. True.

B. False.

Questions 13 to 18 relate to the following information. There are multiple sources of a uniformly mixed pollutant with marginal aggregate abatement cost given by

$$MAAC(e) = \phi(3600 - E)$$

where ϕ could be one of two values: $\phi_L = 1$ or $\phi_H = 3$. The regulator cannot distinguish between these two possibilities, henceforth denoted the "L cost scenario" and the "H cost scenario" respectively. The marginal damage schedule is

$$MD(e) = E$$

13. The full-information Pigouvian tax rates, denoted τ_L^* and τ_H^* for the L cost scenario and H cost scenario respectively, are

A. $\tau_L^* = 1800$ and $\tau_H^* = 2200$

B. $\tau_L^* = 750$ and $\tau_H^* = 1500$

C. $\tau_L^* = 1800$ and $\tau_H^* = 2700$

D. $\tau_L^* = 750$ and $\tau_H^* = 2000$

14. Suppose the regulator sets $\tau = \tau_L^*$. Then aggregate emissions in the H cost scenario are

A. 2000

B. 3000

C. 4000

D. 4500

15. Suppose the regulator sets $\tau = \tau_H^*$. Then aggregate emissions in the L cost scenario are

A. 900

B. 1200

C. 1500

D. 1800

16. If the regulator sets $\tau = \tau_L^*$ and $\phi = \phi_H$ then the loss of social surplus is

A. 225000

B. 180000

C. 175000

D. 230000

17. If the regulator sets $\tau = \tau_H^*$ and $\phi = \phi_L$ then the loss of social surplus is

A. 810000

B. 650000

C. 157500

D. 215000

18. The best explanation for the relationship between your answers to Q17 and Q16 is that

A. the cost of under-taxing is higher than the cost of over-taxing because marginal damage is increasing

B. the cost of over-taxing is higher than the cost of under-taxing because marginal damage is increasing

C. the cost of over-taxing is higher than the cost of under-taxing because the marginal aggregate abatement cost schedules have different slopes under the two cost scenarios

D. None of the above

Questions 19 to 36 relate to the following information. There are multiple sources of a uniformly mixed pollutant with marginal aggregate abatement cost given by

$$MAAC(e) = \phi(3600 - E)$$

where ϕ could be one of two values: $\phi_L = 1$ or $\phi_H = 3$. The regulator cannot distinguish between these two possibilities, henceforth denoted the "L cost scenario" and the "H cost scenario" respectively. Its beliefs are that $\phi = \phi_L$ with probability 1/6. The marginal damage schedule is

$$MD(e) = \frac{3E}{2}$$

19. The full-information Pigouvian tax rates, denoted τ_L^* and τ_H^* for the L cost scenario and H cost scenario respectively, are

A. $\tau_L^* = 1440$ and $\tau_H^* = 4500$

B. $\tau_L^* = 2160$ and $\tau_H^* = 3600$

C. $\tau_L^* = 900$ and $\tau_H^* = 2700$

D. $\tau_L^* = 1350$ and $\tau_H^* = 4050$

20. Suppose the regulator sets $\tau = \tau_L^*$. Then aggregate emissions in the H cost scenario are

A. 3600

B. 2040

C. 2700

D. 2880

21. Suppose the regulator sets $\tau = \tau_H^*$. Then aggregate emissions in the L cost scenario are

A. 0

B. 720

C. 1080

D. 360

22. If the regulator sets $\tau = \tau_L^*$ and $\phi = \phi_H$ then the loss of social surplus is

A. 484000

B. 640000

C. 518400

D. 368000

23. If the regulator sets $\tau = \tau_H^*$ and $\phi = \phi_L$ then the loss of social surplus is

A. 2400000

B. 2592000

C. 1880000

D. 1280000

24. The expected loss of social surplus when $\tau = \tau_L^*$ is

A. 568000

B. 432000

C. 640000

D. 724000

25. The expected loss of social surplus when $\tau = \tau_H^*$ is

A. 568000

B. 432000

C. 640000

D. 724000

26. The relationship between your answers to Q24 and Q25 holds for any example with linear marginal damage and linear marginal aggregate abatement costs.

A. True.

B. False.

27. The probability-weighted average of the full-information taxes, denoted $\mathbf{E}[\tau^*]$, is

A. 2880

B. 3720

C. 3360

D. 2160

28. If the regulator sets $\tau = \mathbf{E}[\tau^*]$ then emissions under the L cost scenario are

A. 120

B. 80

C. 60

D. 240

29. If the regulator sets $\tau = \mathbf{E}[\tau^*]$ then emissions under the H cost scenario are

A. 2480

B. 360

C. 180

D. 2840

30. If the regulator sets $\tau = \mathbf{E}[\tau^*]$ then the expected loss of social surplus is

A. 278000

B. 312000

C. 216000

D. 426000

31. Let $LSS(\tau)_L$ denote the loss of social surplus under the L cost scenario when the tax is set at τ, and the let $LSS(\tau)_H$ denote the loss of social surplus under the H cost scenario when the tax is set at τ. Then the optimal tax, denoted $\tilde{\tau}$, minimizes

A. $\frac{5}{6}LSS(\tau)_H + \frac{1}{6}LSS(\tau)_L$

B. $\frac{1}{6}LSS(\tau)_H + \frac{5}{6}LSS(\tau)_L$

C. $LSS(\frac{5}{6}\tau)_H + LSS(\frac{1}{6}\tau)_L$

D. None of the above

32. It can be shown that the optimal tax in this example is $\tilde{\tau} = 2880$. The relationship between $\tilde{\tau}$ and $\mathbf{E}[\tau^*]$ from Q27 is explained by the fact that the cost of over-taxing is higher than the cost of under-taxing because the marginal aggregate abatement cost schedules have different slopes under the two cost scenarios.

A. True.

B. False.

33. If the regulator sets $\tau = \tilde{\tau}$ then emissions under the L cost scenario are

A. 420

B. 720

C. 980

D. 240

34. If the regulator sets $\tau = \tilde{\tau}$ then emissions under the H cost scenario are

A. 2640

B. 3200

C. 2180

D. 2400

35. If the regulator sets $\tau = \tilde{\tau}$ then the expected loss of social surplus is

A. 278000

B. 312000

C. 216000

D. 426000

36. The relative magnitude of your answers to Q30 and Q35 reflects the symmetry of the losses associated with over-taxing versus under-taxing.

A. True.

B. False.

Questions 37 to 52 relate to the following information. There are multiple sources of a uniformly mixed pollutant with marginal aggregate abatement cost given by

$$MAAC(e) = \phi(2700 - E)$$

where ϕ could be one of two values: $\phi_L = 10$ or $\phi_H = 20$. The regulator cannot distinguish between these two possibilities, henceforth denoted the "L cost scenario" and the "H cost scenario" respectively. Its beliefs are that $\phi = \phi_L$ with probability 3/5. The marginal damage schedule is

$$MD(e) = 10E$$

37. The full-information Pigouvian tax rates, denoted τ_L^* and τ_H^* for the L cost scenario and H cost scenario respectively, are

A. $\tau_L^* = 13500$ and $\tau_H^* = 18000$

B. $\tau_L^* = 24600$ and $\tau_H^* = 36000$

C. $\tau_L^* = 1760$ and $\tau_H^* = 2700$

D. $\tau_L^* = 13500$ and $\tau_H^* = 21400$

38. Suppose the regulator sets $\tau = \tau_L^*$. Then aggregate emissions in the H cost scenario are

A. 3520

B. 1755

C. 1880

D. 2025

39. Suppose the regulator sets $\tau = \tau_H^*$. Then aggregate emissions in the L cost scenario are

A. 560

B. 900

C. 1100

D. 840

40. If the regulator sets $\tau = \tau_L^*$ and $\phi = \phi_H$ then the loss of social surplus is

A. 547005

B. 687955

C. 718600

D. 759375

41. If the regulator sets $\tau = \tau_H^*$ and $\phi = \phi_L$ then the loss of social surplus is

A. 2025000

B. 3594000

C. 1680500

D. 2880050

42. The expected loss of social surplus when $\tau = \tau_L^*$ is

A. 366600

B. 532800

C. 303750

D. 228800

43. The expected loss of social surplus when $\tau = \tau_H^*$ is

A. 1215000

B. 487000

C. 2657800

D. 1846000

44. The probability-weighted average of the full-information taxes, denoted $\mathbf{E}[\tau^*]$, is

A. 26800

B. 15300

C. 34800

D. 14400

45. If the regulator sets $\tau = \mathbf{E}[\tau^*]$ then emissions under the L cost scenario are

A. 960

B. 870

C. 1170

D. 1040

46. If the regulator sets $\tau = \mathbf{E}[\tau^*]$ then emissions under the H cost scenario are

A. 1935

B. 1240

C. 1390

D. 2370

47. If the regulator sets $\tau = \mathbf{E}[\tau^*]$ then the expected loss of social surplus is

A. 324600

B. 289000

C. 215650

D. 303750

48. Let $LSS(\tau)_L$ denote the loss of social surplus under the L cost scenario when the tax is set at τ, and the let $LSS(\tau)_H$ denote the loss of social surplus under the H cost scenario when the tax is set at τ. Then the optimal tax, denoted $\tilde{\tau}$, minimizes

A. $\frac{3}{5}LSS(\tau)_H + \frac{2}{5}LSS(\tau)_L$

B. $\frac{2}{5}LSS(\tau)_H + \frac{3}{5}LSS(\tau)_L$

C. $\frac{3}{5}LSS(\tau)_L + \frac{2}{5}LSS(\tau)_L$

D. $\frac{1}{2}LSS(\tau)_H + \frac{1}{2}LSS(\tau)_L$

49. It can be shown that the optimal tax in this example is $\tilde{\tau} = 14400$. If the regulator sets $\tau = \tilde{\tau}$ then emissions under the L cost scenario are

A. 980

B. 720

C. 1260

D. 1340

50. If the regulator sets $\tau = \tilde{\tau}$ then emissions under the H cost scenario are

A. 1260

B. 1980

C. 1340

D. 1020

51. If the regulator sets $\tau = \tilde{\tau}$ then the expected loss of social surplus is

A. 243000

B. 287600

C. 198460

D. 314880

52. The relative magnitude of your answers to Q51 and Q47 reflects the symmetry of the losses associated with under-taxing versus over-taxing.

A. True.

B. False.

The remaining questions do not review anything not already reviewed in the questions above. They simply provide another example to allow you to test your understanding more one time. You may wish to skip these if you are already feeling confident about your knowledge of the material.

Questions 53 to 69 relate to the following information. There are multiple sources of a uniformly mixed pollutant with marginal aggregate abatement cost given by

$$MAAC(e) = \phi(12600 - E)$$

where ϕ could be one of two values: $\phi_L = 5$ or $\phi_H = 10$. The regulator cannot distinguish between these two possibilities, henceforth denoted the "L cost scenario" and the "H cost scenario" respectively. Its beliefs are that $\phi = \phi_L$ with probability 1/5. The marginal damage schedule is

$$MD(e) = 5E$$

53. The full-information Pigouvian tax rates, denoted τ_L^* and τ_H^* for the L cost scenario and H cost scenario respectively, are

A. $\tau_L^* = 27400$ and $\tau_H^* = 29500$

B. $\tau_L^* = 31500$ and $\tau_H^* = 42000$

C. $\tau_L^* = 11760$ and $\tau_H^* = 21300$

D. $\tau_L^* = 17500$ and $\tau_H^* = 26700$

54. Suppose the regulator sets $\tau = \tau_L^*$. Then aggregate emissions in the H cost scenario are

A. 7890

B. 10670

C. 9450

D. 8600

55. Suppose the regulator sets $\tau = \tau_H^*$. Then aggregate emissions in the L cost scenario are

A. 4200

B. 2340

C. 5400

D. 7650

56. If the regulator sets $\tau = \tau_L^*$ and $\phi = \phi_H$ then the loss of social surplus is

A. 6790500

B. 9654000

C. 8268750

D. 8134650

57. If the regulator sets $\tau = \tau_H^*$ and $\phi = \phi_L$ then the loss of social surplus is

A. 22050000

B. 34698700

C. 19876040

D. 25680000

58. The expected loss of social surplus when $\tau = \tau_L^*$ is

A. 8764500

B. 5328050

C. 7303750

D. 6615000

59. The expected loss of social surplus when $\tau = \tau_H^*$ is

A. 1215000

B. 4410000

C. 2657800

D. 1846000

60. If the regulator had to choose either $\tau = \tau_L^*$ or $\tau = \tau_H^*$ then it would choose $\tau = \tau_H^*$

A. because the cost of under-taxing is higher than the cost of over-taxing

B. because the cost of over-taxing is higher than the cost of under-taxing but the probability of over-taxing here is low because π is low

C. because the cost of under-taxing is higher than the cost of over-taxing and the probability of under-taxing here is high because π is high

D. None of the above

61. The probability-weighted average of the full-information taxes, denoted $\mathbf{E}[\tau^*]$, is

A. 39900

B. 25600

C. 34650

D. 29800

62. If the regulator sets $\tau = \mathbf{E}[\tau^*]$ then emissions under the L cost scenario are

A. 5200

B. 6240

C. 2470

D. 4620

63. If the regulator sets $\tau = \mathbf{E}[\tau^*]$ then emissions under the H cost scenario are

A. 7000

B. 7640

C. 8610

D. 9160

64. If the regulator sets $\tau = \mathbf{E}[\tau^*]$ then the expected loss of social surplus is

A. 3087000

B. 3289000

C. 4215650

D. 2303750

65. Let $LSS(\tau)_L$ denote the loss of social surplus under the L cost scenario when the tax is set at τ, and the let $LSS(\tau)_H$ denote the loss of social surplus under the H cost scenario when the tax is set at τ. Then the optimal tax, denoted $\tilde{\tau}$, minimizes

A. $\frac{1}{5}LSS(\tau)_H + \frac{4}{5}LSS(\tau)_L$

B. $\frac{1}{5}LSS(\tau)_L + \frac{4}{5}LSS(\tau)_H$

C. $\frac{1}{2}LSS(\tau)_H + \frac{1}{2}LSS(\tau)_L$

D. None of the above

66. It can be shown that the optimal tax in this example is $\tilde{\tau} = 37800$. If the regulator sets $\tau = \tilde{\tau}$ then emissions under the L cost scenario are

A. 5980

B. 4720

C. 7260

D. 5040

67. If the regulator sets $\tau = \tilde{\tau}$ then emissions under the H cost scenario are

A. 8820

B. 9980

C. 7340

D. 8020

68. If the regulator sets $\tau = \tilde{\tau}$ then the expected loss of social surplus is

A. 2243000

B. 3287600

C. 2646000

D. 3314880

69. The relative magnitude of your answers to Q68 and Q64 reflects the symmetry of the losses associated with under-taxing versus over-taxing.

A. True.

B. False.

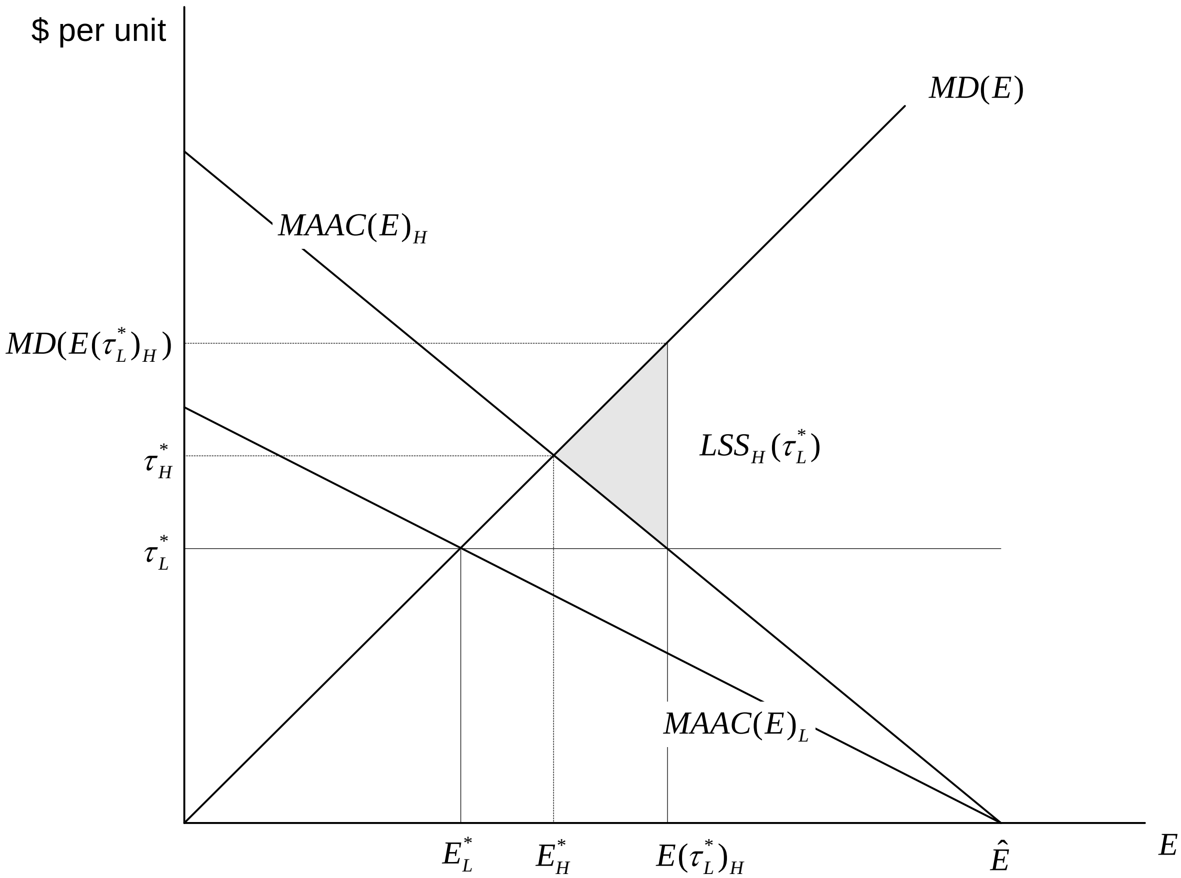


Figure R9-1

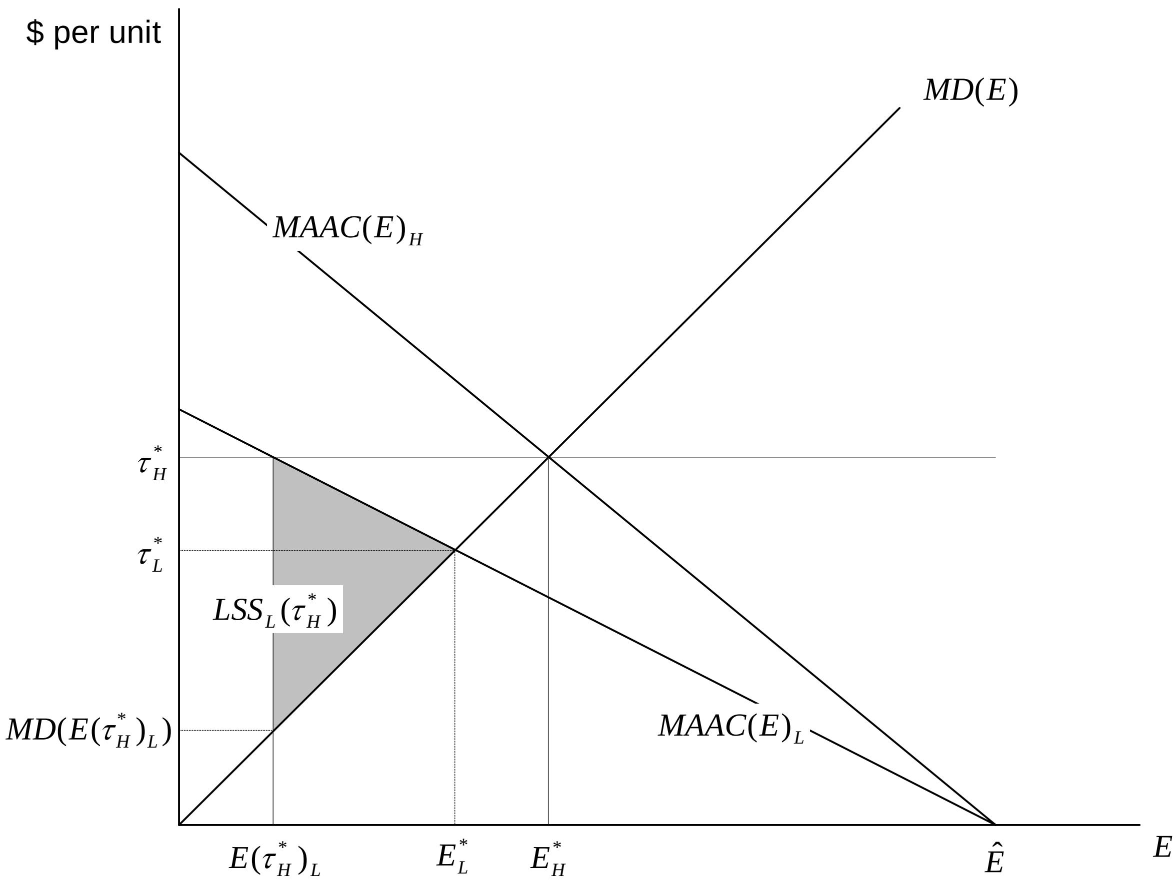


Figure R9-2

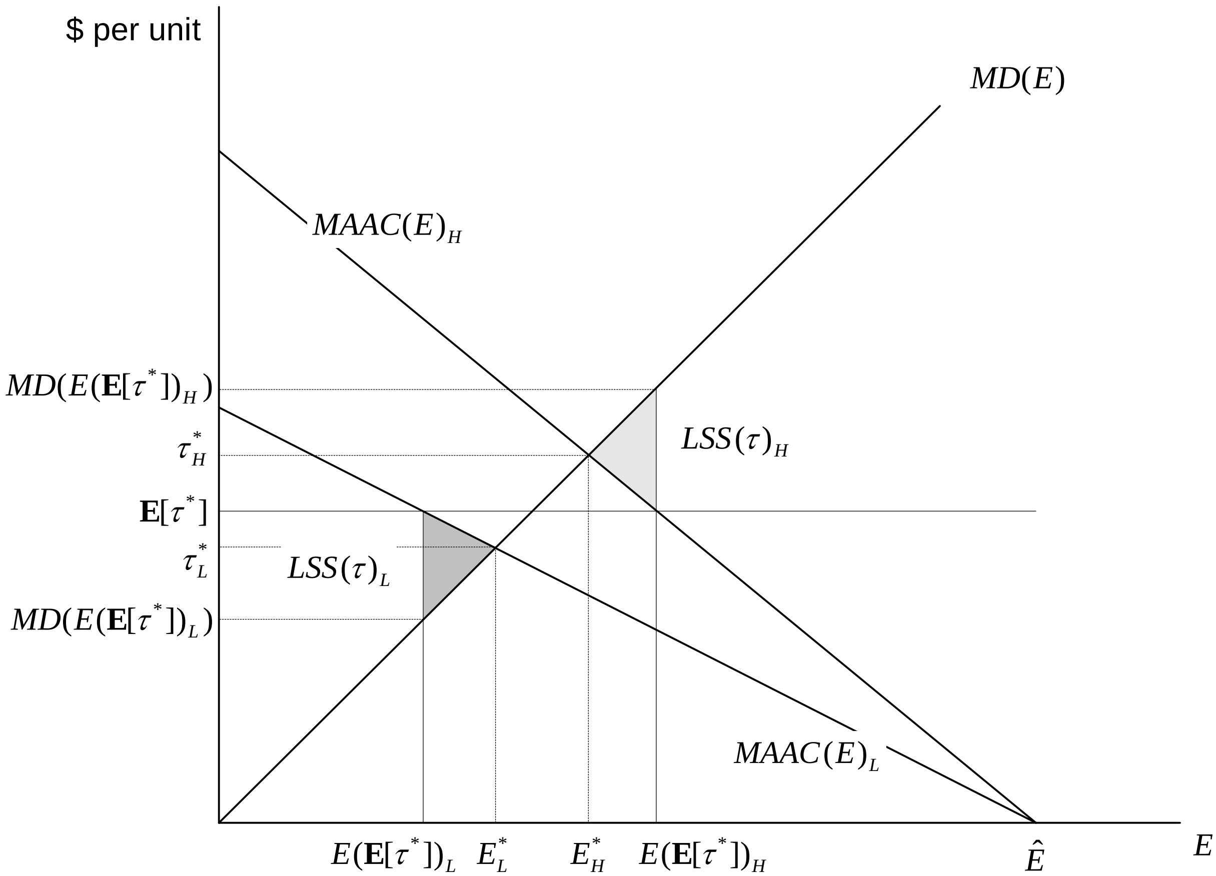

$ per unit
MD(E)
MAAC(E)_H
MD(E(E[τ*])_H)
τ*_H
E[τ*]
τ*_L
MD(E(E[τ*])_L)
LSS(τ)_H
LSS(τ)_L
MAAC(E)_L
E(E[τ*])_L
E*_L
E*_H
E(E[τ*])_H
Ê
E

Figure R9-3

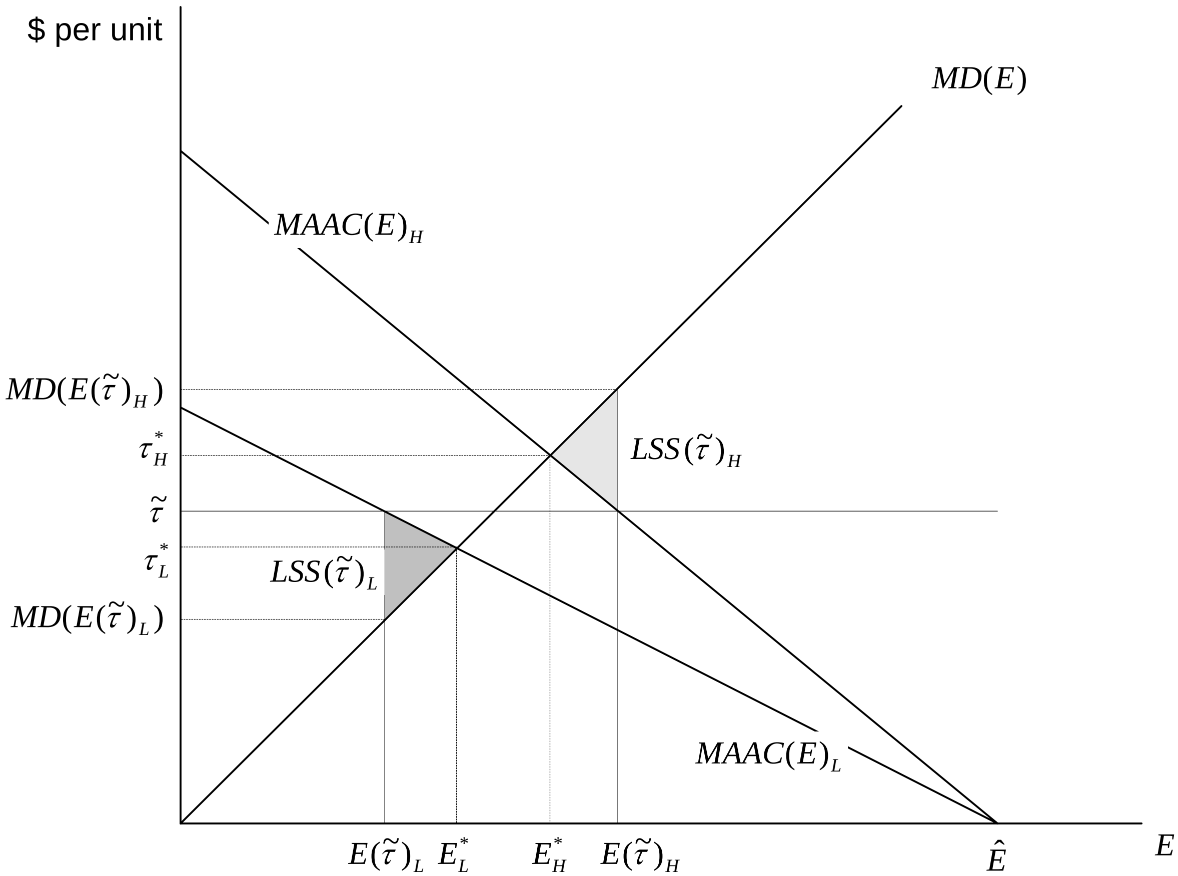

$ per unit
MD(E)
MAAC(E)_H
MD(E(τ̃)_H)
τ*_H
τ̃
τ*_L
MD(E(τ̃)_L)
LSS(τ̃)_H
LSS(τ̃)_L
MAAC(E)_L
E(τ̃)_L
E*_L
E*_H
E(τ̃)_H
Ê
E

Figure R9-4

ANSWER KEY

1. B
2. A
3. D
4. A
5. C
6. A
7. B
8. B
9. C
10. A
11. B
12. A
13. C
14. B
15. A
16. B
17. A
18. C
19. B
20. D
21. A
22. C
23. B
24. B
25. B
26. B
27. C
28. D
29. A
30. B
31. A
32. A
33. B
34. A

35. C
36. B
37. A
38. D
39. B
40. D
41. A
42. C
43. A
44. B
45. C
46. A
47. D
48. B
49. C
50. B
51. A
52. B
53. B
54. C
55. A
56. C
57. A
58. D
59. B
60. B
61. A
62. D
63. C
64. A
65. B
66. D
67. A
68. C
69. B

10. THE CHOICE BETWEEN A TAX AND EMISSIONS TRADING

1

OUTLINE

10.1 Introduction
10.2 Choosing the Supply of Permits when Abatement Cost is Uncertain
10.3 Comparative Policy-Performance
10.4 Equilibrium Permit Prices
10.5 Properties of the Expected Permit Price*

* Advanced Topic

2

10.6 Emissions Trading with a “Safety Valve”

3

10.1 INTRODUCTION

4

Introduction

- In the previous topic we saw that when MACs are uncertain, the level of emissions in response to a tax is also uncertain.
- We also saw that even when the tax rate is set optimally to account for that uncertain response, the uncertainty leads to an expected loss of social surplus.

5

Introduction

- Might it be better to set aggregate emissions directly, using an emissions trading program, and thereby eliminate the uncertainty over the level of emissions induced by the policy?

6

Introduction

- The downside with this alternative approach is that we must then set the supply of permits under uncertainty about the permit price that will emerge in equilibrium.

7

Introduction

- Thus, in choosing between a tax and emissions trading, we face a choice between uncertainty over quantity (in response to a tax) and uncertainty over price (in response to the supply of permits).

8

Introduction

- In this topic we investigate this choice, and characterize the conditions under which a tax is best, and the conditions under which emissions trading is best.

9

Introduction

- We begin this investigation by looking at the policy problem of choosing the supply of permits under an emissions trading program when abatement cost is uncertain.

10

10.2 CHOOSING THE SUPPLY OF PERMITS WHEN ABATEMENT COST IS UNCERTAIN

11

Choosing the Supply of Permits

- We know from Topic 7 that once the regulator sets the supply of permits, trading will ensure that MACs are brought into equality across sources.
- Recall Figure 7-3 from Topic 7.

12

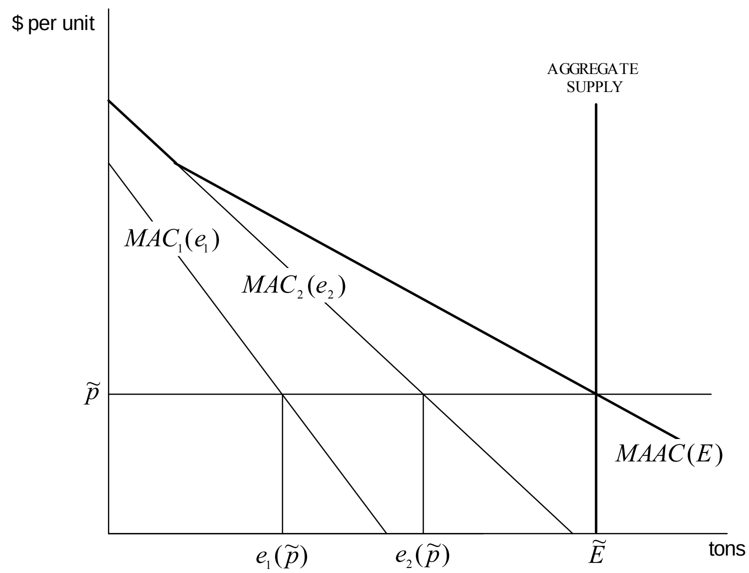


FIGURE 7-3 (repeat)

13

Choosing the Supply of Permits

- This key property of the permit market holds even if the regulator is uncertain about the MACs of the sources.
- However, the regulator now faces uncertainty over what permit supply to set.

14

Choosing the Supply of Permits

- Recall again from Topic 7 that when the individual MACs are known, the regulator can construct the marginal aggregate abatement cost and then choose the permit supply to equate this with marginal damage.

- Recall Figure 7-8 from Topic 7.

15

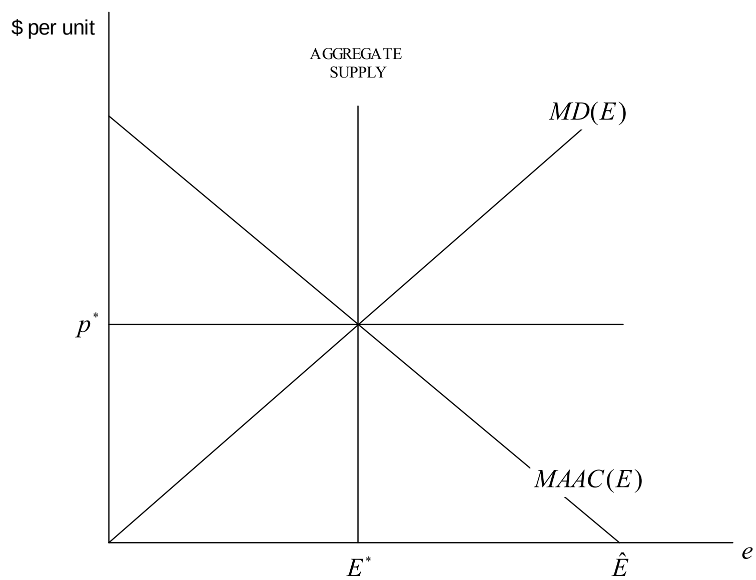


FIGURE 7-8 (repeat)

16

Choosing the Supply of Permits

- However, if the individual MACs are unknown then *MAAC*(*E*) is also unknown, and so the regulator does not know where it crosses *MD*(*E*).

- So how does the regulator choose the supply of permits?

17

Choosing the Supply of Permits

- To investigate this question, we will study the same simple setting we examined in Topic 9, where there are just two possible *MAAC*(*E*) schedules, as depicted in Figure 10-1.

18

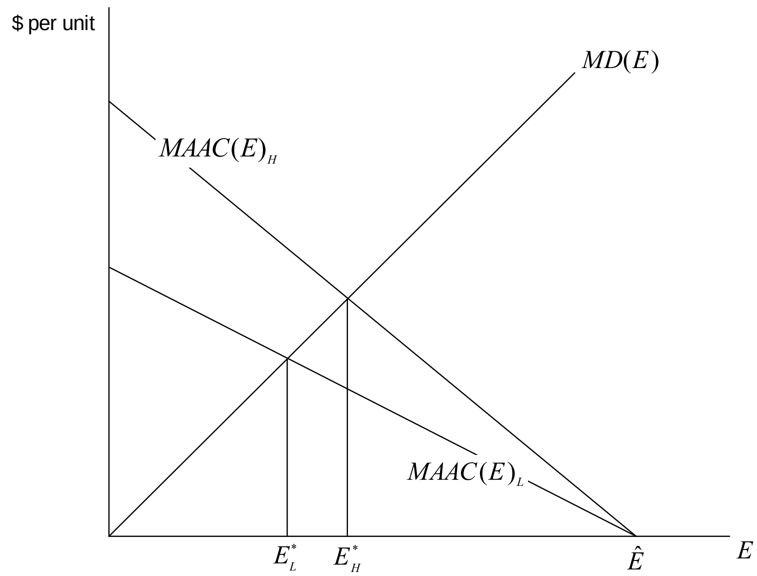


FIGURE 10-1

19

Choosing the Supply of Permits

- The full-information optimal quantities are $E_L{}^*$ and $E_H{}^*$, as depicted in Figure 10-1.

20

Choosing the Supply of Permits

- Suppose the regulator sets supply at E_L^* .
- This will be optimal if the true state of the world is L, but it will be too low if the true state of the world is H, and the associated loss of social surplus is the area $LSS_H(E_L^*)$ in Figure 10-2.

21

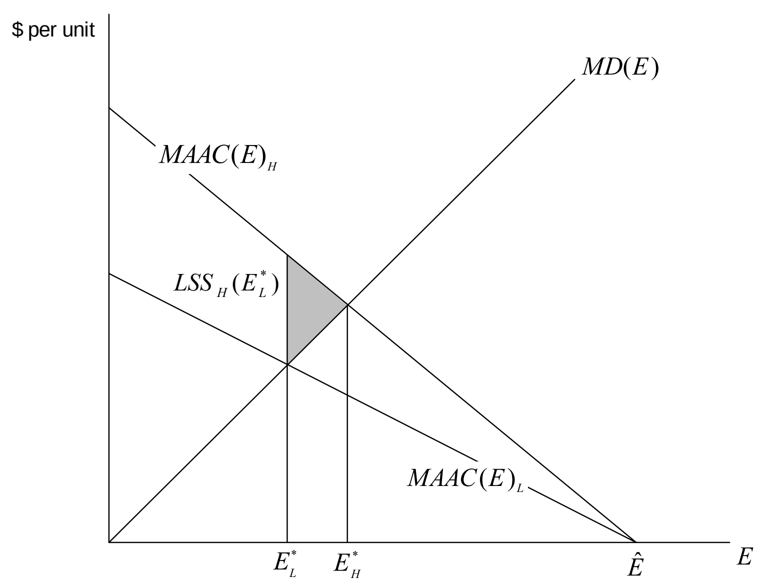


FIGURE 10-2

22

Choosing the Supply of Permits

- Conversely, if the regulator sets emissions at E_H^* then it will be socially optimal if and only if H is the true state of the world.
- If L is the true state of the world, then emissions will be too high, with an associated loss of social surplus equal to the area $LSS_L(E_H^*)$ in Figure 10-3.

