MODELLING THERMAL EDDY DIFFUSIVITY AT CANOPY HEIGHT

L. A. MEHLENBACHER* and D. W. A. WHITFIELD
Department of Botany, University of Alberta, Edmonton, Alberta, Canada

(Received 5 November 1976)

Abstract. A one-dimensional, steady-state plant-atmosphere model using different formulae for the thermal stability function is applied to data for corn crops. There are two general types of formulae available. Those proposed by W. C. Swinbank, A. J. Dyer and B. B. Hicks, and E. K. Webb were derived from measurements taken hundreds or thousands of roughness lengths above grass fields. Formulae recently proposed by A. S. Thom, J. B. Stewart, H. R. Oliver and J. H. C. Gash were derived from measurements taken nine roughness lengths above a pine forest. Use of the latter formulae yields better agreement between predicted and measured values of thermal eddy diffusivity at canopy height in the corn crops. These improved diffusivity values result in improved temperature-profile predictions in the top metre of the canopy.

List of Symbols

A =	Leaf area density	$\mathrm{m^2\,m^{-3}}$
B_1, B_2, B_3 =	Stability parameters	
Cf =	Stability parameter; defined by $0.075(\zeta_{*0} - \zeta_{RH})$	
$CLAI$ =	Cumulative leaf area index	$\mathrm{m^2\,m^{-2}}$
$CNDS$ =	Thermal conductivity of soil; -0.50	
Cp =	Specific heat of air; 1000.0	$\mathrm{J\,kg^{-1}\,{}^\circ C^{-1}}$
D =	Displacement height	m
g =	Acceleration of gravity; 9.8	$\mathrm{m\,s^{-2}}$
k =	von Kármán's constant; 0.41	
Kh =	Thermal eddy diffusivity	$\mathrm{m^2\,s^{-1}}$
$Khmd$ =	Thermal molecular diffusivity	$\mathrm{m^2\,s^{-1}}$
LE =	Latent heat flux	$\mathrm{W\,m^{-2}}$
Lmo =	Monin–Obukhov length	m
Lt =	Latent heat of vaporization	$\mathrm{J\,kg^{-1}}$
LW =	Long-wave radiation intensity	$\mathrm{W\,m^{-2}}$
LWA =	Net absorbed long-wave radiation	$\mathrm{W\,m^{-2}}$
P_{atm} =	Atmospheric pressure; 1.013×10^5	$\mathrm{N\,m^{-2}}$
Q =	Air specific humidity	$\mathrm{kg\,kg^{-1}}$
Q_1 =	Soil surface specific humidity	$\mathrm{kg\,kg^{-1}}$
QL =	Leaf specific humidity	$\mathrm{kg\,kg^{-1}}$
ra =	Leaf boundary-layer resistance	$\mathrm{s\,m^{-1}}$
R_{m-n} =	Diffusion resistance between level m and level n	$\mathrm{s\,m^{-1}}$
RHS =	Soil surface relative humidity	
Ri =	Richardson number	
rs =	Leaf resistance	$\mathrm{s\,m^{-1}}$
SH =	Sensible heat flux	$\mathrm{W\,m^{-2}}$
SW =	Short-wave radiation flux	$\mathrm{W\,m^{-2}}$
SWA =	Absorbed short-wave radiation	$\mathrm{W\,m^{-2}}$
T =	Air temperature	°C
T_1 =	Soil surface temperature	°C
$TS1$ =	Soil temperature at 5 cm	°C
TL =	Leaf temperature	°C

---

* Present Address: Department of Animal Science, University of Alberta.

Boundary-Layer Meteorology **12** (1977) 153–170. *All Rights Reserved*

U = Wind speed $\mathrm{m\,s^{-1}}$
U_* = Friction velocity $\mathrm{m\,s^{-1}}$
V = View factor
Z = Height above soil surface m
Zb = Vertical range of molecular diffusion; 0.001 m
Z_0 = Roughness length m

ε = Stability parameter; defined by $(Z-D)/Lmo$
ζ = Stability parameter; defined by $(Z-D)/Z_o$
ζ_{*0} = Value of ζ for neutral stability above which the surface appears smooth
ρ = Air density $\mathrm{kg\,m^{-3}}$
σ = Stefan's constant; 5.57×10^{-8} $\mathrm{W\,m^{-2}\,K^{-4}}$
ϕe = Stability function for water vapour
ϕh = Stability function for sensible heat
ϕm = Stability function for momentum
ψh = Integrated thermal stability parameter; defined by $\int((1-\phi h)/\varepsilon)\,d\varepsilon$
ψm = Integrated momentum stability parameter defined by $\int((1-\phi m)/\varepsilon)\,d\varepsilon$

Subscripts
CH = Denotes canopy height
RH = Denotes reference height

1. Introduction

Models of vertical heat transfer in plant canopies which are being used to predict temperature and humidity profiles require accurate values of thermal eddy diffusivity at canopy height (Legg and Monteith, 1975, p. 181). These values can be obtained by measurements of temperature and humidity gradients at canopy height (Wright and Brown, 1967), as is done in the models of Waggoner and Reifsnyder (1968) and Goudriaan and Waggoner (1972), or by modelling the boundary layer above the canopy, as in the model of Stewart and Lemon (1969). Since the purpose of the model is to predict the temperature and humidity gradients, we prefer the second approach.

The basis of modelling thermal eddy diffusivity at canopy height is the formula:

$$Kh_{CH} = \frac{U_* k\,(Z_{CH} - D)}{\phi h(\varepsilon_{CH})}$$

(Dyer, 1974), and so we are concerned with determination of the thermal stability function ϕh. Stewart and Lemon (1969) used Swinbank's formula for ϕh (Swinbank, 1964) derived from measurements taken several metres above short grass. As pointed out by Thom *et al.* (1975), this means that a formula derived from measurements taken on the order of hundreds or thousands of roughness lengths above a canopy was being used in a situation where measurements were taken on the order of ten roughness lengths above a canopy. Therefore, it is logically understandable that the thermal eddy diffusivity predicted by this formula was only half of the measured value, although the physical cause of the discrepancy is not established (Thom *et al.*, 1975). Even though the formula for ϕh has been refined in recent years (see Webb, 1970; Dyer and Hicks, 1970; Businger *et al.*, 1971; Pruitt *et al.*, 1973;

and Högström, 1974; or see the review by Dyer, 1974), it should probably be applied only when the reference height is over 25 roughness lengths above the displacement height. For situations in which it is lower, Thom has proposed some adjustments to the formulae of Dyer and Hicks and Webb. We use a plant-atmosphere model similar to Stewart's and find that using the more recent boundary-layer formulae improves its predictions not only for the corn crop of Stewart and Lemon but also for the more intensively investigated corn crop of Brown, Covey, and Wright (Brown and Covey, 1966; Wright and Brown, 1967).

2. Model

The one-dimensional steady-state plant-atmosphere model assumes that the vegetation is homogeneous and of large areal extent. The upper and lower boundaries are, respectively, a reference height above the canopy and a small depth, 5 cm, below the soil surface. At the reference height above the stand, measurements of wind speed, temperature, specific humidity, plus short-wave and long-wave radiation are required. At the lower boundary, the temperature at 5 cm depth is needed. The soil water potential at 5 cm together with a soil-water model could be used to predict the relative humidity of the soil surface. However, we have chosen to use the relative humidity of the soil surface as a driving variable. For each set of boundary variables, the model predicts the wind speed, temperature, humidity, and diffusivity profiles between the soil surface and the reference height, as well as the overall fluxes of sensible and latent heat from the canopy.

