Analysis of Variance

• Reading: Chapter 18

• Repeated-measures (within-subject) ANOVA
  • one-factor version
  • each subject tested at all levels of the factor
  • conceptually similar to related-samples \( t \) test
  • assess effects of independent variable \( \text{within} \) each subject
    • differences \( \text{between} \) subjects not relevant
  • fewer subjects needed
  • greater power than between-subjects design
  • possible concern with order effects
Analysis of Variance

• Example repeated-measures study
  • influence of mood on personality attributions
    • 3 mood conditions: happy, sad, neutral
    • task: rate “friendliness” of pictures of faces
  • all subjects tested under each mood condition
  • dealing with possible order effects
    • counterbalancing of orders (latin-square design)
      1/3 of subjects: H S N
      1/3 of subjects: S N H
      1/3 of subjects: N H S
Analysis of Variance

- Data from the example repeated-measures study
  - 3 mood conditions: happy, sad, neutral

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</table>

\[M = 6.67 \quad 4.33 \quad 4.50\]

\[GM = 5.17\]
Analysis of Variance

- Assess consistency of effect across subjects
Analysis of Variance

- Why does consistency matter?
- indicates how variable sample means can be

$$\mu_A = \mu_B = 7$$

Population 1

Population 2

Population 3
Analysis of Variance

• Consistency is quantified as an interaction between subjects and conditions

• Three sources of variability in repeated-measures design

\[ SS_{total} = \sum (X - GM)^2 \]

- \[ SS_{subjects} \]
- \[ SS_{conditions} \]
- \[ SS_{error(interaction)} \]
Analysis of Variance

\[ SS_{\text{total}} = \Sigma (X - GM)^2 \]

\[ SS_{\text{subjects}} = k \Sigma (M_{\text{subject}} - GM)^2 \quad (k = \text{number of conditions}) \]

\[ SS_{\text{conditions}} = n \Sigma (M_{\text{condition}} - GM)^2 \quad (n = \text{number of subjects}) \]

\[ SS_{\text{error}} = SS_{\text{total}} - SS_{\text{subjects}} - SS_{\text{conditions}} \]
Analysis of Variance

• Between-subject variability

\[ SS_{\text{subjects}} = k \sum (M_{\text{subjects}} - GM)^2 \]

<table>
<thead>
<tr>
<th>Sub#</th>
<th>Happy</th>
<th>Sad</th>
<th>Neutral</th>
<th>( M_{\text{subj}} )</th>
<th>((M_{\text{subjects}} - GM)^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
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<td>4</td>
<td>4</td>
<td>((4 - 5.17)^2)</td>
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<td>2</td>
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<td>6</td>
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<td>4</td>
<td>((4 - 5.17)^2)</td>
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</tbody>
</table>

\[ M = 6.67 \quad 4.33 \quad 4.50 \]

\[ GM = 5.17 \]

\[ \sum (M_{\text{subjects}} - GM)^2 = 12.83 \]

\[ SS_{\text{subjects}} = 3 \times (12.83) = 38.49 \]

\[ df_{\text{subjects}} = n - 1 = 6 - 1 = 5 \]
Analysis of Variance

• Variability between condition means

\[ SS_{\text{conditions}} = n \sum (M_{\text{conditions}} - GM)^2 \]

\[ = 6 \left[ (6.67 - 5.17)^2 + (4.33 - 5.17)^2 + (4.50 - 5.17)^2 \right] \]

\[ = 6 (3.40) = 20.40 \]

\[ df_{\text{conditions}} = k - 1 \]

\[ = 3 - 1 = 2 \]

\[ MS_{\text{conditions}} = \frac{SS_{\text{conditions}}}{df_{\text{conditions}}} \]

\[ = \frac{20.40}{2} = 10.20 \]
Analysis of Variance

- Interaction between conditions and subjects (error)
  \[ SS_{error} = SS_{total} - SS_{subjects} - SS_{conditions} \]

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</tbody>
</table>

\[ SS_{total} = (6-5.17)^2 + (7-5.17)^2 + \ldots + (3-5.17)^2 \]
\[ = 74.56 \]

\[ SS_{error} = 74.56 - 38.49 - 20.40 \]
\[ = 15.67 \]

\[ df_{error} = (k-1)(n-1) \]
\[ = (3-1)(6-1) \]
\[ = 10 \]

\[ MS_{error} = \frac{SS_{error}}{df_{error}} = \frac{15.67}{10} = 1.57 \]
### Analysis of Variance

