Assignment 5
February 20, 2017

\[ s_{M_1-M_2} = \sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}} \quad t = \frac{M_1 - M_2}{s_{M_1-M_2}} \quad \delta = d\sqrt{\frac{n}{2}} \]

\[ SS_{\text{group}} = n\sum (M_j - GM)^2 \quad SS_{\text{error}} = \sum (X_{ij} - M_j)^2 \quad SS_{\text{total}} = \sum (X - GM)^2 \]

\[ SS_{\text{error}} = s_1^2(n-1) + s_2^2(n-1) + \ldots + s_k^2(n-1) = \sum s^2(n-1) \]

\[ df_{\text{group}} = k-1 \quad df_{\text{error}} = k(n-1) \quad df_{\text{total}} = k(n) - 1 \]

\[ MS_{\text{error}} = \frac{SS_{\text{error}}}{df_{\text{error}}} \quad MS_{\text{group}} = \frac{SS_{\text{group}}}{df_{\text{group}}} \quad F = \frac{MS_{\text{group}}}{MS_{\text{error}}} \]

\[ t = \frac{M_1 - M_j}{\sqrt{\frac{MS_{\text{error}}}{n} + \frac{MS_{\text{error}}}{n}}} \quad \eta^2 = \frac{SS_{\text{group}}}{SS_{\text{total}}} \]

1. Which population parameter is estimated by \( MS_{\text{error}} \) in an analysis of variance?

2. The following data are error scores for two hypothetical groups of subjects on a typing test. One group was tested under conditions of high noise and one was tested under conditions of normal office noise. Carry out an independent samples \( t \) test and then carry out an ANOVA. In both cases, you are testing the null hypothesis that noise has no effect on errors. Set \( \alpha = .05 \) in each case and for the \( t \) test, assume a nondirectional alternative hypothesis. Compare the obtained \( t \) ratio and the obtained \( F \) ratio. Notice that \( t^2 = F \). This will always be the case, within rounding error. What value in the ANOVA computation is equal to the pooled estimate of variance obtained when computing the \( t \) ratio?

| High noise: | 6 9 12 6 7 |
| Normal: | 4 7 2 3 4 |

3. Provide the missing values in the following ANOVA summary table which is based on an experiment involving four groups.

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
<td>12.05</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td></td>
<td>44</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>84.12</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4. In a one-factor analysis of variance based on a sample of data, if $SS_{group} = 0$, what does this one fact necessarily imply? Select one answer.
(a) all scores within each group are equal
(b) the null hypothesis is true
(c) the F-ratio will be significant
(d) all obtained sample means are equal
(e) two of the above

5. In a one-factor analysis of variance based on a sample of data, if $SS_{error} = 0$, what does this one fact imply? Select one answer.
(a) all raw scores in the experiment are the same
(b) the null hypothesis is true
(c) the raw scores within a group are equal but scores may differ between groups
(d) all condition means are equal
(e) the F-ratio will be equal to 1
(f) two of the above

6. Three groups of subjects were formed by random assignment. Subjects were asked to read a story about a social event, then to describe it to another person. Each subject in group 1 described the event to someone younger than herself or himself, subjects in group 2 each described the event to someone of approximately the same age, and subjects in group 3 each described the event to someone older. The experimenter measured the number of nonverbal behaviors that each subject used while describing the event. The raw data for each group are shown below. Compute an analysis of variance. Decide whether the null hypothesis can be rejected with $\alpha = .05$. Assuming the null hypothesis is rejected, conduct a Fisher's LSD test to compare each pair of groups. You will find that one of the comparisons is not significant. How much power does this study have to find a large-size difference ($d = .8$) between two groups, assuming a nondirectional test? Based on the outcome of these comparisons, and keeping in mind the results of the power estimate, what conclusions can you draw regarding the relationship between age of the audience relative to the storyteller and the number of nonverbal behaviors used by the story teller?

| Group 1: | 6 | 4 | 3 | 3 |
| Group 2: | 8 | 5 | 9 | 10 |
| Group 3: | 10 | 12 | 8 | 14 |

It is also instructive to apply a Bayesian test that compares the two groups that were found to not be significantly different in the multiple comparison tests you computed above. Enter the four scores for each group into R using these commands (you can even copy and paste them):

```R
> g2 = c(8, 5, 9, 10)
> g3 = c(10, 12, 8, 14)
```

Then call the `BayesFactor` package from the library and use the `ttestBF` command to compare these two groups. What can you conclude from the resulting Bayes factor?