We can divide the model into six sections, which we briefly discuss here. A complete list of the equations is presented in Appendix A.

2.1. RADIATION

A simple radiation model was considered sufficient for our purposes. Exponential attenuation of short-wave radiation with extinction coefficient of 0.65 provides as accurate a fit to the measured radiation as the Duncan model (Stewart and Lemon, 1969, Figure 15). Absorption coefficients on 0.82 and 0.87 were used for corn leaves and soil, respectively, as suggested by Stewart and Lemon (1969, p. 14).

For net long-wave radiation, the formula of Murphy and Knoerr (1972, p. 45) was used, with the view factors being derived empirically for a corn crop using Stewart and Lemon's data for transmission of diffuse radiation (Stewart and Lemon, 1969, Figure 17).

2.2 BOUNDARY LAYER ABOVE THE CANOPY

The boundary layer above the canopy is described by a set of integrated stability functions (Murphy and Knoerr, 1975, Equations 4, 5, 6). Table I shows the different stability formulae used in the model. Although we are concerned only with unstable conditions, the formulae for stable conditions are also presented in Table II. Thom's formulae have been modified slightly to make B_1, B_2, B_3 functions of Ri rather than

TABLE I

Stability functions (unstable)

Author	ϕm	ϕh
Swinbank	$\mathrm{Ri}\,(1-\exp(-\mathrm{Ri}))^{-1}$	ϕm^2
Dyer and Hicks	$(1-16\,\mathrm{Ri})^{-0.25}$	ϕm^2
Thom		
$-0.12<\mathrm{Ri}\leq 0$	$(1-16\,\mathrm{Ri})^{-0.25}$	$\phi m^2/(1+Cf+B_1\,\mathrm{Ri})^{a}$
$-0.70<\mathrm{Ri}\leq -0.12$	$(1-16\,\mathrm{Ri})^{-0.25}$	$\phi m^2/(1+Cf+B_2\mathrm{Ri})^{b}$
$\mathrm{Ri}\leq -.0.7$	$6.523Cf/1.2$	ϕm^2

[a] $Cf=0.075(\zeta_{*0}-\zeta_{RH})$.
$B_1=[6.523\,Cf/1.2/2.23-(Cf+1)]/-0.12$.
[b] $B_2=[6.523\,Cf/1.2/(1\text{-}16\,\mathrm{Ri})^{0.75}-(Cf+1)]/\mathrm{Ri}$.

TABLE II

Stability functions (stable)

Author	ϕm	ϕh
Webb	$(1-5.2\,\mathrm{Ri})^{-1}$	ϕm
Thom		
$0\leq\mathrm{Ri}\leq 0.02$	$(1-5.2\,\mathrm{Ri})^{-1}$	$\phi m(1+Cf+B_3\mathrm{Ri})^{a}$
$0.02<\mathrm{Ri}\leq 0.14$	$(1-5.2\,\mathrm{Ri})^{-1}$	ϕm

[a] $B_3=-Cf/0.02$.

constants. The deviations for rough surfaces from the formulae for smooth surfaces are functions of Cf and Ri or ε, where

$$\varepsilon=\frac{Z_{RH}-D}{Lmo}=\mathrm{Ri}\,\frac{\phi m^2}{\phi h}\,.$$

Therefore, in Figure 1 we have illustrated ϕh as a function of ε in a manner consistent with Businger *et al.* (1971), Pruitt *et al.* (1973), and Dyer (1974); and in Figure 2 we have illustrated $F=(\phi h\,\phi m)^{-1}$ as a function of Ri in a manner consistent with Thom *et al.* (1975, Figure 1). Both figures show the graphs of the formulae of Swinbank, Dyer and Hicks, Webb, and Thom. ζ_{*0} is the number of roughness lengths above displacement height that one must be before the canopy top can be treated as a smooth surface, at which point the formulae of Dyer and Hicks and Webb become applicable. It is dependent upon the roughness of the vegetation surface and is estimated at 25 for a pine forest but is probably less for a crop such as corn (Thom *et al.*, 1975). So we chose $\zeta_{*0}=20$ as a reasonable estimate. Since $\zeta_{RH}=(Z_{RH}-D)/Z_0$, Cf is dependent on two more crop-specific numbers D and Z_0. However, it is unlikely that these are constant (Monteith, 1973, pp. 92–94). Maki (1969) presents the most comprehensive study of D and Z_0 as functions of U_{RH} for a corn crop and gives two sets of data (July 24–25; Aug. 1–9). Therefore we have taken the average of these to

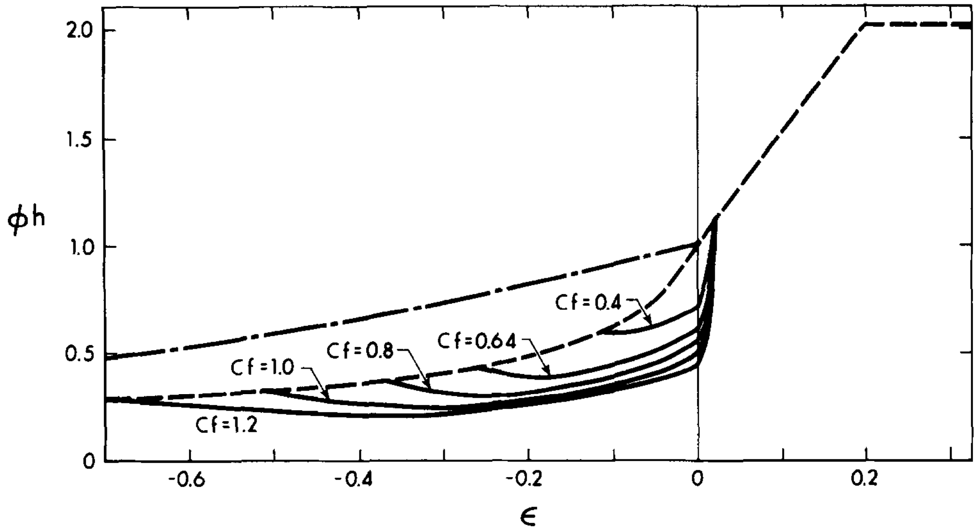

Fig. 1. Thermal stability function ϕh as a function of ε. *Unstable*: — — — Swinbank; ——– —— Dyer and Hicks; ——— Thom.