#### ANOVA Summary Table

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjects</td>
<td>38.49</td>
<td>5</td>
<td>10.20</td>
<td>6.50</td>
</tr>
<tr>
<td>Conditions</td>
<td>20.40</td>
<td>2</td>
<td>10.20</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>15.67</td>
<td>10</td>
<td>1.57</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>74.56</td>
<td>17</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ F = \frac{MS_{\text{conditions}}}{MS_{\text{error}}} = \frac{10.20}{1.57} = 6.50 \]

\[ \alpha = .05, \text{ critical } F (2, 10)= 4.10 \]
Analysis of Variance

• Which means differ from one another?

• Post hoc comparisons
  • Fisher LSD or Bonferroni correction can be used
  • test difference between specific pairs of means
  • use the value of $MS_{error}$ from the ANOVA and the corresponding $df_{error}$

• from the example data set, make two comparisons
  • Happy vs. Sad
  • Sad vs. Neutral
Analysis of Variance

• Happy ($M = 6.67$) vs. Sad ($M = 4.33$)

\[
t = \frac{M_i - M_j}{\sqrt{\frac{MS_{error}}{n} + \frac{MS_{error}}{n}}} = \frac{6.67 - 4.33}{\sqrt{\frac{1.57}{6} + \frac{1.57}{6}}} = \frac{2.34}{.72} = 3.25
\]

\[
df_{error} = 10 \quad \alpha = .05, \text{ critical } t = \pm 2.228, \text{ reject } H_0
\]

Bonferroni correction: $\alpha/2 = .05/2 = .025$

$t$ table offers $.02$, but not $.025$

$\alpha = .02, \text{ critical } t = \pm 2.764, \text{ reject } H_0$
Analysis of Variance

- Sad ($M = 4.33$) vs. Neutral ($M = 4.50$)

\[ t = \frac{M_i - M_j}{\sqrt{\frac{M_{\text{error}}}{n} + \frac{M_{\text{error}}}{n}}} = \frac{4.50 - 4.33}{\sqrt{\frac{1.57}{6} + \frac{1.57}{6}}} = \frac{0.17}{0.72} = 0.24 \]

\[ df_{\text{error}} = 10 \]

\[ \alpha = .05, \text{ critical } t = \pm 2.228, \text{ do not reject } H_0 \]
Analysis of Variance

• Demonstration that $MS_{\text{error}}$ is not affected by whether $H_0$ is true

Population 2

$MS_{\text{error}} = 0.40$

Condition
A    B
4    5
10   9

Condition
A    B
4    5
12   11

Analysis of Variance

• Assumptions of the repeated-measures design
  • sphericity: homogeneity of variance and of variance of difference scores

<table>
<thead>
<tr>
<th>Sub#</th>
<th>Happy</th>
<th>Sad</th>
<th>Neutral</th>
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<th>H-N</th>
<th>S-N</th>
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</table>

\[ s^2 = 4.67 \quad 1.87 \quad 4.30 \quad s^2_D = 4.27 \quad 2.17 \quad 2.97 \]
Analysis of Variance

• One-factor, repeated measures ANOVA
• Effect of encoding task on implicit memory
  • 3 conditions
    • read (young)
    • generate (hot-?)
    • non-studied
  • dependent measure: correct word identification
Analysis of Variance

• Number correct (out of 10 per condition)

<table>
<thead>
<tr>
<th>S#</th>
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<th>Nonst.</th>
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<tbody>
<tr>
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<td>8</td>
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</tbody>
</table>

\[
GM = \frac{\sum X}{N} = \frac{5 + \ldots + 1}{24} = \frac{96}{24} = 4.0
\]

\[
SS_{total} = \sum (X - GM)^2 = (5 - 4)^2 + \ldots + (1 - 4)^2 = 98.0
\]

\[
df_{total} = k(n) - 1 = 3(8) - 1 = 23
\]
Analysis of Variance

• Number correct (out of 10 per condition)

<table>
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<tr>
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<td>8</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>3.00</td>
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</table>

\[
SS_{\text{subjects}} = k \sum (M_{\text{subj}} - GM)^2 \quad \text{GM} = 4.00
\]

\[
= 3[(3.33 - 4)^2 + \ldots + (3 - 4)^2 ]
\]

\[
= 47.33
\]
Analysis of Variance

• Number correct (out of 10 per condition)

