1. Note that there are 36 equally likely outcomes when throwing two dice: 1 through 6 on the first die combined with 1 through 6 on the second die: $6 \times 6 = 36$. A 7 can be obtained in the following ways: 1,6 or 2,5 or 3,4 or 4,3 or 5,2 or 6,1, where the first number in each pair represents the outcome of the first die and the second number is the outcome of the second die. Thus, there are 6 ways to get a 7, each of which is a mutually exclusive outcome with probability $= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. So the probability of rolling a 7 is $\frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{6}{36} = \frac{1}{6} = .17$.

2. The probability of picking the 4 winning numbers in Novo 4/25 would be computed by treating each draw as an independent event and multiplying the probabilities of each event together: $\frac{4}{25}$ for the first number, $\frac{3}{24}$ for the second number (assuming the first draw was one of the four you picked), $\frac{2}{23}$ for the third number (assuming the first two numbers were ones you picked), and $\frac{1}{22}$ for the final number (assuming the first three numbers were ones you picked). This produces $\frac{4}{25} \times \frac{3}{24} \times \frac{2}{23} \times \frac{1}{22} = \frac{24}{303,600} = \frac{1}{12,650} = .000079$.

3. With 4 flips there are 6 different sequences that yield 2 heads: HHTT, HTHT, HTTH, TTHH, THTH, THHT. There are 4 sequences that yield 1 head: HTTT, THTT, TTHT, TTTH. There are 4 sequences that yield 3 heads (HHHT, HHTH, HTHH, THHH), and each sequence occurs with probability $(.5)(.5)(.5)(.5) = .0625$. The probability of obtaining any one of those 4 sequences is the sum of their individual probabilities, or $4(.0625) = .25$.

4. Note that there are 12 face cards in a deck of 52 cards. One way to answer this question is to find out the probability of there being no face card among the two that are dealt, then subtract that probability from 1. Taking that route, we have the probability of the first card not being a face card $= \frac{40}{52}$. Then the probability of the second card not being a face card is $\frac{39}{51}$. The probability of both of these events occurring is $(\frac{40}{52})(\frac{39}{51}) = .5882$. So the probability of dealing at least one face card is $1 - .5882 = .4118$. Another way to answer the question is to compute the probability of getting each of the three sequences of results that meet the definition of at least one face card then add them together: (1) a face card, then not a face card, (2) not a face card, then a face card, (3) two face cards. For the first sequence, the probability is $(\frac{12}{52})(\frac{40}{51}) = .1810$, for the second sequence, the probability is $(\frac{40}{52})(\frac{12}{51}) = .1810$, and for the third sequence, the probability is $(\frac{12}{52})(\frac{11}{51}) = .0498$. Adding these three probabilities together yields $.1810 + .1810 + .0498 = .4118$. 
5. (a) There are 25 different score values in the distribution, each occurring equally often. A score of 14 occurs as often as each of the other 24 score values in the distribution, so the probability of drawing a 14 is $1/25 = 0.04$. The same goes for a score of 15, so the probability of obtaining either a 14 or a 15 is $0.04 + 0.04 = 0.08$.

(b) The relevant range of score values is 20 to 25 inclusive, which contains 6 different score values. The probability of drawing any one of these score values on the first draw is $6/25 = 0.24$. The same goes for the second draw, so the probability of both draws producing a value in the range of interest is $(0.24)(0.24) = 0.0576$.

6. For variance of the sample to be zero, the two scores would have to be the same. The only way that could happen with this population, is for the two 4's to have been drawn to form the sample. How likely is that to happen? On the first draw, the probability of obtaining a 4 would be $2/8$, and then on the second draw the probability of obtaining the remaining 4 would be $1/7$. So the probability of drawing both 4's would be $(2/8)(1/7) = 2/56 = 0.0357$.

7. (a) The probability of one score being 75 or greater is the area in the smaller portion for $z = (75–60)/10 = 1.5$, or $0.0668$. The probability of obtaining a score from this region on two independent draws is the product of the two individual probabilities (multiplication rule), or $0.0668^2 = 0.0045$.