23

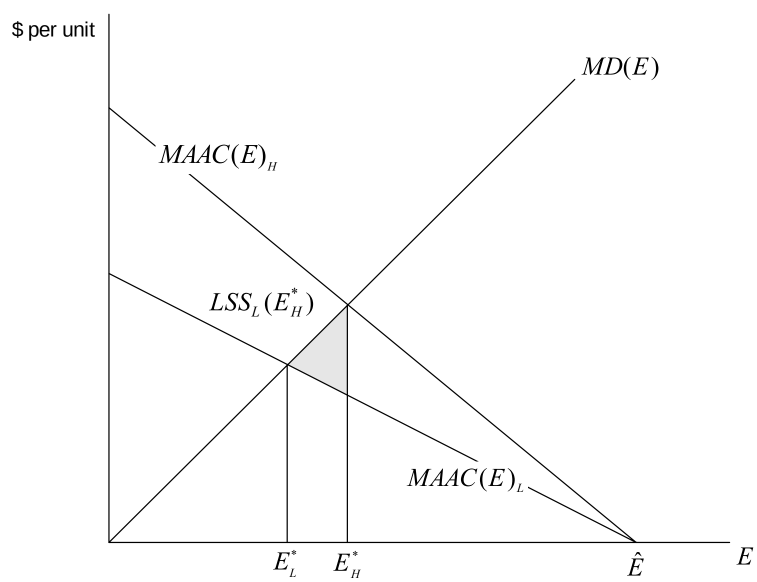


FIGURE 10-3

24

Choosing the Supply of Permits

- As with the tax policy, the goal of the regulator is to strike a balance between these two extreme outcomes, by setting the permits supply somewhere between E_L^* and E_H^*.

25

Choosing the Supply of Permits

- Suppose the regulator sets the quantity at $\tilde{E}$, as in Figure 10-4.

- If L is the true state of the world then $\tilde{E}$ will be too high, and if H is the true state of the world then $\tilde{E}$ will be too low.

26

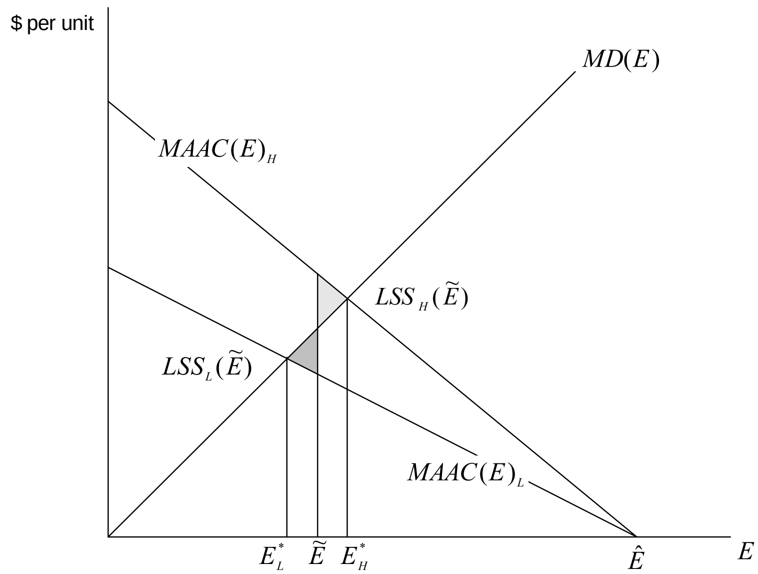


FIGURE 10-4

27

Choosing the Supply of Permits

- Note that the direction of these errors is <u>opposite</u> to those under the tax (recall Figure 9-29), where emissions are too low if the true state is L, and too high if the true state is H.

28

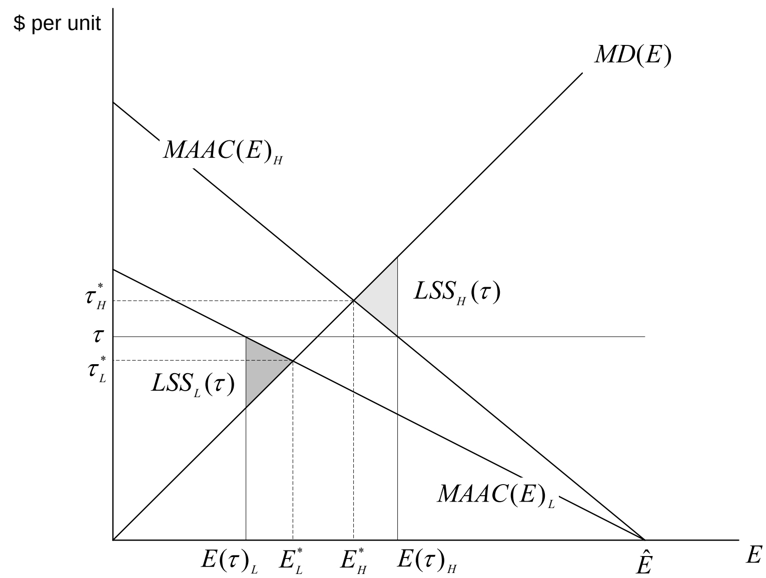


FIGURE 9-29 (repeat)

29

Choosing the Supply of Permits

- The loss of social surplus associated with the two possible errors under the emissions target are $LSS_L(\tilde{E})$ and $LSS_H(\tilde{E})$ respectively, as depicted in Figure 10-4.

30

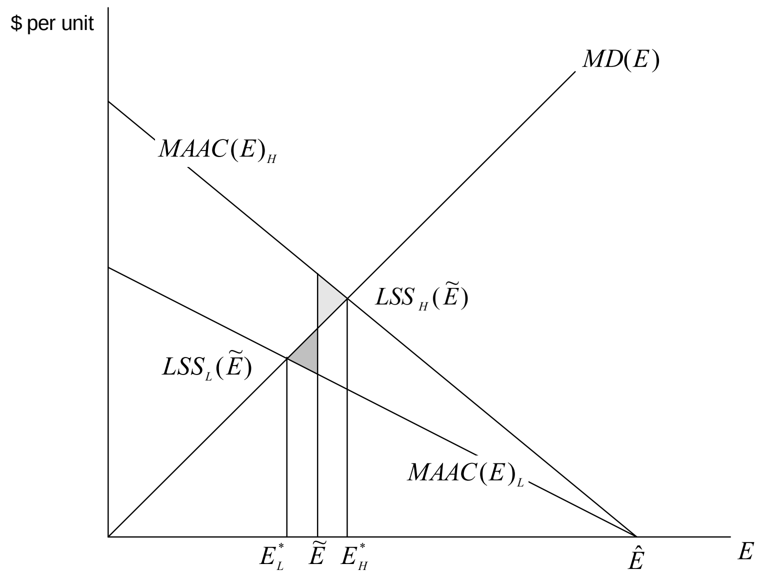


FIGURE 10-4 (repeat)

31

Choosing the Supply of Permits

- As with the tax policy, the goal of the regulator is to minimize the **expected loss** from these errors, defined as the probability-weighted sum of the associated surplus losses:

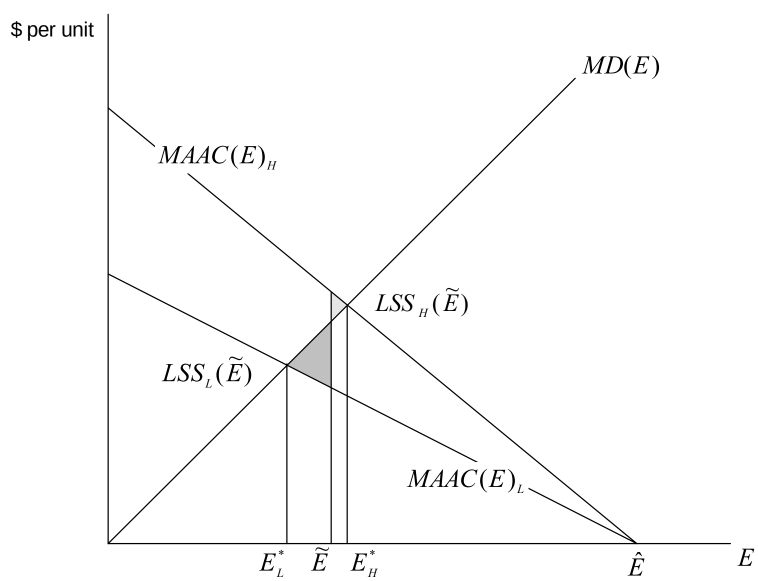
$$\mathbf{E}[LSS(E)] = \pi LSS_L(E) + (1-\pi)LSS_H(E)$$

32

Choosing the Supply of Permits

- Thus, if π is small (there is a high probability that the true state is H) then the regulator should set a high emissions target since this reduces the size of the most likely loss, $LSS_H(\tilde{E})$; see Figure 10-5.

33



FUGURE 10-5

34

Choosing the Supply of Permits

- Conversely, if π is large (there is a high probability that the true state is L) then the regulator should set a low emissions target since this reduces the size of the most likely loss, $LSS_L(\tilde{E})$; see Figure 10-6.

35

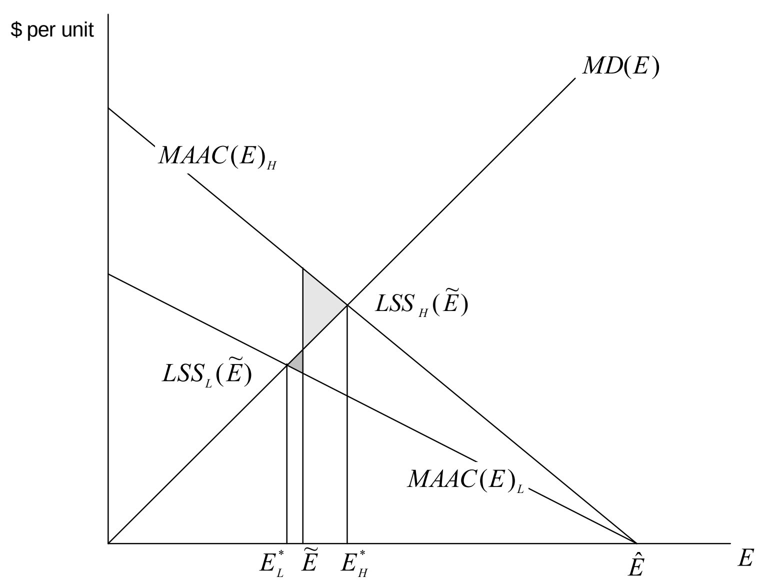


FIGURE 10-6

36

Choosing the Supply of Permits

- Recall from Topic 9 that the optimal tax rate is complicated to calculate without using calculus even for the simple linear case.
- In contrast, the optimal emissions target is straightforward to calculate in the linear case.

37

Choosing the Supply of Permits

- In general, the optimal emissions target is characterized by

$$\mathbf{E}[MAAC(\widetilde{E})] = MD(E)$$

where

$$\mathbf{E}[MAAC(\widetilde{E})] = \pi MAAC(\widetilde{E})_L + (1-\pi) MAAC(\widetilde{E})_H$$

38

Choosing the Supply of Permits

- In the linear case, this optimality condition becomes

$$\pi\varphi_L(\hat{E}-\tilde{E})+(1-\pi)\varphi_H(\hat{E}-\tilde{E})=\delta\tilde{E}$$

- See Figure 10-7.

39

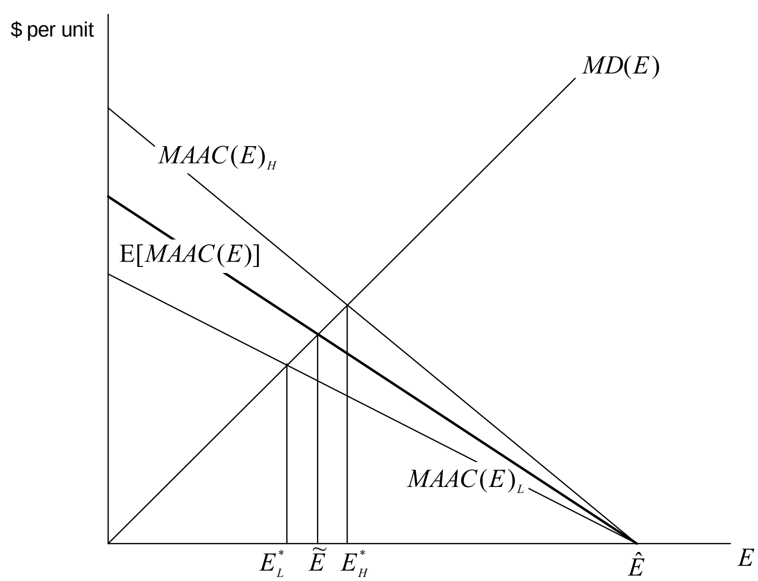


FIGURE 10-7

40

Choosing the Supply of Permits

- Solving the optimality condition yields

$$\tilde{E} = \frac{\overline{\varphi}\hat{E}}{\overline{\varphi} + \delta}$$

where

$$\overline{\varphi} = \pi\varphi_L + (1 - \pi)\varphi_H$$

is the expected value of φ.

41

Choosing the Supply of Permits

- Thus, the optimal emissions target has exactly the same form as optimal emissions in the absence of uncertainty, except that the expected value of φ takes the place of φ.

42

The Asymmetry of Loss

- It is important to note that the optimal quantity of emissions is <u>not</u> simply equal to the probability-weighted average of the quantities that would be optimal for each value of φ under full information.

43

The Asymmetry of Loss

- Specifically, we can show that

$$\widetilde{E} > \mathbf{E}[E^*] \equiv \pi E_L^* + (1-\pi)E_H^*$$

where

$$E_i^* = \frac{\varphi \hat{E}}{\varphi + \delta_i}$$

is the full-information E for i=L and H.

44

The Asymmetry of Loss

- The MAAC schedules in the L and H states diverge (the gap between them grows with more abatement), so the cost of emissions being too low is greater than the cost of emissions being too high.

45

Choosing the Supply of Permits

- The optimal quantity reflects this asymmetry, and so $\tilde{E}$ is closer to E_H^* than a simple probability-weighted averaging of E_L^* and E_H^* would dictate.

- That is, optimal emissions are higher than the expected value of the full-information quantities.

46

Choosing the Supply of Permits

- Note the link between this relationship and that between the optimal tax and the expected value of the full-information tax rates (recall Figure 9-33):

$$\tilde{\tau} < \pi\tau_L^* + (1-\pi)\tau_H^*$$

- Why? A higher tax rate induces lower emissions.

47

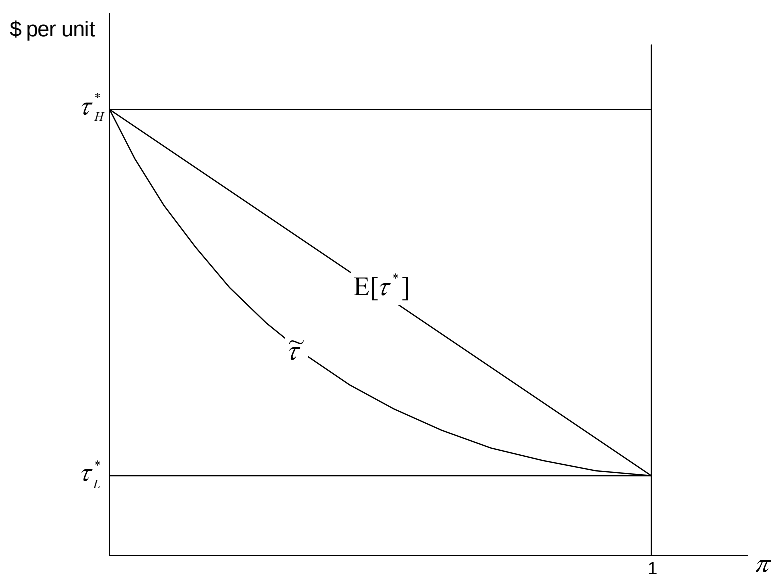


FIGURE 9-33 (repeat)

48

10.3 COMPARATIVE POLICY-PERFORMANCE

49

Comparative Policy-Performance

- Now that we have characterized the optimal permit supply under an emissions trading program, we can compare that policy with the tax policy.

50

Comparative Policy-Performance

- The basis for that comparison is expected loss of social surplus.
- In particular, we will say that the tax policy outperforms the quantity policy if

$$\mathbf{E}[LSS(\tilde{\tau})] < \mathbf{E}[LSS(\tilde{E})]$$

and vice versa.

51

Comparative Policy-Performance

- We will not work through the mathematics here, but based on this criterion it can be shown that the tax policy outperforms the quantity policy if and only if

$$\pi > \frac{\delta - \varphi_L}{\varphi_H - \varphi_L}$$

and vice versa.

52

Comparative Policy-Performance

- More generally, if we plot

$$\mathbf{E}[LSS(\tilde{\tau})] - \mathbf{E}[LSS(\tilde{E})]$$

against π, for given values of δ, φ_L and φ_H, it has the shape depicted in Figure 10-8.

53

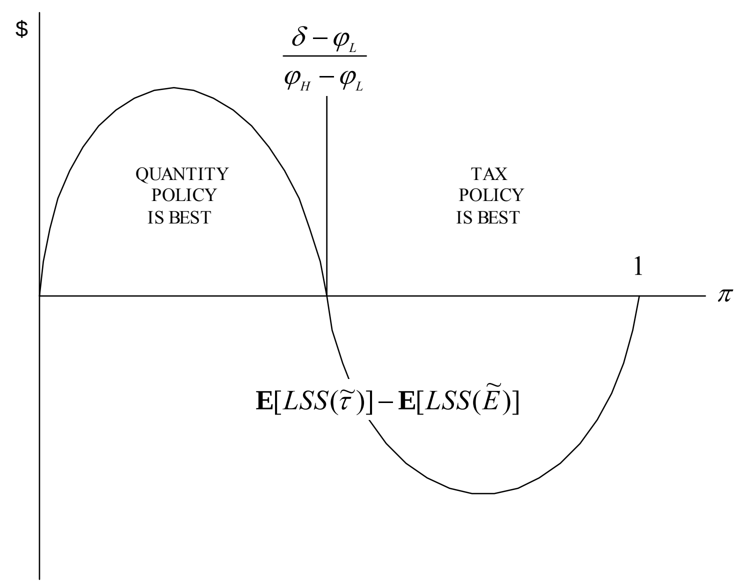


FIGURE 10-8

54

Comparative Policy-Performance

- Thus, at the extreme values of π (at π=0 and π =1), the two policies have the same performance; that is, in the absence of any uncertainty, the two policies are equivalent.

55

Comparative Policy-Performance

- For positive but low values of π (the L state is unlikely, and the H state is most likely), the quantity policy is best.
- For higher values of π (above the threshold value in Figure 10-8) the tax policy is best.
- What is the source of this relationship?

56

Comparative Policy-Performance

- The key difference between the tax policy and the fixed-quantity policy is that under the latter, the quantity of emissions actually emitted is independent of the true MAAC; the quantity emitted is determined solely by the beliefs of the regulator.

57

Comparative Policy-Performance

- In contrast, actual emissions under the tax policy are determined partly by the tax (which is determined solely by the beliefs of the regulator) and partly by the true MAAC, since sources response to the tax based on their true MACs.

58

Comparative Policy-Performance

- This is a critical difference between the policies, and one that does not arise in a world with full information.
- This difference underlies the relationship in Figure 10-8, in the following way.

59

Comparative Policy-Performance

- Suppose the regulator believes that $MAAC_H$ is most likely (low π)
- It will set a relatively high tax rate under a tax policy, and a relatively high emissions quantity under a quantity policy.
- Now suppose the regulator is wrong, and the true state is $MAAC_L$; see Figure 10-9.

60

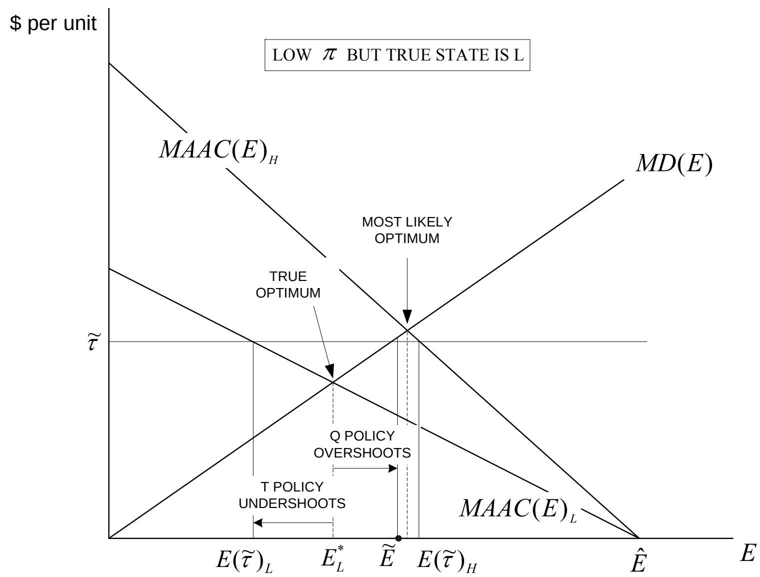


FIGURE 10-9

61

Comparative Policy-Performance

- Due to this error, the tax policy will induce an emissions level that is too low relative to the true (full-information) optimum.
- In this sense, the tax policy "undershoots" the true optimum; see Figure 10-9.
- In contrast, the quantity policy "overshoots" the true optimum; see Figure 10-9.

62

Comparative Policy-Performance

- The cost of these errors is <u>not</u> the same: the loss of social surplus is higher under the tax policy; see Figure 10-10.

63

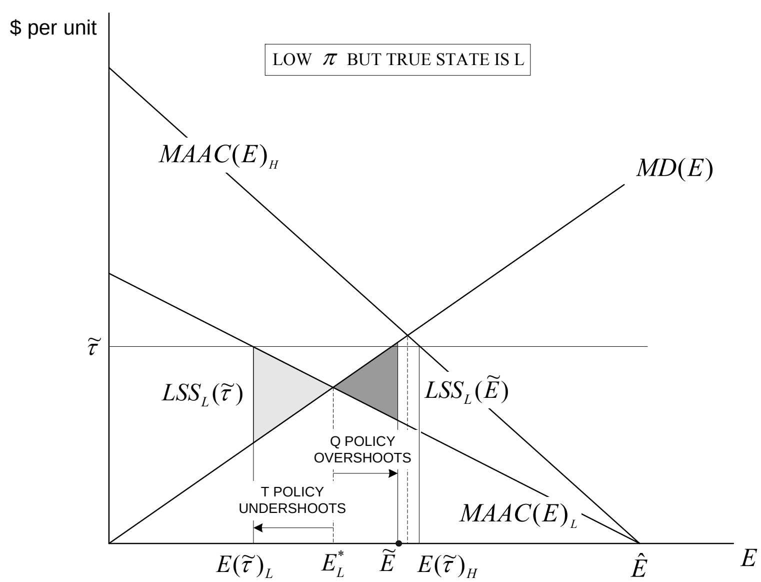


FIGURE 10-10

64

Comparative Policy-Performance

- The cost of undershooting (under the tax policy) is higher than the cost of overshooting (under the quantity policy) because $MAAC_L$ and $MAAC_H$ diverge as emissions decline.
- That is, errors which lead to emissions being too low are more costly than errors which lead to emissions being too high.

65

Comparative Policy-Performance

- This difference between the policies means that the quantity policy performs better when π is relatively low; recall Figure 10-8.

66

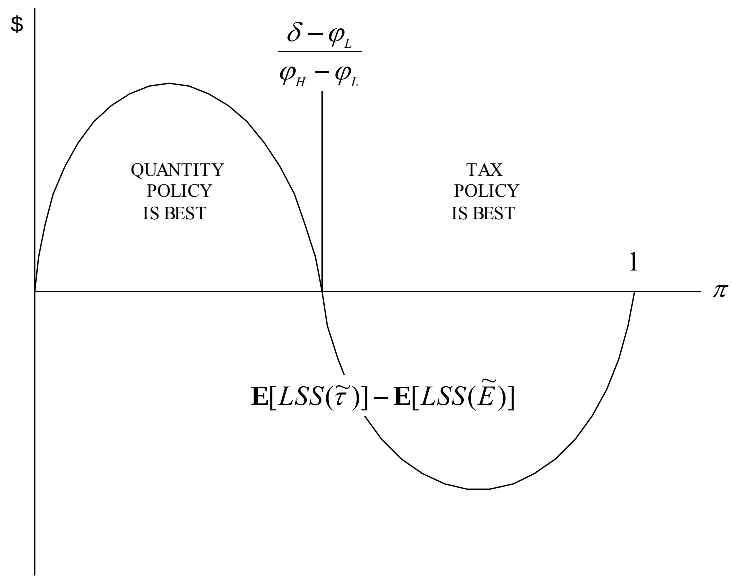


FIGURE 10-8 (repeat

67

Comparative Policy-Performance

- Now consider a situation where the regulator believes that $MAAC_L$ is most likely (high π)
- It will it set a relatively low tax rate under a tax policy, and a relatively low emissions quantity under a quantity policy.
- Now suppose the regulator is wrong, and the true state is $MAAC_H$; see Figure 10-11.

68

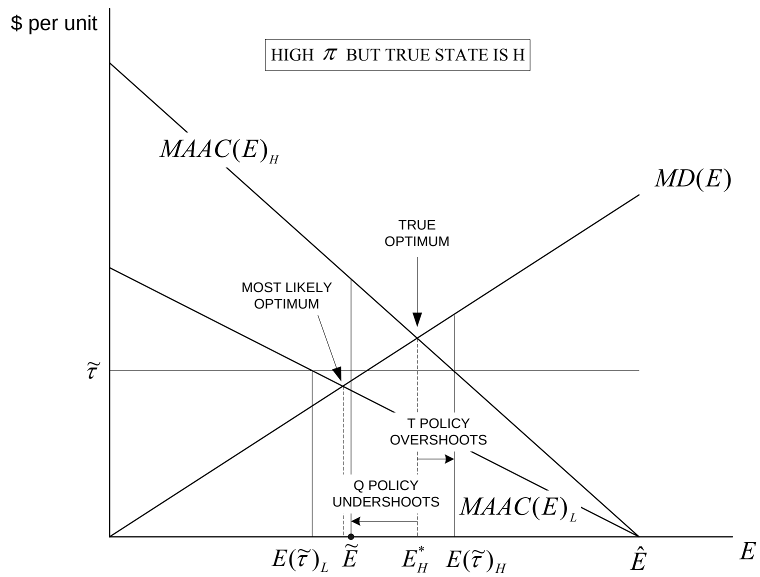


FIGURE 10-11

69

Comparative Policy-Performance

- Due to this error, the tax policy will induce an emissions level that is too high relative to the true (full-information) optimum.
- That is, the tax policy now "overshoots" the true optimum; see Figure 10-11.
- In contrast, the quantity policy now "undershoots" the true optimum; see Figure 10-11.

70

Comparative Policy-Performance

- Once again, the cost of these errors is <u>not</u> the same, but now the loss of social surplus is higher under the quantity policy; see Figure 10-12.

71

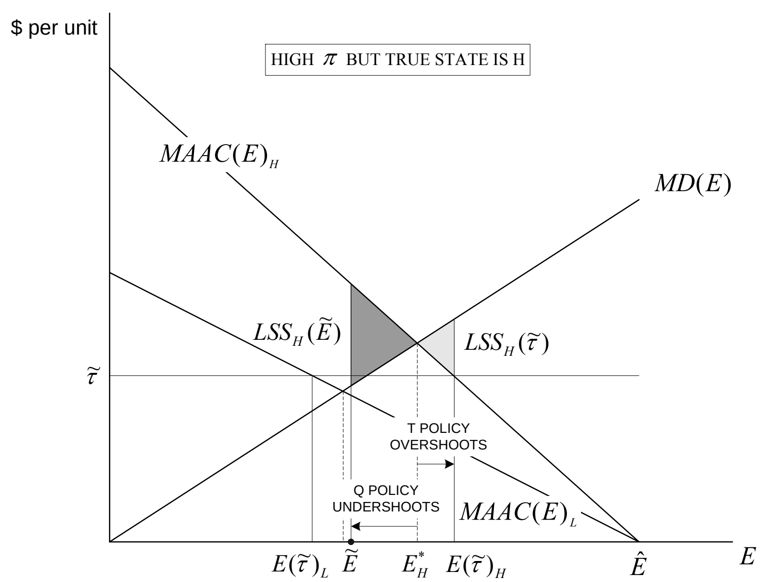


FIGURE 10-12

72

Comparative Policy-Performance

- Thus, the relative performance of the two policies is now reversed: at relatively high values of π, the tax policy outperforms the quantity policy (recall Figure 10-8).

73

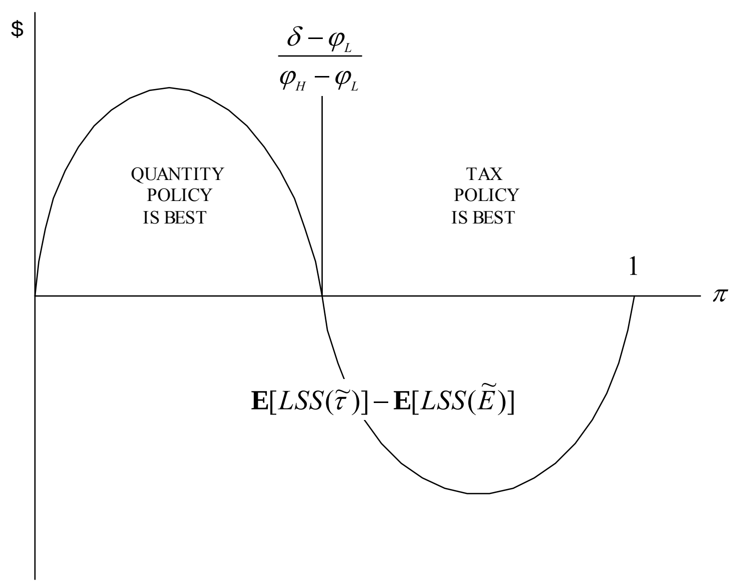


FIGURE 10-8 (repeat

74

Comparative Policy-Performance

- Note the important role that δ plays in the performance ranking of the two policies.
- All else equal, a higher δ calls for a higher tax rate, and this works against the tax policy, for the reason just explained.
- Hence, the threshold value of π, above which the tax policy is best, rises with δ.

75

Comparative Policy-Performance

- Note that if δ is high enough ($\delta > \varphi_H$) then the quantity policy is better than tax policy at any value of π, because in that case the threshold in Figure 10-8 is greater than one.

76

10.4 EQUILIBRIUM PERMIT PRICES

77

Equilibrium Permit Prices

- Recall from the previous section that the regulator sets the supply of permits at $\tilde{E}$, to equate $MD(E)$ and $\mathbf{E}[MAAC(E)]$, and issues that many permits.
- Recall Figure 10-7.

78

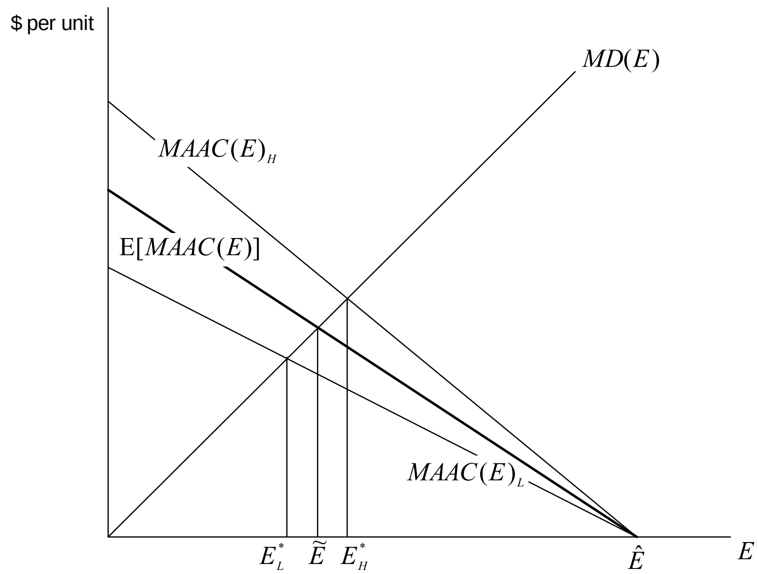


FIGURE 10-7 (repeat)

79

Equilibrium Permit Prices

- The equilibrium price of permits will be

$$\tilde{p}_L = MAAC(\tilde{E})_L \qquad \underline{\text{or}} \qquad \tilde{p}_H = MAAC(\tilde{E})_H$$

depending on whether the true MAAC is L or H respectively; see Figure 10-13.

80

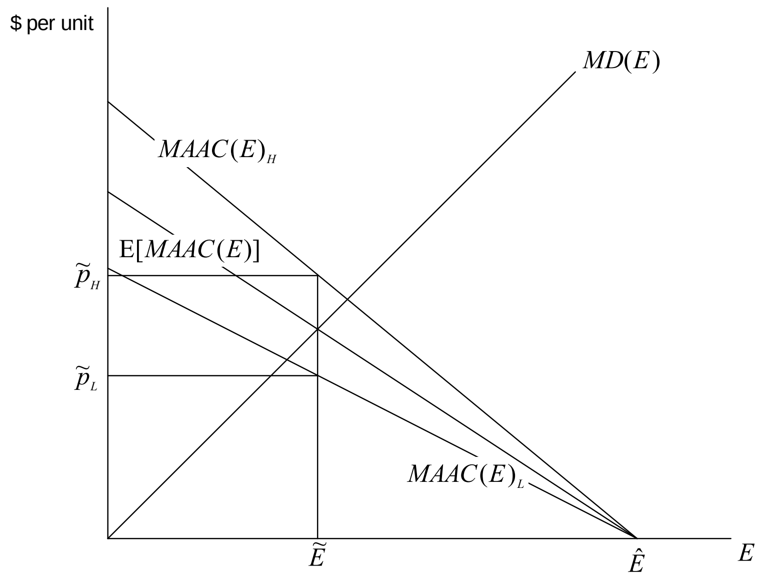


FIGURE 10-13

81

Equilibrium Permit Prices

- The <u>expected value</u> of the equilibrium price is

$$\mathbf{E}[\tilde{p}] = \pi\tilde{p}_L + (1-\pi)\tilde{p}_H$$

82

Equilibrium Permit Prices

- Since

$$\tilde{p}_i = MAAC(\tilde{E})_i$$

for i = L and H, it follows that

$$\mathbf{E}[\tilde{p}] = \mathbf{E}[MAAC(\tilde{E})]$$

83

Equilibrium Permit Prices

- Finally, since Ẽ has been set to equate **E**[$MAAC(E)$] and $MD(E)$, it follows that

$$\mathbf{E}[\tilde{p}] = MD(\tilde{E})$$

- See Figure 10-14.

84

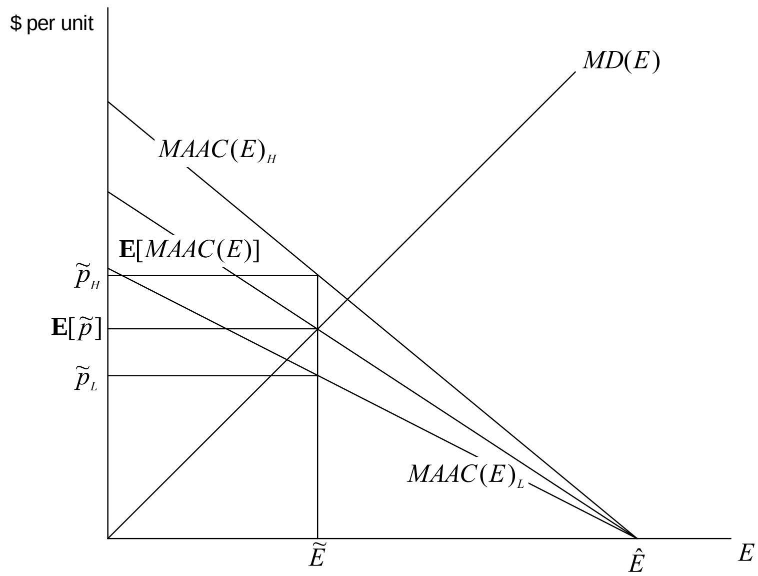


FIGURE 10-14

85

10.5 PROPERTIES OF THE EXPECTED PERMIT PRICE*

* Advanced Topic

86

Properties of the Expected Permit Price

- We know from Section 10-2 that in the linear case,

$$\tilde{E} = \frac{\bar{\varphi}\hat{E}}{\bar{\varphi} + \delta}$$

where

$$\bar{\varphi} = \pi\varphi_L + (1-\pi)\varphi_H$$

87

Properties of the Expected Permit Price

- Thus, we can calculate the equilibrium prices by setting $E=\tilde{E}$ in the MAAC schedule,

$$\varphi_i(\hat{E} - \tilde{E})$$

for i = L and H.

88

Properties of the Expected Permit Price

- This yields

$$\tilde{p}_L = \frac{\varphi_L \delta \hat{E}}{\bar{\varphi} + \delta} \quad \text{and} \quad \tilde{p}_H = \frac{\varphi_H \delta \hat{E}}{\bar{\varphi} + \delta}$$

89

Properties of the Expected Permit Price

- Note that both of these prices are increasing in π, since φ^{bar} is in the denominator of both, and φ^{bar} itself is decreasing in π, as the following rearrangement of φ^{bar} shows:

$$\bar{\varphi} = \varphi_H - \pi(\varphi_H - \varphi_L)$$

- See Figure 10-15.

90

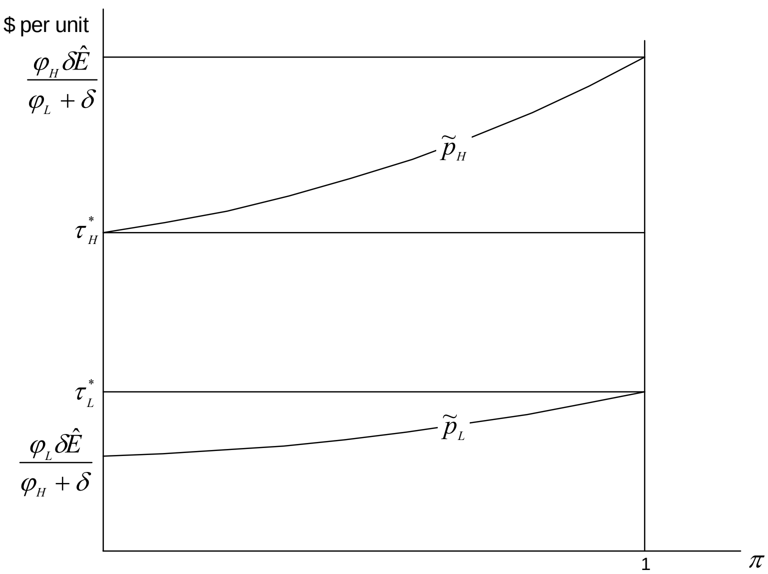


FIGURE 10-15

91

Properties of the Expected Permit Price

- To understand these relationships, consider Figure 10-16 in which a higher value of π causes **E**[*MAAC*(*E*)] to pivot closer to $MAAC_L$.

