Probability

• Chapter 7 (omit 7.3)

• Why learn about probability?
  • inferential statistics
  • sampling from a population
Probability

- Outcome = result of some event
  - probability of an outcome defined as number of ways that outcome could be produced relative to total number of possible, equally likely, outcomes
    - \( p(\text{tail when flipping a fair coin}) = \frac{1}{2} = .5 \)
    - \( p(\text{rolling a 5 on a fair die}) = \frac{1}{6} = .1667 \)

\[
p = \frac{\text{relevant outcomes}}{\text{all possible outcomes}}
\]

- 6/49 lottery example
  - \( p(1\text{st draw is one of your numbers}) = \frac{6}{49} = .1224 \)
Probability

• Probability of not obtaining an outcome is just

\[ 1 - p(\text{outcome}) \Rightarrow \text{complement} \]

• 6/49 lottery example

\[ p(1\text{st draw is not one of your numbers}) = 1 - \frac{6}{49} \]

\[ = 1 - 0.1224 \]

\[ = 0.8776 \]
Probability

• Two useful rules of probability

• Addition rule: probability of obtaining one or another of two or more *mutually exclusive* outcomes is the sum of their individual probabilities

• deck of cards: \( p(\text{drawing King or Ace}) \)

\[
p(\text{King}) = \frac{4}{52} \quad p(\text{Ace}) = \frac{4}{52}
\]

\[
p(\text{King or Ace}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = .1538
\]

\[
p(\text{face card}) = \frac{4}{52} + \frac{4}{52} + \frac{4}{52} = \frac{12}{52} = .2308
\]
Probability

• Multiplication rule: probability of two or more *independent* outcomes occurring jointly is the product of their individual probabilities

• *independent* means the outcome of one event provides no information about the outcome the other event produces

• \( p(2 \text{ heads in a row}) = .5(.5) = .25 \)

• \( p(\text{two face cards}) = \left( \frac{12}{52} \right) \left( \frac{11}{51} \right) = .0498 \)

• note that the second outcome \((11/51)\) is a conditional probability

• but independence is maintained
Probability

• Multiplication rule
• $p$(winning 6/49)
• Which set of numbers is more likely to win?

8, 12, 19, 32, 41, 48  or  1, 2, 3, 4, 5, 6
Probability

- Multiplication rule
- $p(\text{winning } 6/49)$
- six independent events
- independent in the sense that having drawn the first $N$ numbers, tells us nothing about which one of the remaining numbers will be picked on draw $N + 1$

$$p(\text{winning}) = p(1\text{st } #) \times p(2\text{nd } #) \times p(3\text{rd } #) \times p(4\text{th } #) \times p(5\text{th } #) \times p(6\text{th } #)$$

$$= \frac{6}{49} \times \frac{5}{48} \times \frac{4}{47} \times \frac{3}{46} \times \frac{2}{45} \times \frac{1}{44}$$

$$= .000000071 \text{ or } \frac{1}{13,983,816}$$
Probability

• Multiplication rule
  • $p$(winning 6/49) $\approx 1/14$ million

  Which set of numbers is more likely to win?
  8, 12, 19, 32, 41, 48 or 1, 2, 3, 4, 5, 6

$p(2$ heads in $3$ flips):

$p(10$ heads in $10$ flips):
Probability and the Normal Distribution

• Random sampling from a population
  • theoretical basis for much of inferential statistics
  • random method is one means of achieving *independence* in selection of sample members
    • inclusion of one individual in the sample provides no information about who else will be included
Probability and the Normal Distribution

• Using the normal distribution to compute probabilities
  • normally distributed population with $\mu = 75$, $\sigma = 10$
  • probability of randomly drawing a subject with a score of 80 or larger?

\[ z = \frac{80 - 75}{10} = 0.50 \]

smaller portion = 0.3085
Probability and the Normal Distribution

• Using the normal distribution to compute probabilities
  • normally distributed population with $\mu = 75$, $\sigma = 10$
    • probability of randomly drawing, without replacement, a subject with a score of 95 or larger and then a subject with a score of 55 or smaller?

\[
z = \frac{95 - 75}{10} = 2.00
\]

smaller portion: 0.0228

\[
p = 0.0228 \times 0.0228 = 0.0005
\]

• probability of randomly drawing a subject with a score of 95 or larger or a score of 55 or smaller?

\[
p = 0.0228 + 0.0228 = 0.0456
\]
Probability Demonstration

- Monty Hall problem
Uniting the most conscientious students from several sections of psyc 300. The purpose is to discuss the more challenging aspects of the course. All participants are asked to have a firm grasp of the basics before attending. The Facebook page will serve as a platform for online discussion as well as scheduling meeting times on campus.

**UVICPSYC300STUDYCLUB (all caps)**

**Link:**
[https://m.facebook.com/groups/1107024419345833?tsid=0.8325790522431188&source=typeahead](https://m.facebook.com/groups/1107024419345833?tsid=0.8325790522431188&source=typeahead)

For students interested in preparing, or currently preparing, for the GRE exam.

**UVICGRECLUB (all caps)**

**Link:**
[https://m.facebook.com/groups/103749016760007?tsid=0.7869767547362209&source=typeahead](https://m.facebook.com/groups/103749016760007?tsid=0.7869767547362209&source=typeahead)