7. A developmental psychologist carried out a study to test the null hypothesis that there are no age differences among young children with respect to their tendency to defy their parents. The researcher tested three age groups: 4, 6, and 8 yrs. An analysis of variance was computed and a significant $F$ ratio was obtained. The researcher's assistant then informed her that there was an error in the data for two of
the 6-year old subjects. When the erroneous scores were replaced by the correct scores it was found that the mean for the 6-year old group was unchanged, but the variance for that group increased. When the data are analyzed again using ANOVA, what change (increase or decrease), if any, will be found in $MS_{\text{group}}$, $MS_{\text{error}}$, and the $F$ ratio?

8. A study is carried out to test the effectiveness of four different treatments (A, B, C, and D) for manic-depression. The mean behavioral adjustment score, based on 16 subjects in each condition, and the variance of scores within each condition are shown below.
(a) Compute an analysis of variance and construct an ANOVA summary table.
(b) Test the $F$ ratio for significance using $\alpha = .05$.
(c) Compute $\eta^2$ and state what it means.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>6.0</td>
<td>6.0</td>
<td>5.0</td>
<td>7.0</td>
</tr>
<tr>
<td>$s^2$</td>
<td>4.2</td>
<td>4.6</td>
<td>4.0</td>
<td>4.2</td>
</tr>
</tbody>
</table>

9. A psychologist tests three different methods of improving the degree of social interaction exhibited by autistic children to determine which method or methods should be recommended for use in a clinical setting. A sample of 36 autistic children is randomly assigned to three conditions, with 12 children in each treatment group. The mean degree of social interaction for each group upon completion of training is shown below (higher scores correspond to more interaction). The researcher conducts an ANOVA and finds $F(2, 33) = 8.26$, with $MS_{\text{error}} = 8.08$. The researcher then conducts pairwise comparisons between each pair of methods using the Bonferroni procedure.

<table>
<thead>
<tr>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.4</td>
<td>11.6</td>
<td>13.0</td>
</tr>
</tbody>
</table>

(a) Compute the $t$ test for the comparison of Method 1 vs. Method 2. The $t$ ratios for the other two comparisons are: Method 1 vs. Method 3 $t(33) = -3.96$; Method 2 vs. Method 3 $t(33) = -1.21$.
(b) According to the Bonferroni correction, what comparison-wise $\alpha$ must be used for each of these comparisons to keep familywise $\alpha = .05$?
(c) Using the Bonferroni correction, determine which of these three comparisons is significant. If the comparison-wise $\alpha$ you need does not appear in the $t$ table, then round comparison-wise $\alpha$ to the next lower $\alpha$ value that is in the $t$ table to do these tests. Keeping in mind the value of $\alpha$ for each comparison under the Bonferroni correction, how much power does this test have to detect a large effect size ($d = .8$) using the present sample size of $n = 12$? You will have to deal with comparison-wise $\alpha$ when referring to Howell's power table in the same way you did when evaluating the $t$ test. You can also check your work and use a more precise value for $\alpha$ by using the `pwr.t.test` function in R.
(d) On the basis of these results, and assuming that more social interaction is a desirable outcome, which method or methods of treatment should the researcher recommend?
(e) What would your answers to parts (a), (c), and (d) be if there had been 40 subjects in each condition, rather than just 12? Assume the means and $MS_{\text{error}}$ remain unchanged. This means that the $t$ ratios for the pairwise comparisons between Methods 1 and 3 would be -7.24 and between Methods 2 and 3 would be -2.20.
10. Assume that the $F$ distribution in the diagram below is the one that corresponds to 2 $df$ for the numerator and 30 $df$ for the denominator, under the assumption that the null hypothesis is true.
(a) Draw a second distribution in this diagram that would roughly correspond to an $F$ distribution with 2 and 30 $df$ when the null hypothesis is false.
(b) Now suppose that the null hypothesis is true, but a fourth group is added to the design. Draw a new $F$ distribution in this diagram based on 3 and 40 $df$ when the null hypothesis is true. Hint: Examine the table of critical $F$ values to see how the $F$ distribution changes as the $df$ values for the numerator and denominator increase.