(b) The probability of obtaining one score of 75 or more and another of 45 or less (in either order) is the product of the individual probabilities comprising one sequence or order (e.g., low first, followed by high) plus the product of probabilities for the outcomes of the other possible sequence (e.g., high first, then low). The probability of obtaining a score of 45 or lower is based on $z = (45–60)/10 = -1.50$; the area in the smaller portion is again $0.0668$. So the probability of obtaining one score in the lower region and one in the higher region, in either order, is $0.0668^2 + 0.0668^2 = 0.0089$.

(c) The relevant outcome is a score that is among the upper 0.05 of the distribution of scores. To find the corresponding raw score, begin by looking for an area of 0.0500 in the $z$ table under the heading Smaller portion. The closest we can come is an area of 0.0505 or 0.0495. These two areas are equally close to what we seek, so we can work with either one of them. Arbitrarily choosing 0.0495, we see that the corresponding $z$ score is 1.65. So the raw score equivalent is $X = 60 + 1.65(10) = 76.5$.

8. (a) The area between 30 and 35 will correspond to the probability of drawing a score from this range. The relevant $z$ scores are: $z = (30–25)/5 = 1.0$ and $z = (35–25)/5 = 2.0$. The smaller portion of the distribution for $z = 1.0$ is $0.1587$ and the smaller portion for $z = 1.0$ is $0.0228$. Taking the difference between these two areas will result in the area between the two cutoffs ($y = 1.0$ and $z = 2.0$). So we have $0.1587 – 0.0228 = 0.1359$ as the probability of drawing a score between 30 and 35. Note that this problem also could have been solved by using either the area between $z$ and the mean or the larger portion.
(b) The probability of one randomly drawn score being less than or equal to 32 is ($z = 32 - 25/5 = 1.4; \text{area in larger portion}) .9192$. For all 8 scores to be in this region, the probability would be $0.9192^8 = 0.5097$, because each draw is an independent event and the probability of the joint occurrence of these 8 outcomes is the product of their individual probabilities.

(c) To be at least two and a half standard deviations away from the mean, a score would have to have a corresponding $z$ value of $\pm 2.5$. For the first person drawn to have such a score, the person could be drawn from either the upper or lower end of the distribution. The proportion of scores in one of these ends (2.5 standard deviations away from the mean) is given by the standard normal distribution table for a $z$ score of 2.5, which is 0.0062. So the probability that the first score drawn comes from one OR the other of these two regions is $0.0062 + 0.0062 = 0.0124$. The probability that the second person comes from one of these two regions would also be 0.0124. So the probability that BOTH people (two independent events each producing a particular outcome) come from one of those regions would be $(0.0124)(0.0124) = 0.00015$. [Note that more than the usual 4 decimal places of accuracy are given here because there are so many leading zeros.]

9. To use R to find the probability of getting 3 heads in 4 flips of a fair coin, we can use the `dbinom` command as follows.

```r
dbinom(3, 4, .5)
```

> [1] 0.25

So the answer is .25. The [1] that you see in the result line just indicates that this is the first element of the result (of course, this result has only one element).

To find the probability of each of the possible outcomes, 0 through to 4 heads, we can use the same command, but specify the full range of outcomes as the first argument. We will place the output into a data variable called `result` so that later we can plot the probabilities. First we will just display the probabilities by printing them out.

```r
result = dbinom(0:4, 4, .5)
> result
[1] 0.0625 0.2500 0.3750 0.2500 0.0625
```

The probabilities are listed in the order corresponding to increasing number of heads, from 0 to 4. You can see that outcomes in the middle of the range are the most likely ones. This can be seen by plotting the set of probabilities.

```r
> plot(result)
```
This plot is symmetrical and even looks a bit like a normal distribution. Repeating this exercise with 20 tosses we have the following.

```r
> result = dbinom(0:20, 20, .5)
> plot(result)
```

The distribution is even more like a normal distribution.

10. The data variable that will hold the sample means is defined and set to be empty.

```r
> means = NULL
```

Now we draw 500 random samples from a normally distributed population with \( \mu = 50 \) and \( \sigma = 10 \), and put the mean of each sample into the variable we called `means`.

```r
> means = replicate(500, c(means, mean(round(rnorm(20, mean=50, sd=10)))))
```

The histogram of the means will look something like this. Your results may differ because of differences in random sampling.

```r
> hist(means, 20)
```
This distribution of sample means is very much like a normal distribution. The smallest mean is around 42 and the largest is around 59.