92

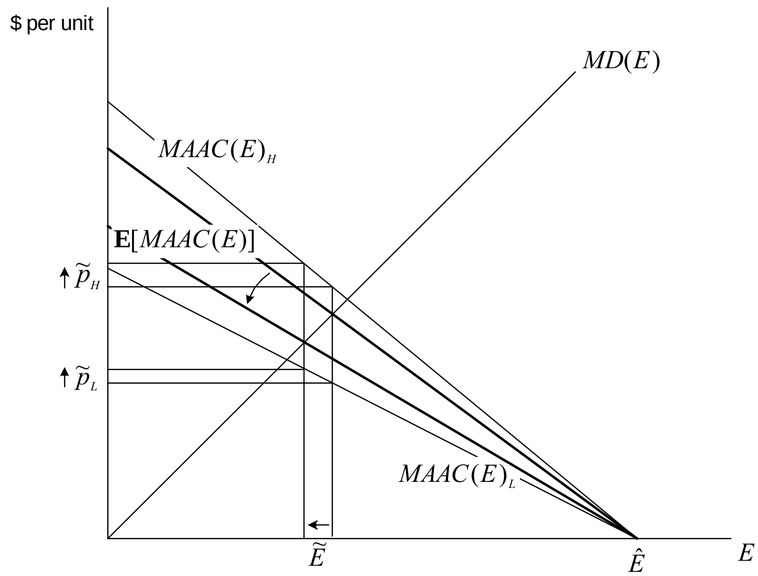


FIGURE 10-16

93

Properties of the Expected Permit Price

- As a consequence of this pivot, the optimal quantity of emissions falls – reflecting the higher probability that abatement costs are low – and this reduction in the supply of permits puts upward pressure on price, regardless of which state of the world is true.

94

Properties of the Expected Permit Price

- In the limit, as we reach π=1, we are effectively in a world with full information where the true state is L, and so

$$\tilde{p}_L = \tau_L^*$$

as depicted in Figure 10-15.

95

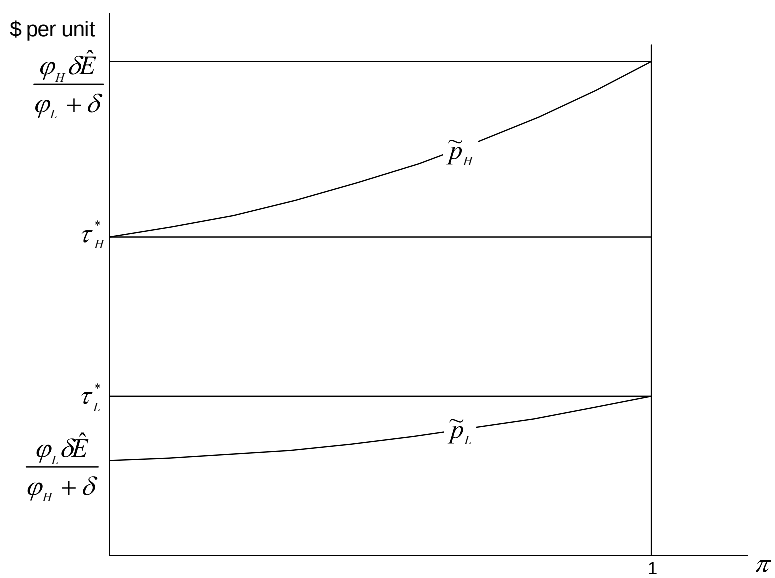


FIGURE 10-15 (repeat)

96

Properties of the Expected Permit Price

- Conversely, when π=0, we are effectively in a world with full information where the true state is H, and so

$$\tilde{p}_H = \tau^*_H$$

again, as depicted in Figure 10-15.

97

Properties of the Expected Permit Price

- Importantly, note from Figure 10-15 that $\tilde{p}_H$ rises at a faster rate than $\tilde{p}_H$ as π rises; that is, the two prices diverge as π rises.

- Why?

98

Properties of the Expected Permit Price

- The divergence of $MAAC_L$ and $MAAC_H$ as E falls means that as π rises (and hence, $\tilde{E}$ falls), $\tilde{p}_L$ and $\tilde{p}_H$ also diverge.
- This is evident from Figure 10-16, and is highlighted in Figure 10-17, which focuses on the relationship between π, $\tilde{E}$ and permit prices in the two states.

99

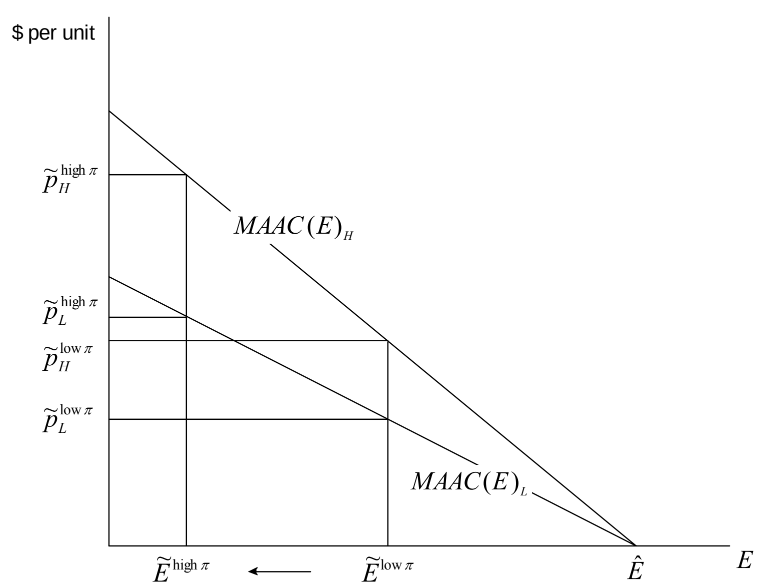


FIGURE 10-17

100

Properties of the Expected Permit Price

- Figure 10-17 illustrates that as π rises, and $\tilde{E}$ falls, the gap between $\tilde{p}_L$ and $\tilde{p}_H$ rises, as reflected in Figure 10-15.

101

Properties of the Expected Permit Price

- Now consider the expected value of the permit price:

$$\mathbf{E}[\tilde{p}] = \pi\tilde{p}_L + (1-\pi)\tilde{p}_H = \frac{\bar{\varphi}\delta\hat{E}}{\bar{\varphi}+\delta}$$

- Note that this is equal to $MD(\tilde{E})$.

102

Properties of the Expected Permit Price

- This expected price is <u>decreasing</u> in π, as depicted in Figure 10-18.

103

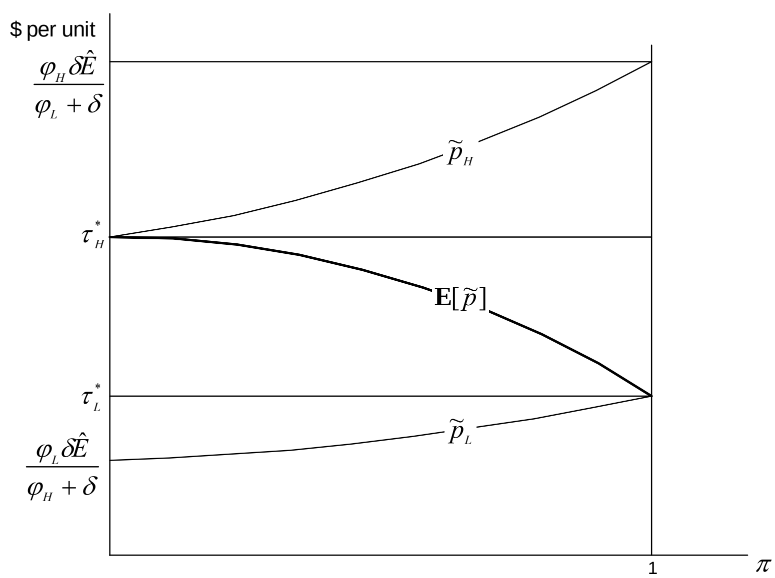


FIGURE 10-18

104

Properties of the Expected Permit Price

- How can the expected price be decreasing in π when both possible prices are increasing in π?
- Mathematically, a higher value of π puts more weight on p_L and less weight on p_H in the expected value, and p_L is smaller than p_H, so the expected value falls.

105

Properties of the Expected Permit Price

- In terms of the economics, a higher value of π means a lower optimal level of emissions – reflecting the higher probability that abatement costs are low – and this means that MD at the optimum is lower.
- This in turn means that $\mathbf{E}[\tilde{p}]$ is lower since

$$\mathbf{E}[\tilde{p}] = MD(\tilde{E})$$

106

Properties of the Expected Permit Price

- We can also see this from Figure 10-19, in which **E**[$MAAC(E)$] pivots towards $MAAC_L$ as π rises.

107

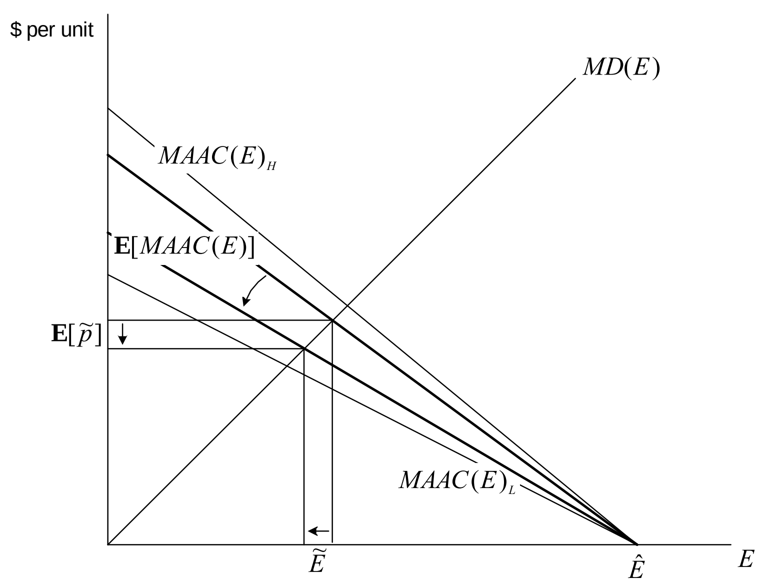


FIGURE 10-19

108

Properties of the Expected Permit Price

- Note that from Figure 10-19 that $\mathbf{E}[\tilde{p}]$ falls at an <u>increasing rate</u> as π rises.
- This reflects the fact that the difference between $\tilde{p}_L$ and $\tilde{p}_H$ rises as π rises, as explained in reference to Figure 10-17.

109

Properties of the Expected Permit Price

- Note that this property of $\mathbf{E}[\tilde{p}]$ means that

$$\mathbf{E}[\tilde{p}] > \mathbf{E}[\tau^*] = \pi\tau_L^* + (1-\pi)\tau_H^*$$

as illustrated in Figure 10-20, where $\mathbf{E}[\tau^*]$ declines linearly as π rises.

110

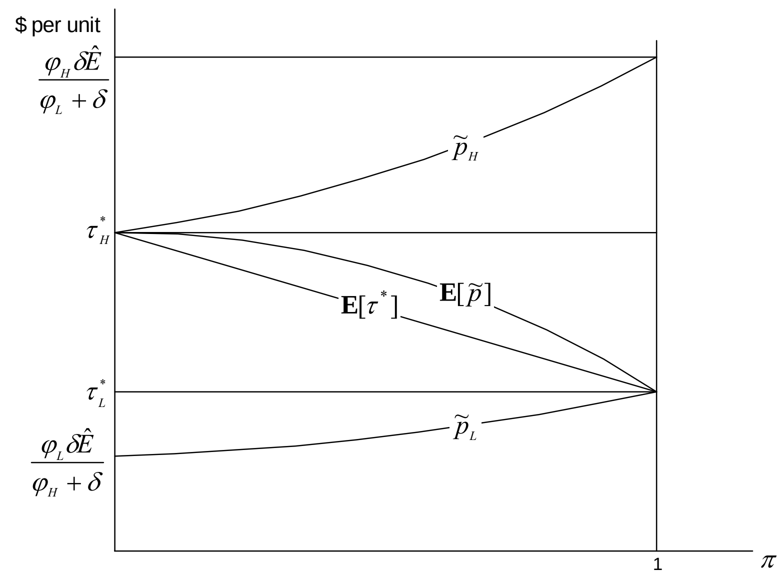


FIGURE 10-20

111

Properties of the Expected Permit Price

- Now let us compare the expected permit price with the optimal tax rate from Topic 9-4 denoted $\tilde{\tau}$.

- The relationship between the two is depicted in Figure 10-21, where both are plotted against π.

112

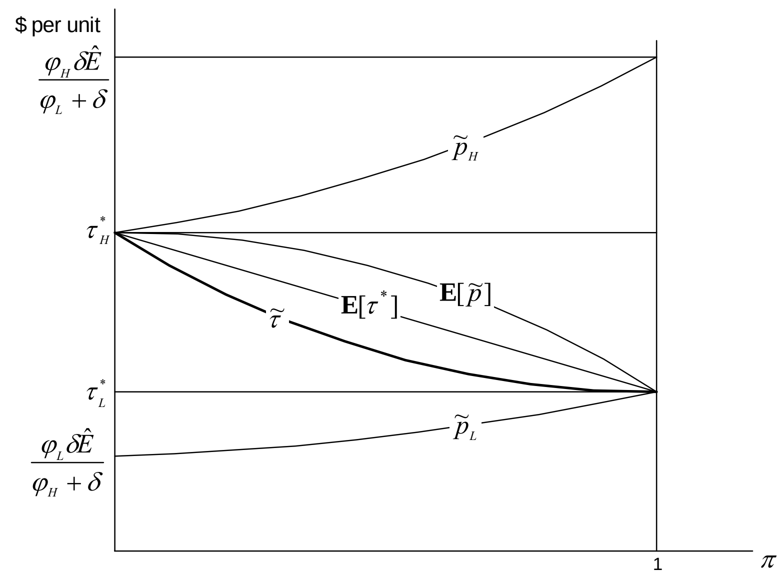


FIGURE 10-21

113

Properties of the Expected Permit Price

- Note that

$$\mathbf{E}[\tilde{p}] > \mathbf{E}[\tau^*] > \tilde{\tau}$$

- except when π=0 or π=1.

114

Properties of the Expected Permit Price

- These relationships reflect the divergence of the MAAC schedules as aggregate emissions fall.

115

Properties of the Expected Permit Price

- It is important to remember that $\tilde{\tau}$ is not uncertain – it is chosen by the regulator – but the permit price is uncertain.

- The permit price is never actually equal to $\mathbf{E}[p]$; it is equal to either p_L or p_H, depending on the true state of the world.

116

Properties of the Expected Permit Price

- Moreover, recall that the gap between p_L and p_H rises as π rises, reflecting that fact that the gap between $MAAC_L$ and $MAAC_H$ rises as optimal emissions fall in response to a rising π; recall Figure 10-17.

117

Properties of the Expected Permit Price

- This rising gap between p_L and p_H is a useful measure of the rising risk associated with a quantity-based policy as π rises.
- Recall from Section 10-2 that if π is high enough then the quantity-based policy is outperformed by a tax policy for precisely this reason; recall Figure 10-17.

118

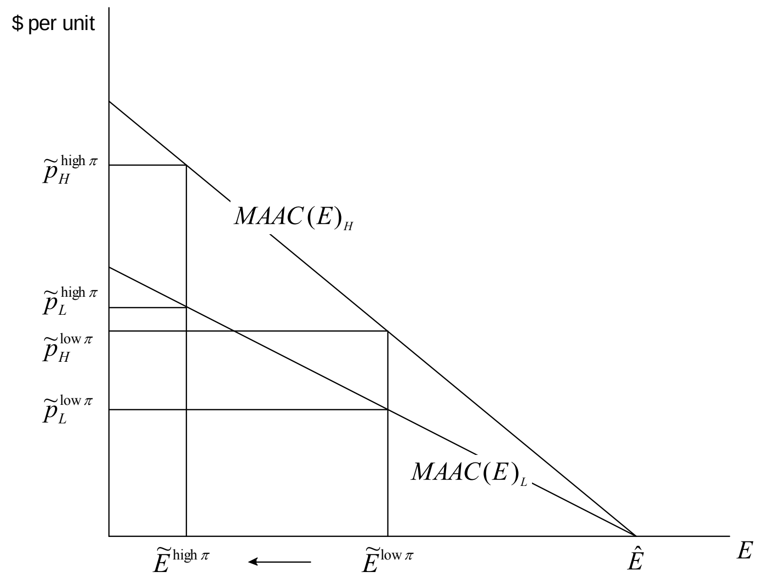


FIGURE 10-17 (repeat)

119

Properties of the Expected Permit Price

- Figure 10-22 depicts the critical threshold for π alongside the permit prices and the tax.
- The figure serves as a useful summary of some of the main points we have covered so far.

120

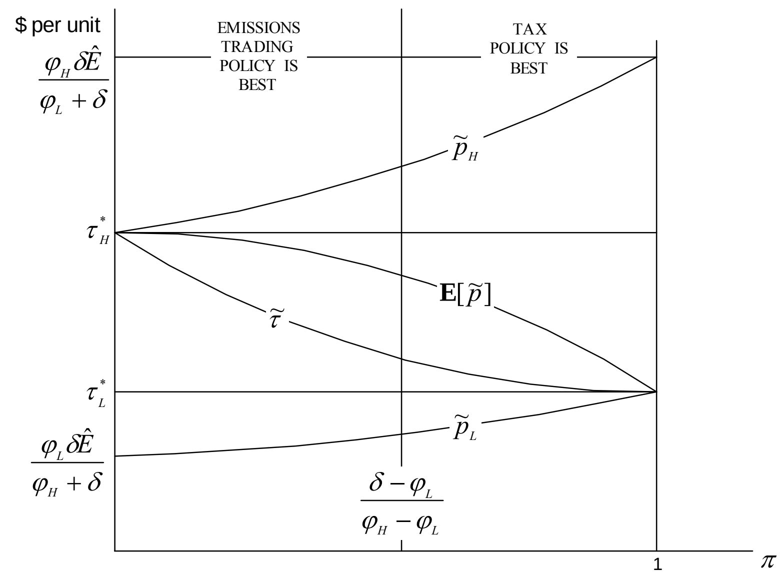


FIGURE 10-22

121

10.6 EMISSIONS TRADING WITH A "SAFETY VALVE"

122

Emissions Trading with a "Safety Valve"

- Our analysis to date has shown that a quantity-based policy with emissions trading is better than a tax policy if there is a low probability that MAAC is low; the converse holds if there is a high probability that MAAC is low.

123

Emissions Trading with a "Safety Valve"

- This raises a natural question:
  - can we use a combination of the two policies in a way that is better than using just one policy or the other?

124

Emissions Trading with a "Safety Valve"

- In this section we investigate a combined policy that uses a quantity-based policy with a "safety valve".
- We will examine this policy in the context of our simple model with two possible MAAC schedules, L and H.

125

Emissions Trading with a "Safety Valve"

- The combined policy works as follows.
- The regulator issues a quantity of permits E_L^* such that

$$MAAC(E_L^*) = MD(E_L^*)$$

- See Figure 10-23.

126

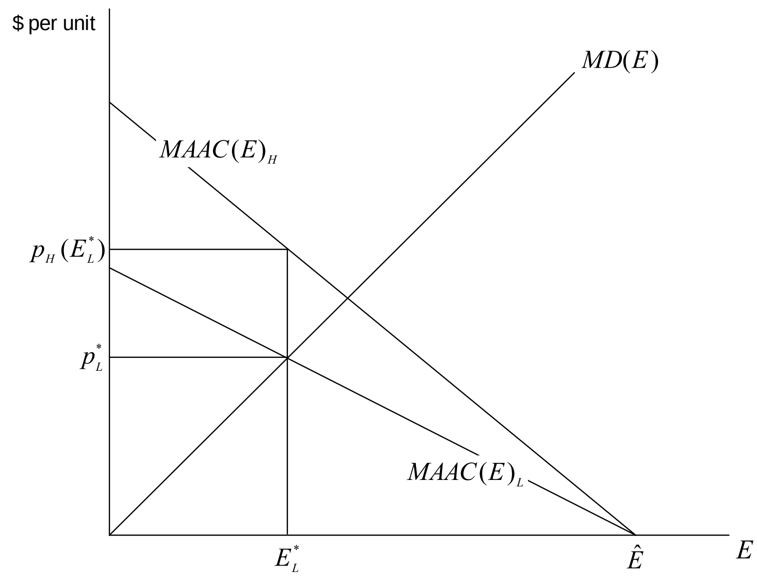


FIGURE 10-23

127

Emissions Trading with a "Safety Valve"

- If permits trade at $p_L{}^*$ in Figure 10-23 then the regulator knows that the true state of nature is L.

- If instead permits trade at $p_H(E_L{}^*)$ in Figure 10-23 then the regulator infers correctly that H is the true state of nature.

128

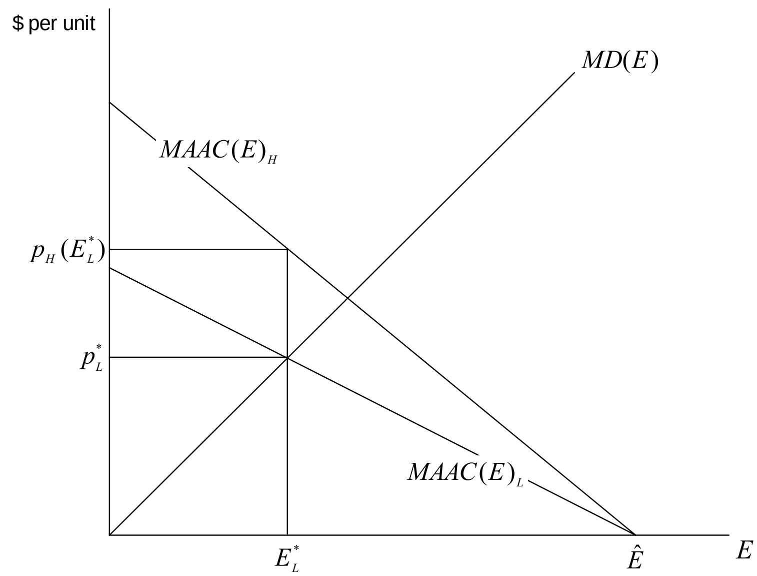


FIGURE 10-23 (repeat)

129

Emissions Trading with a "Safety Valve"

- Note the importance of a competitive permit market here:
  - no source can be so large relative to the market that it can manipulate the permit price and thereby send a false signal to the regulator.

130

Emissions Trading with a "Safety Valve"

- Upon observing $p_H(E_L^*)$, the regulator could in principle issue more permits, raising the total supply to E_H^* in Figure 10-22, thereby inducing a reduction in the trading price to p_H^* in Figure 10-24.

131

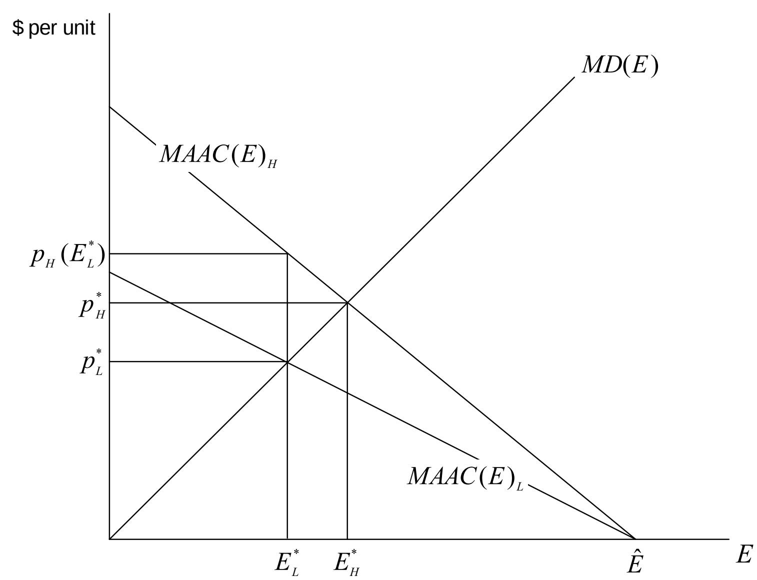


FIGURE 10-24

132

Emissions Trading with a "Safety Valve"

- The problem with this *ex post* adjustment approach in practice is that many of the abatement actions taken in response to the initial high price may be irreversible.
- This is especially true of investments in new technology.
- The costs of these investments generally cannot be recovered by "undoing" them.

133

Emissions Trading with a "Safety Valve"

- Thus, the regulator would like to prevent these sub-optimal investments from happening before they are made.
- A natural solution here is to announce ahead of time that unlimited additional permits can be purchased from the regulator at a price of p_H^* should the demand arise.

134

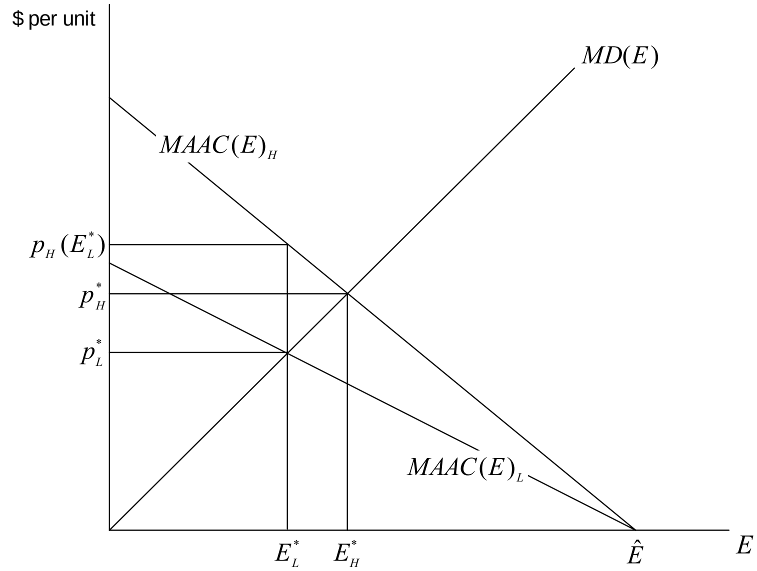


FIGURE 10-24 (repeat)

135

Emissions Trading with a "Safety Valve"

- This means that the trading price will never rise above $p_H{}^*$, and that no sub-optimal irreversible investments will be made in response to a temporary price higher than $p_H{}^*$.

136

Emissions Trading with a "Safety Valve"

- This commitment to sell additional permits at a fixed price works like a "safety valve":
  - if pressure on the permit price pushes it above p_H^* then the sale of additional permits immediately relieves that pressure.

137

Emissions Trading with a "Safety Valve"

- This solution to the informational problem works perfectly if there are only two possible states of nature, since the regulator knows exactly how many permits to issue initially, and at what price to set the safety valve.

138

Emissions Trading with a "Safety Valve"

- In a setting with more than two possible states of nature, the problem cannot be solved so easily.
- In such a setting, it is generally not possible to achieve the full-information outcome, and the optimal safety-valve policy must be chosen under uncertainty about the outcome.

139

Emissions Trading with a "Safety Valve"

- This optimization problem is complicated, and we will not investigate it further here.
- However, the simple two-state example serves to illustrate that a combined policy – emissions trading coupled with a safety valve – can generally do better than either a fixed-quantity policy or a tax policy alone.

END

140

TOPIC 10 REVIEW QUESTIONS

These review questions begin with a solved example. It is recommended that you work through this example before commencing the questions. The example here continues the example used in the Topic 9 Review. You may want to work through that again before proceeding.

A SOLVED EXAMPLE

The example relates to the following information. There are multiple sources of a uniformly mixed pollutant with marginal aggregate abatement cost given by

$$MAAC(e) = \phi(2160 - E)$$

where ϕ could be one of two values: $\phi_L = 1$ or $\phi_H = 2$. The regulator cannot distinguish between these two possibilities, henceforth denoted the "L-cost scenario" and the "H-cost scenario" respectively. Its beliefs are that $\phi = \phi_L$ with probability 3/10.

The marginal damage schedule is

$$MD(e) = \delta E$$

where $\delta = 1$.

The analysis of this example in Topic 9 Review derived a total of 14 results. We begin our analysis here leading off from where that analysis concluded. (The equation numbers here also carry on from that point).

15. Calculate the full-information optimal quantities, denoted E_L^* and E_H^* for the L-cost scenario and H-cost scenario respectively.

We calculated these in Result 1 from the Topic 9 Review, when we calculated the full-information taxes. They are

(42) $E_L^* = 1080$

and

(43) $E_H^* = 1440$

16. Suppose the regulator sets $E = E_L^*$. Calculate the loss of social surplus in the H-cost scenario.

We need to calculate the shaded area in Figure 10-2, reproduced here as Figure R10-1. This area is

(44) $$LSS(E_L^*)_H = \int_{E_L^*}^{E_H^*} MAAC(E)_H \, dE - \int_{E_L^*}^{E_H^*} MD(E) dE$$

To calculate this area without using calculus, we first need to find $MAAC(E_L^*)_H$; see Figure R8-5. This is equal to

(45) $$MAAC(E_L^*)_H = 2(2160 - E_L^*) = 2160$$

since $\phi = 2$ in the H-cost scenario. We can then calculate the area of the shaded triangle as

(46) $$\begin{aligned} LSS(E_L^*)_H &= \frac{[MAAC(E_L^*)_H - \tau_L^*][E_H^* - E_L^*]}{2} \\ &= \frac{[2160 - 1080][1440 - 1080]}{2} \\ &= 194400 \end{aligned}$$

17. Suppose the regulator sets $E = E_L^*$. Calculate the expected loss of social surplus.

The L-cost scenario occurs with probability 3/10, and under that scenario E_L^* is the social optimum, so there no associated loss of social surplus. The H-cost scenario occurs with probability 7/10, and under that scenario E_L^* creates a loss of social surplus equal to $LSS(E_L^*)_H$, as calculated in Result 16 above.

Thus, the expected loss of social surplus is

(47) $$\mathbf{E}[LSS(E_L^*)] = \frac{3}{10} 0 + \frac{7}{10} LSS(E_L^*)_H = 0 + \frac{7}{10} 194400 = 136080$$

18. Suppose the regulator sets $E = E_H^*$. Calculate the loss of social surplus in the L-cost scenario.

We need to calculate the shaded area in Figure 10-3, reproduced here as Figure R10-2. This area is

(48) $$LSS(E_H^*)_L = \int_{E_L^*}^{E_H^*} MD(E)dE - \int_{E_L^*}^{E_H^*} MAAC(E)_L dE$$

To calculate this area without using calculus, we first need to find $MAAC(E_H^*)_L$; see Figure R10-2. This is equal to

(49) $$MAAC(E_H^*)_L = 1(2160 - E_H^*) = 720$$

since $\phi = 1$ in the L-cost scenario. We can then calculate the area of the shaded triangle as

(50) $$LSS(E_H^*)_L = \frac{[\tau_H^* - MAAC(E_H^*)_L][E_H^* - E_L^*]}{2}$$

$$= \frac{[1440 - 720][1440 - 1080]}{2}$$

$$= 129600$$

19. Suppose the regulator sets $E = E_H^*$. Calculate the expected loss of social surplus.

The L-cost scenario occurs with probability 3/10, and under that scenario E_H^* creates a loss of social surplus equal to $LSS(E_H^*)_L$, as calculated in Result 18 above. The H-cost scenario occurs with probability 7/10, and under that scenario E_H^* is the social optimum, so there is no associated loss of social surplus. Thus, the expected loss of social surplus is

(51) $$\mathbf{E}[LSS(E_H^*)] = \frac{3}{10}LSS(E_H^*)_L + \frac{7}{10}0 = \frac{3}{10}129600 + 0 = 38880$$

20. Calculate the probability-weighted average of the full-information quantities, denoted $\mathbf{E}[E^*]$.

Using (42) and (43),

(52) $$\mathbf{E}[E^*] = \frac{3}{10}E_L^* + \frac{7}{10}E_H^* = \frac{3}{10}1080 + \frac{7}{10}1440 = 1332$$

21. Suppose the regulator sets $E = \mathbf{E}[E^*]$. Calculate the expected loss of social surplus.

We need to calculate the probability-weighted average of the shaded areas in Figure 10-4 evaluated at $E = \mathbf{E}[E^*]$. The figure is reproduced here as Figure R10-3. First calculate $LSS(\mathbf{E}[E^*])_L$:

(53) $$LSS(\mathbf{E}[E^*])_L = \int_{E_L^*}^{\mathbf{E}[E^*]} MD(E)dE - \int_{E_L^*}^{\mathbf{E}[E^*]} MAAC(E)_L \, dE$$

To calculate this area without using calculus, we first need to find $MD(\mathbf{E}[E^*])$ and $MAAC(\mathbf{E}[E^*])_L$; see Figure R10-3. These are equal to, respectively,

(54) $$MD(\mathbf{E}[E^*]) = \delta\mathbf{E}[E^*] = 1332$$

since $\delta = 1$ in this example, and

(55) $$MAAC(\mathbf{E}[E^*])_L = 1(2160 - \mathbf{E}[E^*]) = 2160 - 1332 = 828$$

since $\phi = 1$ in the L-cost scenario. We can then calculate the area of the shaded triangle as

(56) $$LSS(\mathbf{E}[E^*])_L = \frac{[MD(\mathbf{E}[E^*]) - MAAC(\mathbf{E}[E^*])_L][\mathbf{E}[E^*] - E_L^*]}{2}$$

$$= \frac{[1332 - 828][1332 - 1080]}{2}$$

$$= 63504$$

Now calculate $LSS(\mathbf{E}[E^*])_H$, from Figure R8-7:

(57) $$LSS(\mathbf{E}[E^*])_H = \int_{\mathbf{E}[E^*]}^{E_H^*} MAAC(E)_H \, dE - \int_{\mathbf{E}[E^*]}^{E_H^*} MD(E)dE$$

To calculate this area without using calculus, we first need to find $MAAC(\mathbf{E}[E^*])_H$; see Figure R10-3. This is equal to

(58) $$MAAC(\mathbf{E}[E^*])_H = 2(2160 - \mathbf{E}[E^*]) = 2(2160 - 1332) = 1656$$

since $\phi = 2$ in the H-cost scenario. We can then calculate the area of the shaded triangle as

(59) $$LSS(\mathbf{E}[E^*])_H = \frac{[MAAC(\mathbf{E}[E^*])_H - MD(\mathbf{E}[E^*])][E_H^* - \mathbf{E}[E^*]]}{2}$$

$$= \frac{[1656-1332][1440-1332]}{2}$$

$$= 17496$$

We can now calculate the probability-weighted average of $LSS(\mathbf{E}[E^*])_L$ and $LSS(\mathbf{E}[E^*])_H$ to find the expected loss of social surplus when $E = \mathbf{E}[E^*]$:

(60) $$\mathbf{E}[LSS(\mathbf{E}[E^*])] = \frac{3}{10} LSS(\mathbf{E}[E^*])_L + \frac{7}{10} LSS(\mathbf{E}[E^*])_H$$

$$= \frac{3}{10} 63504 + \frac{7}{10} 17496$$

$$= 31298$$

Compare this result with Results 17 & 19 above. The expected loss of social surplus from using $E = \mathbf{E}[E^*]$ is less than from using either E_L^* or E_H^* because the former policy takes into account the relative probabilities of the tow possible cost scenarios.

However, we can do even better than setting $E = \mathbf{E}[E^*]$ because this policy does not take into account the asymmetry of losses under the L-cost and H-cost scenarios that arises from the different slopes of $MAAC(E)_L$ and $MAAC(E)_H$. The next few questions relate to an <u>optimal</u> quantity policy, which does take account of this asymmetry. In particular, the optimal quantity, minimizes

(61) $$\mathbf{E}[LSS(E)] = \frac{3}{10} LSS(E)_L + \frac{7}{10} LSS(E)_H$$

where $LSS(E)_L$ denotes the loss of social surplus under the L-cost scenario, and $LSS(E)_H$ denotes the loss of social surplus under the H-cost scenario.

We can find this optimal quantity (without calculus) by solving

(62) $\mathbf{E}[MAAC(E)] = MD(E)$

where $\mathbf{E}[MAAC(E)]$ is expected marginal aggregate abatement cost.

22. Calculate $\mathbf{E}[MAAC(E)]$.

Expected marginal aggregate abatement cost is the probability-weighted average of $MAAC(E)_L$ and $MAAC(E)_H$:

(63) $$\mathbf{E}[MAAC(E)] = \frac{3}{10}MAAC(E)_L + \frac{7}{10}MAAC(E)_H$$

$$= \frac{3}{10}(2160 - E) + \frac{7}{10}2(2160 - E)$$

$$= 3672 - \frac{17E}{10}$$

23. Calculate the optimal quantity-based policy, denoted $\tilde{E}$.

$\tilde{E}$ is characterized by $\mathbf{E}[MAAC(E)] = MD(E)$. Using (63), we have

(64) $$3672 - \frac{17E}{10} = \delta E$$

where $\delta = 1$ in this example. Solving (64) yields

(65) $$\tilde{E} = 1360$$

Note that this optimal quantity is higher than $\mathbf{E}[E^*]$. This reflects the fact that the MAAC schedules in the L and H scenarios diverge, so the error associated with emissions being too high is less costly than the error associated with emissions being too low.