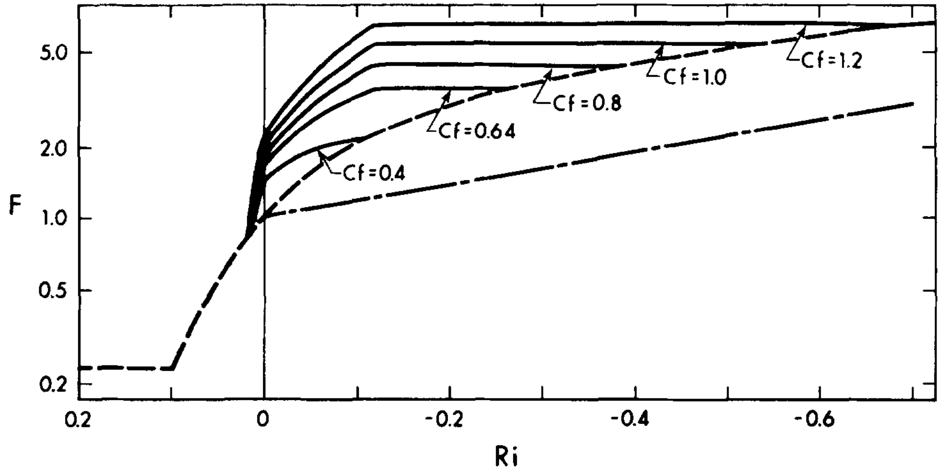

Fig.2. Stability factor $F = (\phi h \phi m)^{-1}$ as a function of Ri. *Unstable*: — — — Swinbank; ——– —— Dyer and Hicks; ——— Thom. *Stable*: ——– —— Webb; ——— Thom.

apply to the corn crop of Brown, Covey, and Wright. Unfortunately, we do not have a complete set of wind speed data, so most of the U_{RH} are interpolated. Figure 3 shows the variation of U_{RH}, D/Z_{CH} and Z_0/Z_{CH} throughout the day for the corn crop of Brown, Covey, and Wright. For the corn crop of Stewart and Lemon we have used $D/Z_{CH} = 0.75$ and $Z_0/Z_{CH} = 0.08$ as suggested by Thom *et al.* (1975, p. 97).

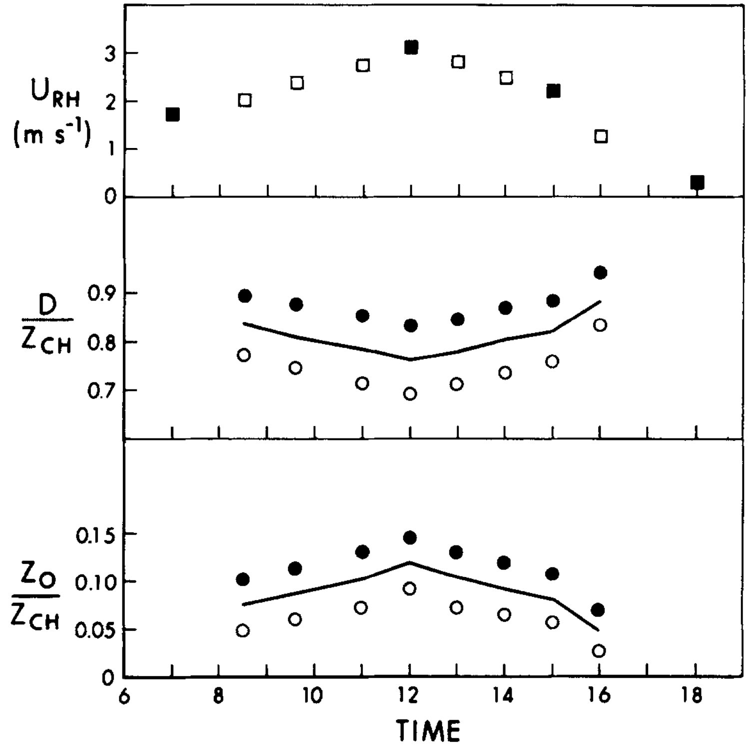

Fig.3. U_{RH}, D/Z_{CH}, and Z_0/Z_{CH} as functions of time. U_{RH}: ■ measurements; □ interpolations. D/Z_{CH} and Z_0/Z_{CH}: ● July 24–25; ○ August 1–9; ——— average.

2.3. AIR IN THE CANOPY

Wind speed and eddy diffusivity are described by exponential relationships such as those given by Cionco (1972). Temperature and specific humidity are described by diffusion equations (Stewart and Lemon, 1969, Equation 120; Murphy and Knoerr, 1975, Equations 1, 2, 15 and 16). For the eddy diffusivity of water vapour we use thermal eddy diffusivity, since Högström (1974) has shown that $\phi h = \phi e$ in unstable conditions.

2.4. CORN LEAVES

The temperature of the leaves is found as the solution of a steady-state energy balance equation at each level, and humidity inside the leaves is considered to be the saturation specific humidity at the leaf temperatures. Stomatal resistance is a key factor in the correct prediction of fluxes (Lemon *et al.*, 1973). Raschke (1970, p. 420) has shown that resistance to water vapour loss in corn leaves was 5 to 9 times higher in dry air than in moist air. Therefore, we use the basic equation of Stewart and Lemon (1969, p. 55) and multiply it by $(QL(Z) - Q(Z))/\Delta Q_{\text{crit}}$ where ΔQ_{crit} is the lowest value of the leaf-air humidity gradient which affects the stomatal resistance.

We found that $\Delta Q_{\text{crit}} = 0.004$ for the corn crop of Stewart and Lemon and $\Delta Q_{\text{crit}} = 0.006$ for the corn crop of Brown, Covey, and Wright resulted in good agreement between heat flux predictions and measurements.

2.5. AIR ABOVE SOIL SURFACE

Stewart and Lemon (1969) and Murphy and Knoerr (1975) couple the differential equations describing the turbulent processes in the canopy directly to the soil or litter surface. This method forces the profiles of temperature and humidity to be unrealistically dependent upon the conditions of the soil surface. A more realistic approach which accounts for the boundary layer above the soil surface was used by Goudriaan and Waggoner (1972, p. 112). They used the height of the boundary layer Zb as an intermediate level between the soil surface and the canopy. We use a higher level, on the order of centimetres rather than millimetres, to allow a potential for comparison with measurements. We uncouple the air in the canopy from the soil surface by expressing the conditions of the first level above the soil surface as functions of the conditions of the soil surface, the conditions of the lowest level in the stand, and the resistances between these levels and the intermediate level above the soil surface.

2.6. SOIL SURFACE

The soil surface is described by an energy balance equation (Stewart and Lemon, 1969, Equation 4).

3. Data

Data to use in the model and to test the results of the model were obtained from the literature. Data for the corn crop of Stewart and Lemon for Aug. 18, 1968 (12:00) were obtained from Stewart and Lemon (1969) and Lemon *et al.* (1973). The 4.2 m level was used as the reference height, since measurements were available in both

TABLE III

Driving variables

Variable	Stewart and Lemon (18 August 1968)	Brown, Covey, and Wright (12 September, 1962)							
		8:25	9:35	11:08	12:00	13:05	14:02	15:03	16:02
U_{RH}	2.2	2.05	2.4	2.75	3.1	2.8	2.5	2.2	1.2
I_{RH}	19.7	14.3	16.5	17.6	18.6	19.1	20.0	20.2	20.6
Q_{RH}	0.007	0.008	0.0077	0.0074	0.0072	0.0069	0.0076	0.0079	0.0084
SW_{RH}	907	195	515	840	860	815	710	560	350
LW_{RH}	339	309	345	351	350	368	365	361	346
$TS1$	24.5	12.5	14.5	15.0	16.5	17.0	17.5	18.0	18.0
RHS	0.51	0.92	0.92	0.90	0.85	0.75	0.75	0.75	0.75

publications. In case of discrepancy, the 1973 publication was used. Leaf area was obtained from Stewart and Lemon (1969, Figure 8), radiation from Lemon *et al.* (1973, p. 73), profiles of wind, humidity, temperature, and diffusivity from Stewart and Lemon (1969, Figures 31(b), 33(a), 33(b)) and Lemon *et al.* (1973, Figures 4, 5), and heat fluxes from Stewart and Lemon (1969, Table II). Data for the corn crop of

