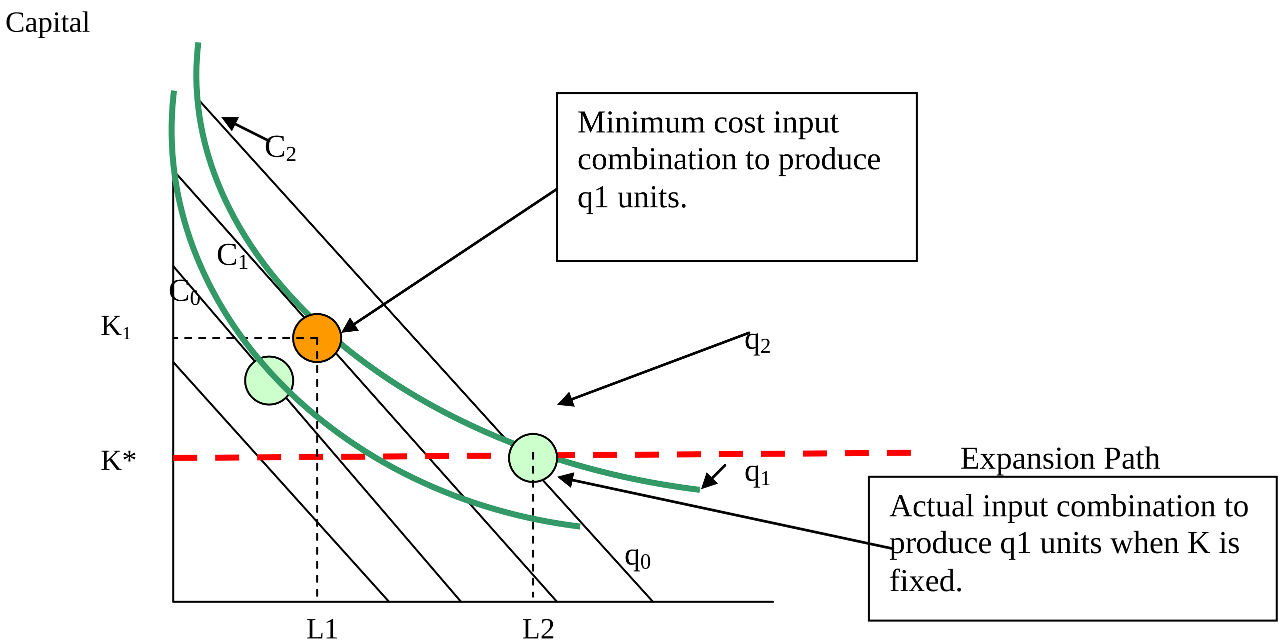
Economics 203: Intermediate Microeconomics I

Lab Exercise #3

Section 1: Discussion:

Explain the why the short-run minimum cost of producing a certain output may differ from the long-run minimum cost. Illustrate your explanation with a diagram.

In the short-run, at least one factor of production is assumed fixed. When this happens and the firm must produce a specific quantity of output, the firm may not be able to use the input combination that attains minimum cost relative to the long-run situation. In the long run, all factors of production are variable. Hence, a firm will produce a specific output where the isocost line is tangent to the isoquant (the specific quantity that must be produced). In the short-run, this may not be possible for a specific quantity.



If capital is fixed at K* units, and the firm must produce q1 units, L2 units of labour will be used with this fixed amount of capital to produce q1 units of output. The total cost is C2. If the firm were able to use any combination of K and L, it would use L1 and K1 units of labour and capital and lower its cost to C1. This would be the long-run solution. Total cost is C1.

Section 2: Application:

For the Bridges-to-Shores Corporation, the relationship between output (Q) and the number of hours of specialized manual labour (S) and machine-operated labour (M) is:

$$Q=650S + 145M - 0.45\,S^2 - 0.65M^2$$

The hourly wage of specialised manual labour is $42, and the hourly wage of machine-operated labour is $25. The firm can hire as much labour as it wants at these wage rates.

A) The vice president of manufacturing recommends that the firm hire 50 hours of manual labour and 95 hours of machine-operated labour. Evaluate this recommendation.

To find the optimal input combinations, choose where:

$$\boxed{\frac{MP_S}{P_S} = \frac{MP_M}{P_M}}$$

The marginal products are:

MP_S= 650 - 0.9S
MP_M =145 – 1.3M

$$\frac{MP_S}{P_S} = \frac{MP_M}{P_M}$$

$$\frac{650 - 0.9S}{42} = \frac{145 - 1.3M}{25} \Leftarrow \textit{cross multiply}$$

$$25(650 - .9S) = 42(145 - 1.3M)$$

$$16250 - 22.5S = 6090 - 54.6M$$

$$22.5S = 10160 + 54.6M$$

$$S = 451.5556 + 2.426667M \leftarrow \textit{equation}$$

Enter in S=50 and M=95 into either equation we see that this is not the optimal input combination.

$$50 = 451.5556 + 2.42667(95)$$

$$50 \neq 682.08925$$

B) If the Bridges-to-Shores company decides to spend a total of $25,000 on inputs (specialized manual and machine-operated labour), how many hours of each type of labour should it hire?

TC=25,000

$$TC = P_s S + P_M M$$

$$25{,}000 = 42S + 25M$$

$$25{,}000 = 42(451.5556 + 2.42667M) + 25M$$

$$25{,}000 = 18965.3352 + 101.92014M + 25M$$

$$6034.6648 = 126.92014M$$

$$M = 47.5469$$

$$S = 451.5556 + 2.42667M = 566.9363$$

Must use 567 hours of specialized labour and 47.5 hours of machine-operated labour.

Question

Consider the Swipe-It Company. The relationship between output per hour (Q) and the amount of labour (L) and the number of machines (K) used per hour is:

$$Q = 3L^{0.1}K^{0.9}$$

The price of labour is $10 per hour, and the price of a machine is $12 per hour. If the Swipe-It Company produces 1000 units of output per hour, how much labour and machines should be used?

The Swipe-It Company should choose an input combination such that:

$$\frac{MP_L}{P_L} = \frac{MP_K}{P_K}$$

Since $Q = 3L^{0.1}K^{0.9}$ then

$$MP_L = \frac{\partial Q}{\partial L} = 0.3L^{-0.9}K^{0.9}$$

$$MP_K = \frac{\partial Q}{\partial K} = 2.7L^{0.1}K^{-0.1}$$

Hence, if $\frac{MP_L}{P_L} = \frac{MP_K}{P_K}$. Then

$$\frac{0.3L^{-0.9}K^{0.9}}{10} = \frac{2.7L^{0.1}K^{-0.1}}{12}$$

$$\frac{0.3K^{0.9}}{L^{0.9}} = \frac{27L^{0.1}}{K^{0.1}}$$

$$3.6K^{0.9} \times K^{0.1} = 27L^{0.1} \times L^{0.9}$$

$$3.6K = 27L$$

$$K = 7.5L$$

Since Q=1000:

$$1000 = 3L^{0.1}K^{0.9}$$

$$1000 = 3L^{0.1}(7.5L)^{0.9}$$

$$1000 = 3(7.5^{0.9})L$$

$$1000 = 3L(6.1313)$$

$$1000 = 18.394L$$

$$L = 54.36556$$

$$K = 7.5L = 407.7417$$

$$Check: 3(54.36556)^{0.1}(407.7417)^{0.9} = 1000.0005 \approx 1000$$

To minimize cost, the company should hire 54.36 workers and 408 machines (when rounding up).