24. Suppose the regulator sets $E = \tilde{E}$. Calculate the expected loss of social surplus.

We need to calculate the probability-weighted average of the shaded areas in Figure 10-4 evaluated at $E = \tilde{E}$. The figure is reproduced here as Figure R10-4. First calculate $LSS(\tilde{E})_L$:

(66) $$LSS(\tilde{E})_L = \int_{E_L^*}^{\tilde{E}} MD(E)dE - \int_{E_L^*}^{\tilde{E}} MAAC(E)_L\, dE$$

To calculate this area without using calculus, we first need to find $MD(\tilde{E})$ and $MAAC(\tilde{E})_L$; see Figure R10-4. These are equal to, respectively,

(67) $MD(\tilde{E}) = \delta\tilde{E} = 1360$

since $\delta = 1$ in this example, and

(68) $MAAC(\tilde{E})_L = 1(2160 - \tilde{E}) = 2160 - 1360 = 800$

since $\phi = 1$ in the L-cost scenario. We can then calculate the area of the shaded triangle as

(69) $$LSS(\tilde{E})_L = \frac{[MD(\tilde{E}) - MAAC(\tilde{E})_L][\tilde{E} - E_L^*]}{2}$$

$$= \frac{[1360 - 800][1360 - 1080]}{2}$$

$$= 78400$$

Now calculate $LSS(\tilde{E})_H$, from Figure R-8:

(70) $$LSS(\tilde{E})_H = \int_{\tilde{E}}^{E_H^*} MAAC(E)_H \, dE - \int_{\tilde{E}}^{E_H^*} MD(E) dE$$

To calculate this area without using calculus, we first need to find $MAAC(\tilde{E})_H$; see Figure R10-4. This is equal to

(72) $MAAC(\tilde{E})_H = 2(2160 - \tilde{E}) = 2(2160 - 1360) = 1600$

since $\phi = 2$ in the H-cost scenario. We can then calculate the area of the shaded triangle as

(73) $$LSS(\tilde{E})_H = \frac{[MAAC(\tilde{E})_H - MD(\tilde{E})][E_H^* - \tilde{E}]}{2}$$

$$= \frac{[1600 - 1360][1440 - 1360]}{2}$$

$$= 9600$$

We can now calculate the probability-weighted average of $LSS(\tilde{E})_L$ and $LSS(\tilde{E})_H$ to find the expected loss of social surplus when $E = \tilde{E}$:

(74) $$\mathbf{E}[LSS(\tilde{E})] = \frac{3}{10} LSS(\tilde{E})_L + \frac{7}{10} LSS(\tilde{E})_H$$
$$= \frac{3}{10} 78400 + \frac{7}{10} 9600$$
$$= 30240$$

Compare this result with Result 21 above. The expected loss of social surplus under the optimal quantity-based policy is about 3.4% lower than the expected loss of social surplus when $E = \mathbf{E}[E^*]$. This superior performance of the optimal policy reflects the fact that it takes into account the asymmetry of losses associated with setting emissions too low versus setting them too high.

We now want to compare the optimal quantity-based policy with the optimal tax policy that we derived in Result 14 from the Topic 9 Review.

25. Based on Results 24 and 14, determine which policy performs better in this setting.

From (41) in the Topic 9 Review, the expected loss of social surplus under the optimal tax policy is

(75) $\mathbf{E}[LSS(\tilde{\tau})] = 18144$

From (74), the expected loss of social surplus under the optimal quantity-based policy is

(76) $\mathbf{E}[LSS(\tilde{E})] = 30240$

Thus, the tax policy performs much better than the quantity-based policy. This reflects the fact that in this setting, δ is very low so the condition on π that we identified in Topic 10.3 (see Figure 10-8) is met even though π is relatively low.

REVIEW QUESTIONS

Questions 1 to 14 relate to the following information. There are multiple sources of a uniformly mixed pollutant with marginal aggregate abatement cost given by

$$MAAC(e) = \phi(3600 - E)$$

where ϕ could be one of two values: $\phi_L = 1$ or $\phi_H = 3$. The regulator cannot distinguish between these two possibilities, henceforth denoted the "L cost scenario" and the "H cost scenario" respectively. Its beliefs are that $\phi = \phi_L$ with probability 1/6. The marginal damage schedule is

$$MD(e) = \frac{3E}{2}$$

This policy setting in the same as the one we examined in Questions 19 – 36 from Topic 9 Review. Question 13 below will relate back to that material.

1. The full-information optimal quantities, denoted E_L^* and E_H^* for the L cost scenario and H cost scenario respectively, are

A. $E_L^* = 1660$ and $E_H^* = 1880$

B. $E_L^* = 1440$ and $E_H^* = 2400$

C. $E_L^* = 1860$ and $E_H^* = 2740$

D. $E_L^* = 1120$ and $E_H^* = 3260$

2. If the regulator sets $E = E_L^*$ then the loss of social surplus under the L cost scenario is

A. 2073600

B. 1152000

C. 1728000

D. 0

3. If the regulator sets $E = E_L^*$ then the loss of social surplus under the H cost scenario is

A. 2073600

B. 1152000

C. 1728000

D. 0

4. If the regulator sets $E = E_H^*$ then the loss of social surplus under the L cost scenario is

A. 2073600

B. 1152000

C. 1728000

D. 0

5. If the regulator sets $E = E_L^*$ then the expected loss of social surplus is

A. 2073600

B. 1152000

C. 1728000

D. 0

6. If the regulator sets $E = E_H^*$ then the expected loss of social surplus is

A. 192000

B. 215200

C. 340600

D. 184200

7. The probability-weighted average of the full-information optimal quantities, denoted $\mathbf{E}[E^*]$, is

A. 3600

B. 2240

C. 1880

D. 4120

8. The optimal quantity-based policy when $0 < \pi < 1$ is to set $E = \mathbf{E}[E^*]$.

A. True.

B. False.

9. Expected marginal aggregate abatement cost, denoted $\mathbf{E}[MAAC(E)]$, is

A. $9600 - \frac{8E}{3}$

B. $3600 - \frac{8E}{3}$

C. $7200 - \frac{E}{3}$

D. $3600 - \frac{5E}{6}$

10. The optimal quantity-based policy, denoted $\tilde{E}$, is

A. 2400

B. 2700

C. 1896

D. 2304

11. Consider the relationship between your answers to Q10 and Q7. The difference between these two quantities arises because the MAAC schedules in the L and H states diverge, so the error associated with emissions being too high is less costly than the error associated with emissions being too low.

A. True.

B. False.

12. If the regulator sets $E = \tilde{E}$ then the expected loss of social surplus is

A. 196200

B. 164000

C. 172800

D. 214400

We now want to compare the optimal quantity-based policy with the optimal tax policy that we examined in Questions 19 – 36 from Topic 9 Review.

13. Based on your answers to Q12 here and Q35 from the Topic 9 Review, the quantity-based policy performs better than the tax in this setting.

A. True.

B. False.

14. In general, the tax policy performs best when

A. the probability of the L cost scenario is high because in that case the optimal tax is high.

B. the probability of the L cost scenario is low because in that case the optimal tax is low.

C. the probability of the L cost scenario is high because in that case the optimal tax is low.

D. the probability of the L cost scenario is low because in that case the optimal tax is high.

Questions 15 to 26 relate to the following information. There are multiple sources of a uniformly mixed pollutant with marginal aggregate abatement cost given by

$$MAAC(e) = \phi(2700 - E)$$

where ϕ could be one of two values: $\phi_L = 10$ or $\phi_H = 20$. The regulator cannot distinguish between these two possibilities, henceforth denoted the "L cost scenario" and the "H cost scenario" respectively. Its beliefs are that $\phi = \phi_L$ with probability 3/5. The marginal damage schedule is

$$MD(e) = 10E$$

This policy setting in the same as the one we examined in Questions 37 – 52 from Topic 9 Review. Question 25 below will relate back to that material.

15. The full-information optimal quantities, denoted E_L^* and E_H^* for the L cost scenario and H cost scenario respectively, are

A. $E_L^* = 1650$ and $E_H^* = 1780$

B. $E_L^* = 2140$ and $E_H^* = 3200$

C. $E_L^* = 1350$ and $E_H^* = 1800$

D. $E_L^* = 980$ and $E_H^* = 3260$

16. If the regulator sets $E = E_H^*$ then the loss of social surplus under the H cost scenario is

A. 3075600

B. 2154000

C. 2764000

D. 0

17. If the regulator sets $E = E_L^*$ then the loss of social surplus under the H cost scenario is

A. 3075600

B. 2154000

C. 4768000

D. 3037500

18. If the regulator sets $E = E_H^*$ then the loss of social surplus under the L cost scenario is

A. 2025000

B. 3408600

C. 2235460

D. 1968000

19. If the regulator had to choose either $E = E_L^*$ or $E = E_H^*$ then it would be indifferent between the two policies.

A. True.

B. False.

20. In general, the regulator is always indifferent between $E = E_L^*$ and $E = E_H^*$.

A. True.

B. False.

21. The probability-weighted average of the full-information optimal quantities, denoted $\mathbf{E}[E^*]$, is

A. 1600

B. 2640

C. 1530

D. 1120

22. Expected marginal aggregate abatement cost, denoted $\mathbf{E}[MAAC(E)]$, is

A. $2700 - \frac{2E}{5}$

B. $62000 - 10E$

C. $2700 - \frac{3E}{5}$

D. $37800 - 14E$

23. The optimal quantity-based policy, denoted $\tilde{E}$, is

A. 1575

B. 1530

C. 1780

D. 1645

24. If the regulator sets $E = \tilde{E}$ then the expected loss of social surplus is

A. 236600

B. 897600

C. 607500

D. 243000

25. Based on your answers to Q24 here and Q51 from the Topic 9 Review, the quantity-based policy performs better than the tax in this setting.

A. True.

B. False.

26. In general, the tax policy performs best when

A. the probability of the H cost scenario is high because in that case the optimal tax is high.

B. the probability of the H cost scenario is low because in that case the optimal tax is low.

C. the probability of the H cost scenario is high because in that case the optimal tax is low.

D. the probability of the H cost scenario is low because in that case the optimal tax is high.

The remaining questions do not review anything not already reviewed in the questions above. They simply provide another example to allow you to test your understanding more one time. You may wish to skip these if you are already feeling confident about your knowledge of the material.

Questions 27 to 35 relate to the following information. There are multiple sources of a uniformly mixed pollutant with marginal aggregate abatement cost given by

$$MAAC(e) = \phi(12600 - E)$$

where ϕ could be one of two values: $\phi_L = 5$ or $\phi_H = 10$. The regulator cannot distinguish between these two possibilities, henceforth denoted the "L cost scenario" and the "H cost scenario" respectively. Its beliefs are that $\phi = \phi_L$ with probability 1/5. The marginal damage schedule is

$$MD(e) = 5E$$

This policy setting in the same as the one we examined in Questions 53 – 69 from Topic 9 Review. Question 34 below will relate back to that material.

27. The full-information optimal quantities, denoted E_L^* and E_H^* for the L cost scenario and H cost scenario respectively, are

A. $E_L^* = 5650$ and $E_H^* = 6780$

B. $E_L^* = 6300$ and $E_H^* = 8400$

C. $E_L^* = 4350$ and $E_H^* = 7800$

D. $E_L^* = 2980$ and $E_H^* = 9260$

28. If the regulator sets $E = E_L^*$ then the loss of social surplus under the H cost scenario is

A. 33075600

B. 42154000

C. 33075000

D. 23037500

29. If the regulator sets $E = E_H^*$ then the loss of social surplus under the L cost scenario is

A. 42025000

B. 23408600

C. 22050000

D. 11968000

30. The probability-weighted average of the full-information optimal quantities, denoted $\mathbf{E}[E^*]$, is

A. 8600

B. 6640

C. 7530

D. 7980

31. Expected marginal aggregate abatement cost, denoted $\mathbf{E}[MAAC(E)]$, is

A. $112700 - \frac{E}{5}$

B. $113400 - 9E$

C. $112700 - \frac{4E}{5}$

D. $137800 - 9E$

32. The optimal quantity-based policy, denoted $\tilde{E}$, is

A. 8100

B. 7530

C. 8780

D. 7645

33. If the regulator sets $E = \tilde{E}$ then the expected loss of social surplus is

A. 2236600

B. 4897600

C. 3607500

D. 3780000

34. Based on your answers to Q33 here and Q68 from the Topic 9 Review, the quantity-based policy performs better than the tax in this setting.

A. True.

B. False.

35. In general, the tax policy performs best when

A. the probability of the H cost scenario is high because in that case the optimal tax is high

B. the probability of the H cost scenario is high because in that case the optimal tax is low

C. the probability of the H cost scenario is low because in that case the optimal tax is high

D. None of the above*

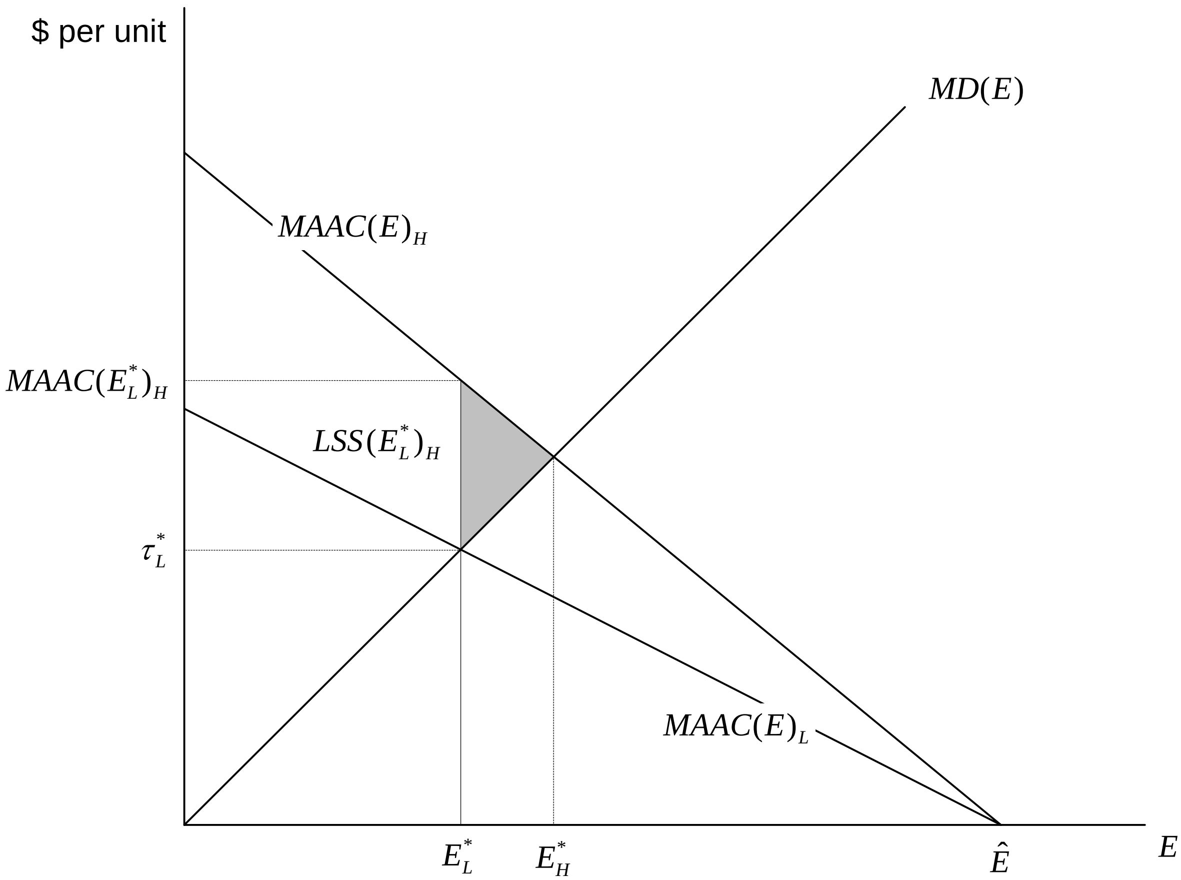


Figure R10-1

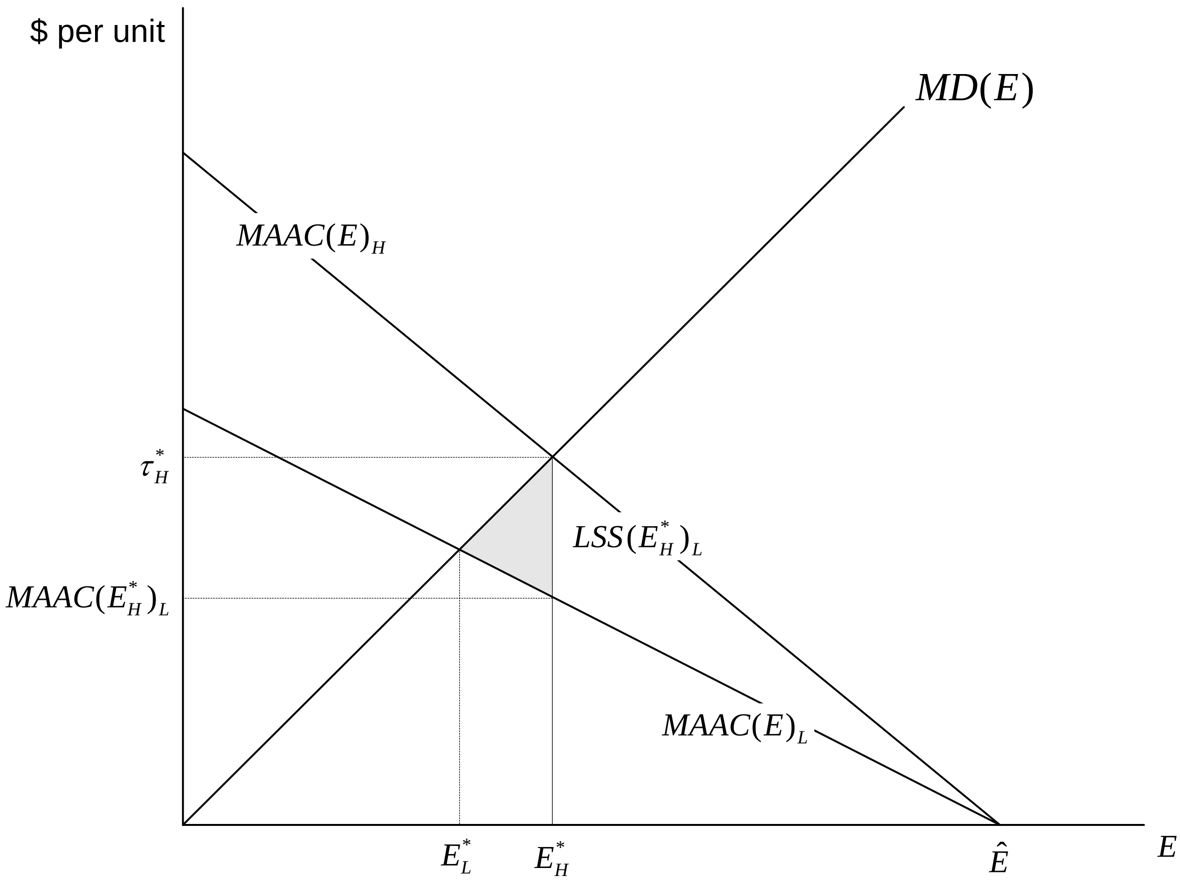


Figure R10-2

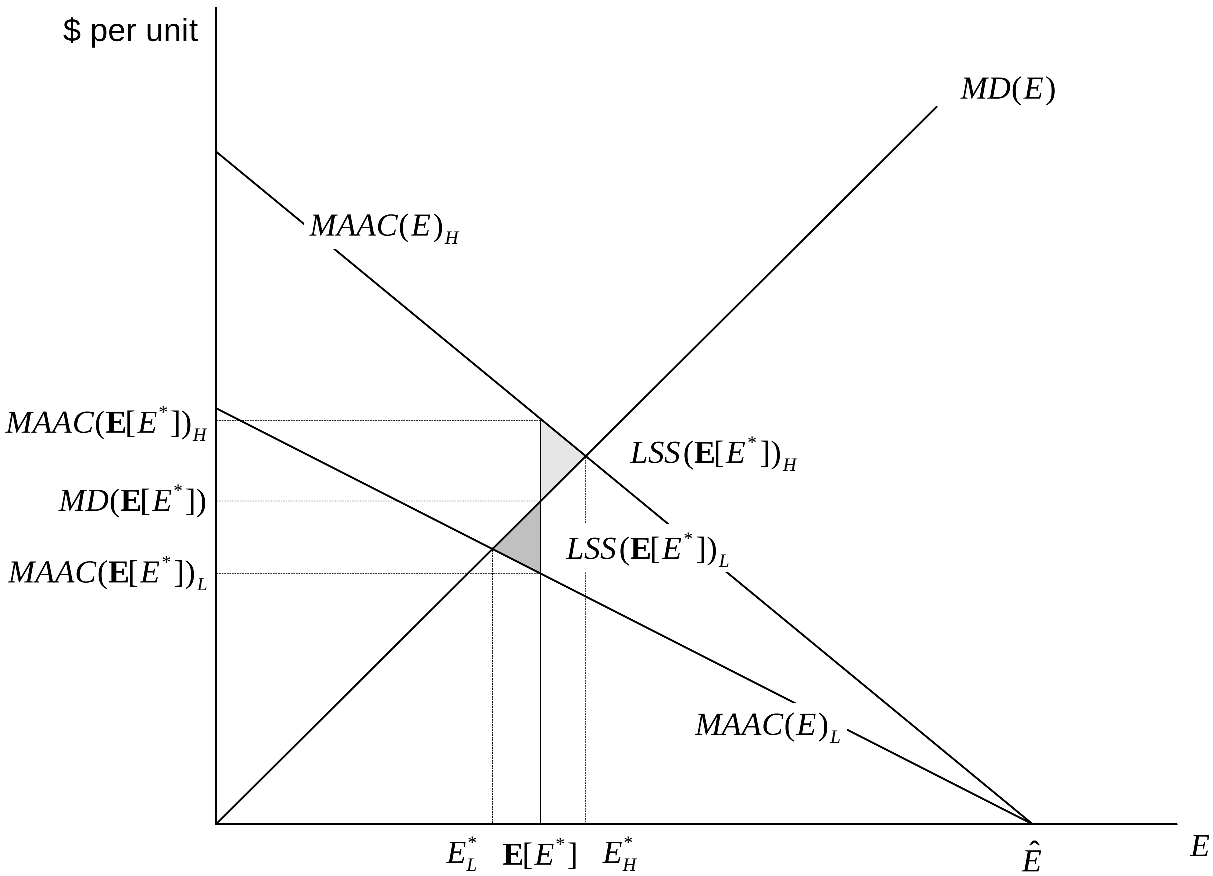

$ per unit
MD(E)
MAAC(E)_H
MAAC(E[E*])_H
MD(E[E*])
MAAC(E[E*])_L
LSS(E[E*])_H
LSS(E[E*])_L
MAAC(E)_L
E*_L
E[E*]
E*_H
Ê
E

Figure R10-3

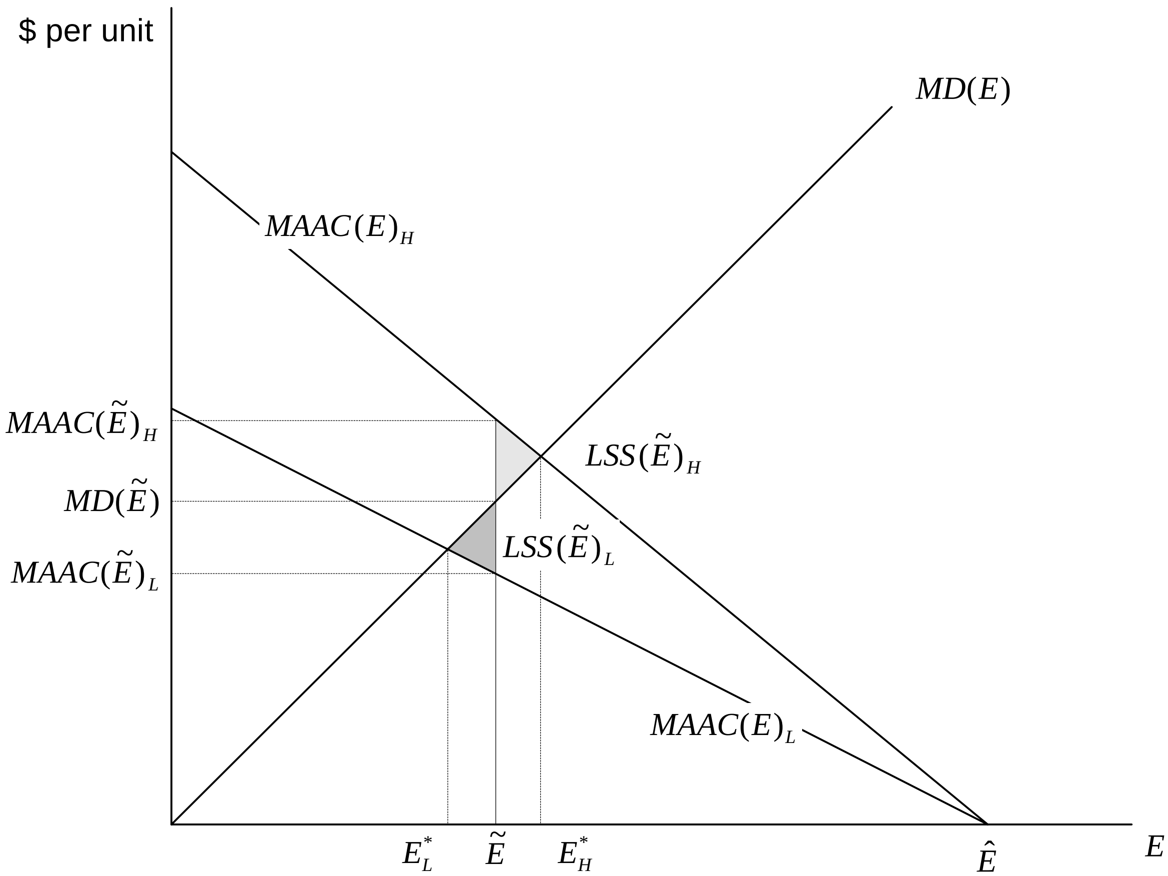

$ per unit
MD(E)
MAAC(E)_H
MAAC(Ẽ)_H
LSS(Ẽ)_H
MD(Ẽ)
LSS(Ẽ)_L
MAAC(Ẽ)_L
MAAC(E)_L
E*_L
Ẽ
E*_H
Ê
E

Figure R10-4

ANSWER KEY

1. B
2. D
3. A
4. B
5. C
6. A
7. B
8. B
9. A
10. D
11. A
12. C
13. A
14. C
15. C
16. D
17. D
18. A
19. A
20. B
21. C
22. D
23. A
24. C
25. B
26. B
27. B
28. C
29. C
30. D
31. B
32. A
33. D
34. B
35. D

11. TRANSBOUNDARY POLLUTION AND CLIMATE CHANGE

OUTLINE

2

3

11.1 INTRODUCTION

Introduction

- We have so far dealt only with a policy setting where the regulator has jurisdiction over all sources of the regulated pollutant.

- Policy becomes much more complicated when pollution flows across jurisdictional boundaries.

5

Introduction

- In that more complicated setting – where pollution is **transboundary** – the regulator cannot control the pollution flowing in from another jurisdiction but it must take account of that in-flowing pollution when setting its own policies.

6

Introduction

- Conversely, the regulator has no incentive to take account of the pollution that flows out of its jurisdiction, even when that out-flowing pollution causes damage to another jurisdiction.

7

Introduction

- One of the most important categories of transboundary pollutant is greenhouse gases (GHGs), which are responsible for global **climate change**.

- We will cast most of our discussion of transboundary pollution in that context.

8

11.2 CLIMATE CHANGE

Climate Change

- There are six main greenhouse gases (GHGs) but the most important in terms of overall impact – and the one most closely related to energy use – is carbon-dioxide (CO_2).

10

Climate Change

- CO_2 concentrations in the atmosphere have risen from around 280 parts per million (ppm) during the mid 1700s to over 400 ppm today.
- Ice core samples indicate that concentrations have not been that high for over 500,000 years.

11

Climate Change

- At the highest projected emission rates – along a “business-as-usual” path – atmospheric concentrations will reach 800ppm by 2100.
- Most climate models predict that this will cause a significant long-run warming of the planet.

12

The Impacts of Climate Change

- The nature and extent of climate change is not known with certainty but most climate models predict a number of costly impacts, including:
  - an overall warming but with possible exceptions in which some regions become cooler

13

The Impacts of Climate Change

- changes in local weather patterns and a likely increase in the variance within those patterns (leading to less predictable weather)
- an increase in storm intensities
- changes in the distribution of wild plants and animals, including pests, and changes in the growing conditions for agricultural crops

14

The Impacts of Climate Change

- a rise in sea levels due to the thermal expansion of water, and to the melting of the ice sheets that cover Greenland and Antarctic.

15

The Impacts of Climate Change

- The uncertainty over these impacts reflects an incomplete understanding of the complex dynamic processes that govern
  - the terrestrial and oceanic **carbon cycles** and how they interact with the climate.
  - the **thermal capacity** of the oceans.
- These are extremely complex systems.

16

The Economics of Climate Change

- In some respects, the economics of climate change is no different from the economics of any other pollution problem.
- In particular, we can characterize the climate-related damage in terms of a marginal damage function, and the cost of cutting emissions in terms of a marginal abatement cost function.

17

The Economics of Climate Change

- However, there are three key properties of the climate-change problem that make its economics more complicated:
  - emissions are transboundary
  - damage is due to the stock of accumulated emissions (not the flow itself)
  - damage is lagged
- Consider each of these in turn.

18

Emissions are Transboundary

- GHGs are a transboundary pollutant because they are uniformly-mixed on a global level.
- Recall from Topic 5.1 that a uniformly-mixed pollutant is one that spreads out evenly within the receptive region, leading to measured pollutant concentrations that are approximately equal across the region.

19

Emissions are Transboundary

- This means that the climate-change damage incurred by any one country is a function of aggregate global emissions.
- No single country has complete control over its own climate because it cannot control the emissions from other countries.
- Small emitters have essentially no unilateral control at all.

20

Damage is Due to the Stock

- Climate change is associated with the stock of accumulated GHGs in the atmosphere.
- Even if the flow of emissions was reduced to zero tomorrow, the stock of atmospheric GHGs would persist for decades, returning to pre-industrial levels only after several centuries.

21

Damage is Lagged

- The link between climate change and atmospheric GHGs is a lagged one, due to inertia in the natural system.
- Current warming is due primarily to the stock as it was about 30 years ago, and the effects of the existing stock will not be felt for another 30 years.

22

Damage is Lagged

- The primary source of this inertial lag is the **thermal lag** associated with oceanic warming, and this is one of the most complex aspects of climate-modeling.

23

Damage is Lagged

- This lagged property of the climate change process has four important implications:
  - cutting emissions today will not yield benefits until many years into the future
  - averting significant climate change in the future will require the removal of vast amounts existing GHGs from the atmosphere

24

Damage is Lagged

- some climate change will inevitably occur over the coming decades due to the elevated stock of carbon dioxide during past decades
- an optimal response to the climate-change problem should consider adaptation and defensive action to be equally important as abatement.

25

Our Focus in this Topic

- We cannot build all of these features of the climate-change problem into a simple model that we can analyze within the scope of this course.
- In this topic we will focus exclusively on the transboundary nature of the problem.

26

Our Focus in this Topic

- We will later study a stock pollutant with lagged damage in Topic 12, but without the complications of a transboundary problem.

27

11.3 ABATEMENT AS A PUBLIC GOOD

Abatement as a Public Good

- To understand the economics of a globally transboundary pollutant, it is useful to think of abatement by any one country as a contribution to a **global public good**.

29

Abatement as a Public Good

- **Public goods** are characterized by two features:
  - – joint consumption possibilities
  - – high exclusion costs

30

Abatement as a Public Good

- **Joint consumption possibilities:** the benefits of the good can be enjoyed by more than one agent simultaneously.
- **High exclusion costs:** it is very costly to prevent agents from consuming the good once it is provided.

31

Abatement as a Public Good

- These two features of a public good mean that its non-cooperative private provision tends to be inefficient.
- Why?

32

Abatement as a Public Good

- Each contributor to the public good bestows a benefit on the other users but that external benefit that cannot be priced because the good is non-excludable.
- This means that each agent has an incentive to **free-ride** on the contributions made by other agents, and in equilibrium, no agent provides enough.

33

Abatement as a Public Good

- GHG abatement is a **global public good** because
  - all countries benefit, in terms of reduced future climate change, when any one country cuts its emissions
  - non-abating countries cannot be excluded from those benefits

34

Abatement as a Public Good

- Public goods are a special kind of **reciprocal positive externality**:
  - the provision of the good by one agent bestows a positive benefit on other agents

35

Abatement as a Public Good

- The reciprocal nature of the externality means that the framework we used for analyzing unilateral externalities (in Topic 2) cannot fully capture the complexity of the climate change problem.
- We need a richer framework for analysis, provided by **game theory.**

36

11.4 INTRODUCTION TO GAME THEORY

Introduction to Game Theory

- Game theory is the appropriate analytical framework in a setting with **strategic interaction** (where the actions of one agent affect the payoff of another agent).

38

Example: The "Prisoners' Dilemma" Game

- In this game there are two countries (player 1 and player 2), and each country can either emit (**E**) on a business-as-usual basis, or abate (**A**).
- Payoffs under the possible combinations are reported in Table 11-1, where the first number is the payoff to the row player.

39

The "Prisoners' Dilemma" Game

COUNTRY 1 \ COUNTRY 2	A	E
A	* 4 , 4	2 , 5
E	5 , 2	NE 3 , 3

TABLE 11-1

40

The "Prisoners' Dilemma" Game

- The **Nash Equilibrium** (NE) of this game is an outcome in which each country chooses its strategy to maximize its own payoff, given the equilibrium strategy of the other country.

- By definition, no country has an incentive to deviate from the NE.

41

The "Prisoners' Dilemma" Game

- The NE of the PD game is {**E**,**E**} despite the fact that both countries would be better off if they could commit to choosing {**A**,**A**}.[+]
- This inefficiency of the equilibrium is due to the public-good nature of abatement.

+ See Appendix A11 for a step-by-step solution method.

42

General Representation of a Game

- Let s_i denote the strategy of player *i*, and let s_{-i} denote the vector (or list) of strategies of all other players.
- In the PD game, $s_i = A$ or E
- Let $u_i(s_i,s_{-i})$ denote the payoff to player *i*.

43

General Representation of a Game

- Payoffs in the PD game:

$u_1(A,A) = 4$ $\quad u_2(A,A) = 4$

$u_1(A,E) = 2$ $\quad u_2(A,E) = 2$

$u_1(E,A) = 5$ $\quad u_2(E,A) = 5$

$u_1(E,E) = 3$ $\quad u_2(E,E) = 3$

44

General Representation of a Game

- A **Nash equilibrium** is a vector of strategies $\{\hat{s}_i, \hat{s}_{-i}\}$ such that

$$u_i(\hat{s}_i, \hat{s}_{-i}) \geq u_i(s_i, \hat{s}_{-i}) \quad \forall s_i, \ \forall i$$

45

General Representation of a Game

- This definition just a restatement of the informal definition from s.39, rephrased below.
  - a NE is an outcome of the game in which each player chooses her strategy to maximize her own payoff, given the equilibrium strategies of all other players.

46

Special Properties of the PD Game

- The PD game is a very simple game, in four respects.
- First, there are only **two players**.
- Second, strategies in the PD game are **binary**:
  – a strategy involves taking one of two available actions (*A* or *E*)

47

Special Properties of the PD Game

- Third, payoffs are **symmetric** (the same for each player)
- Recall the payoffs in the PD game:

$u_1(A,A) = 4$	$u_2(A,A) = 4$
$u_1(A,E) = 2$	$u_2(A,E) = 2$
$u_1(E,A) = 5$	$u_2(E,A) = 5$
$u_1(E,E) = 3$	$u_2(E,E) = 3$

48

Special Properties of the PD Game

- Fourth, each player has a **dominant strategy:**
  - the best strategy for each player is independent of what the other player does: $s_i = E$ is a dominant strategy for player i
- Recall Table 11-1.

49

Special Properties of the PD Game

		COUNTRY 2	
		A	E
COUNTRY 1	A	4 , 4 *	2 , 5
	E	5 , 2	3 , 3 NE

TABLE 11-1 (repeat)

50

Properties of the Climate-Change Game

- The climate-change game is much more complicated than the PD game because
  - there are more than two players
  - strategies are not binary
  - payoffs are not symmetric
  - the strategy space is multi-dimensional
  - there are no dominant strategies
- Consider each of these in turn.

51

Properties of the Climate-Change Game

- The climate-change game is more complicated than the PD game because
  - **there are more than two players**
  - strategies are not binary
  - payoffs are not symmetric
  - the strategy space is multi-dimensional
  - there are no dominant strategies

52

More than Two Players

- Each sovereign nation has jurisdiction over its own climate policy.
- There are over 190 members of the United Nations.
- EU members coordinate their climate policies but they are nonetheless sovereign nations.

53

Properties of the Climate-Change Game

- The climate-change game is more complicated than the PD game because
  - there are more than two players
  - **strategies are not binary**
  - payoffs are not symmetric
  - the strategy space is multi-dimensional
  - there are no dominant strategies

54

Strategies are Not Binary

- The choice over emissions is not binary; it is a choice along a continuum (with zero as its lower bound)

55

Properties of the Climate-Change Game

- The climate-change game is more complicated than the PD game because
  - there are more than two players
  - strategies are not binary
  - **payoffs are not symmetric**
  - the strategy space is multi-dimensional
  - there are no dominant strategies

56

Payoffs are Not Symmetric

- Countries differ with respect to
  - current emissions levels
  - abatement costs
  - current and future damage from climate change
- These differences mean that payoffs in the game differ across countries.

57

Properties of the Climate-Change Game

- The climate-change game is more complicated than the PD game because
  - there are more than two players
  - strategies are not binary
  - payoffs are not symmetric
  - **the strategy space is multi-dimensional**
  - there are no dominant strategies

58

The Strategy Space is Multi-Dimensional

- In the context of the climate change game, a strategy for country *i* has three potential components:
  - – an **emissions** choice;
  - – an **adaptation** investment choice; and
  - – a **geoengineering** choice.

59

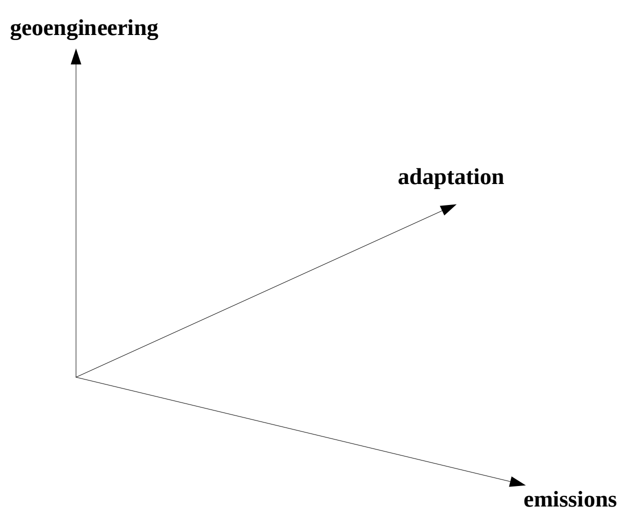


FIGURE 11-1

60

The Strategy Space is Multi-Dimensional

- This multi-dimensionality makes the climate game very complex because
  - emissions are a public bad (abatement is a public good)
  - adaptation is a private good
  - geoengineering is a public good (if it works)
- Here we will abstract from adaptation and geoengineering, and focus on emissions.