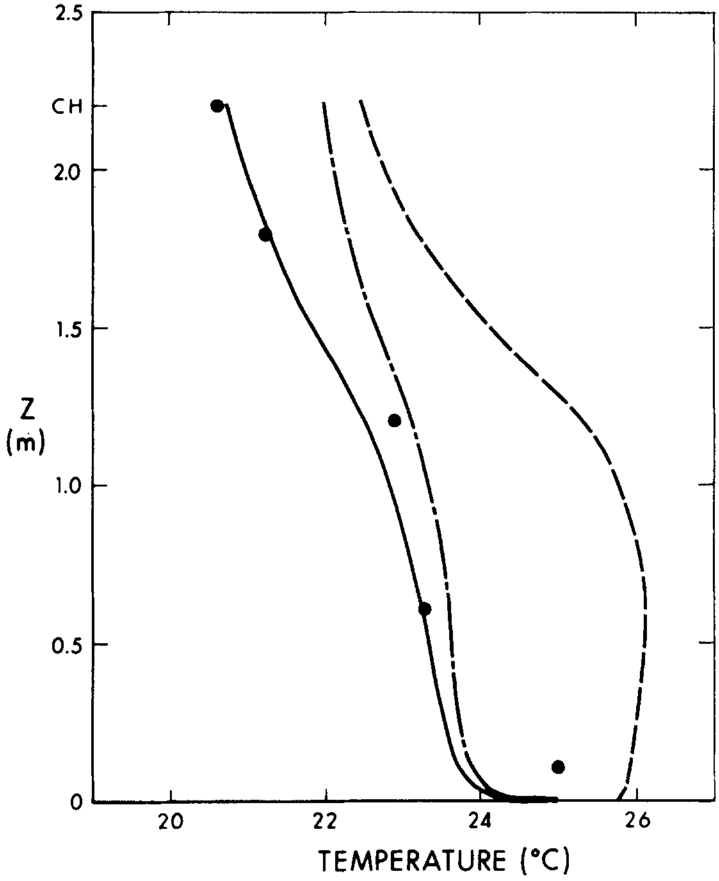

Fig.4. Air-temperature profile for the corn crop of Stewart and Lemon (12:00 noon, August 18, 1968). CH denotes canopy height. ● measurements; — — — Swinbank; — — — Dyer and Hicks; ——— Thom.

TABLE IV

Modelled and measured fluxes and Kh_{CH} (corn crop of Stewart and Lemon)

	Measured	Swinbank	Dyer–Hicks	Thom
KH_{CH}	0.34	0.19	0.54	0.34
SH	377	341	362	363
LE	279	219	243	235

Brown, Covey, and Wright for Sept. 12, 1962 (8:25, 9:35, 11:08, 12:00, 13:05, 14:02, 15:03, 16:02) were obtained from Wright and Brown (1967) and Brown and Covey (1966). Leaf area was obtained from Brown and Covey (1966, Figure 2), radiation from Brown and Covey (1966, Figure 5), profiles of wind, humidity, temperature, and diffusivity from Wright and Brown (1967, Figures 1, 2, 3, and Table 1), and heat fluxes from Brown and Covey (1966, Figure 5). Neither set of data provided us with *TS*1, *RHS*, or LW_{RH}.

For *TS*1 and *RHS*, we chose values which resulted in reasonable agreement with the soil sensible heat fluxes given in the publications (Brown and Covey, 1966, Figs. 10, 11; Shawcroft, 1970, Table 3). For LW_{RH}, the formulae given in Monteith (1973, pp. 34–37) were used. Table III presents a summary of the driving variables used in the simulations.

4. Results

4.1. Corn crop of Stewart and Lemon

Table IV contains the results of running the model for the corn crop of Stewart and Lemon. The predictions of the fluxes of sensible and latent heat are relatively unrelated to the type of stability functions used. The value for *Kh* at canopy height obtained using Swinbank's formula is similar to that obtained by Stewart and Lemon (1969) and is about half the measured value. Using Dyer and Hicks' formula results

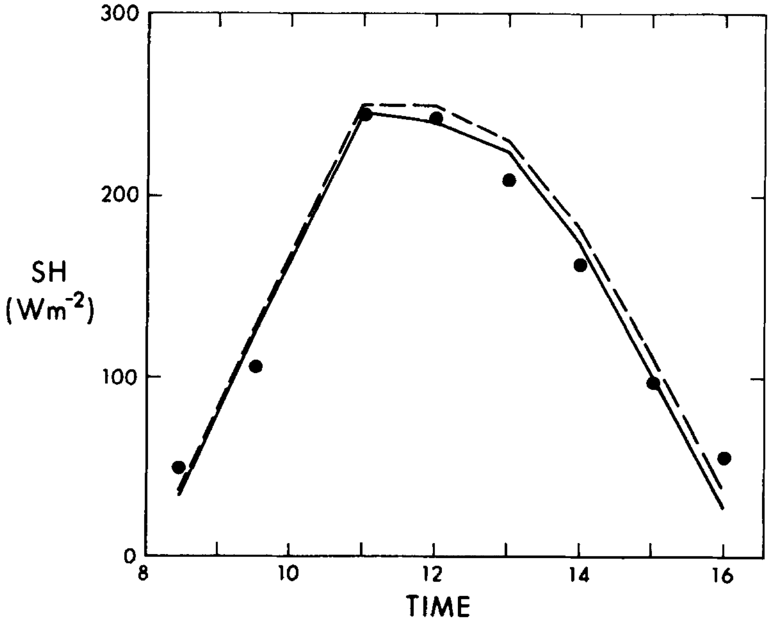

Fig.5. Sensible heat flux as a function of time for a corn crop of Brown, Covey, and Wright (September 12, 1962). ● measurements; — — — Swinbank and Dyer and Hicks; ——— Thom.

in a value over half again as large as the measured value, while Thom's formulae yield the measured value. Figure 4 shows how important a good Kh_{Ch} is in accurately predicting temperature profiles. Stability functions that give an inaccurate Kh_{CH} value also give an inaccurate temperature at canopy height. Moreover, the Kh_{CH} value itself, when it is too large or too small, yields a profile that is too shallow or too steep. We note, however, that the model profile using Thom's formula is significantly more accurate only in the top of the canopy. We will discuss this later.

4.2. Corn crop of Brown, Covey, and Wright

Figures 5 and 6 show that the heat-flux predictions are relatively unaffected by the use of different stability functions, although Thom's gives better overall results, especially for *LE*.