11. A data file called learn.txt is available on the course web site for this question. This data file contains data from a hypothetical drug study involving four groups of rats that learned to navigate through a maze to earn a reward. The first column, labeled cond, indicates the conditions and the second column, labeled learn, indicates the scores. One group (labeled "a" in the data file) was tested after being administered Drug A and another group (labeled "b" in the data file) was tested under Drug B. For both groups, the drug was administered through an injection. A third group (labeled "c" in the data file) was given a placebo injection and a fourth group (labeled "d" in the data file) was given no treatment at all. For all four groups, the researcher measured how many trials each animal needed to learn the maze perfectly (i.e., travel the maze from start to end without making an error). Note that fewer trials needed to learn means better learning.

Using the steps illustrated in the instructions on using R (section 11a) for the one-factor ANOVA, compute a one-factor ANOVA for these data using the aov function. You will see that the effect of condition is significant, so proceed to the Fisher's LSD multiple comparison procedure using the pairwise.t.test function. It will also be helpful to see the means for each condition, so load the psych library and use the describeBy function to get the descriptive statistics for this study. To answer each question below, indicate which two conditions should be compared and indicate the outcome of the test.
(a) Is there evidence here to support the claim that performance is better under one drug than under the other?
(b) Is there evidence for a placebo effect?
(c) Is there evidence that Drug A is effective? Keep in mind the outcome of test (b) above when deciding which conditions to compare here.
(d) Is there evidence that Drug B is effective? Keep in mind the outcome of test (b) above when deciding which conditions to compare here.

Now suppose that instead of using Fisher's LSD test, the researcher used the Bonferroni test for the four pairwise comparisons above. Note that the total number of comparisons the researcher is making is four, not all six possible comparisons. What, if any, changes in the outcome of the pairwise comparisons
would occur, relative to what was found with the LSD test? You can use the output from the `pairwise.t.test` function that you have already run, but apply the Bonferroni correction this time, keeping in mind that four comparisons are being made.

12. Apply a Bayesian analysis to the data in the `learn.txt` file. The `anovaBF` function can be used for the overall comparison among the four groups. The form of the command (after calling `BayesFactor` from the library) would be (assuming the data have been read into an R data frame called `maze`):

```r
> anovaBF(learn ~ cond, data = maze)
```

What is the Bayes factor produced by this analysis and what does it allow you to conclude?

You can also do pairwise comparisons with a Bayesian analysis, making use of the `ttestBF` function. You will first need to isolate the data from the two conditions being compared by using the `subset` function. Try this out with the following two pairwise comparisons: Drug A vs. Drug B, and Drug B vs. placebo. To get you started, here is how the `subset` command can be used to select from the full data frame the data for the two groups to be included in the first pairwise comparison:

```r
> DrugA = subset(maze, cond == "a")
> DrugB = subset(maze, cond == "b")
```

Then the `ttestBF` command can be applied to these two sets of data, `DrugA` and `DrugB`. But notice that these two variables each include two columns, one for the condition label and one for the number of trials needed to learn, just like the original data frame. So the `ttestBF` command has to be set up as follows to specify the relevant column of data to be used (i.e., the column containing the scores):

```r
> ttestBF(DrugA$learn, DrugB$learn)
```

Next, follow the same steps for the Drug B vs. placebo comparison. What can you conclude from these two comparisons? How do these conclusions differ, if at all, from the conclusions reached when using null-hypothesis significance testing to make these two comparisons?