61

Properties of the Climate-Change Game

- The climate-change game is more complicated than the PD game because
  - there are more than two players
  - strategies are not binary
  - payoffs are not symmetric
  - the strategy space is multi-dimensional
  - **there are no dominant strategies**

62

No Dominant Strategies

- In the PD game, $s_i = E$ is the best choice for player *i* regardless of what the other does; it is a dominant strategy.
- In the climate change game, the optimal emissions choice for each country depends on what every other country emits; there are no dominant strategies.

63

No Dominant Strategies

- Why?
  – we will return to this question in a moment (and see that emissions are **strategic substitutes**).
- Before we do so, we need to revisit the interpretation of the Nash equilibrium, in terms of **beliefs**.

64

Beliefs and the Interpretation of the NE

- In a game with dominant strategies, neither player needs to think about how the other players will act; those other actions have no bearing on the best choice for any given player.

65

Beliefs and the Interpretation of the NE

- If instead, the best action for player *i* depends on the actions of other players, then player *i* must form a **belief** about how those other players will act.

66

Beliefs and the Interpretation of the NE

- This is straightforward in a **sequential-move game** (like chess).

- In such a game, there is a temporal order to moves, and each player observes the actions of the players who moved before her.

67

Beliefs and the Interpretation of the NE

- Things are more complicated in a **simultaneous-move game**, in which all players act at the same time (or equivalently, act without first observing the actions of the other players).

68

Beliefs and the Interpretation of the NE

- In such games, the best strategy for player *i* must be a **best-response** to the strategies that she *expects* the others to play.

- How are those expectations formed?

69

Beliefs and the Interpretation of the NE

- Key assumption underlying the Nash equilibrium concept: **common knowledge of rationality**
  - all players believe that all other players are rational agents, and all players are aware of those beliefs.

70

Beliefs and the Interpretation of the NE

- This means that every player expects every other player to choose their own best strategy.
- In a game with **complete information** – where all agents know the payoffs of all other agents – then the only possible outcome is the Nash equilibrium.

71

Beliefs and the Interpretation of the NE

- Aside: in a game with **incomplete information**, we need a stronger equilibrium concept:
  - the **Bayesian Nash equilibrium** (which embodies a logic for how players construct beliefs).

72

11.5 THE CLIMATE-CHANGE GAME

The Climate-Change Game

- We begin with a brief recap of the domestic policy problem for a local pollutant (a non-strategic setting).

74

The Climate-Change Game

- In the local pollutant problem, the socially optimal level of emissions (the level that maximizes social surplus) is e^* such that

$$MAC(e^*) = MD(e^*)$$

75

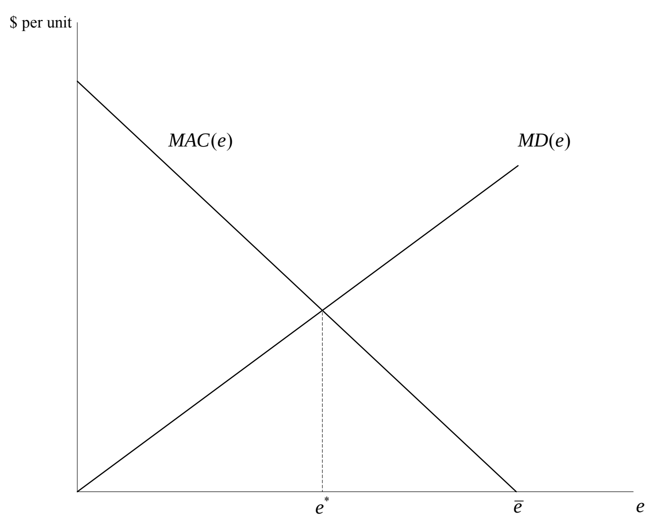


FIGURE 11-2

76

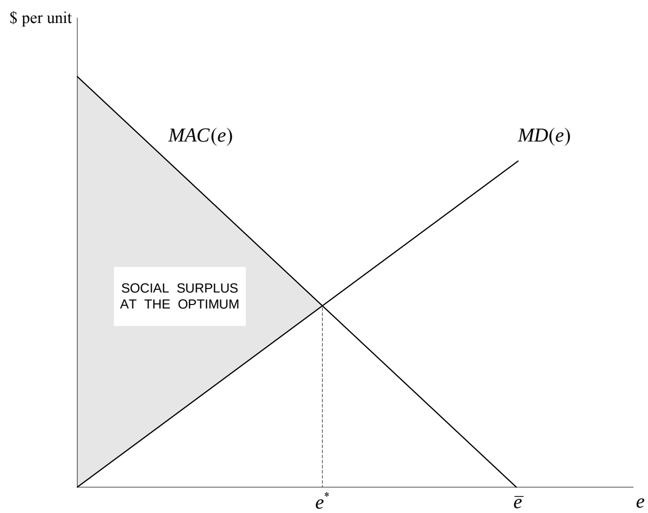


FIGURE 11-3

77

The Climate-Change Game

- Now let us reinterpret the policy problem as one where this country (country 1) chooses domestic GHGs in the context of the climate change game with two countries.
- To begin, suppose emissions from the other country are zero, and draw the MD schedule for country 1 accordingly; see Figure 11-4.

78

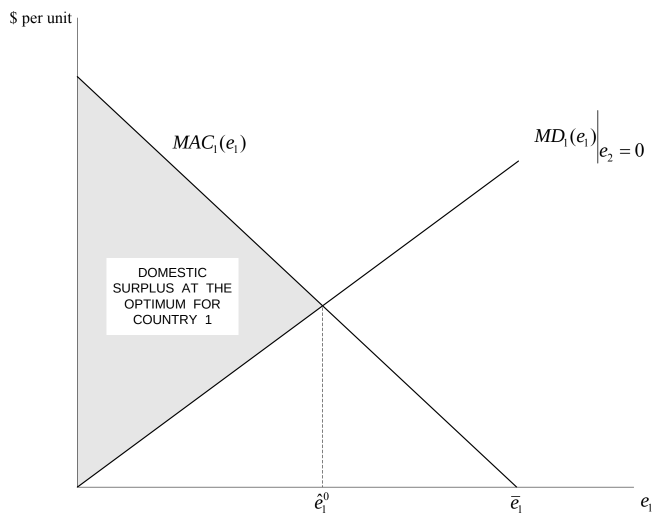


FIGURE 11-4

79

The Climate-Change Game

- Now suppose that country 2 is emitting a positive level of emissions.
- Emissions from country 1 now <u>add</u> to the emissions from country 2, and recall that the damage <u>to country 1</u> is a function of **aggregate emissions**:

$$D_1(E) \qquad \text{where} \qquad E = e_1 + e_2$$

80

Strictly Convex Damage

- The science and the economics suggest that the damage function is **strictly convex** in emissions (more on why in a moment)
- This means that the damage caused by emissions from country 1 depend on the level of emissions from country 2.
- See Figures 11-5 and 11-6.

81

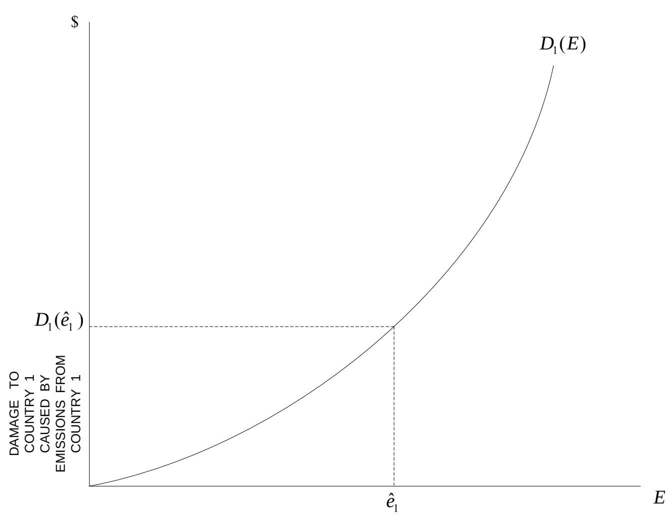


FIGURE 11-5

82

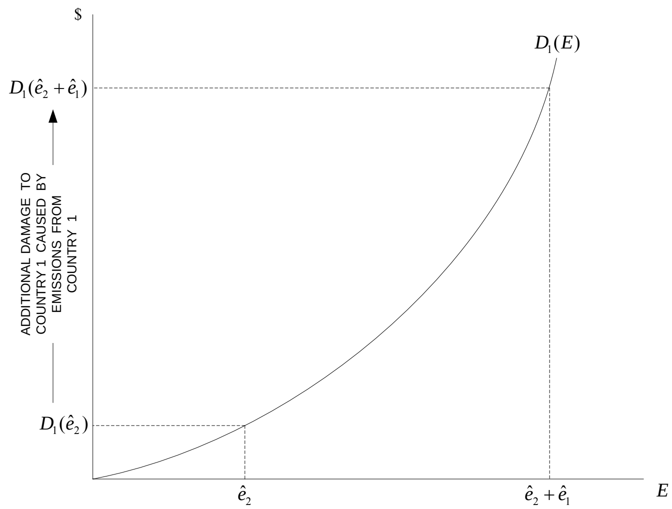


FIGURE 11-6

83

Strictly Convex Damage

- The strict convexity of the damage function means that

$$D_1(\hat{e}_2 + \hat{e}_1) - D_1(\hat{e}_2) > D_1(\hat{e}_1)$$

- We will see in a moment that this means that emissions are **strategic substitutes**.

84

Strictly Convex Damage

- Suppose instead we believe that damage is <u>linear</u> in aggregate emissions (contrary to what the science and economics tell us).
- In that case:

$$D_1(\hat{e}_2 + \hat{e}_1) - D_1(\hat{e}_2) = D_1(\hat{e}_1)$$

- See Figures 11-7 and 11-8.

85

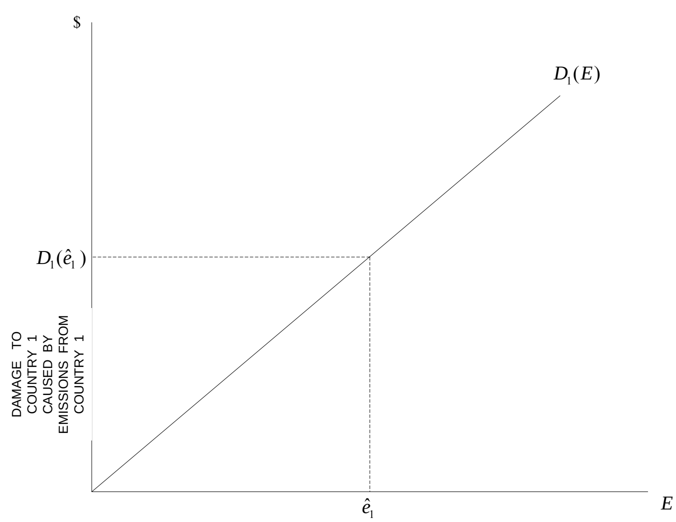


FIGURE 11-7

86

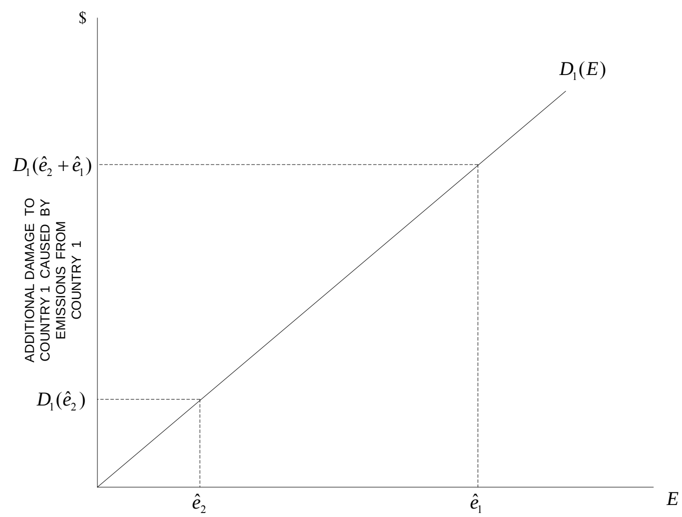


FIGURE 11-8

87

Strictly Convex Damage

- We will see in a moment that a linear damage function would mean that there are dominant strategies in the climate change game (just as there are in the PD game).

88

Strictly Convex Damage

- Why do we believe that the damage function is strictly convex?
  - from the science: self-reinforcing feedback loops
  - from the economics: risk aversion coupled with the unpredictability of an altered climate

89

Strictly Convex Damage

- A **self-reinforcing (or positive) feedback loop** is a repeating cycle of events such that

$\Delta X>0$ causes ΔY, and ΔY then causes $\Delta X>0$

or

$\Delta X<0$ causes ΔY, and ΔY then causes $\Delta X<0$

90

Strictly Convex Damage

- Global warming is likely to set-off self-reinforcing processes that will exacerbate the warming trend through
  - reduced albedo of land surfaces
  - release of methane and carbon dioxide due to warmer ocean water and thawing of permafrost
  - change in cloud cover

91

Strictly Convex Damage

- How does strict convexity show up in our standard diagrammatic description of the policy problem?

- Recall the policy problem from Figure 11-4.

92

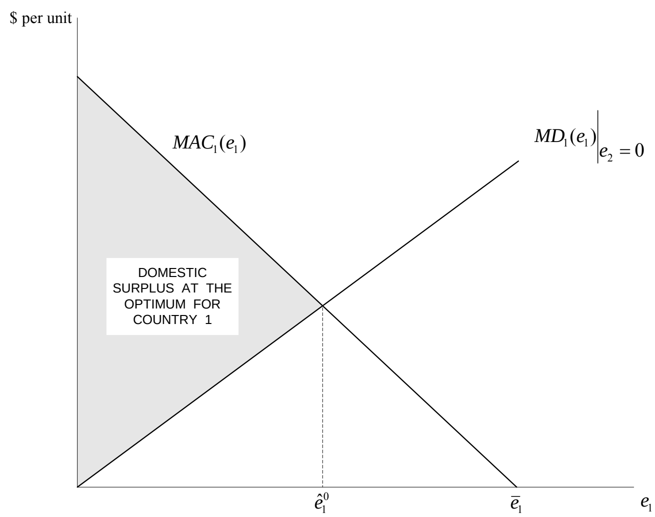


FIGURE 11-4 (repeat)

93

Strictly Convex Damage

- Strict convexity of the damage function means that the **marginal damage schedule**
  - is positively-sloped in own emissions; and
  - is shifted to the left by an increase in emissions from the other country by the amount of those emissions.

94

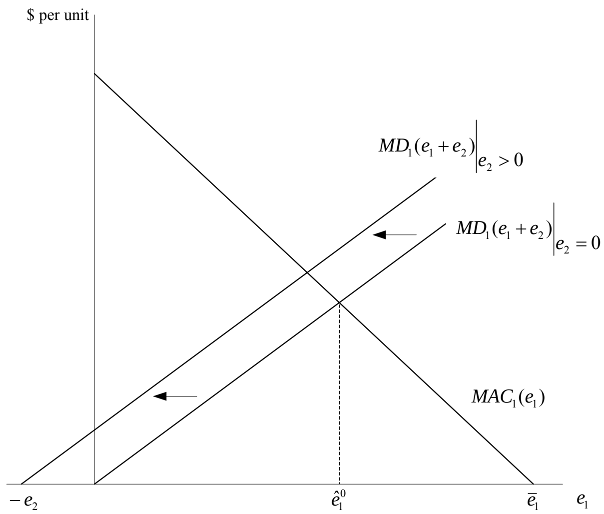


FIGURE 11-9

95

Strictly Convex Damage

- Why? The marginal damage schedule measures the **slope** of the damage function, and strict convexity of the damage function means that the slope is increasing as aggregate emissions rise.
- Recall Figures 11-5 and 11-6.

96

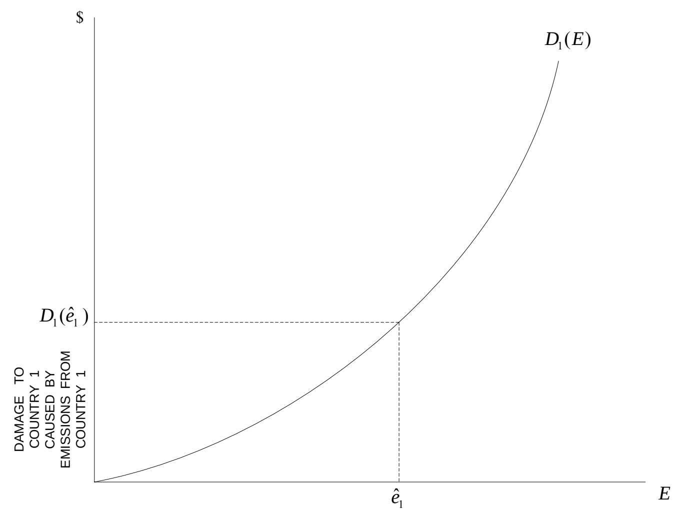


FIGURE 11-5 (repeat) 97

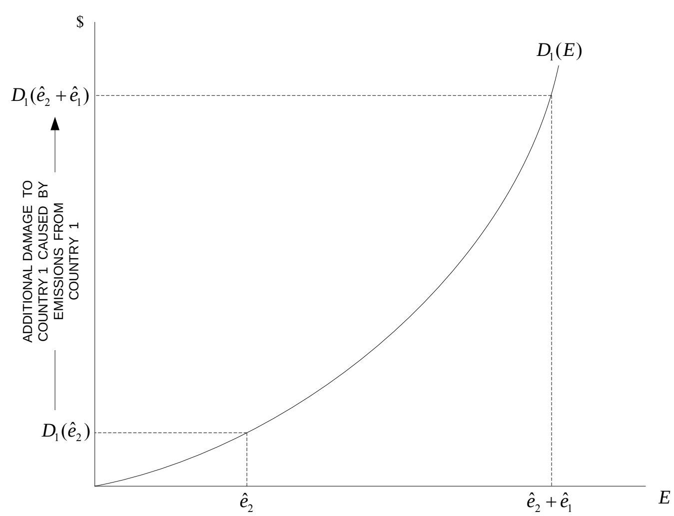


FIGURE 11-6 (repeat) 98

Strictly Convex Damage

- The shift in the marginal damage schedule for country 1 reflects the fact that it faces the effect of the emissions from country 2 even if it emits nothing itself.
- Its own emissions then add to the total level of emissions that cause it damage.

99

Strictly Convex Damage

- Thus, its marginal damage schedule effectively starts at $-e_2$; it would have to have <u>negative</u> emissions itself in order to face zero aggregate emissions overall.

100

Strictly Convex Damage

- This shift in the marginal damage schedule for country 1 when emissions from country 2 rise means that country 1 must re-optimize in response to that change:
  - optimal emissions for country 1 fall

- See Figure 11-10.

101

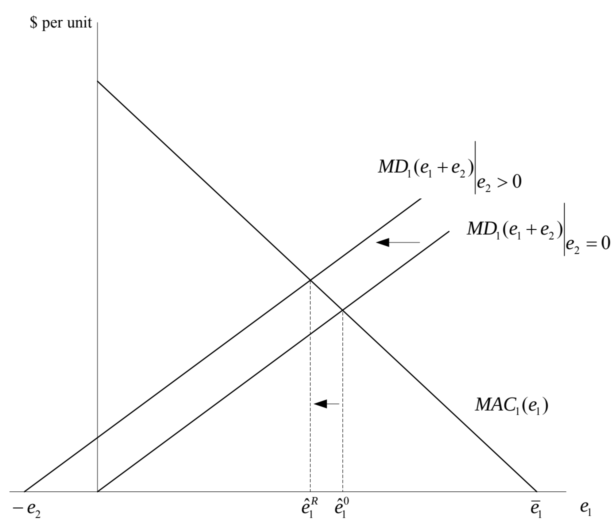


FIGURE 11-10

102

Strategic Substitutes

- This negative response by country 1 to an increase in emissions from country 2 means that emissions are (by definition) **strategic substitutes** in the climate change game.
- This property of the game stems from the strict convexity of the damage function.

103

Strategic Substitutes

- If instead the damage function is linear (meaning that it has constant slope) then the marginal damage function is perfectly flat in own emissions, and independent of emissions from the other country.
- See Figure 11-11.

104

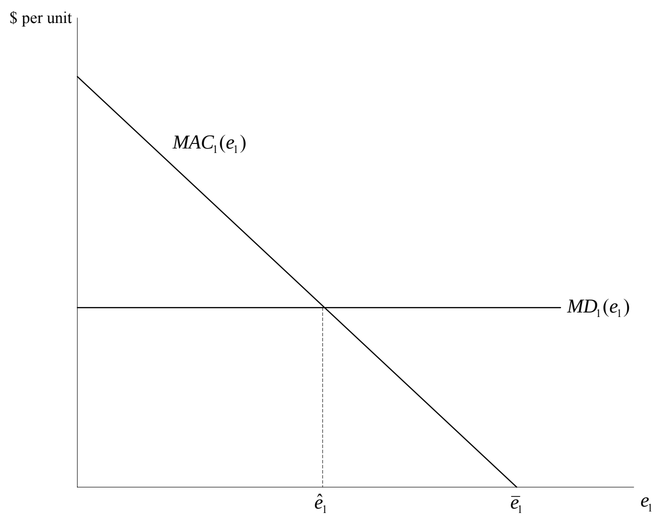


FIGURE 11-11

105

Strategic Substitutes

- In the case of linear damage, the optimal emissions choice for country 1 does not depend on the level of emissions from country 2.
- This means (by definition) that country 1 has a **dominant strategy**.
- This is not a realistic scenario.

106

Best-Response Functions

- We now want to use the logic of Figure 11-10 to derive a **best-response function** (BRF) for country 1 (when damage is strictly convex).

- The BRF tells us what level of emissions country 1 will optimally choose at any given level of emissions from country 2.

107

Best-Response Functions

- By convention, we plot the BRF in (e_1,e_2) space, as in Figure 11-12.

108

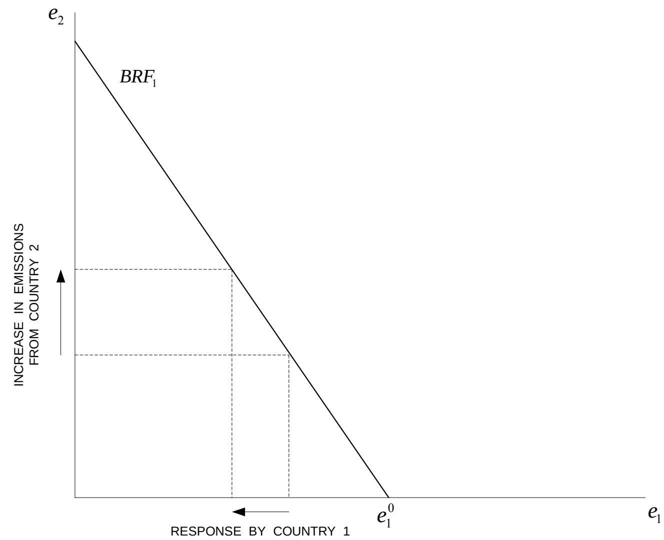


FIGURE 11-12

109

Best-Response Functions

- It is important to stress that the term "best-response" is not meant to suggest that country 1 observes an action by country 2 and then responds in a sequential-move sense.

110

Best-Response Functions

- The correct interpretation of the BRF is as a description of how country 1 will behave in response to how it **anticipates** country 2 will behave.
- The negative slope of the BRF reflects the fact that emissions are strategic substitutes; recall Figure 11-10.

111

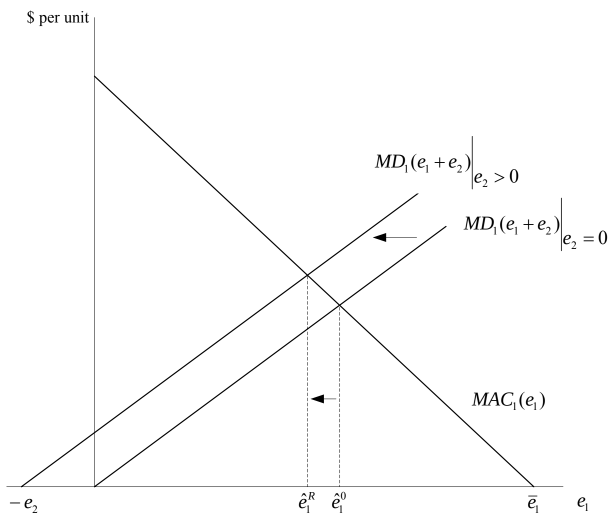


FIGURE 11-10 (repeat)

112

Best-Response Functions

- The horizontal intercept of the BRF identifies the "**sole-agent**" level of emissions, which is the level of emissions that country 1 would choose if there was no country 2 (or if country 2 has zero emissions).

113

Best-Response Functions

- The vertical intercept identifies the level of emissions from country 2 that would just cause country 1 to set its own emissions to zero.

- That limiting outcome is depicted in Figure 11-13.

114

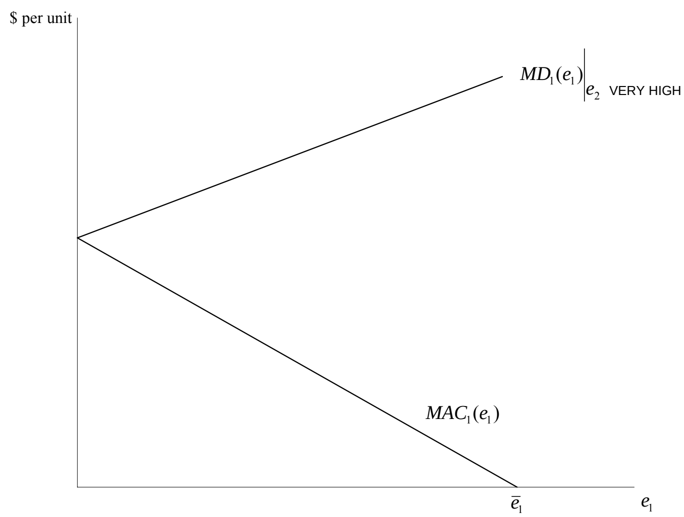


FIGURE 11-13

115

Best-Response Functions

- What happens to the payoff to country 1 as we move along its BRF?
- Figure 11-14 reproduces Figure 11-10, and illustrates the payoff to country 1 (as measured by domestic surplus) when facing positive emissions from country 2.
- That payoff is area (**A** - **B**) in Figure 11-14.

116

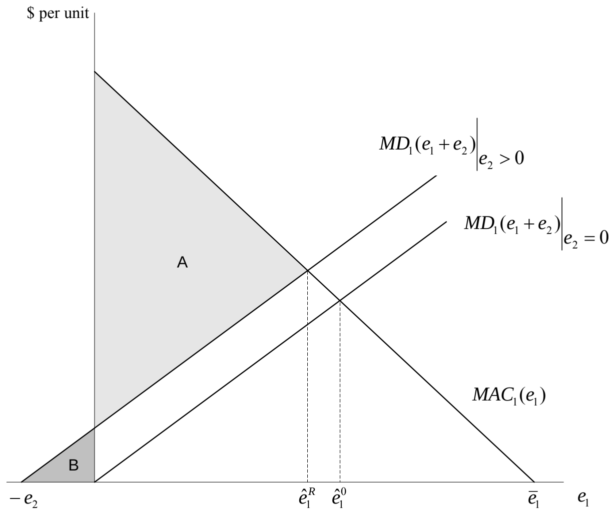


FIGURE 11-14

117

Best-Response Functions

- Compare this payoff with the payoff when $e_2 = 0$, as depicted in Figure 11-4.

118

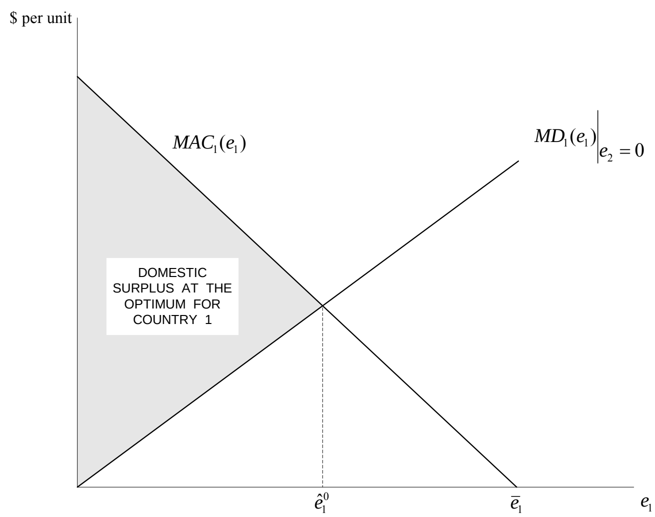


FIGURE 11-4 (repeat)

119

Best-Response Functions

- It is clear from Figures 11-4 and 11-14 that the payoff to country 1 falls as emissions from country 2 rise.
- Thus, the payoff to country 1 declines as we move upwards along its BRF, as depicted in Figure 11-15.

120

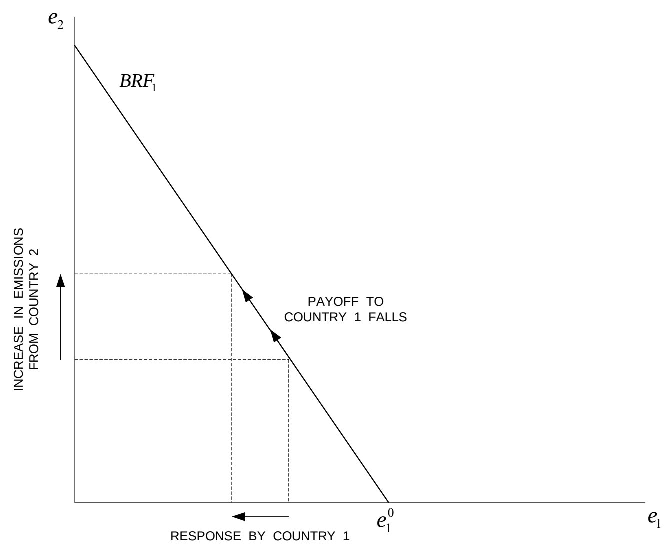


FIGURE 11-15

121

Best-Response Functions

- Why? Higher emissions from country 2 impose a direct cost on country 1 via the damage caused by those emissions.
- Country 1 responds optimally by reducing its own emissions, but it must nonetheless suffer a loss of surplus compared with when $e_2 = 0$.

122

Iso-Surplus Contours

- We can depict the payoff to country 1 under different values of e_2 directly on Figure 11-15 using iso-surplus contours.
- An **iso-surplus contour** for country 1 is a locus of points in (e_1,e_2) space along which its payoff (as measured by domestic social surplus) is constant.

123

Iso-Surplus Contours

- More familiar iso-payoff contour:
  - indifference curves for consumers

- A representative iso-surplus contour for country 1 is depicted in Figure 11-16.

124

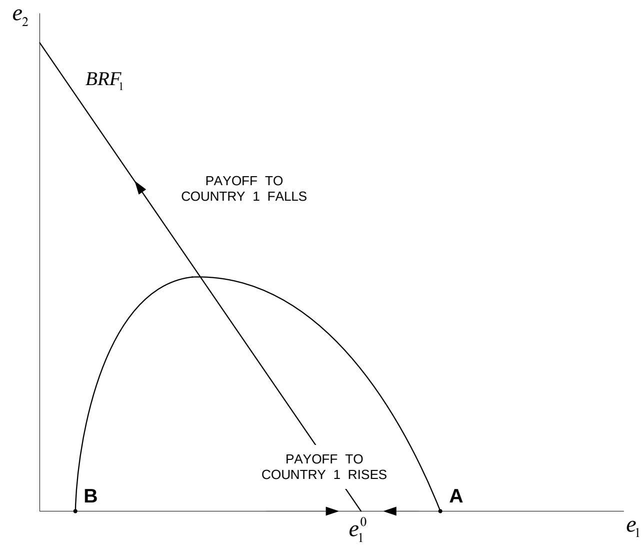


FIGURE 11-16

125

Iso-Surplus Contours

- To understand its shape, suppose we start at point **A** (where e_2=0) and move along the axis towards e_1^0 . Surplus would <u>increase</u> (since we know that e_1^0 is surplus-maximizing when e_2=0).

126

Iso-Surplus Contours

- Thus, for surplus to remain constant – as it must along the contour – e_2 would have to rise to offset the gain from reducing e_1 towards e_1^0.

127

Iso-Surplus Contours

- Conversely, suppose we start at point **B** (where e_2=0) and move along the axis towards e_1^0. Again, surplus would <u>increase</u> (since we know that e_1^0 is surplus-maximizing when e_2=0).

128

Iso-Surplus Contours

- Thus, for surplus to remain constant, e_2 would have to rise to offset the gain from reducing e_1 towards e_1^0.

129

Iso-Surplus Contours

- Figure 11-17 depicts three different iso-surplus contours for country 1, each one corresponding to an increasingly lower level of surplus as we move upwards along the BRF.

130

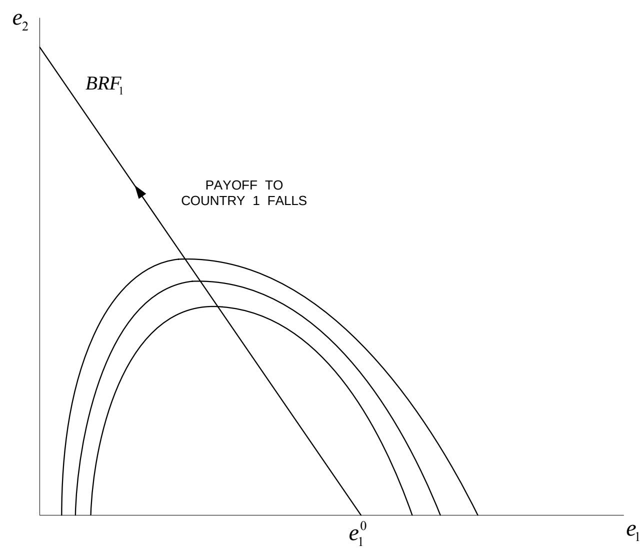


FIGURE 11-17

131

Iso-Surplus Contours

- Note that the iso-surplus contours are flat (zero slope) where they cross the BRF.
- Why?

132

Iso-Surplus Contours

- Recall that the BRF is the surplus-maximizing response to a given level of emissions from country 2.
- Thus, we can think of a point on the BRF as identifying the choice of e_1 that achieves the lowest possible iso-surplus contour for the given value of e_2 at that point on the BRF; see Figure 11-18.

133

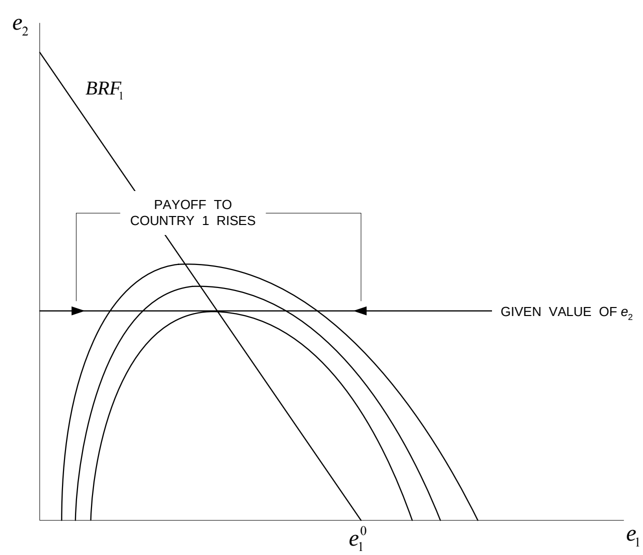


FIGURE 11-18

134

Iso-Surplus Contours

- The logic of Figure 11-18 tells us that an iso-surplus contour must be just tangent to the “given e_2” line at the point where that line crosses the BRF.
- Since the “given e_2” line has zero slope, so too must the iso-surplus contours where they cross the BRF.

135

The Choice Problem for Country 2

- We now want to analyze the choice problem for country 2.
- This country behaves in the same optimizing way as country 1:
  - it chooses a level of e_2 to maximize its own domestic surplus, given the level of emissions chosen by country 1.

136

The Choice Problem for Country 2

- We can depict this choice problem for country 2 in (e_1,e_2) space, just as we did for country 1.

137

The Choice Problem for Country 2

- In the special case where the countries are identical with respect to abatement cost and damage, the graphical depiction for country 2 is an exact mirror image of that for country 1 (reflected in the 45^0 line); see Figure 11-19.

138

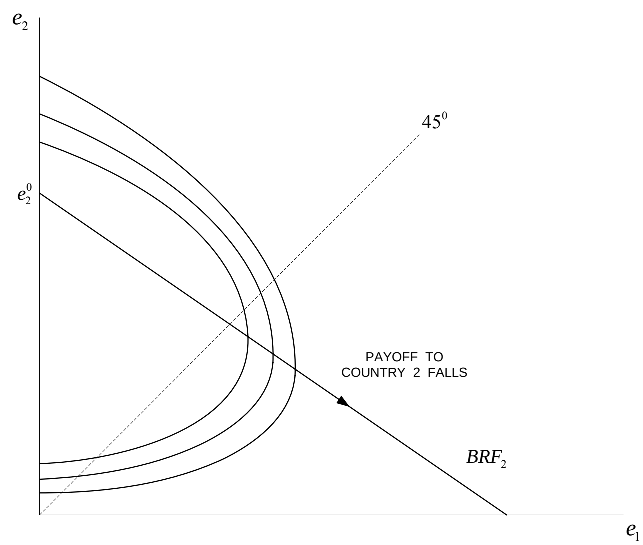


FIGURE 11-19

139

The Choice Problem for Country 2

- Note from Figure 11-19 that the iso-surplus contours are “flat” – when viewed “sideways” – where they cross BRF_2.
- This reflects the same logic that underlay the discussion of Figure 11-18 for country 1; that same logic is depicted in Figure 11-20 for country 2.