Figure 7 and Table V show the Kh_{CH} values obtained using the different stability functions. Neglecting stability altogether, i.e., assuming constant neutral conditions,

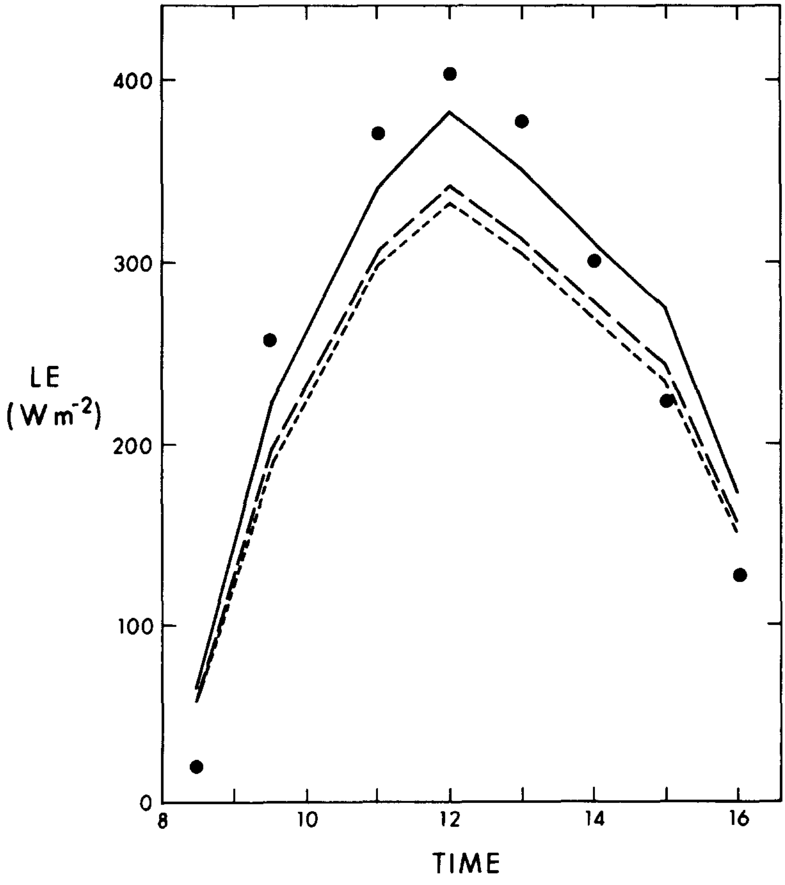

Fig.6. Latent heat flux as a function of time for corn crop of Brown, Covey, and Wright (September 12, 1962). ● measurements; — — — Swinbank; - - - - Dyer and Hicks; ——— Thom.

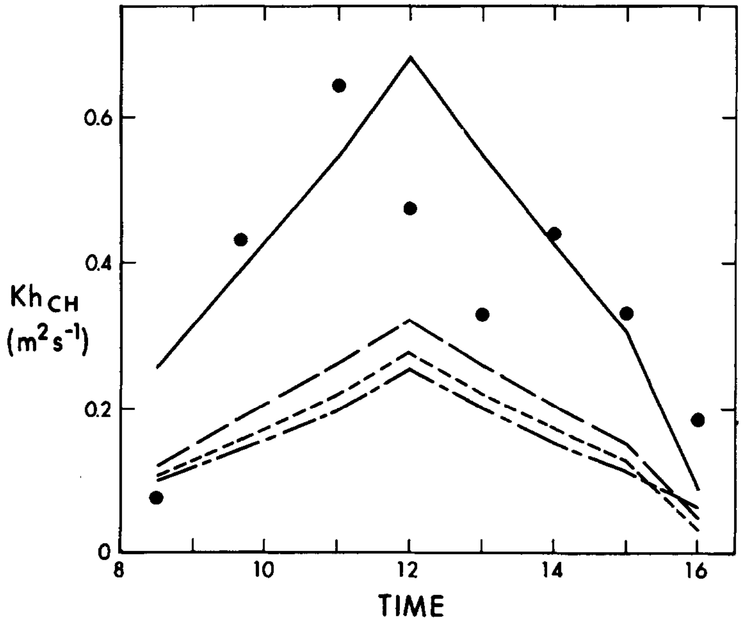

Fig.7. Kh_{CH} as a function of time for corn crop of Brown, Covey, and Wright (September 12, 1962). ● measurements; — — — no stability; - - - - Swinbank; — — — Dyer and Hicks; ——— Thom.

TABLE V

Modelled and measured Kh_{CH} (corn crop of Brown, Covey, and Wright)

12 September 1962	Measured	No stability		Swinbank		Dyer–Hicks		Thom	
Time	*Kh*	*Kh*	error	*Kh*	error	*Kh*	error	*Kh*	error
8:25	0.076	0.102	0.026	0.108	0.032	0.121	0.045	0.255	0.179
9:35	0.430	0.144	0.286	0.157	0.273	0.187	0.243	0.388	0.042
11:00	0.640	0.196	0.444	0.216	0.424	0.261	0.379	0.543	0.097
12:00	0.472	0.257	0.215	0.278	0.194	0.326	0.146	0.684	0.212
13:00	0.329	0.204	0.125	0.223	0.106	0.263	0.066	0.551	0.222
14:00	0.439	0.158	0.281	0.173	0.266	0.206	0.233	0.427	0.012
15:00	0.331	0.119	0.212	0.130	0.201	0.152	0.179	0.315	0.016
16:00	0.185	0.065	0.120	0.038	0.147	0.050	0.135	0.094	0.091
Average over time			0.214		0.205		0.178		0.109

yields values that are about half of the measured values, with an average deviation of 0.214 s m^{-2}. Using the formula of Swinbank reduces the deviation slightly to 0.205, while Dyer and Hicks' formula diminishes the deviation a bit more to 0.178. A significant improvement in the accuracy of the prediction is obtained using Thom's formulae. The average deviation is only 0.109 s m^{-2}; and in Figure 7 we see that the predictions of Thom's formulae give a best fit throughout the day. We should note that the value of Kh_{CH} obtained using any stability formula is very sensitive to D and Z_0 while using Thom's formula also involves sensitivity to ζ_{*0}. Figure 8 shows the results of using different ζ_{*0} values while using the D, Z_0 results of Maki for July, August and the average.

Temperature profiles at eight times throughout the day are presented in Figure 9. Recalling that Swinbank's formula yields the best prediction of Kh_{CH} at 8:25, it is not surprising that it also yields the best temperature profile. Similarly, at 12:00 and 13:05, using Dyer and Hicks' formula results in the best Kh_{CH} values and temperature profiles. For the remaining five times, Thom's formulae give the best results. Table VI shows the model temperature-profile deviations from measured values,