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</table>

$SS_{\text{conditions}} = n \sum (M_{\text{cond}} - GM)^2$

$= 8[(5.5 - 4)^2 + (3 - 4)^2 + (3.5 - 4)^2] = 28.00$

$GM = 4.00$

$df_{\text{conditions}} = k - 1$

$= 3 - 1 = 2$

$MS_{\text{conditions}} = \frac{28}{2} = 14$
Analysis of Variance

• Number correct (out of 10 per condition)

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</table>

\[
SS_{\text{error}} = SS_{\text{total}} - SS_{\text{subjects}} - SS_{\text{conditions}} = 98 - 47.33 - 28 = 22.67
\]

\[
df_{\text{error}} = (k - 1)(n - 1) = 2(7) = 14
\]

\[
MS_{\text{error}} = \frac{22.67}{14} = 1.62
\]
Analysis of Variance

ANOVA summary table

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjects</td>
<td>7</td>
<td>47.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conditions</td>
<td>2</td>
<td>28.00</td>
<td>14.00</td>
<td>8.64</td>
</tr>
<tr>
<td>Error</td>
<td>14</td>
<td>22.67</td>
<td>1.62</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>23</td>
<td>98.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \alpha = .05, \text{ critical } F (2, 14) = 3.74 \)

<table>
<thead>
<tr>
<th></th>
<th>Read</th>
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<th>Nonst.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M )</td>
<td>5.5</td>
<td>3.0</td>
<td>3.5</td>
</tr>
</tbody>
</table>
**Analysis of Variance**

- **Fisher LSD test**
  \[ t = \frac{M_i - M_j}{\sqrt{\frac{MS_{error}}{n} + \frac{MS_{error}}{n}}} \]
  \[ = \frac{5.5 - 3.5}{\sqrt{\frac{1.62}{8} + \frac{1.62}{8}}} = \frac{2}{0.636} = 3.14 \]

- **Power to detect a large effect size** -- \( d = .8 \)
  \[ \delta = d \sqrt{\frac{n}{2(1 - \rho)}} = .8 \sqrt{\frac{8}{2(1 -.71)}} = 2.97 \]
  Power = .85

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<tr>
<td></td>
<td>5.5</td>
<td>3.0</td>
<td>3.5</td>
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\( df_{error} = 14 \quad \alpha = .05 \)

Critical \( t = \pm 2.145 \)
Analysis of Variance

• Repeated-measures ANOVA in R

```r
> word = read.table(file.choose(new=F),header=T)
> head(word)
        subj score cond
1        s1     5  read
2        s1     3  gen
3        s1     2  new
4        s2     4  read
5        s2     2  gen
6        s2     3  new
```
Analysis of Variance

• Repeated-measures ANOVA in R

```r
> res=aov(score~cond+Error(subj/cond),word)
> summary(res)

Error: subj
     Df Sum Sq Mean Sq F value Pr(>F)
Residuals  7  47.33   6.762

Error: subj:cond
     Df Sum Sq Mean Sq    F value Pr(>F)
cond       2  28.00  14.000    8.647  0.00359 **
Residuals 14  22.67   1.619
```
Analysis of Variance

• Repeated-measures ANOVA in R

```r
> pairwise.t.test(word$score, word$cond, p.adjust = "none", paired = T)

Pairwise comparisons using paired t tests

data:  word$score and word$cond

    gen new
new 0.4869 -
read 0.0016 0.0255

P value adjustment method: none
```
Analysis of Variance

• Bayesian analysis of repeated-measures design

```r
> anovaBF(score~cond+subj, data=word, whichRandom="subj")

  Bayes factor analysis
  ------------
[1]  cond + subj : 15.80473 ±0.62%

Against denominator:
  score ~ subj
---

Bayes factor type: BFlinearModel, JZS
Analysis of Variance

• Bayesian analysis of repeated-measures design
  • pairwise comparison, new vs. read

```r
> new = subset(word, cond=="new")
> read = subset(word, cond=="read")
> ttestBF(new$score, read$score, paired = T)
Bayes factor analysis
-------------
[1] Alt., r=0.707 : 3.162649 ±0%

Against denominator:
  Null, mu = 0
---
Bayes factor type: BFoneSample, JZS