140

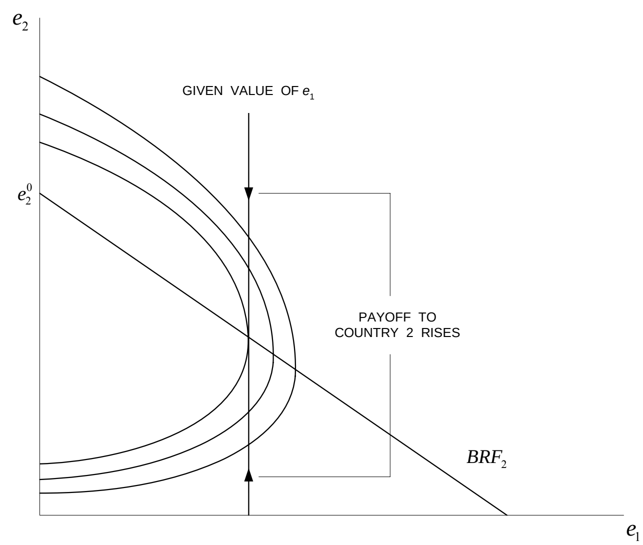


FIGURE 11-20 141

11.6 The Non-Cooperative Equilibrium

The Non-Cooperative Equilibrium

- We now wish to characterize the Nash equilibrium in this climate change game.
- We will henceforth refer to this equilibrium as the **non-cooperative equilibrium** (NCE).

143

The Non-Cooperative Equilibrium

- In the context of this climate change game, the NCE is a pair of emissions levels

$$\{\hat{e}_1, \hat{e}_2\}$$

such that $\hat{e}_1$ is a best response to $\hat{e}_2$, and $\hat{e}_2$ is a best response to $\hat{e}_1$.

144

The Non-Cooperative Equilibrium

- The graphical interpretation of the NCE is the intersection of the BRFs, as depicted in Figure 11-21.

145

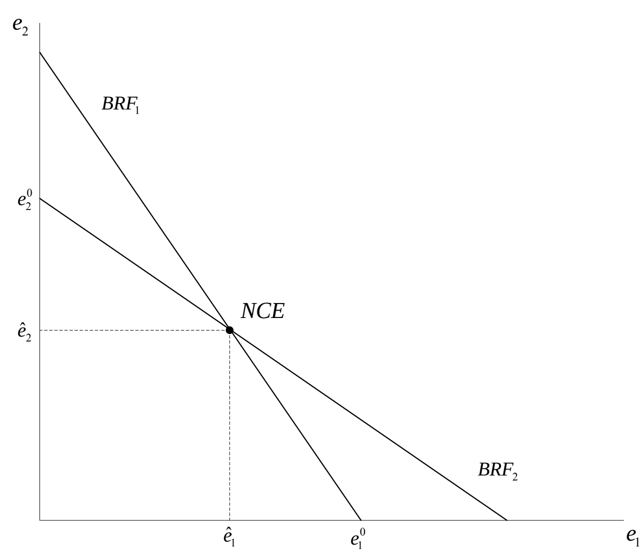


FIGURE 11-21

146

11.7 A NUMERICAL EXAMPLE

A Numerical Example

- Consider an example with two countries.
- Suppose that country 1 has marginal abatement cost and marginal damage given respectively by

$$MAC_1(e_1) = 900 - e_1$$

$$MD_1(E) = E$$

148

A Numerical Example

- Recall that

$$E = e_1 + e_2$$

so the position of $MD_1(E)$ depends on the magnitude of e_2.

149

A Numerical Example

- Figure 11-22 depicts $MAC_1(e_1)$ and $MD_1(E)$, where the latter is drawn for two different values of e_2 (e_2 = 0 and e_2 = 260).

150

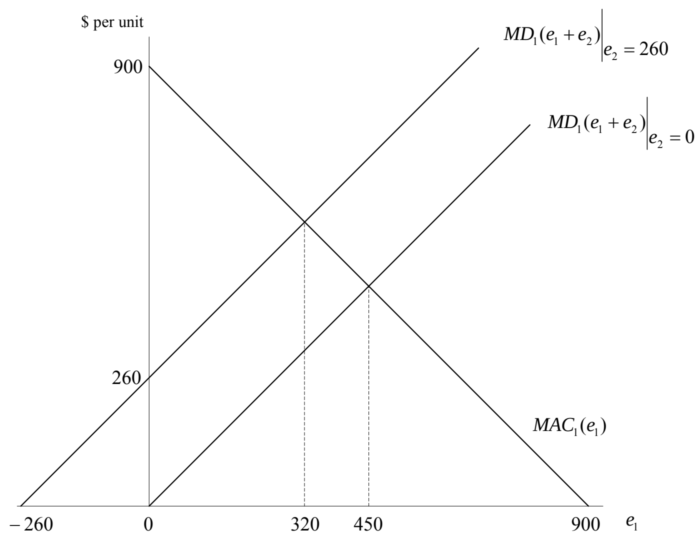


FIGURE 11-22

151

A Numerical Example

Suppose that country 2 has marginal abatement cost and marginal damage given respectively by

$$MAC_2(e_2) = 1680 - 2e_2$$

$$MD_2(E) = 2E$$

152

A Numerical Example

- Figure 11-23 depicts $MAC_2(e_2)$ and $MD_2(E)$, where the latter is drawn for two different values of e_1 (e_1 = 0 and e_2 = 320)

153

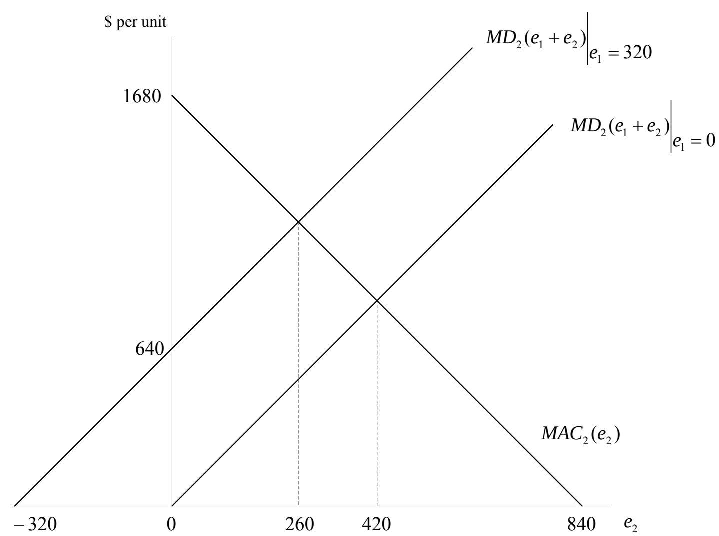


FIGURE 11-23

154

A Numerical Example

- We derive the BRF for country 1 as the solution to

$$MAC_1(e_1) = MD_1(e_1 + e_2)$$

155

A Numerical Example

- Making the substitutions from s.148 and solving for e_1 yields BRF_1:

$$e_1(e_2) = 450 - \frac{e_2}{2}$$

156

A Numerical Example

To plot this BRF in (e_1,e_2) space, we need to make e_2 the subject of the equation by taking its inverse, which yields

$$e_1^{-1}(e_2) = 900 - 2e_1$$

- This equation is plotted in Figure 11-24, where it is labeled BRF_1.

157

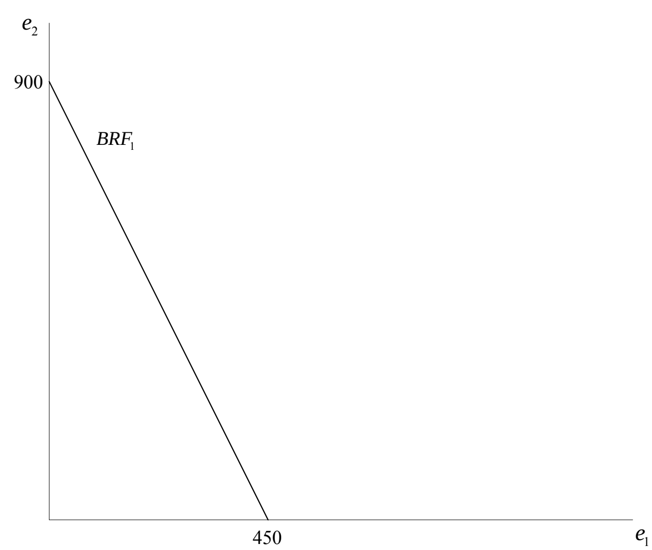


FIGURE 11-24

158

A Numerical Example

- We derive the BRF for country 2 as the solution to

$$MAC_2(e_2) = MD_2(e_1 + e_2)$$

159

A Numerical Example

- Making the substitutions from s.152 and solving for e_2 yields BRF_2:

$$e_2(e_1) = 420 - \frac{e_1}{2}$$

160

A Numerical Example

- This equation is plotted in Figure 11-25, where it is labeled BRF_2.

161

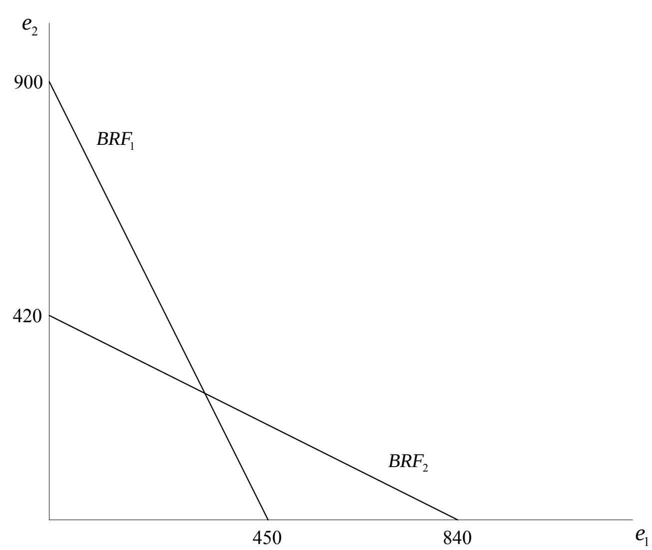


FIGURE 11-25

162

A Numerical Example

- We can now solve for the NCE by finding the intersection of BRF_1 and BRF_2:

$$e_1^{-1}(e_2) = e_2(e_1)$$

163

A Numerical Example

- Making the substitutions from s.157 and s.160 yields

$$900 - 2e_1 = 420 - \frac{e_1}{2}$$

164

A Numerical Example

- Solving for e_1 yields the NCE value of e_1:

$$\hat{e}_1 = 320$$

165

A Numerical Example

- Making this substitution for e_1 in BRF_2 from s.156 then yields the NCE value of e_2 :

$$\hat{e}_2 = 260$$

- This NCE is depicted in Figure 11-26.

166

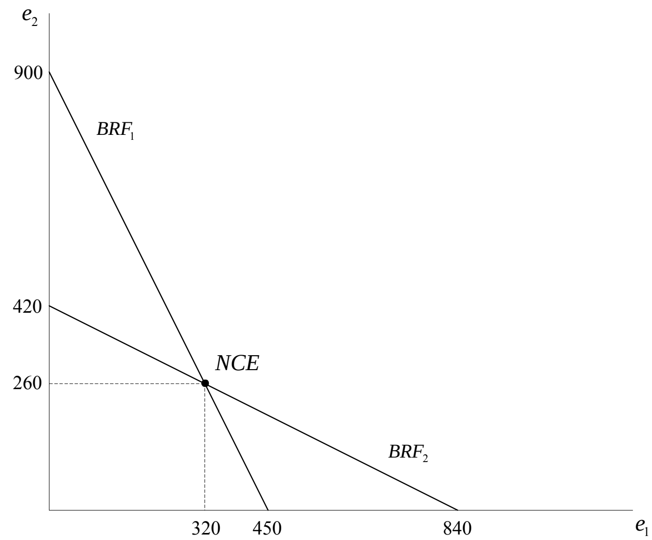


FIGURE 11-26

167

A Numerical Example

- We can now see that Figures 11-22 and 11-23 were deliberately constructed with the marginal damage schedules evaluated at the NCE emission levels.
- They are reproduced as Figures 11-27 and 11-28 respectively, highlighting the payoffs (measured as domestic surplus) to the two countries at the NCE.

168

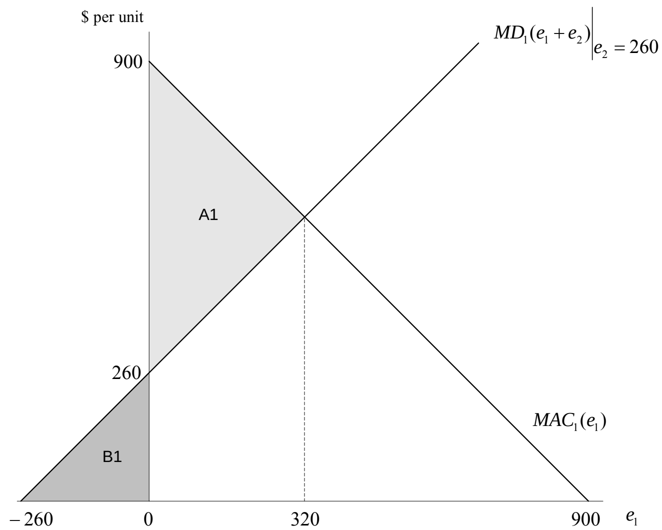


FIGURE 11-27 169

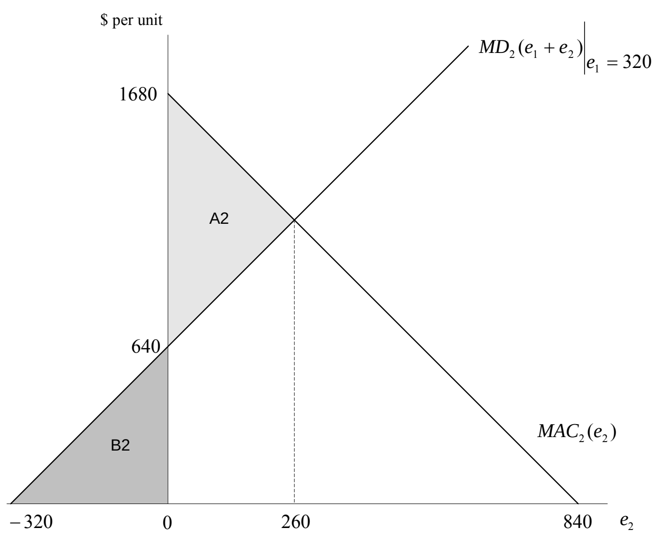


FIGURE 11-28 170

A Numerical Example

- It is straightforward to calculate these domestic surplus measures from the areas of the shaded triangles.

171

A Numerical Example

- For country 1:

$$A_1 = \frac{(900-260)320}{2} = 102,400$$

$$B_1 = -\frac{(260)260}{2} = 33,800$$

172

A Numerical Example

- Thus, domestic surplus at the NCE for country 1 is

$$DS_1 = 102{,}400 - 33{,}800 = 68{,}600$$

173

A Numerical Example

- For country 2:

$$A_2 = \frac{(1680 - 640)260}{2} = 135{,}200$$

$$B_1 = -\frac{(640)320}{2} = 102{,}400$$

174

A Numerical Example

- Thus, domestic surplus at the NCE for country 2 is

$$DS_2 = 135{,}200 - 102{,}400 = 32{,}800$$

175

11.8 WELFARE PROPERTIES OF THE NON-COOPERATIVE EQUILIBRIUM

Welfare Properties of the Non-Cooperative Equilibrium

- We now want to examine the welfare properties of the NCE.
- We begin by examining the surplus values for each country at the NCE, as represented by their iso-surplus contours in (e_1,e_2) space; see Figure 11-29.

177

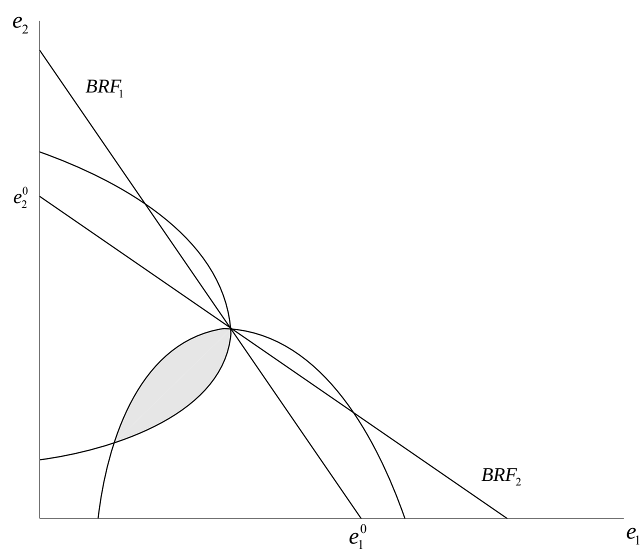


FIGURE 11-29

178

Welfare Properties of the Non-Cooperative Equilibrium

- The shaded region in Figure 11-29 depicts the set of allocations that Pareto-dominate the NCE.
- Both countries would be better off at a point in the interior of this region, relative to the NCE.

179

Welfare Properties of the Non-Cooperative Equilibrium

- This region is sometimes called the **lens of mutual benefit**.
- Note that emissions are lower for both countries in this lens than at the NCE.
- This reflects the free-rider problem associated with the global public good:
  - each country under-contributes to abatement

180

Welfare Properties of the Non-Cooperative Equilibrium

- Are some allocation in the lens better than others?

- To answer this question we first need to characterize the Pareto frontier in this game.

181

Welfare Properties of the Non-Cooperative Equilibrium

- The **Pareto frontier** is the set of Pareto-efficient allocations.
- In the context of the two-country game, it is the locus of points in (e_1,e_2) space at which the iso-surplus contours for the two countries are **tangential** to other, as depicted by the heavy curve labeled *PF* in Figure 11-30.

182

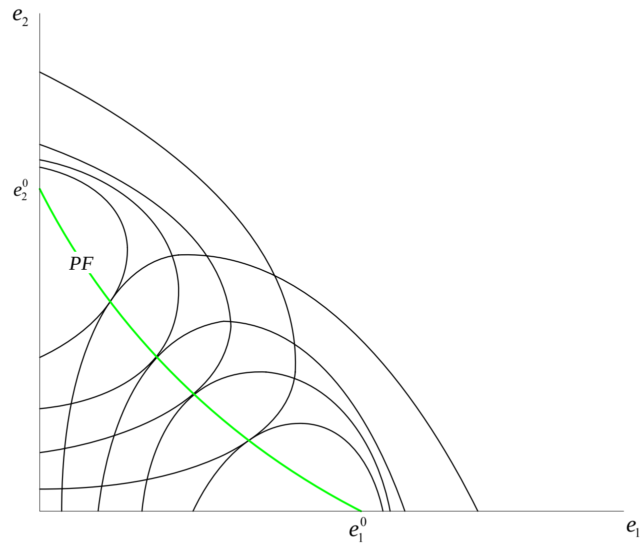


FIGURE 11-30

183

Welfare Properties of the Non-Cooperative Equilibrium

- Why is this locus the Pareto frontier?
- Recall that a Pareto-efficient allocation is one from which it is not possible deviate in a way that makes one agent better off without making the other agent worse off.

184

Welfare Properties of the Non-Cooperative Equilibrium

- In the context of our game, this means that for any given level of surplus for one country (say, country 1), it should not be possible to keep country 1 on the iso-surplus contour corresponding to that level of surplus and move along that contour in a way that makes country 2 better.

185

Welfare Properties of the Non-Cooperative Equilibrium

- To visualize this, suppose we are currently at point **A** in Figure 11-31.

186

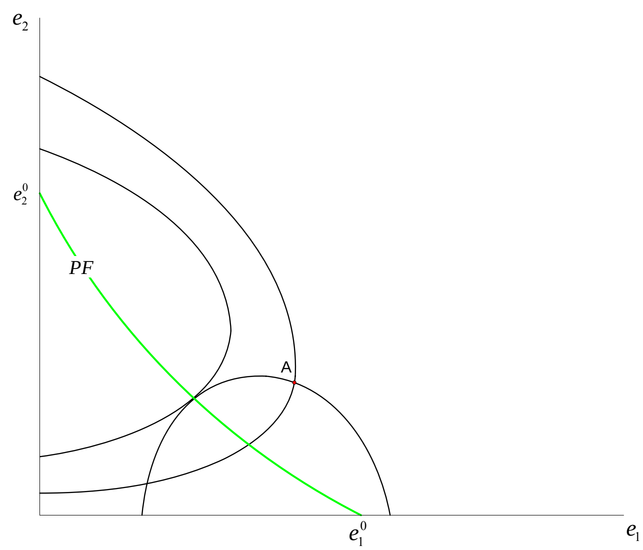


FIGURE 11-31

187

Welfare Properties of the Non-Cooperative Equilibrium

- From point **A**, it is clearly possible to move along the same iso-surplus contour for country 1 (following the red arrow in Figure 11-32), and at the same time make country 2 better off.

188

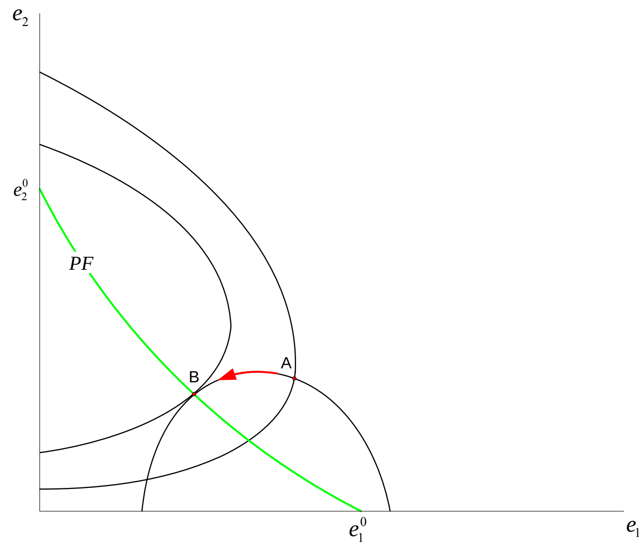


FIGURE 11-32

189

Welfare Properties of the Non-Cooperative Equilibrium

- It follows that point **A** cannot be Pareto efficient.
- The same is true for any other point on that iso-surplus contour for country 1, except point **B**, where we reach a tangency with an iso-surplus contour for country 2.

190

Welfare Properties of the Non-Cooperative Equilibrium

- The same argument applies to any point on any iso-surplus contour for country 1 that is not tangential to an iso-surplus contour for country 2.

191

Welfare Properties of the Non-Cooperative Equilibrium

- Thus, only those points that lie at a tangency of the iso-surplus contours can be Pareto efficient, and the locus of all such points is then, by definition, the Pareto frontier.

192

Welfare Properties of the Non-Cooperative Equilibrium

- Now let us overlay the Pareto frontier on Figure 11-29, which highlighted the lens of mutual benefit; see Figure 11-33.

193

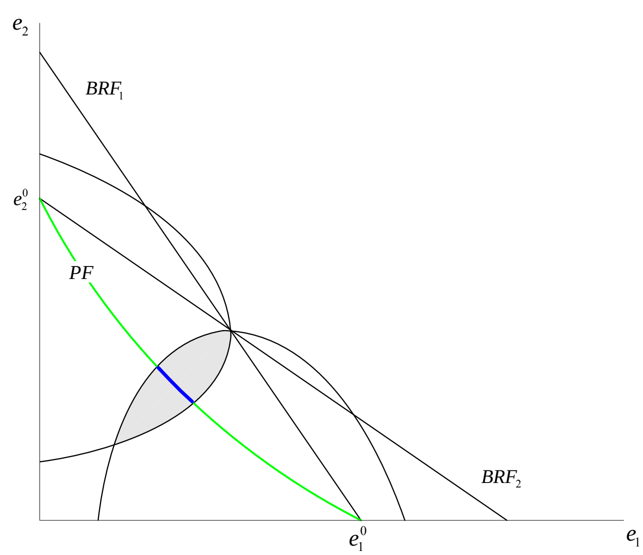


FIGURE 11-33

194

Welfare Properties of the Non-Cooperative Equilibrium

- The portion of the Pareto frontier that passes through the lens is called the core.
- The **core** is the set of Pareto efficient allocations that Pareto-dominate the NCE.
- Note that all points in the core involve lower emissions (for both countries) than the NCE.

195

Welfare Properties of the Non-Cooperative Equilibrium

- As noted earlier, this inefficiency of the NCE reflects the externality between the two countries.
- Neither country takes into the account the external benefit bestowed on the other country when deciding how much abatement to undertake.

196

Welfare Properties of the Non-Cooperative Equilibrium

- It is important to note that our welfare analysis so far has implicitly assumed that the two countries are symmetric: the NCE lies on the 45^0 line.

- In a world with asymmetry, it is possible that the NCE does not involve excessive emissions for <u>both</u> countries.

197

Welfare Properties of the Non-Cooperative Equilibrium

- However, it is possible to show – if we bring some more mathematics into the analysis – that <u>aggregate</u> emissions in the non-cooperative equilibrium are always higher than the level that would maximize global surplus.

198

APPENDIX A11: SOLVING THE PRISONERS' DILEMMA GAME

COUNTRY 2

COUNTRY 1	A	E
A	4 , 4	2 , 5
E	5 , 2	3 , 3

TABLE 11-1 (repeat) 200

Solving the PD Game

- First consider the choice problem for country 1 if it anticipates that country 2 will play **A**.

- See Tables A11-1 through A11-3.

201

COUNTRY 2

COUNTRY 1	A	E
A	4 , 4	2 , 5
E	5 , 2	3 , 3

TABLE A11-1

202

COUNTRY 2

COUNTRY 1	A	E
A	4 , 4	2 , 5
E	5 , 2	3 , 3

TABLE A11-2 203

COUNTRY 2

COUNTRY 1	A	E
A	4 , 4	2 , 5
E	5 , 2	3 , 3

TABLE A11-3 204

Solving the PD Game

- Thus, country 1 will choose **E** if it anticipates that country 2 will play **A**.
- Record that choice (boxed in Table A11-3).

205

Solving the PD Game

- Next consider the choice problem for country 1 if it anticipates that country 2 will play **E**.
- Country 1 will choose **E**; see Table A11-4.

206

COUNTRY 2

COUNTRY 1	A	E
A	4 , 4	→ 2 , 5
E	5 , 2	→ 3 , 3

TABLE A11-4

207

Solving the PD Game

- Now consider the choice problem for country 2 if it anticipates that country 1 will play **A**.
- Country 2 will choose **E**; see Table A11-5.

208

COUNTRY 2

COUNTRY 1	A	E
A	4 , 4	2 , 5
E	5 , 2	3 , 3

TABLE A11-5

209

Solving the PD Game

- Finally, consider the choice problem for country 2 if it anticipates that country 1 will play **E**.
- Country 2 will choose **E**; see Table A11-6.

210

COUNTRY 2

COUNTRY 1	A	E
A	4 , 4	2 , 5
E	5 , 2	3 , 3

TABLE A11-6 211

Solving the PD Game

- Thus, there is a unique NE to this game in which both countries play **E** even though they would both be better playing {**A**,**A**}.
- See Table A11-7.

212

COUNTRY 2

COUNTRY 1	A	E
A	* 4 , 4	2 , 5
E	5 , 2	3 , 3 NE

TABLE A11-7

213

TOPIC 11 REVIEW QUESTIONS

1. Climate change is a transboundary pollution problem because

A. the climate in any one country has an economic impact on all countries via international trade.

B. climatic regions do not necessarily correspond to national territories.

C. greenhouse gases are a globally uniformly-mixed pollutant.

D. temperature differentials around the globe are gradually eliminated by air flow.

2. Significant reductions in greenhouse gas emissions would cause an immediate slowing of climate change.

A. True.

B. False.

3. A public good is characterized by two key features:

A. joint production possibilities and high exclusion costs.

B. congestibility and high exclusion costs.

C. joint consumption possibilities and low exclusion costs.

D. None of the above.

4. Greenhouse gas abatement is a global public good because

A. all countries benefit from abatement by any one country, and non-abating countries cannot be excluded from those benefits.

B. all countries must pay for their own abatement but all countries benefit from the abatement by other countries.

C. countries that undertake abatement can free-ride on other countries, and thereby reduce the cost that abatement

D. All of the above.

5. Consider **Table R11-1**. The table describes a game between two countries when their emissions are transboundary. *S* indicates strict environmental standards; *L* indicates lax environmental standards. The Nash equilibrium outcome in this game is

A. country A plays S and country B plays L

B. both countries play L

C. both countries play S

D. None of the above.

		COUNTRY A	
		S	L
COUNTRY B	S	6, 6	2, 7
	L	7, 2	3, 3

Table R11-1

6. Consider **Table R11-2**. The table describes a game between two countries when their emissions are transboundary. *S* indicates strict environmental standards; *L* indicates lax environmental standards. The Nash equilibrium in this game is

A. country A plays S and country B plays L

B. both countries play L

C. both countries play S

D. None of the above.

		COUNTRY A	
		S	L
COUNTRY B	S	6, 5	1, 8
	L	7, 4	4, 3

Table R11-2

7. Consider **Table R11-3**. The table describes a game between two countries when their emissions are transboundary. *S* indicates strict environmental standards; *L* indicates lax environmental standards. The Nash equilibrium in this game is

A. country A plays S and country B plays L

B. both countries play L

C. both countries play S

D. None of the above.

		COUNTRY A: S	COUNTRY A: L
COUNTRY B	S	3, 4	2, 5
	L	2, 4	4, 3

Table R11-3

Questions 8 – 18 relate to the following information. Consider a climate-change game between two countries. Country 1 has marginal abatement cost given by

(3) $$MAC_1(e_1) = 7200 - 8e_1$$

and marginal damage given by

(4) $$MD_1(E) = 4E$$

Country 2 has marginal abatement cost given by

(5) $$MAC_2(e_2) = 3360 - 4e_2$$

and marginal damage given by

(6) $$MD_1(E) = 2E$$

Figure R11-1 illustrates the emissions choice problem for country 1, under two different scenarios with respect to emissions from country 2. (The figure is not drawn to scale).

8. The sole agent optimum for country 1 is

A. 405

B. 465

C. 600

D. 900

9. If $e_2 = 405$ then the best-response by country 1 is to set emissions equal to

A. 405

B. 465

C. 600

D. 900

10. If country 2 raises its emissions from $e_2 = 0$ to $e_2 = 405$ then under an optimal response by country 1, damage to country 1 rises by

A. area (B – A)

B. area (A + C)

C. area (A + C – F)

D. None of the above.

11. If country 2 raises its emissions from $e_2 = 0$ to $e_2 = 405$ then under an optimal response by country 1, abatement cost for country 1 rises by

A. area (E + F)

B. area (E + F + G)

C. area G

D. area D

12. The best-response function for country 1 is

A. $e_2(e_1) = 900 - \frac{e_1}{3}$

B. $e_1(e_2) = 900 - \frac{e_2}{3}$

C. $e_1(e_2) = 7200 - 900e_2$

D. $e_1(e_2) = 600 - \frac{e_2}{3}$

13. The best-response function for country 2 is

A. $e_2(e_1) = 560 - \frac{e_1}{3}$

B. $e_2(e_1) = 3360 - 840e_1$

C. $e_1(e_2) = 3360 - 840e_2$

D. $e_2(e_1) = 405 - \frac{e_1}{5}$

14. The non-cooperative equilibrium (NCE) is

A. $\hat{e}_1 = 455$ and $\hat{e}_2 = 415$

B. $\hat{e}_1 = 465$ and $\hat{e}_2 = 405$

C. $\hat{e}_1 = 405$ and $\hat{e}_2 = 405$

D. $\hat{e}_1 = 465$ and $\hat{e}_2 = 415$

15. Domestic surplus for country 1 at the NCE is

A. \$1297350 and corresponds to area (B – A) in Figure R11-1.

B. \$969300 and corresponds to area B in Figure R11-1.

C. \$2160000 and corresponds to area (B + C + E) in Figure R11-1.

D. None of the above.

16. Figure R11-2 depicts a candidate set of iso-surplus contours for this game in (e_1, e_2) space. These contours

A. depict surplus values for country 1 and are labeled plausibly with respect to their numerical values.

B. depict surplus values for country 1 but are not labeled plausibly with respect to their numerical values.

C. depict surplus values for country 2 and are labeled plausibly with respect to their numerical values.

D. depict surplus values for country 2 but are not labeled plausibly with respect to their numerical values.

17. The Pareto frontier in this game is

A. the locus of points where the iso-surplus contours intersect the BRFs in (e_1, e_2) space.

B. the set of emissions pairs that Pareto-dominate the NCE.

C. the locus of Pareto efficient points in (e_1, e_2) space.

D. the set of emissions pairs in which emissions are lower for both countries that at the NCE.

18. The core of this game is

A. the set of emissions pairs that Pareto-dominate the NCE.

B. the set of Pareto-efficient emissions pairs that Pareto-dominate the NCE.

C. the set of Pareto-efficient emissions pairs.

D. the set of Pareto-efficient emissions pairs that are Pareto-dominated the NCE.

19. "In a setting with symmetric countries, all points in the core involve fewer emissions for both countries, and lower aggregate emissions, than at the NCE".

A. True.

B. False.

Questions 20 – 28 relate to the following information. Consider a climate-change game between two countries. Country 1 has marginal abatement cost given by

(3) $MAC_1(e_1) = 384 - 4e_1$

and marginal damage given by

(4) $MD_1(E) = 4E$

Country 2 has marginal abatement cost given by

(5) $MAC_2(e_2) = 336 - 4e_2$

and marginal damage given by

(6) $MD_2(E) = 2E$

Figure R11-3 illustrates the emissions choice problem for country 1, under two different scenarios with respect to emissions from country 2. (The figure is not drawn to scale).

20. The sole agent optimum for country 1 is

A. 48

B. 56

C. 96

D. None of the above.

21. If $e_2 = 56$ then the best-response by country 1 is to set emissions equal to

A. 20

B. 24

C. 48

D. 56

22. If country 2 reduces its emissions from $e_2 = 56$ to $e_2 = 0$ then under an optimal response by country 1, damage to country 1 falls by

A. area (B – A)

B. area (A + C)

C. area (A + C – F)

D. None of the above.

23. If country 2 reduces its emissions from $e_2 = 56$ to $e_2 = 0$ then under an optimal response by country 1, abatement cost for country 1 rises by

A. area (E + F)

B. area (E + F + G)

C. area G

D. None of the above.

24. The best-response function for country 1 is

A. $e_1(e_2) = 48 - \frac{e_2}{4}$

B. $e_1(e_2) = 96 - 2e_2$

C. $e_1(e_2) = 48 - \frac{e_2}{2}$

D. $e_1(e_2) = 48 + \frac{e_2}{4}$

25. The best-response function for country 2 is

A. $e_2(e_1) = 56 - \frac{e_1}{3}$

B. $e_2(e_1) = 48 - \frac{e_1}{4}$

C. $e_2(e_1) = 96 - \frac{2e_1}{3}$

D. $e_2(e_1) = 24 - \frac{e_1}{2}$

26. The non-cooperative equilibrium (NCE) is

A. $\hat{e}_1 = 48$ and $\hat{e}_2 = 56$

B. $\hat{e}_1 = 24$ and $\hat{e}_2 = 26$

C. $\hat{e}_1 = 20$ and $\hat{e}_2 = 56$

D. $\hat{e}_1 = 24$ and $\hat{e}_2 = 48$

27. Domestic surplus for country 1 <u>at the NCE</u> is

A. – \$2304 and corresponds to area (B – A) in Figure R11-3.

B. – \$4672 and corresponds to area (B – A) in Figure R11-3.

C. \$3624 and corresponds to area (B – A) in Figure R11-3.

D. None of the above.

28. Figure R11-4 depicts a candidate set of iso-surplus contours for this game in (e_1, e_2) space. These contours

A. depict surplus values for country 1 and are labeled plausibly with respect to their numerical values.

B. depict surplus values for country 1 but are not labeled plausibly with respect to their numerical values.

C. depict surplus values for country 2 and are labeled plausibly with respect to their numerical values.

D. depict surplus values for country 2 but are not labeled plausibly with respect to their numerical values.

29. “In a setting with <u>asymmetric</u> countries, all points in the core involve lower aggregate emissions than at the NCE”.

A. True.

B. False.

The remaining questions do not review anything not already reviewed in the questions above. They simply provide another example to allow you to test your understanding more one time. You may wish to skip these if you are already feeling confident about your knowledge of the material.

Questions 30 – 38 relate to the following information. Consider a climate-change game between two countries. Country 1 has marginal abatement cost given by

(3) $MAC_1(e_1) = 600 - 3e_1$

and marginal damage given by

(4) $MD_1(E) = 4E$

Country 2 has marginal abatement cost given by

(5) $MAC_2(e_2) = 1200 - 4e_2$

and marginal damage given by

(6) $MD_2(E) = 4E$

Figure R11-5 illustrates the emissions choice problem for country 2, under two different scenarios with respect to emissions from country 1. (The figure is <u>not</u> drawn to scale).