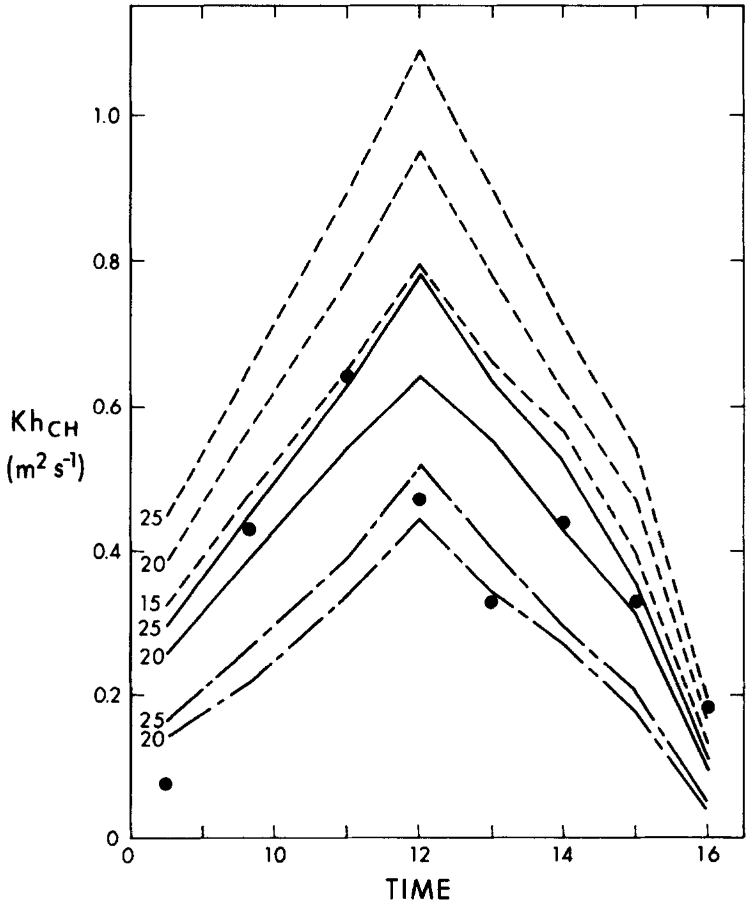

Fig.8. Kh_{CH} as a function of time for corn crop of Brown, Covey and Wright (September 12, 1962). ● measurements; — — — no stability; — — — Swinbank; — — — Dyer and Hicks; ——— Thom.

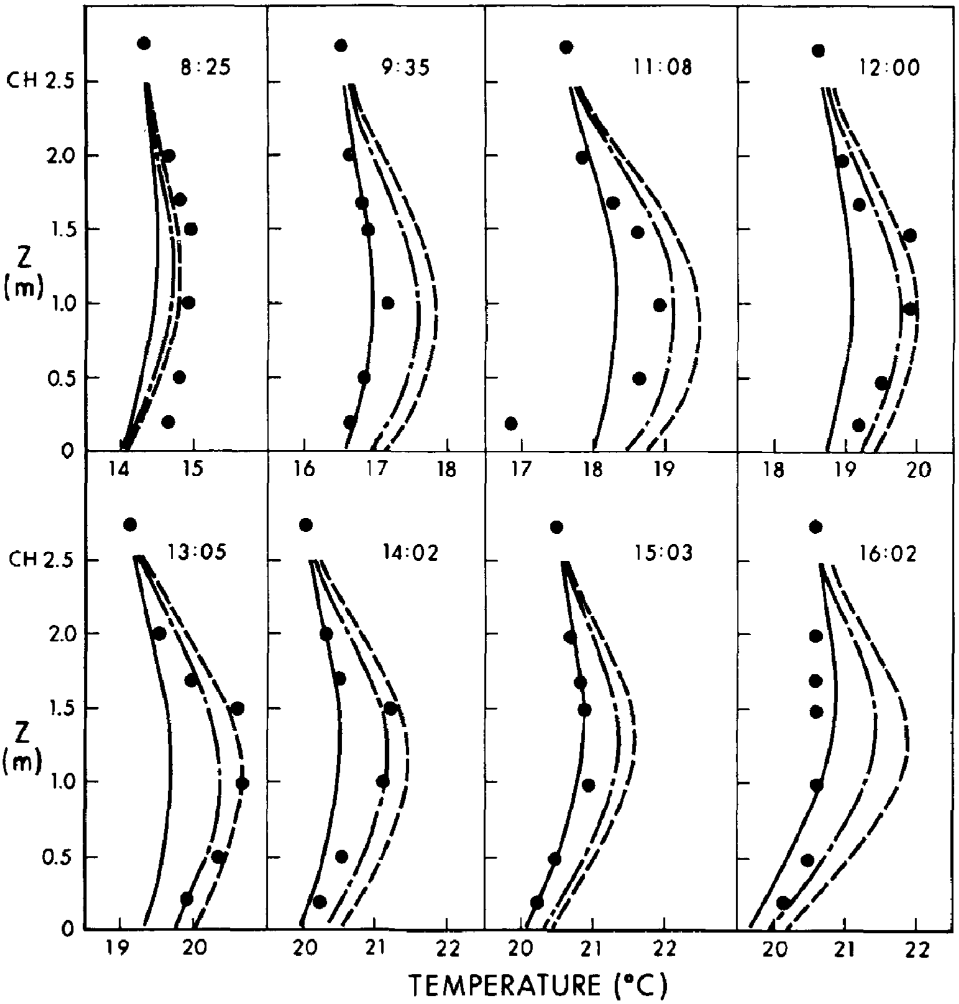

Fig. 9. Air-temperature profiles for the crop of Brown, Covey and Wright at eight times throughout the day (September 12, 1962). CH Denotes canopy height. ● measurements; — — — Swinbank; — — — Dyer and Hicks; ——— Thom.

averaged over all six data points and over the top two data points. The averages over time of the deviations of the model profiles from the six measured values are 0.54, 0.36 and 0.33, indicating only slight overall improvement using Thom's formulae, not comparable with the 0.205, 0.178 and 0.109 improvement in Kh_{CH}. However, if we note that we are using a simple exponential attenuation of eddy diffusivity into the canopy, then we can surmise that this may be an additional source of error. Since we have discussed and attempted improvement of the value of Kh only at canopy height, we can realistically expect improvement of the profile only in the top of the canopy. These results show average deviations over time of 0.51, 0.35, and 0.16 which are more comparable to the improvements in Kh_{CH}.

TABLE VI

Model temperature-profile deviations from measured values

12 September, 1962	Swinbank		Dyer–Hicks		Thom	
Time	6 Points	Top 2 Points	6 Points	Top 2 Points	6 Points	Top 2 Points
8:25	0.2	0.1	0.3	0.2	0.4	0.3
9:35	0.7	0.6	0.5	0.4	0.1	0.03
11:08	0.9	0.6	0.6	0.5	0.5	0.1
12:00	0.3	0.4	0.2	0.2	0.5	0.2
13:05	0.2	0.4	0.2	0.2	0.6	0.3
14:02	0.5	0.6	0.3	0.4	0.3	0.04
15:03	0.5	0.5	0.3	0.3	0.04	0.02
16:02	0.9	0.9	0.5	0.6	0.2	0.3
Average over time	0.54	0.51	0.36	0.35	0.33	0.16

5. Summary

In order to obtain an accurate prediction for a profile, it is important to be able to predict the profile of the corresponding diffusivity. The first step in this effort is to obtain the eddy diffusivity at the top of the canopy. Since the formulae of Thom *et al.* (1975) were derived under conditions more comparable to conditions above a corn canopy, it was hypothesized that a plant-atmosphere model utilizing these formulae would yield better predictions of thermal eddy diffusivity at canopy height. We have seen that this is the case and that the improved predictions of thermal eddy diffusivity at canopy height result in improved temperature predictions in the top of the canopy.

Acknowledgements

We are grateful to Dr K. D. Hage for reviewing the manuscript and to him and Dr J. M. Mayo for advice during development of the model.

The study was sponsored in part by Alberta Environment contract 74-25 directed by Dr P. R. Gorham and by the Alberta Oil Sands Environmental Research Program Vegetation Project VE6.1 directed by Dr L. C. Bliss.

Appendix A: Equations of the Model

A.1. RADIATION

(1) $SW(Z) = SW_{RH} \exp(-0.65\, CLAI(Z))$.

(2) $SWA(Z) = 0.82(0.65\, SW(Z))$

(3)
$$LWA(Z)=\sigma\sum_{y}\{(TL(y)+273.16)^4-(TL(Z)+273.16)^4\}V(Z,y)$$
$$+V(Z,Z_{RH})\{LW_{RH}-\sigma(TL(Z)+273.16)^4\}$$
$$+V(Z,0)\sigma\{(T_1+273.16)^4-(TL(Z)+273.16)^4\}.$$

A.2. BOUNDARY LAYER ABOVE CANOPY

(4)
$$\frac{kZ}{U_*}\frac{\partial U(Z)}{\partial Z}=\phi_m(\varepsilon)\,.$$

Integrating from Z_{CH} to Z_{RH} yields:

$$U_{CH}=U_{RH}-\frac{U_*}{k}\left[\ln\left(\frac{Z_{RH}-D}{Z_{CH}-D}\right)-(\psi_m(\varepsilon_{RH})-\psi_m(\varepsilon_{CH}))\right].$$

(5)
$$\frac{-\rho(Z)C_pU_*kZ}{SH}\frac{\partial I(Z)}{\partial Z}=\phi_h(\varepsilon)\,.$$

Integrating from Z_{CH} to Z_{RH} yields:

$$T_{CH}=T_{RH}+\frac{SH}{\rho_{RH}C_pU_*k}\left[\ln\left(\frac{Z_{RH}-D}{Z_{CH}-D}\right)-(\psi_h(\varepsilon_{RH})-\psi_h(\varepsilon_{CH}))\right].$$

(6)
$$\frac{-\rho(Z)Lt(Z)U_*kZ}{LE}\frac{\partial Q(Z)}{\partial Z}=\phi_h(\varepsilon)\,.$$

Integrating from Z_{CH} to Z_{RH} yields:

$$Q_{CH}=Q_{RH}+\frac{LE}{\rho_{RH}Lt_{RH}U_*k}\left[\ln\left(\frac{Z_{RH}-D}{Z_{CH}-D}\right)-(\psi_h(\varepsilon_{RH})-\psi_h(\varepsilon_{CH}))\right].$$

(7)
$$Kh_{CH}=\frac{U_*k(Z_{CH}-D)}{\phi h(\varepsilon_{CH})}\,.$$

Integrating (4) from $D+Z_0$ to Z_{RH} yields:

(8)
$$U_*=\frac{kU_{RH}}{\ln[(Z_{RH}-D)/Z_0]-\psi_m(\varepsilon_{RH})}\,.$$

(9)
$$Lmo=-\frac{U_*^3T_{RH}\rho_{RH}C_p}{KgSH}\,.$$

A.3. AIR IN THE CANOPY

(10)
$$U(Z)=U_{CH}\exp\left(3.0\left(\frac{Z}{Z_{CH}}-1.0\right)\right).$$

(11)
$$Kh(Z)=Kh_{CH}\exp\left(4.0\left(\frac{Z}{Z_{CH}}-1.0\right)\right).$$

(12)
$$\rho(Z)=0.34838\,P_{\mathrm{atm}}/(T(Z)+273.16)$$

(empirical formulation based on measurements in List (1966)).

(13) $$\frac{-\partial}{\partial Z}\left(Kh(Z)\frac{\partial T(Z)}{\partial Z}\right)=\frac{A(Z)(TL(Z)-T(Z))}{ra(Z)}.$$

Finite-differencing the left side yields:

$$Kh_{i+\frac{1}{2}}\frac{(T_{i+1}-T_i)}{h^2}-Kh_{i-\frac{1}{2}}\frac{(T_i-T_{i-1})}{h^2}.$$

Letting $Kh_{i+\frac{1}{2}}\approx\frac{Kh_i+Kh_{i+1}}{2}$ yields the following approximation to (13):

$$T_{i-1}(Kh_{i-1}+Kh_i)-T_i\left(Kh_{i-1}+2Kh_i+Kh_{i+1}+2h^2\frac{A_i}{ra_i}\right)$$
$$+T_{i+1}(Kh_i+Kh_{i+1})=-2h^2\frac{A_1TL_i}{ra_i}.$$

(14) $$-\frac{\partial}{\partial Z}\left(Kh(Z)\frac{\partial Q(Z)}{\partial Z}\right)=A(Z)\frac{(QL(Z)-Q(Z))}{ra(Z)+rs(Z)}.$$

Approximation same as that for (13).

A.4. LEAVES

(15) $$ra(Z)=60.0\left(\frac{0.05}{U(Z)}\right)^{0.5}$$

(from Stewart and Lemon (1969, pp. 75–78)).

(16) $$rs(Z)=DRY\left(146.0+\frac{3.6}{4.59\times10^{-4}\,SW(Z)+0.0015}\right),$$

where

$$DRY=\begin{cases}\dfrac{QL(Z)-Q(Z)}{\Delta Q_{\text{crit}}}, & \text{if}\quad QL(Z)-Q(Z)>\Delta Q_{\text{crit}},\\ 1, & \text{if}\quad QL(Z)-Q(Z)\leq\Delta Q_{\text{crit}}.\end{cases}$$

(17) $$Lt(Z)=4187(595-0.567\,TL(Z)).$$

(Adapted from Murphy and Knoerr (1972, p. 17).)

(18) $$QL(Z)=\frac{62.2}{P_{\text{atm}}}6.1078\exp\left(\frac{17.269\,388\cdot 2\,TL(Z)}{TL(Z)+237.3}\right).$$

(Adapted from Murray (1967, Equation 6) and Webb (1965, Equation 2).)

(19) $$LWA(Z)+SWA(Z)=\rho(Z)C_p\frac{(TL(Z)-T(Z))}{ra(Z)}$$
$$+\rho(Z)\frac{Lt(Z)(QL(Z)-Q(Z))}{ra(Z)+rs(Z)}.$$

A.5. AIR ABOVE SOIL

(20) $$T_2=\frac{R_{2-3}}{R_{1-2}+R_{2-3}}T_1+\frac{R_{1-2}}{R_{1-2}+R_{2-3}}T_3\,.$$

(21) $$Q_2=\frac{R_{2-3}}{R_{1-2}+R_{2-3}}Q_1+\frac{R_{1-2}}{R_{1-2}+R_{2-3}}Q_3\,.$$

(22) $$R_{1-2}=\frac{Zb}{Khmd}+\int_{Zb}^{Z_2}\frac{\mathrm{d}Z}{(Kh_2-Khmd)[(Z-Z_b/(Z_2-Z_b)]+Khmd}\,.$$

Integrating yields:

$$R_{1-2}=\frac{Zb}{Khmd}+\frac{Z_2-Z_b}{Kh_2-Khmd}(\ln(Z_2-Zb+Khmd)-\ln(Khmd))\,.$$

(23) $$R_{2-3}=\int_{Z_2}^{Z_3}\frac{\mathrm{d}Z}{Kh(Z)}\,.$$

A.6. SOIL SURFACE

(24) $$LWA_1+SWA_1=\frac{\rho_1C_p(T_1-T_2)}{R_{1-2}}+\rho_1Lt_1\frac{(Q_1-Q_2)}{R_{1-2}}+\frac{CNDS}{0.05}(T_1-TS1)\,.$$