30. The sole agent optimum for country 2 is

A. 86

B. 150

C. 300

D. None of the above.

31. If $e_1 = 86$ then the best-response by country 2 is to set emissions equal to

A. 86

B. 107

C. 144

D. 150

32. If country 1 <u>reduces</u> its emissions from $e_1 = 86$ to $e_1 = 0$ then under an optimal response by country 2, the change in damage for country 2 is

A. area (B – A)

B. area (A + C)

C. area (F – A – C)

D. None of the above.

33. If country 1 <u>raises</u> its emissions from $e_1 = 0$ to $e_1 = 86$ then under an optimal response by country 2, the change in abatement cost for country 2 is

A. area (E + F)

B. area (E + F + G)

C. area G

D. None of the above.

34. The best-response function for country 1 is

A. $e_1(e_2) = 150 - \frac{e_2}{2}$

B. $e_1(e_2) = 300 - 4e_2$

C. $e_1(e_2) = \frac{600 - 4e_2}{7}$

D. $e_1(e_2) = 600 - \frac{7e_2}{4}$

35. The best-response function for country 2 is

A. $e_2(e_1) = 300 - 4e_1$

B. $e_2(e_1) = 150 - \frac{e_1}{2}$

C. $e_2(e_1) = 150 - \frac{e_1}{4}$

D. $e_2(e_1) = 600 - \frac{7e_1}{4}$

36. The non-cooperative equilibrium (NCE) is

A. $\hat{e}_1 = 86$ and $\hat{e}_2 = 107$

B. $\hat{e}_1 = 86$ and $\hat{e}_2 = 150$

C. $\hat{e}_1 = 86$ and $\hat{e}_2 = 0$

D. $\hat{e}_1 = 0$ and $\hat{e}_2 = 150$

37. Domestic surplus for country 2 <u>at the NCE</u> is

A. $90000 and corresponds to area (A – B) in Figure R11-5.

B. $90000 and corresponds to area (B – A) in Figure R11-5.

C. $90000 and corresponds to area (B + C + E) in Figure R11-5.

D. None of the above.

38. Figure R11-6 depicts a candidate set of iso-surplus contours for this game in (e_1, e_2) space. These contours

A. depict surplus values for country 1 and are labeled plausibly with respect to their numerical values.

B. depict surplus values for country 1 but are not labeled plausibly with respect to their numerical values.

C. depict surplus values for country 2 and are labeled plausibly with respect to their numerical values.

D. depict surplus values for country 2 but are not labeled plausibly with respect to their numerical values.

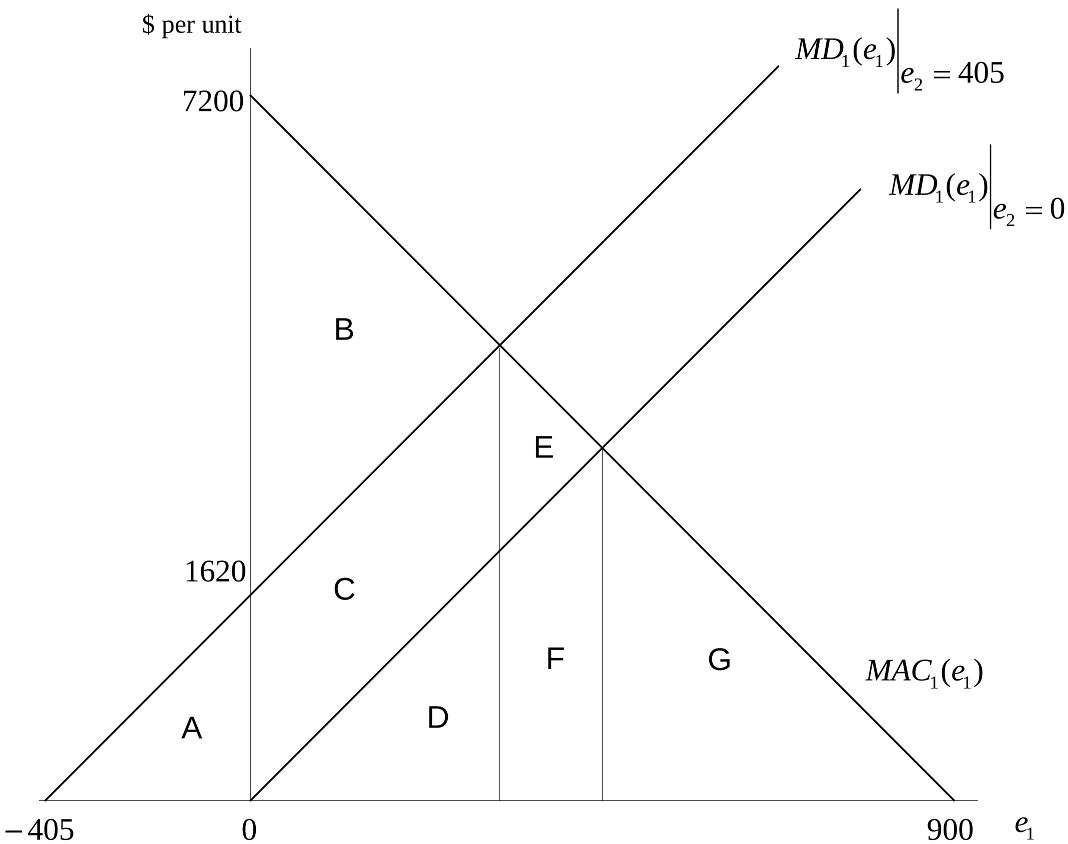

$ per unit
7200
1620
$MD_1(e_1)\big|_{e_2=405}$
$MD_1(e_1)\big|_{e_2=0}$
B
E
C
F
G
A
D
$MAC_1(e_1)$
−405
0
900
e_1

Figure R11-1

Figure R11-2

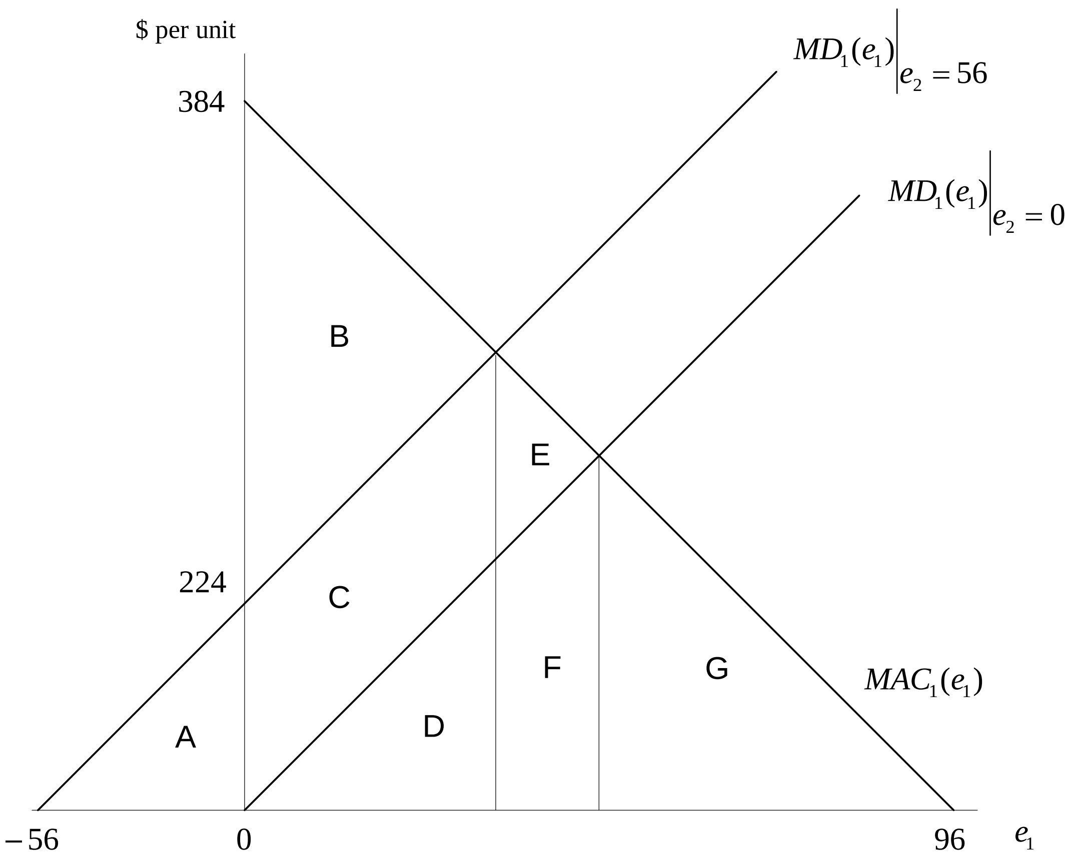

$ per unit
384
224
B
E
C
F
G
A
D
$MD_1(e_1)\big|_{e_2=56}$
$MD_1(e_1)\big|_{e_2=0}$
$MAC_1(e_1)$
– 56
0
96
e_1

Figure R11-3

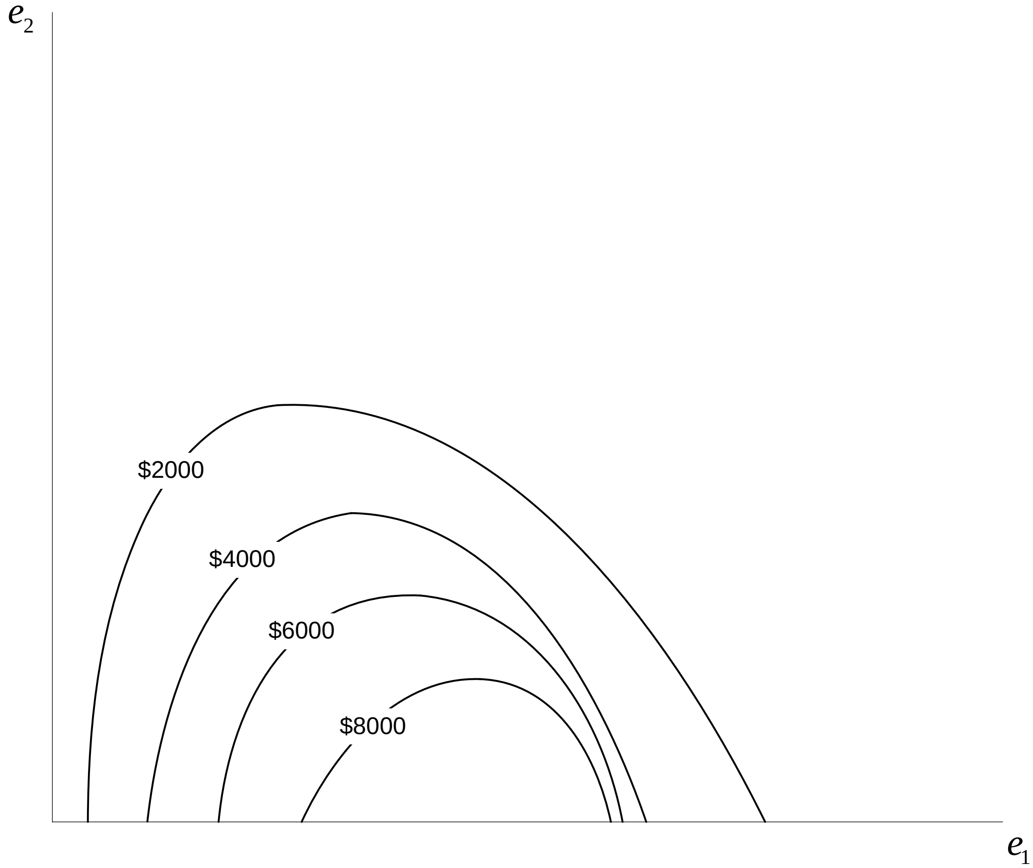


Figure R11-4

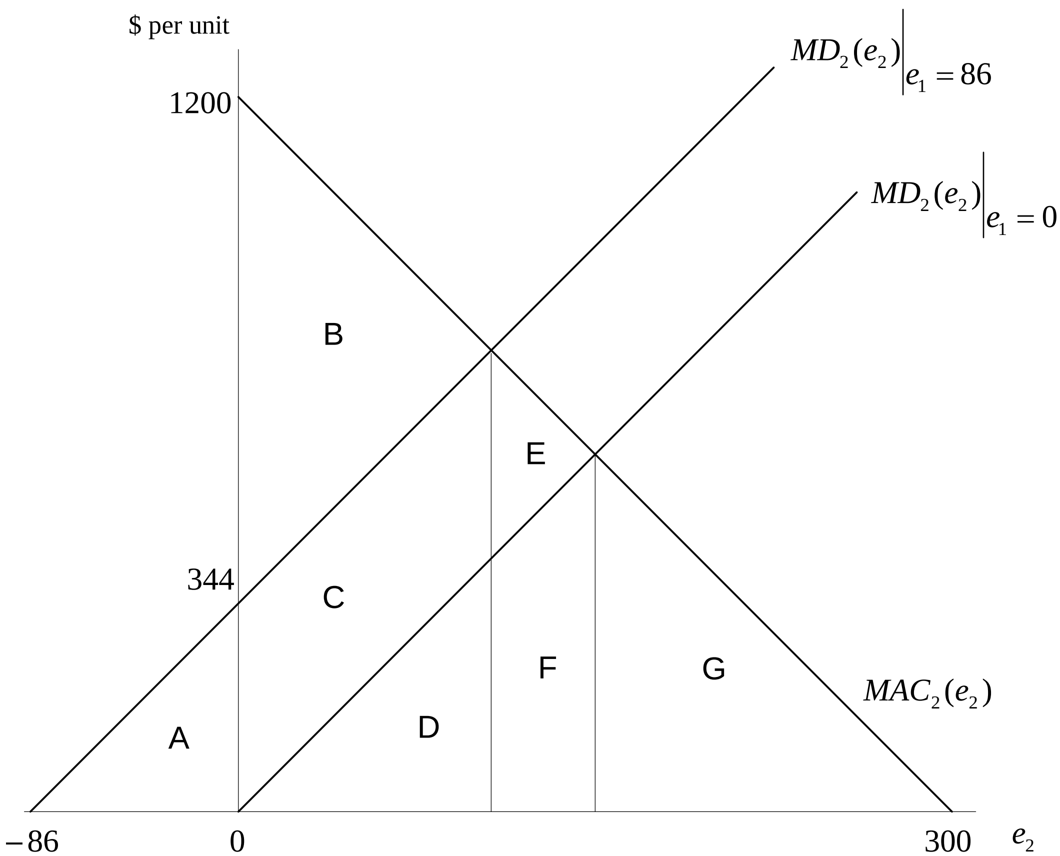

$ per unit
1200
344
$MD_2(e_2)\big|_{e_1=86}$
$MD_2(e_2)\big|_{e_1=0}$
$MAC_2(e_2)$
A
B
C
D
E
F
G
-86
0
300
e_2

Figure R11-5

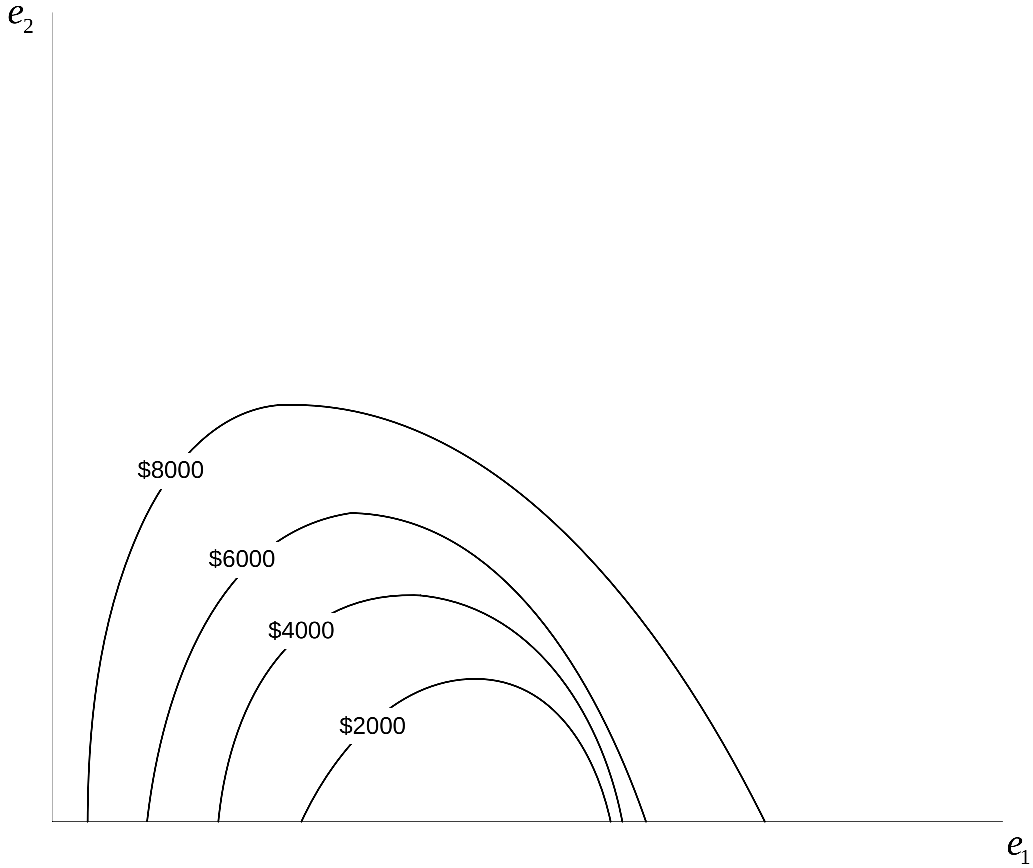


Figure R11-6

ANSWER KEY

1.	C	
2.	B	
3.	D	
4.	A	
5.	B	
6.	A	
7.	D	
8.	C	
9.	B	
10.	C	
11.	A	
12.	D	
13.	A	
14.	B	
15.	D	domestic surplus for country 1 is $969300 and corresponds to area (B – A) in Figure R11-1.
16.	**C**	the values should <u>fall</u> as we move away from the e_2 axis (and they do).
17.	C	
18.	B	
19.	A	
20.	A	
21.	A	
22.	C	
23.	D	abatement cost <u>falls</u> by area (D + F)
24.	C	
25.	A	
26.	D	
27.	D	domestic surplus for country 1 is **–** $2304 but it does <u>not</u> correspond to area (B – A) in Figure R11-3 because the MD schedule in the figure is <u>not</u> drawn for the NCE emissions level for country 2.
28.	A	
29.	A	
30.	B	
31.	B	
32.	C	

33. A
34. C
35. B
36. D
37. C
38. B

12. STOCK POLLUTANTS

OUTLINE

2

3

12.1 INTRODUCTION

Introduction

- We have so far restricted attention to **dissipative pollutants**; that is, pollutants that do not accumulate in the environment over time.
- For these pollutants, the optimal level of pollution in any year is independent of pollution levels in previous years since damage is confined to the year of discharge.

5

Introduction

- Accordingly, when defining the optimal level of pollution in any given year, we do not need to consider the future beyond that year.
- This means that our analysis can be **atemporal**; we do not need a time dimension to the analysis.

6

Introduction

- In contrast, damage from a **stock pollutant** (or persistent pollutant) in any given year depends on the history of emissions.

7

Introduction

- This means that we need **intertemporal analysis**:
  - choices made today have potential implications for future welfare, and those implications must be accounted for in the determination of optimal pollution today.

8

Introduction

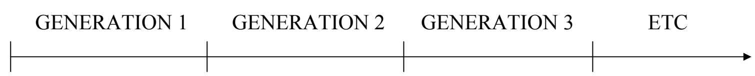
- Intertemporal analysis is complicated, and a thorough treatment requires the use of more mathematics than is appropriate for this course.
- However, some of the key insights that a formal analysis provides can also be gleaned from a graphical treatment and some simple examples.

9

12.2 A SEQUENTIAL GENERATIONS MODEL

A Sequential Generations Model

- Consider a simple model in which there is a sequence of finitely-lived generations, where each generation lives for one "period".



11

A Sequential Generations Model

- Let S_t denote the stock of the pollutant in the environment in period t, and let β denote the **decay factor** for the pollutant.

12

A Sequential Generations Model

- This means that a only fraction (1-*β*) of the stock in any period is still in the environment in the next period.
- The remaining fraction *β* decays and is no longer a source of harm.

13

A Sequential Generations Model

- Thus, the stock of the pollutant in period *t* is

$$S_t = (1-\beta)S_{t-1} + e_t$$

where e_t is the quantity of emissions in period *t*.

- This equation is called the **stock transition equation**.

14

A Sequential Generations Model

- We will examine the choice problem for generation *t* under two different scenarios regarding intergenerational altruism.

15

12.3 NO INTERGENERATIONAL ALTRUISM

No Intergenerational Altruism

- We begin with a setting in which no individual in generation *t* cares about the welfare of future generations.
- We will refer to this case as one with **no intergenerational altruism**.

17

No Intergenerational Altruism

- In this case, damage to generation *t* is entirely due to the impact of S_t in period *t*.
- The harm caused to the next generation by the undecayed portion of S_t that carries over into period t+1 is irrelevant to the pollution choice problem for generation *t*.

18

No Intergenerational Altruism

- Consider the optimal emissions choice for generation *t* in this setting.
- We still cast that choice problem in terms of MD and MAC but with a modification to our graphical framework.

19

No Intergenerational Altruism

- In particular, damage in period *t* is now a function of the stock in period *t*, where that stock is determined partly by emissions in period *t* and partly by the stock in the previous period, as per the stock transition equation:

$$S_t = e_t + (1-\beta)S_{t-1}$$

20

No Intergenerational Altruism

- In contrast, abatement cost in period *t* is a function only of emissions in period *t*.
- Thus, we need to measure both stock <u>and</u> emissions on our graphs.

21

No Intergenerational Altruism

- We deal with this complication by measuring S_t on the horizontal axis, and measuring emissions starting at $(1-\beta)\, S_{t-1}$ on that same axis.
- See Figure 12-1.

22

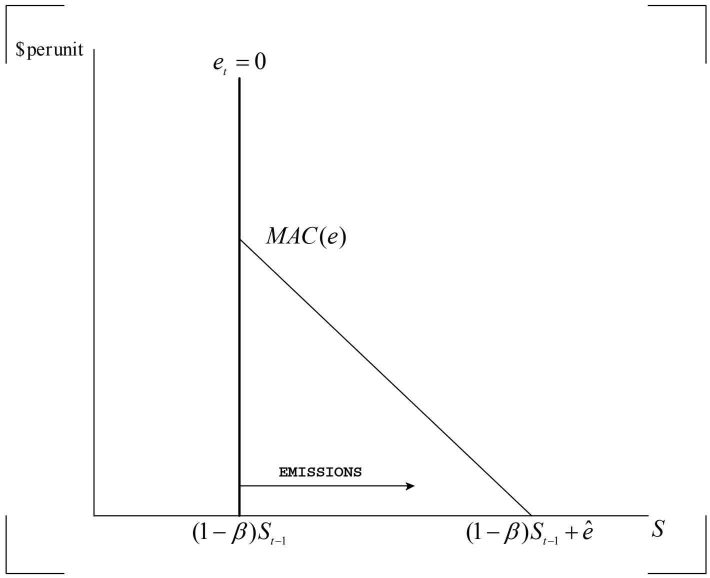


FIGURE 12-1

23

No Intergenerational Altruism

- We can now add $MD(S)$ to our graph and identify the optimal value of S_t, and the corresponding optimal value of e_t.
- See Figure 12-2.

24

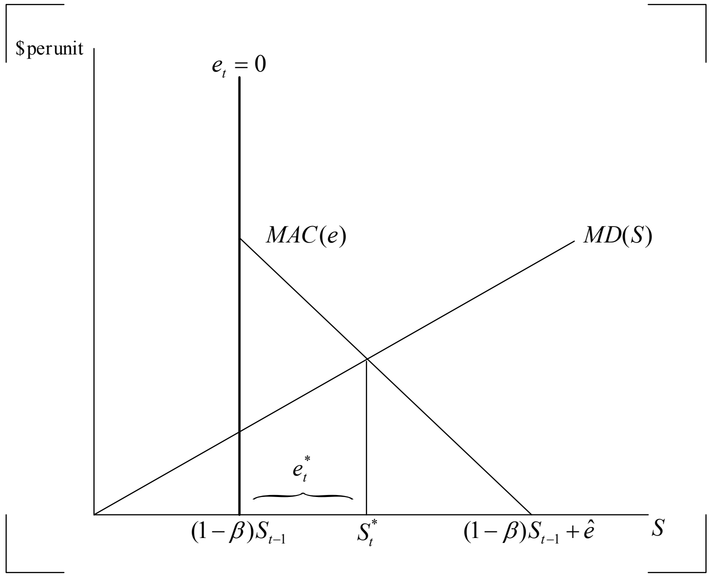


FIGURE 12-2

25

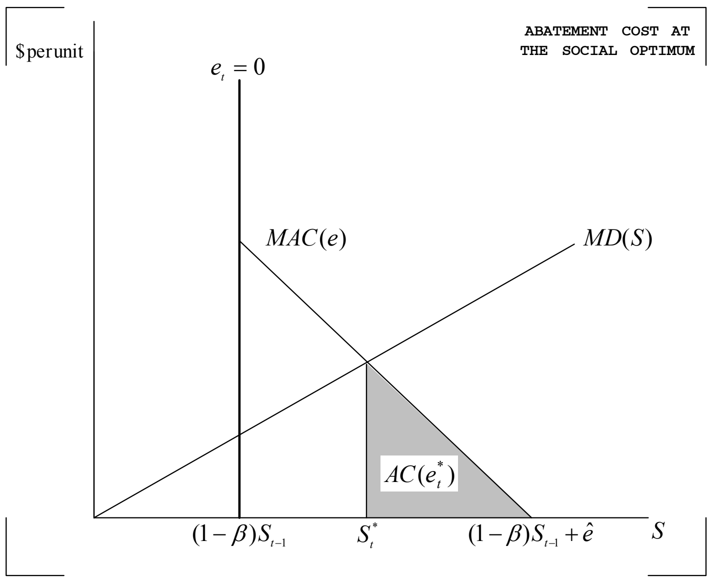
No Intergenerational Altruism

- Abatement cost at the optimal choice is

$$C(e_t^*) = \int_{e_t^*}^{\hat{e}} MAC(e)de$$

- See Figure 12-3.

26



FIFURE 12-3

27

No Intergenerational Altruism

- Damage at this optimal choice is

$$D(S_t^*) = \int_0^{S_t^*} MD(S)dS$$

- See Figure 12-4.

28

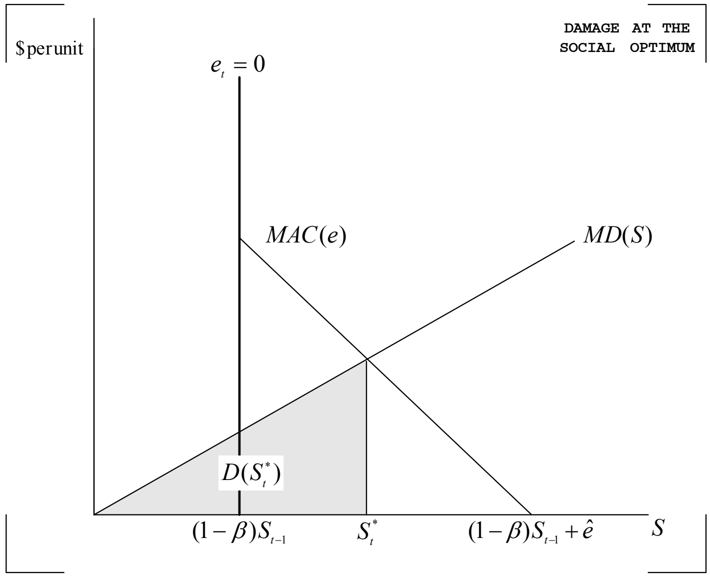


FIGURE 12-4

29

No Intergenerational Altruism

- This damage can be decomposed into two parts:
  - damage due to the stock inherited from generation *t*-1 (see region D1 in Figure 12-5)
  - damage due to new emissions (see region D2 in Figure 12-5)

30

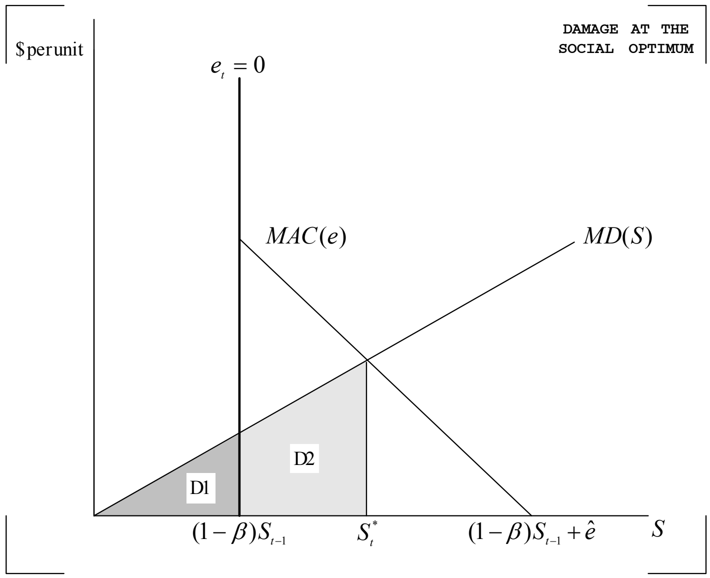


FIGURE 12-5

31

No Intergenerational Altruism

- We will return to the measurement of the intergenerational externality associated with the inherited stock in Section 12.6.

32

12.4 AN EXAMPLE WITH LINEAR MARGINAL COSTS

An Example with Linear Marginal Costs

- Let us modify the linear model from Topic 3.4 to suit the stock-pollutant setting.
- We retain the same MAC function but we now assume that MD is a function of *S* rather than *e*, just as we did in our graphical analysis in Figure 12-2.

34

An Example with Linear Marginal Costs

- In particular, suppose MAC and MD for generation *t* are

$$MAC(e_t) = \gamma(\hat{e} - e_t)$$

and

$$MD(S_t) = \delta S_t$$

respectively.

35

An Example with Linear Marginal Costs

- If generation *t* inherits stock $(1\text{-}\beta)S_{t\text{-}1}$ then the stock in period *t* is

$$S_t = (1-\beta)S_{t-1} + e_t$$

36

An Example with Linear Marginal Costs

- Equating MAC and MD yields the optimal emissions choice for generation *t*, denoted e_t^*:

$$\gamma(\hat{e} - e_t^*) = \delta[(1-\beta)S_{t-1} + e_t^*]$$

37

An Example with Linear Marginal Costs

- This solves for

$$e_t^* = \left(\frac{\gamma}{\gamma+\delta}\right)\hat{e} - \left(\frac{\delta}{\gamma+\delta}\right)(1-\beta)S_{t-1}$$

38

An Example with Linear Marginal Costs

- Note that if β=1 then this solution for the optimum reduces to the same solution we derived in Topic 3.4.
- Why? If β=1 then the stock decays completely within one period, and hence the pollutant is effectively <u>not</u> a stock pollutant at all.

39

An Example with Linear Marginal Costs

- Conversely, if β<1 then some of the stock from period t-1 remains in period t, and this means that optimal emissions in period t are <u>lower</u> than they otherwise would be.
- Why lower?

40

The Importance of Increasing MD

- To understand why, consider Figure 12-6.
- The figure illustrates the damage from a given quantity of emissions when added to two different existing stocks, S_1 and $S_2 > S_1$.

- These damages are the shaded areas A1 and A2 respectively.

41

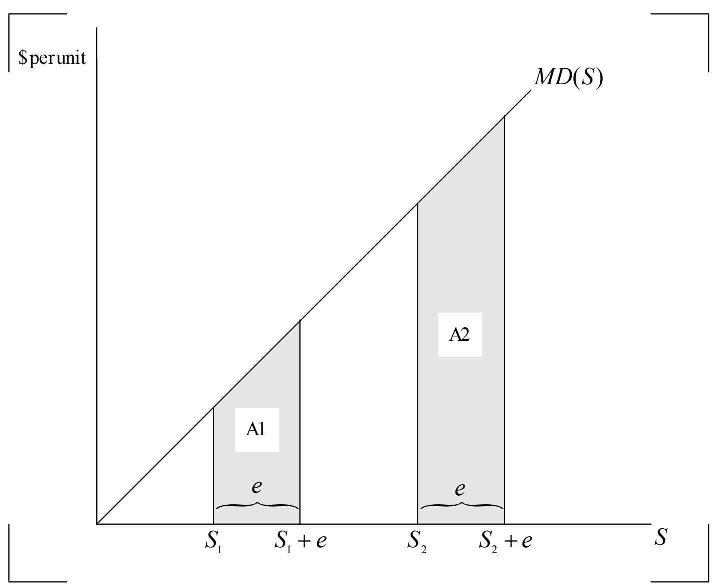


FIGURE 12-6

42

The Importance of Increasing MD

- Note that the same quantity of emissions does more damage when added to S2 than when added to S1, because MD is increasing.
- Only in the special case where MD is constant is the damage due to additional emissions independent of the existing stock; see Figure 12-7.

43

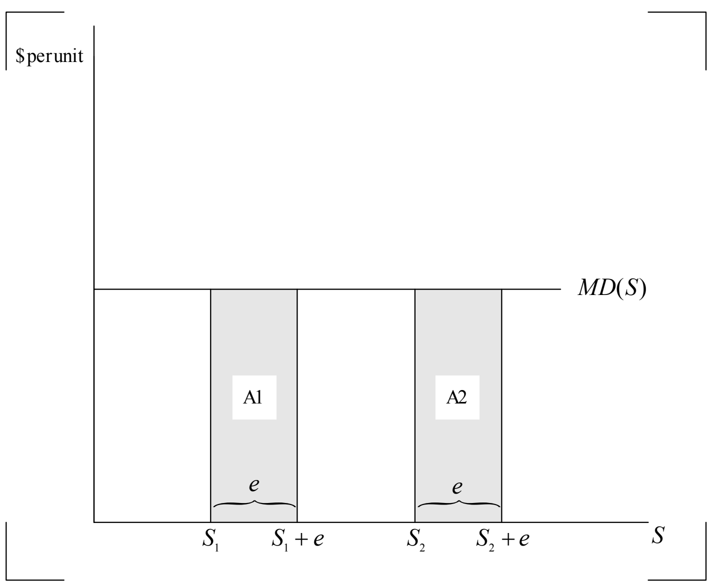


FIGURE 12-7

44

The Importance of Increasing MD

- When MD is increasing, the inflating impact of the existing stock on the damage caused by new emissions – as illustrated in Figure 12-6 – must be reflected in the optimal emissions choice.
- Thus, a larger inherited stock leads to a lower optimal quantity of emissions.

45

- As an aside, recall from Topic 11 that the optimal level of emissions for one country is decreasing in the emissions level of the other country, because marginal damage is increasing.

- Exactly the same reasoning underlies the relationship here between optimal emissions and the undecayed stock.

46

12.5 DYNAMICS AND THE STEADY STATE

Dynamics and the Steady State

- If each generation behaves optimally – in the non-altruistic manner we have so far described – what happens to the stock of the pollutant over time?

48

Dynamics and the Steady State

- To answer this question with any degree of generality requires more mathematical analysis than is appropriate for this course.
- However, we can investigate the question usefully in the context of our linear model.

49

Dynamics and the Steady State in the Linear Model

- Recall from s.38 the optimal emissions choice for generation *t*:

$$e_t^* = \left(\frac{\gamma}{\gamma+\delta}\right)\hat{e} - \left(\frac{\delta}{\gamma+\delta}\right)(1-\beta)S_{t-1}$$

50

Dynamics and the Steady State in the Linear Model

- We can calculate the implied stock in period t using the transition equation:

$$S_t^* = (1-\beta)S_{t-1} + e_t^*$$

51

Dynamics and the Steady State in the Linear Model

- Making the substitution for e_t^* yields

$$S_t^* = \left(\frac{\gamma(1-\beta)}{\gamma+\delta}\right)S_{t-1} + \frac{\gamma\hat{e}}{\gamma+\delta}$$

- This is a **first-order linear difference equation**; it tells us how the stock evolves over time in response to optimal choices by successive generations.

52

Dynamics and the Steady State in the Linear Model

- This equation can be transformed into an **explicit solution** for S_t as a function of t and the initial stock, S_0.
- The mathematics of that transformation is beyond the scope of this course but the result is interesting and worth describing even without the associated mathematics.

53

Dynamics and the Steady State in the Linear Model

- In general, a difference equation of the form

$$S_t = aS_{t-1} + b$$

with $a \neq 1$ has **explicit solution**

$$S_t = (S_0 - S_{SS})a^t + S_{SS}$$

54

Dynamics and the Steady State in the Linear Model

where $S_0 \geq 0$ is the stock at t=0 and

$$S_{SS} = \frac{b}{1-a}$$

is the **steady-state** stock.

55

Dynamics and the Steady State in the Linear Model

- If a = 1 then the stock rises (when b>0) or falls (when b<0) linearly over time:

$$S_t = S_0 + bt$$

- There is no steady state in this case.

56

Dynamics and the Steady State in the Linear Model

- We will restrict attention here to settings in which a>0 (which rules out oscillating behavior) and b>0 (which limits the set of possible time paths).

57

Dynamics and the Steady State in the Linear Model

- To understand the steady state, return to the difference equation:

$$S_t = aS_{t-1} + b$$

- In the steady state, $S_t = S_{t-1}$.
- To find that steady state, we set $S_t = S$ and $S_{t-1} = S$ and solve for S. The solution is S_{SS} .

58

Dynamics and the Steady State in the Linear Model

- Since the steady state is defined by the condition $S_t = S_{t-1}$, it is clear that if the stock <u>starts</u> at the steady state – that is, if $S_0 = S_{SS}$ – then it will <u>stay</u> at that steady state forever.

59

Dynamics and the Steady State in the Linear Model

- We can also see this by examining the explicit solution to the difference equation:

$$S_t = (S_0 - S_{SS})a^t + S_{SS}$$

- If $S_0 = S_{SS}$ then $S_t = S_{SS}$ in every period, for any value of $a \neq 1$.

60

Dynamics and the Steady State in the Linear Model

- Now suppose the stock does not start at the steady state. Will it move towards the steady state over time?
- That depends critically on the value of *a* in the difference equation.
- There are two distinct cases of interest in our context.

61

Dynamics and the Steady State in the Linear Model

Divergent Case: $a>1$

- In this case $S_{SS} < 0$ and a^t grows continually over time.
- Thus, S_t also grows continually over time, and never converges to a steady state.

- See Figure 12-8.

62

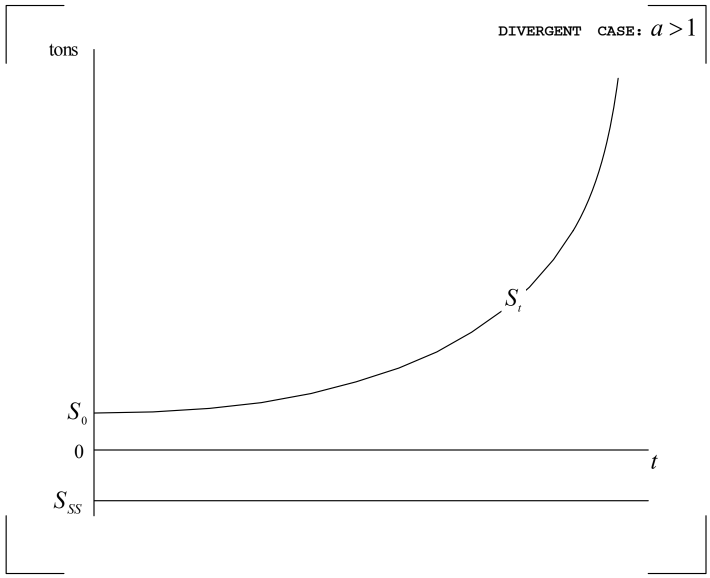


FIGURE 12-8

63

Dynamics and the Steady State in the Linear Model

Convergent Case: a<1

- In this case $S_{SS} > 0$ and a^t shrinks continually over time.
- Thus, S_t converges towards the steady state.