(25) $$Q_1=RHS\frac{62.2}{P_{\mathrm{atm}}}6.1078\exp\left(\frac{17.269\,388\,2T_1}{T_1+237.3}\right)\,.$$

A.7. FLUXES

(26) $$SH=C_p\int_{Z_3}^{Z_{CH}}\frac{\rho(Z)A(Z)(TL(Z)-T(Z))}{ra(Z)}\mathrm{d}Z+Cp\rho_2\frac{(T_1-T_2)}{R_{1-2}}\,.$$

(27) $$LE=\int_{Z_3}^{Z_{CH}}\frac{\rho(Z)Lt(Z)A(Z)(QL(Z)-Q(Z))}{ra(Z)+rs(Z)}\mathrm{d}Z+\rho_1Lt_1\frac{(Q_1-Q_2)}{R_{1-2}}\,.$$

This system of equations is solved in the following manner: Boundary values are read in, variables are initialized, the short-wave radiation profile is calculated, stability functions are initialized to neutral conditions, and then steps 2–6 are iterated until the leaf temperatures converge. Then steps 1–6 are iterated to convergence.

(1) Choose the stability functions and integrated stability functions from a table.

(2) Solve the equations of Section A.2. and Equations (10)–(12) at every level.

(3) Solve Equations (3), (24), and (25) iteratively for soil-surface longwave radiation, temperature, and humidity.

(4) Use simplified Gaussian elimination to solve the band matrices representing Equations (13) and (14) at every level for the air temperature and humidity profiles.

(5) Solve Equations (15) and (16) at every level and then solve Equations (3) and (17)–(19) iteratively for long-wave radiation, leaf temperature and humidity at every level.

(6) Solve Equations (26) and (27) for fluxes of sensible and latent heat.

For our simulations, we used fourteen levels: soil surface, first level above soil surface, ten levels in the canopy, canopy, and reference heights. Using this number of levels, five to six iterations were needed for the first convergence and one to six for the second, using convergence criteria of 0.05 and 0.01 degrees respectively.

References

Brown, K. W. and Covey, W.: 1966, 'The Energy-Budget Evaluation of the Micrometeorological transfer Processes within a Cornfield', *Agr. Meteorol.* **3**, 73–96.

Businger, J. A., Wyngaard, J. C., Izumi, Y., and Bradley, E. F.: 1971, 'Flux-Profile Relationships in the Atmospheric Surface Layer', *J. Atmos. Sci.* **28**, 181–189.

Cionco, R. M.: 1972, 'A Wind-Profile Index for Canopy Flow', *Boundary-Layer Meteorol.* **3**, 255–263.

Dyer, A. J.: 1974, 'A review of Flux-Profile relationships, *Boundary-Layer Meteorol.* **7**, 363–372.

Dyer, A. J. and Hicks, B. B.: 1970, 'Flux-Gradient Relationships in the Constant Flux Layer', *Quart. J. Roy. Meteorol. Soc.* **96**, 715–721.

Goudriaan, J. and Waggoner, P. E.: 1972, 'Simulating Both Aerial Microclimate and Soil Temperature from Observations Above the Folia Canopy', *Neth. J. Agric. Sci.* **20**, 104–124.

Högström, U.: 1974, 'A Field Study of the Turbulent Fluxes of Heat, Water Vapour and Momentum at a Typical Agricultural Site', *Quart. J. Roy. Meteorol. Soc.* **100**, 624–639.

Legg, B. and Monteith, J. L.: 1975, 'Heat and Mass Transfer Within Plant Canopies', in D. A. de Vries and N. H. Afgan (eds.), *Heat and Mass Transfer in the Biosphere*, Chapter 11, Scripta Book Company, Washington, D.C.

Lemon, E., Stewart, D. W., Shawcroft, R. W., and Jensen, S. E.: 1973, 'Experiments in Predicting Evapotranspiration by Simulation with a Soil-Plant-Atmosphere Model', in *Field Soil Water Regime*, Chapter 4, Soil Science Society of America.

List, R. J.: 1966, *Smithsonian Meteorological Tables*, Smithsonian Institution, Washington, D. C.

Maki, T.: 1969, 'On Zero-Plane Displacement and Roughness Length in the Wind Velocity Profile Equation over a Corn Canopy', *Nogyo-Kisho* **25**, 13–18.

Monteith, J. L.: 1973, *Principles of Environmental Physics*, E. Arnold, London.

Murphy, C. E. and Knoerr, K. R. 1972, *Modeling the Energy Balance Process of Natural Ecosystems*, Eastern Deciduous Forest Biome – IBP Res. Rep. 72–10, Oak Ridge Nat. Lab., Oak Ridge, Tenn., and Duke Univ., Durham, N.C., 164 pp.

Murphy, C. E. and Knoerr, K. R.: 1975, 'The Evaporation of Intercepted Rainfall from a Forest Stand: an Analysis by Simulation', *Water Resour. Res.* **11**, 273–280.

Murray, F. W.: 1967, 'On the Computation of Saturation Vapor Pressure', *J. Appl. Meteorol.* **6**, 203–204.

Pruitt, W. O., Morgan, D. L., and Lourence, F. J.: 1973, 'Momentum and Mass Transfers in the Surface Boundary Layer', *Quart. J. Roy. Meteorol. Soc.* **99**, 370–386.

Raschke, K.: 1970, 'Stomatal Responses to Pressure Changes and Interruptions in the Water Supply of Detached Leaves of *Zea Mays* L.', *Plant Physiol.* **45**, 415–423.

Shawcroft, R. W.: 1970, *Water Relations and Stomatal Responses in a Corn Field*, Ph.D. Thesis, Cornell University, 127 pp.

Stewart, D. W. and Lemon, E. R.: 1969, *The Energy Budget at the Earth's Surface: a Simulation of Net Photosynthesis of Field Corn*, Microclimate Investigations Interim Rpt. 69-3, U.S.D.A. Cornell Univ., Ithaca, New York.

Swinbank, W. C.: 1964, 'The Exponential Wind Profile', *Quart. J. Roy. Meteorol. Soc.* **90**, 119–135.

Thom, A. S., Stewart, J. B., Oliver, H. R., and Gash, J. H. C.: 1975, 'Comparison of Aerodynamic and Energy Budget Estimates of Fluxes over a Pine Forest', *Quart. J. Roy. Meteorol. Soc.* **101**, 93–105.

Waggoner, P. E., and Reifsnyder, W. E.: 1968, 'Simulation of the Temperature, Humidity and Evaporation Profiles in a Leaf Canopy', *J. Appl. Meteorol.* **7**, 400–409.

Webb, E. K.: 1965, 'Aerial Microclimate', in P. E. Waggoner (ed.) *Agricultural Meteorology*, Chapter 2, American Meteorological Society, Boston.

Webb, E. K.: 1970, 'Profile Relationships: the Log-Linear Range, and Extension to Strong Stability', *Quart. J. Roy. Meteorol. Soc.* **96**, 67–90.

Wright, J. L. and Brown, K. W.: 1967, 'Comparison of Momentum and Energy Balance Methods of Computing Vertical Transfer Within a Crop', *Agronomy J.* **59**, 427–432.