64

Dynamics and the Steady State in the Linear Model

- The convergent path can take one of two forms, depending on where it begins:
  1. If $S_0 < S_{SS}$ then S_t begins at S_0 and rises asymptotically towards S_{SS} from below; see Figure 12-9.
  2. If $S_0 > S_{SS}$ then S_t begins at S_0 and declines asymptotically towards S_{SS} from above; see Figure 12-10.

65

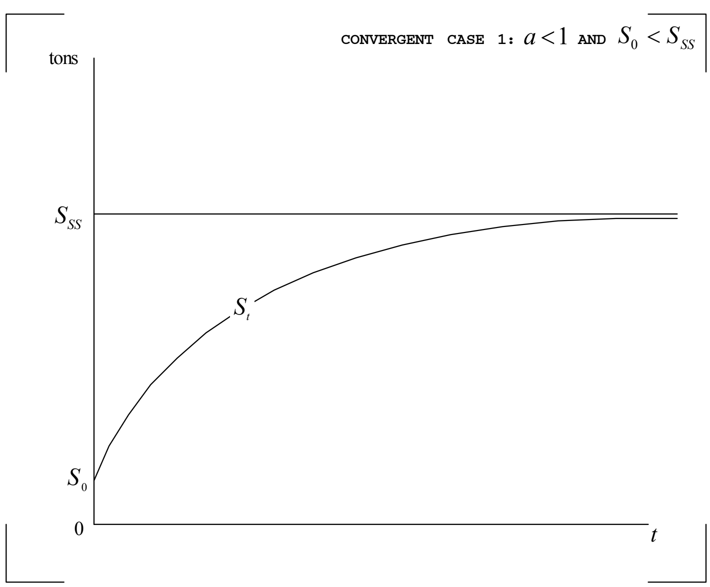


FIGURE 12-9

66

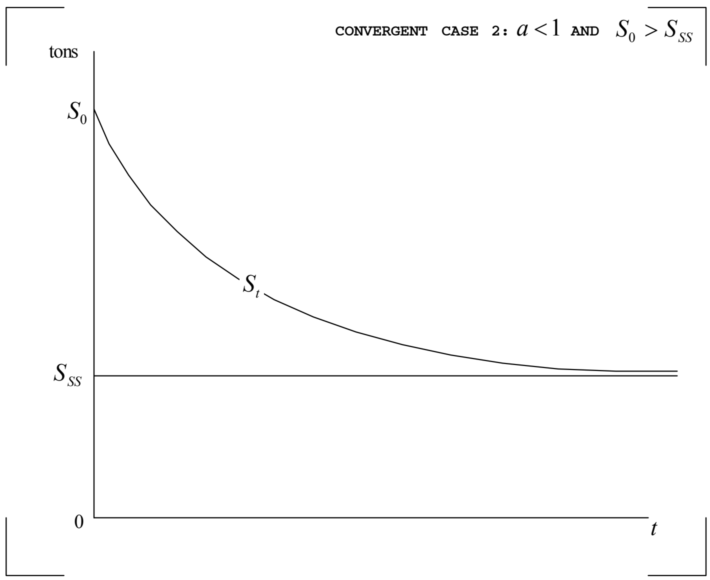


FIGURE 12-10

67

Dynamics and the Steady State in the Linear Model

- Note that S_t never actually reaches the steady state, even in the convergent case, if it does not start there.
- However, in the convergent case S_t moves arbitrarily close to S_{SS} as time passes.
- We can think of this as a stabilization of the stock over time.

68

Dynamics and the Steady State in the Linear Model

- Now let us return to the specific linear example and examine the time path of S_t.

- Recall the difference equation from s.52:

$$S_t^* = \left(\frac{\gamma(1-\beta)}{\gamma+\delta} \right) S_{t-1} + \frac{\gamma\hat{e}}{\gamma+\delta}$$

69

Dynamics and the Steady State in the Linear Model

- It follows that in this example:

$$a = \frac{\gamma(1-\beta)}{\gamma+\delta}$$

and

$$b = \frac{\gamma\hat{e}}{\gamma+\delta}$$

70

Dynamics and the Steady State in the Linear Model

- The time path for S_t is therefore given by

$$S_t = (S_0 - S_{SS})\left(\frac{\gamma(1-\beta)}{\gamma+\delta}\right)^t + S_{SS}$$

where

$$S_{SS} = \frac{\gamma\hat{e}}{\beta\gamma+\delta}$$

71

Dynamics and the Steady State in the Linear Model

- Since (1-β) < 1, it follows that $a < 1$.
- Thus, the <u>convergent case</u> applies here: S_t converges towards the steady state.

72

Dynamics and the Steady State in the Linear Model

- Whether S_t converges from below or above depends on S_0, as depicted in Figures 12-9 and 12-10 respectively.
- We might be inclined to assume that S_0=0 when thinking about a pollutant stock but this might not always be appropriate.

73

Dynamics and the Steady State in the Linear Model

- For example, suppose that society has been emitting for some time but only begins to optimize according to e_t^* after it learns of the relationship between the stock and damage.
- In that case, S_0 measures the stock at the date that informed optimizing behavior begins, and so S_0 may be positive.

74

Dynamics and the Steady State in the Linear Model

- The time path for <u>emissions</u> is found by substituting the solution for S_t from s.70 (lagged one period) into e_t^* from s.51 to yield

$$e_t^* = \frac{\beta\gamma\hat{e}}{\delta + \beta\gamma} - \frac{\delta}{\gamma}(S_0 - S_{SS})\left(\frac{\gamma(1-\beta)}{\gamma+\delta}\right)^t$$

75

Dynamics and the Steady State in the Linear Model

- Note that if we set β=1 in this expression for e_t^* then we recover the same result we derived for optimal emissions for a dissipative pollutant (recall our results from Topic 3).

- Why?

76

Dynamics and the Steady State in the Linear Model

- As noted earlier, if β=1 then the stock decays completely within one period, and hence the pollutant is effectively not a stock pollutant at all.

77

Dynamics and the Steady State in the Linear Model

- Note too that as t approaches infinity, e_t^* approaches e_{SS}, where

$$e_{SS} = \frac{\beta\gamma\hat{e}}{\delta + \beta\gamma} = \beta S_{SS}$$

is the steady-state quantity of emissions.

78

Dynamics and the Steady State in the Linear Model

- Interpretation of e_{SS} :
  - if emissions in each period are just equal to the amount by which the previous stock decayed, then the new stock will be just equal to the previous stock.

79

Dynamics and the Steady State in the Linear Model

- The time paths for emissions and the pollutant stock are depicted in Figures 12-11 and 12-12 for two different cases with respect to the size of the <u>initial stock</u>, S_0.

80

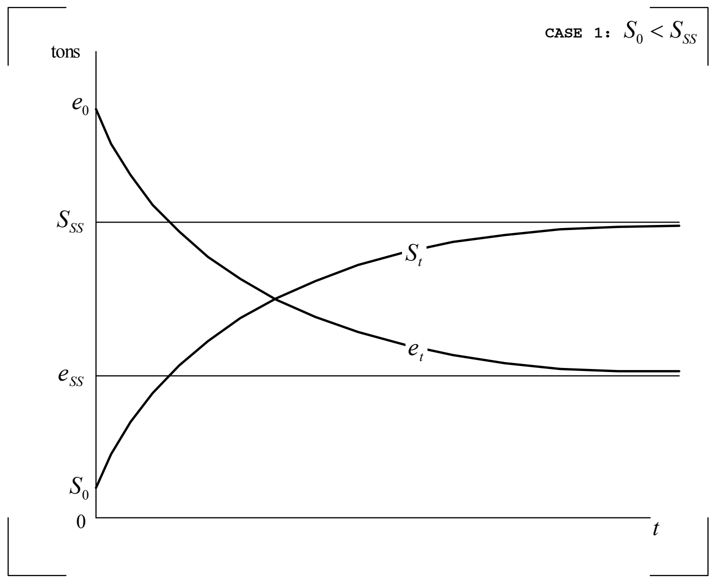


FIGURE 12-11 81

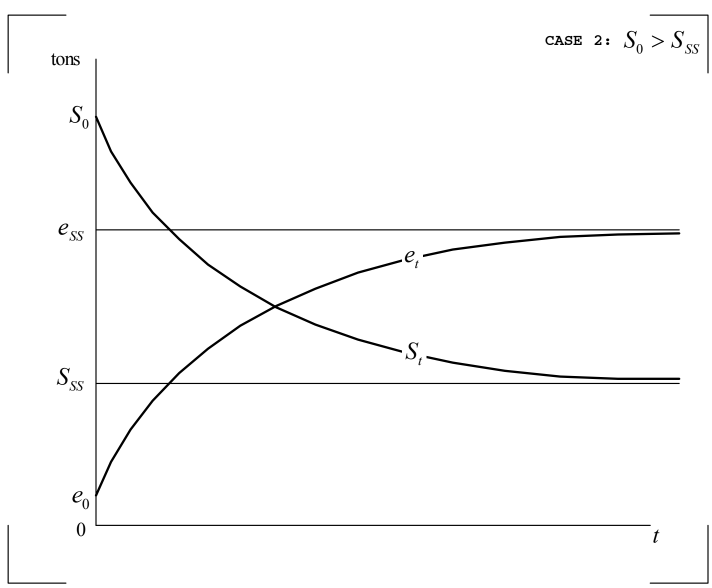


FIGURE 12-12 82

Dynamics and the Steady State in the Linear Model

- Figure 12-11 depicts the case where $S_0 < S_{SS}$.
- In this case, the stock rises over time and converges asymptotically towards S_{SS}.
- Conversely, emissions fall over time and converge asymptotically towards e_{SS}.
- Why do emissions and the stock move in opposite directions?

83

Dynamics and the Steady State in the Linear Model

- The rising stock means that damage from any given quantity of emissions rises over time – recall the logic behind Figure 12-6 – and so the optimal quantity of emissions falls over time.

84

Dynamics and the Steady State in the Linear Model

- Figure 12-12 depicts the case where $S_0 > S_{SS}$.
- In this case, the stock falls over time and converges asymptotically towards S_{SS}.
- Conversely, emissions rise over time and converge asymptotically towards e_{SS}.

85

Dynamics and the Steady State in the Linear Model

- In this case, the falling stock means that damage from any given quantity of emissions falls over time – recall again the logic behind Figure 12-6 – and so the optimal quantity of emissions rises over time.

86

Dynamics and the Steady State in the Linear Model

- The paths depicted in Figure 12.11, where emissions fall over time and the stock rises over time towards the steady state, could conceivably describe the optimal response to the climate change problem, now that we understand the link between the atmospheric stock of GHGs and damage.

87

Dynamics and the Steady State in the Linear Model

- Of course, the analogy is not exact because we know that the link between stock and damage is a lagged one in the case of climate change.
- We consider that important possibility later, in section 12.7.

88

12.6 AN INTERGENERATIONAL EXTERNALITY

An Intergenerational Externality

- Along the time paths in Figures 12-11 and 12-12, each generation passes a positive stock of pollution on to the next generation.
- What is the cost of this inherited stock to any given generation?

90

An Intergenerational Externality

- To calculate this intergenerational externality, recall the optimal choice for generation *t* in response to the stock inherited from generation *t*-1, as depicted in Figure 12-2 (and repeated on the next slide).

91

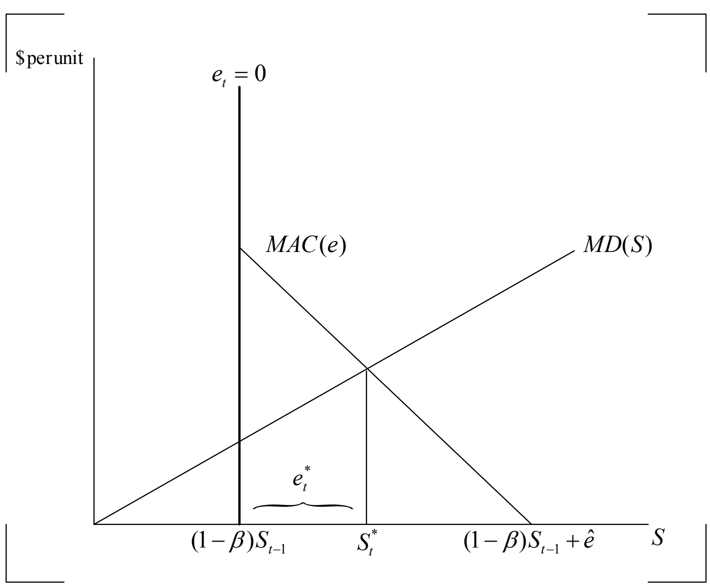


FIGURE 10-2 (repeat)

92

An Intergenerational Externality

- Recall also the total social cost for generation t at this optimum, measured in Figures 12-3 and 12-5, and combined here in Figure 12-13.

93

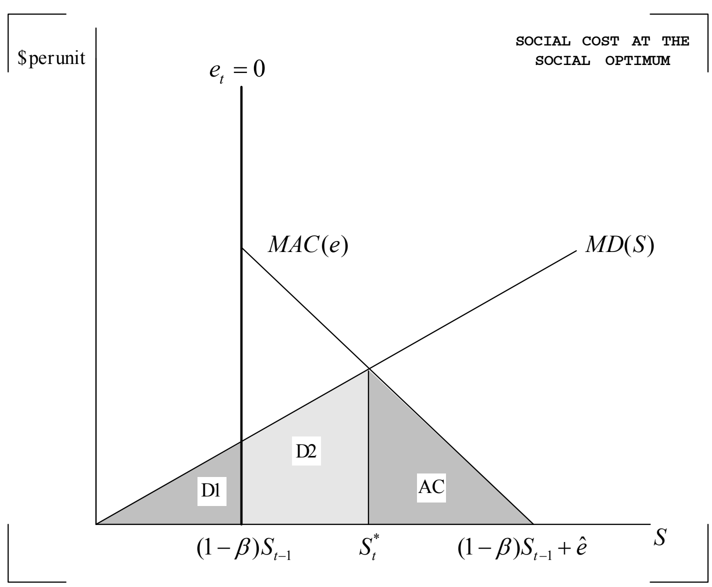


FIGURE 12-13

94

An Intergenerational Externality

- The shaded area in Figure 12-13 measures the sum of damage and abatement cost for generation *t* at the optimum, and it comprises three parts:
  - damage due to the inherited stock itself (D1)
  - damage due to pollution emitted in period t (D2)
  - total abatement cost (AC)

95

An Intergenerational Externality

- Now let us calculate what portion of this cost is attributable to the inherited stock.

- To preview the result, it is more than D1.

96

An Intergenerational Externality

- To see why, consider a hypothetical setting in which the stock inherited by generation *t* is zero.
- The optimal choice for generation *t* in this setting is denoted e^0_t, and is illustrated in Figure 12-14, where $MAC(e)_0$ is measured from $S = 0$.

97

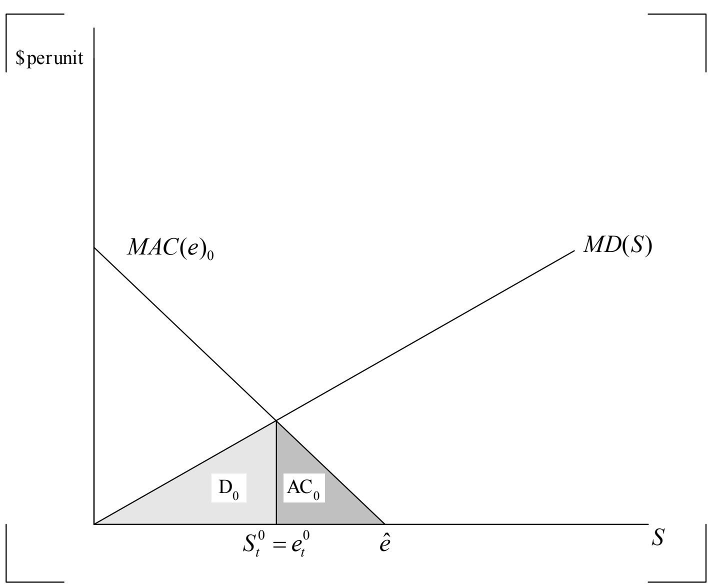


FIGURE 12-14

98

An Intergenerational Externality

- Note that $S_t^0 = e_t^0$ here because $S_t = 0$ by assumption.

- The shaded areas labeled D_0 and AC_0 in Figure 12-14 measure damage and abatement cost respectively in this hypothetical setting.

99

An Intergenerational Externality

- These areas in Figure 12-14 are comparable to those labeled D2 and AC respectively in Figure 12-13.

- There is no area in Figure 12-14 comparable to area D1 in Figure 12-13 because there is no inherited stock in this case.

100

An Intergenerational Externality

- Now overlay Figure 12-14 on Figure 12-13 to identify the additional social cost incurred by generation t due to the inherited stock.
- See Figure 12-15.

101

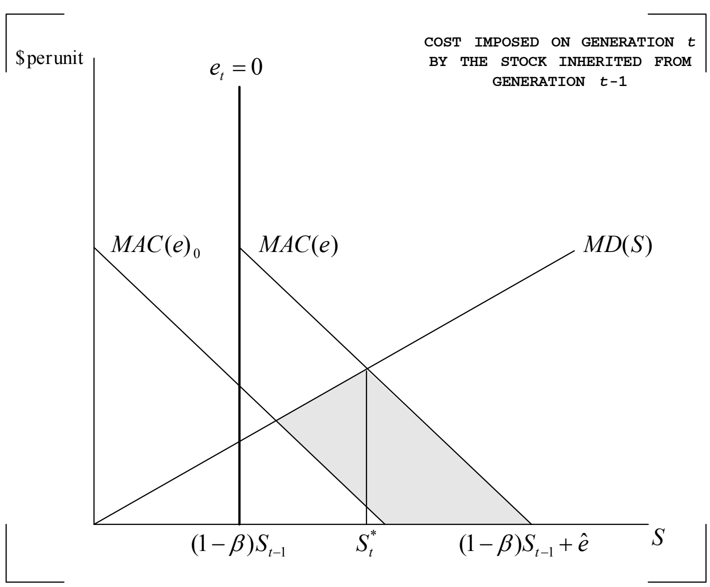


FIGURE 12-15

102

An Intergenerational Externality

- Comparing this area with area D1 from Figure 12-13, it is clear that the cost imposed on generation *t*+1 due to the inherited stock is greater than the damage done by the inherited stock itself.

- Why?

103

An Intergenerational Externality

- The inherited stock inflates the damage caused by emissions in period *t* because MD is increasing (recall the logic behind Figure 12-6).
- In addition, generation *t* optimally undertakes more abatement because of that higher MD, and this in turn causes an increase in abatement cost at the optimum.

104

An Intergenerational Externality

- Who is responsible for this cost imposed on generation *t*?
- Remember that part of the stock carried forward from generation *t*-1 was inherited from generation *t*-2, who in turn inherited a stock from generation *t*-3, and so on.

105

An Intergenerational Externality

- One might reasonably argue that generation *t*-1 is responsible only for the undecayed fraction of its own emissions that are passed on to generation *t*, and not for the entire undecayed stock that generation *t* inherits.

106

An Intergenerational Externality

- By that argument, every generation prior to generation t has imposed some external cost on generation *t*.
- Does this intergenerational externality create inefficiency?

107

An Intergenerational Externality

- Recall from our treatment of negative externalities in Topic 2 that the external agents in that setting could in principle compensate the source agent for cutting back on the offending action, and still be better off.

108

An Intergenerational Externality

- The implicit assumption in that setting is that external agents and the source agent are contemporaries, and so a payment in principle between the two sides is possible.
- This is not true in our sequential generations model: future generations cannot make a payment to the current one.

109

An Intergenerational Externality

- There exists no mechanism through which a PPI can be achieved in this setting, despite the externality.

110

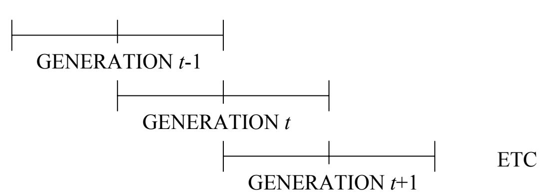
An Intergenerational Externality

- The conclusion is potentially less stark in a setting with **overlapping generations**.
- In the simplest form of an overlapping generations (OLG) model, each agent lives for two periods; they are young in the first period of their life and old in the second.

111

An Intergenerational Externality

- Thus, the young from generation *t* are contemporaries with the old from generation *t*-1:



112

An Intergenerational Externality

- If a **debt contract** can be struck and enforced, such a contract could require the young in period *t* to pay the old in period *t* for abatement undertaken by those old agents when they were young, in period *t*-1.
- This would allow for Pareto-improving trade across generations.

113

An Intergenerational Externality

- We will not pursue this issue further because the analysis gets quite complicated.
- However, the issue highlights the importance of assessing the impact of externalities with reference to the feasibility of trade between the parties involved.

114

12.7 INTERGENERATIONAL ALTRUISM

Intergenerational Altruism

- We have so far assumed that current agents are unconcerned about the welfare of future agents.
- What happens if we introduce intergenerational altruism (IA) into our model?

116

Intergenerational Altruism

- To investigate this question we will focus on a special setting in which the physical impact of the stock is lagged one period.
- In such a setting, IA is the only motivation for abatement since the physical damage from emissions is not felt at all by the generation responsible for those emissions.

117

Intergenerational Altruism

- This setting approximates the reality of the climate change problem because the impact of the atmospheric GHG stock on climate is subject to long time lags (thought to be around 30 years).

118

Intergenerational Altruism

- There are a variety of ways that IA can be modeled, but the simplest approach is to assume that generation t suffers some indirect harm from the stock in period t because of the physical impact this stock will have on generation t+1.

119

Intergenerational Altruism

- Approaching IA in this way allows us to use the same basic model we have used to date but with a slight reinterpretation of the marginal damage function.
- In particular, $MD(S_t)$ now reflects the marginal indirect harm suffered by generation t as a consequence of the stock passed on to generation t+1

120

Altruism in the Linear Model

- Recall the marginal damage function from our simple linear model

$$MD(S_t) = \delta S_t$$

- We now interpret δ as the degree of IA.

121

Altruism in the Linear Model

- In this context, δ reflects the extent to which each generation cares about the harm caused to the next generation as a consequence of the stock passed on.

122

Altruism in the Linear Model

- Applying this reinterpretation of δ, we can obtain some useful insights into the effect of altruism using the results we derived in Section 12.4.

123

Altruism in the Linear Model

- In particular, recall from s.71 that the steady-state stock is

$$S_{SS} = \frac{\gamma \hat{e}}{\beta\gamma + \delta}$$

- This is decreasing in δ:
  - a higher degree of altruism leads to a lower steady-state pollutant stock.

124

Altruism in the Linear Model

- Recall also the time path for the stock:

$$S_t = (S_0 - S_{SS})\left(\frac{\gamma(1-\beta)}{\gamma+\delta}\right)^t + S_{SS}$$

- This too is decreasing in δ:
  - a higher degree of altruism leads to a lower pollutant stock in all periods.

125

Altruism in the Linear Model

- These results are as we would expect:
  - greater concern for future generations leads each generation to reduce the cost imposed on those future generations via the stock that is passed on.

126

Altruism in the Linear Model

- However, it is important to note that even a high degree of altruism does not necessarily mean that the stock will fall over time.
- If S_0 is low enough then the stock will still rise towards S_{SS} over time, as depicted in Figure 12-11, except in the extreme case where $\delta = \infty$.

127

Altruism in the Linear Model

- Thus, intergenerational altruism does not eliminate the intergenerational externality (unless that altruism is infinite).
- The balancing of abatement cost and damage that underlies optimal choices still causes each generation to impose a cost on the next generation even when every generation is altruistic.

128

12.8 SELF-REINFORCING FEEDBACK LOOPS

Self-Reinforcing Feedback Loops

- In the examples we have examined so far – with and without IA – all growth paths converge to a steady state.
- We now want to consider an example where a divergent path can arise as a consequence of decisions that are nonetheless optimal for each generation along that path.

130

Self-Reinforcing Feedback Loops

- Suppose the pollutant stock causes damage to the productive capacity of the economy.
- In particular, imagine a setting in which additional energy must be used to achieve a given level of output to compensate for the damaging effect of climate change on productivity.

131

Self-Reinforcing Feedback Loops

- That extra energy use in turn adds to the stock of atmospheric greenhouse gases, which in turn causes further climate change.
- Thus, climate change today effectively leads to more climate change tomorrow.

132

Self-Reinforcing Feedback Loops

- Recall from Topic 11 that this type of relationship is called a **self-reinforcing feedback loop**.
- In general, such a loop is a repeating cycle of events such that
  - $\Delta X>0$ causes ΔY, and ΔY then causes $\Delta X>0$, or
  - $\Delta X<0$ causes ΔY, and ΔY then causes $\Delta X<0$

133

Self-Reinforcing Feedback Loops

- In contrast, recall that a **self-moderating feedback loop** is a repeating cycle of events such that
  - $\Delta X>0$ causes ΔY, and ΔY then causes $\Delta X<0$, or
  - $\Delta X<0$ causes ΔY, and ΔY then causes $\Delta X>0$

134

The Linear Model with Self-Reinforcing Feedback

- Let us incorporate a positive feedback loop into our linear model from Section 12.6.

- Recall the basic structure of that model:
  - S_t causes no direct harm in period t, but generation t is altruistic and so suffers damage via the impact of S_t on generation t+1

135

The Linear Model with Self-Reinforcing Feedback

- The marginal damage function is

$$MD(S_t) = \delta S_t$$

where δ is the altruism parameter, and

$$S_t = (1-\beta)S_{t-1} + e_t$$

136

The Linear Model with Self-Reinforcing Feedback

- The marginal abatement cost function is

$$MAC(e_t) = \gamma(\hat{e} - e_t)$$

where $\hat{e}$ is the no-abatement quantity of emissions.

137

The Linear Model with Self-Reinforcing Feedback

- We will retain all elements of this structure but with one key modification.

138

The Linear Model with Self-Reinforcing Feedback

- In particular, the no-abatement quantity of emissions is now time-variant, and given by

$$\hat{e}_t = \hat{e}_0 + \lambda S_{t-1}$$

where λ measures the strength of the impact of $S_{t\text{-}1}$ on productivity in period t, and $\hat{e}_0$ reflects “baseline” productivity.

139

The Linear Model with Self-Reinforcing Feedback

- This equation tells us that the emissions needed to produce a given level of output (via energy use, for example) is increasing in the pollutant stock from the previous period, because that stock degrades productivity in the current period.
- The strength of that effect is measured by λ.

140

The Linear Model with Self-Reinforcing Feedback

- The baseline value $\hat{e}_0$ is the no-abatement quantity of emissions corresponding to λ=0.
- Thus, setting λ=0 returns us to the original example from Section 12.6, where $\hat{e}_0 = \hat{e}$.

141

The Linear Model with Self-Reinforcing Feedback

- In graphical terms, the productivity effect of the pollutant stock causes the MAC to shift over time as the stock grows or shrinks.
- Figure 12-16 illustrates this shift in the MAC between two periods, *t* and *t*+1, for a case where the stock is growing over time.

142

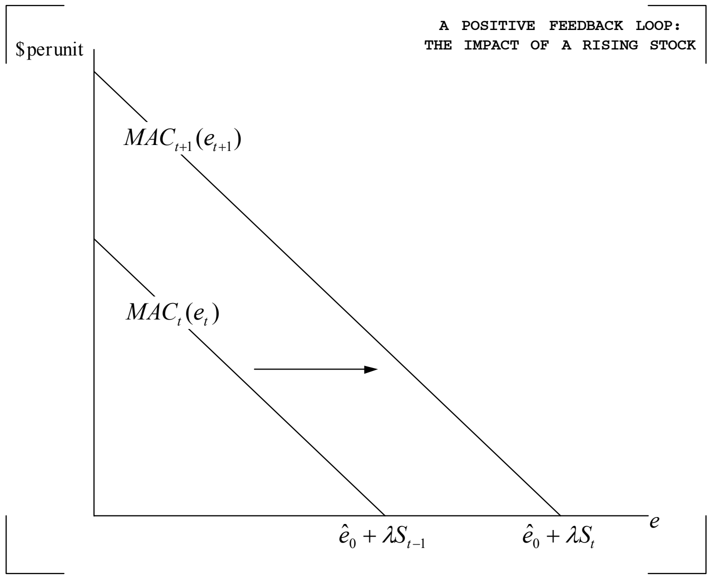


FIGURE 12-16

143

The Linear Model with Self-Reinforcing Feedback

- The shift in MAC depicted in Figure 12-16 means that the cost of abatement for achieving any given level of emissions, say $\tilde{e}$, is higher in period t+1 than in period t; see Figure 12-17 and 12-18.

144

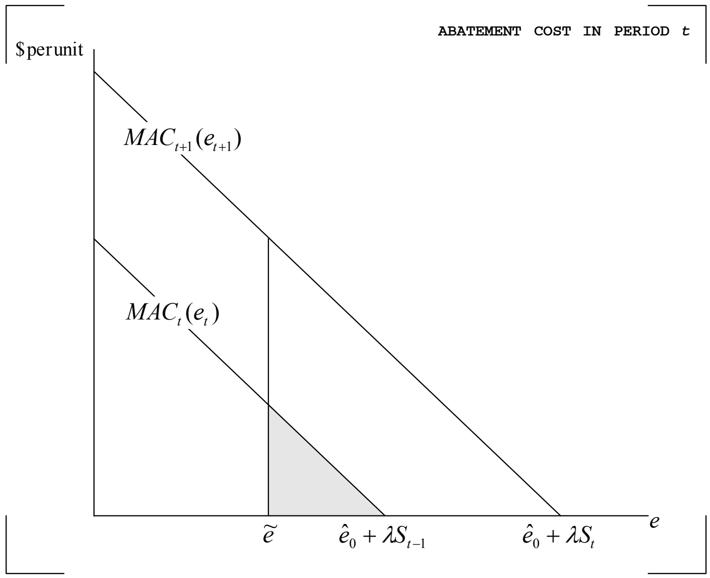


FIGURE 12-17 145

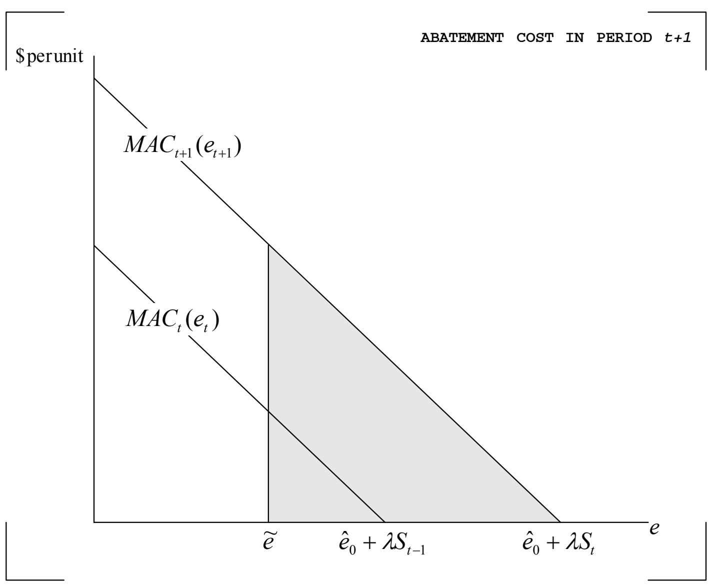


FIGURE 12-18 146

The Linear Model with Self-Reinforcing Feedback

- What underlies this increase in cost?
- Lower productivity in period t+1 relative to period t, due to the damaging impact of the lagged stock, means that more output must be given up (or more abatement measures undertaken) to achieve a given quantity of emissions.
- Thus, abatement is more costly.

147

The Linear Model with Self-Reinforcing Feedback

- Marginal abatement cost in period t is now

$$MAC_t(e_t) = \gamma(\hat{e}_t - e_t) = \gamma(\hat{e}_0 + \lambda S_{t-1} - e_t)$$

and marginal damage in period t is

$$MD(S_t) = \delta S_t = \delta[(1-\beta)S_{t-1} + e_t]$$

148

The Linear Model with Self-Reinforcing Feedback

- Equating MAC to MD and solving for e_t yields the optimal emissions choice for generation *t*:

$$e_t^* = \frac{\gamma \hat{e}_0}{\gamma + \delta} - \left(\frac{\delta(1-\beta) + \gamma\lambda}{\gamma + \delta} \right) S_{t-1}$$

149

The Linear Model with Self-Reinforcing Feedback

- We can calculate the implied stock in period *t* using the transition equation:

$$S_t = (1-\beta) S_{t-1} + e_t^*$$

150

The Linear Model with Self-Reinforcing Feedback

- Making the substitution for e_t^* yields

$$S_t^* = \left(\frac{\gamma(1-\beta+\lambda)}{\gamma+\delta} \right) S_{t-1} + \frac{\gamma \hat{e}_0}{\gamma+\delta}$$

151

The Linear Model with Self-Reinforcing Feedback

- Recall from Section 12.3 that this is a difference equation of the form

$$S_t = aS_{t-1} + b$$

that has explicit solution

$$S_t = (S_0 - S_{SS})a^t + S_{SS}$$

152

The Linear Model with Self-Reinforcing Feedback

where $S_0 \geq 0$ is the stock at t=0 and

$$S_{SS} = \frac{b}{1-a}$$

is the steady-state stock.

153

The Linear Model with Self-Reinforcing Feedback

- Thus, in our current example:

$$a = \frac{\gamma(1-\beta+\lambda)}{\gamma+\delta}$$

and

$$b = \frac{\gamma \hat{e}_0}{\gamma+\delta}$$

154

The Linear Model with Self-Reinforcing Feedback

- The time path for S_t is therefore given by

$$S_t = (S_0 - S_{SS})\left(\frac{\gamma(1-\beta+\lambda)}{\gamma+\delta}\right)^t + S_{SS}$$

where

$$S_{SS} = \frac{\gamma \hat{e}}{(\beta-\lambda)\gamma+\delta}$$

155

The Linear Model with Self-Reinforcing Feedback

- Note that if λ=0 then the solution reduces to the same solution we derived in Section 12.6 (see s.125).
- Thus, we can think of the setting in Section 12.6 as a special case, in which there is no feedback loop.

156

The Linear Model with Self-Reinforcing Feedback

- If there is a feedback loop (λ>0) then the dynamics can be very different from those in Section 12.6.
- In particular, we now have the possibility that *a*>1, in which case the path for S_t will be divergent: the stock could increase explosively away from steady state.

157

The Linear Model with Self-Reinforcing Feedback

- Under what conditions will this arise?
- Setting *a*=1 (from s.154) and solving for δ yields a critical value of δ:

$$\delta_C = (\lambda - \beta)\gamma$$

158

The Linear Model with Self-Reinforcing Feedback

- If $\delta > \delta_C$ then $a<1$, and S_t moves towards a steady state at S_{SS}.
- Conversely, if $\delta \leq \delta_C$ then $a \geq 1$, and S_t follows a divergent path: it rises continually, away from the steady state.

159

The Linear Model with Self-Reinforcing Feedback

- These results tells us that the stock will converge towards a steady state only if the degree of IA is sufficiently high.
- Otherwise, the stock will keep rising and the economy will eventually run out of the resources required to continually compensate for the associated damage to productivity.

160

The Linear Model with Self-Reinforcing Feedback

- Recall the critical value of δ that marks the threshold between these two types of path:

$$\delta_C = (\lambda - \beta)\gamma$$

- This is increasing in γ and $(\lambda$-$\beta)$, where the latter term can be interpreted as the net effect of the stock on productivity, once stock decay is taken into account.

161

The Linear Model with Self-Reinforcing Feedback

- Thus, convergence towards a steady state requires that IA be high relative to the cost of abatement (as reflected in γ) and relative to the strength of the net productivity effect of the pollutant stock.

162

12.9 SUSTAINABILITY REVISITED

Sustainability Revisited

- Recall the discussion of sustainability from Topic 1, where we argued that issues relating to resource management over time are best addressed in the context of a standard framework based on efficiency and distributional considerations.
- Let us now explore that argument further in the context of our stock pollutant model.

164

Sustainability Revisited

- In the feedback-loop model from Section 12.7, the degree of IA is a critical determinant of whether or not the stock converges to a steady-state.
- What does “the” degree of IA mean in a society in which preferences, and hence individual degrees of altruism, might differ across the individuals in that society?

165

Sustainability Revisited

- Suppose individual *i* within generation *t* has an individual degree of IA given by δ_i.
- Then the marginal damage she suffers from the stock passed on to the next generation is

$$md_i(S_t) = \delta_i S_t$$

166

Sustainability Revisited

- Aggregate marginal damage for that generation is then equal to the sum of the individual marginal damages:

$$MD(S_t) = \sum_{i=1}^{n} md_i(S_t) = \left(\sum_{1=1}^{n} \delta_i \right) S_t$$

where n is the number individuals in each generation (assumed to be constant here).

167

Sustainability Revisited

- Note that we can aggregate individual md_i values here because they are all measured in dollars (reflecting WTP); we do not face the immeasurability problem discussed in the context of sustainability.

168

Sustainability Revisited

- Thus, the δ for the generation as a whole can be constructed as

$$\delta = \sum_{i=1}^{n} \delta_i$$

- The optimal policy is then based on this aggregate δ.

169

Sustainability Revisited

- Now think about the determinants of δ_i for a specific individual.

- Those determinants are her own preferences and her own wealth, since WTP (and WTA) is determined jointly by both, and md_i is measured as WTP (or WTA).

170

Sustainability Revisited

- This means that the optimal policy with respect to abatement is determined partly by the distribution of wealth within a generation.

171

Sustainability Revisited

- For example, suppose

$$\delta_i = \rho_i w_i$$

where w_i is the wealth of individual i, and ρ_i is a preference parameter for that individual (reflecting the importance she places on the environmental state passed on to the next generation).

172

Sustainability Revisited

- Then

$$\delta = \sum_{i=1}^{n} \rho_i w_i$$

- Thus, we can think of δ as a weighted sum of preference parameters, where the weights are wealth levels.

173

Sustainability Revisited

- If wealth is redistributed away from individuals with a low ρ_i and given to individuals with a high ρ_i, then δ will rise; high values of ρ_i effectively receive increased weight in the calculation of δ.
- Conversely, δ falls if wealth is redistributed in the opposite direction.

174

Sustainability Revisited

- Critically, this means that we can relate the dynamics of the stock across generations to preferences and the distribution of wealth within generations.

175

Sustainability Revisited

- Thus, issues pertaining to "intergenerational equity" or "intergenerational justice" can be addressed fully in terms of standard efficiency and distributional considerations; we do not need a separate framework based on "sustainability" or some equally dubious alternative foundation.

END

176