Chapter 10 – Costs.

Goals:

+ Understand various concepts of costs.

+ Distinguish between short run and long run cost.

General Concept.

- Fixed Cost: the cost that does not vary with the level of output in the short run.
  - $FC = rK_0$
- Variable Cost: the cost that varies with the level of output in the short run.
  - $VC_{Q1} = wL_1$
- Total Cost: the cost of all the factors of production employed.
  - $TC_{Q1} = FC + VC_{Q1} = rK_0 + wL_1$

General Concept

- Example:
  - Production function $Q=3KL$.
  - Shortrun $K=4$, $r=2$ and $w=24$
  - Derive and graph TC, VC and FC?

<u>The Isocost Function</u>

The production function summarizes the technological options facing the firm. Unfortunately it is not enough to be just aware of these options when making the output and factor input decision.

- The **<u>price</u>** of a factor of production is extremely important in this decision.

- In order to minimize costs and produce efficiently, the firm must know exactly what its costs will be.

Let ‘w’ be the annual cost of each unit of labour.

Let ‘M’ be the price of capital that never needs to be replaced.

Let ‘i’ be the interest rate.

Let r=iM= the opportunity cost of maintaining one unit of capital.

<u>Notation:</u>

P_K=r = price of capital
P_L=w = price of labour
q=Q= output

The total annual cost of producing the good is:

Total cost = Cost of labour + Cost of Capital

$$\mathbf{C = wL + rK}$$

Rearranging we have an expression for K:

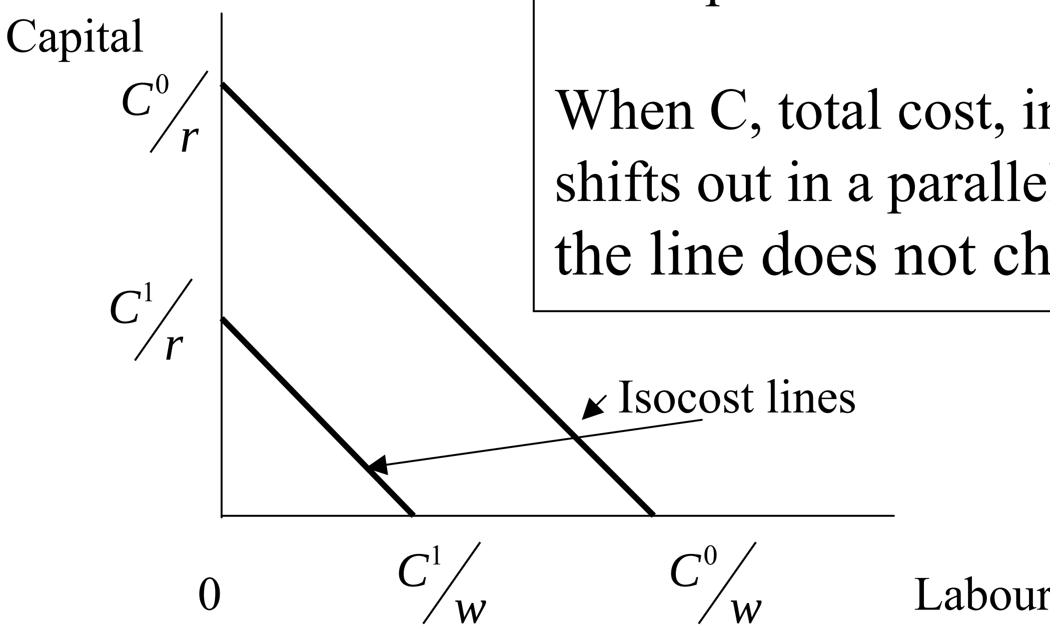
$$\boxed{K = \frac{C}{r} - \frac{w}{r}L \quad \Leftarrow \text{Isocost Line}}$$

The isocost line represents the total cost C as constant for all K-L combinations satisfying the equation.

"An isocost line shows the different combinations of factors of production that can be employed with a given total cost."

For a given cost C, the vertical intercepts of these lines are C/r.
C/r is the amount of capital that can be employed when no labour is used.
The slope of the line is -w/r = the negative of the factor price ratio.

When C, total cost, increases, the isocost line shifts out in a parallel fashion, but the slope of the line does not change.



When C, total cost, increases, the isocost line shifts out in a parallel fashion, but the slope of the line does not change.

The Short-Run Total Cost Function

In the short run, one factor is fixed. Hence, the cost of this fixed factor does not change as quantity produced changes.

The short-run total cost function consists of two components:
Short-run total cost = total fixed cost + total variable cost

$$TC_S = TC_S(q) = F + V(q)$$

The short-run total cost function shows the lowest total cost of producing each quantity when one factor is fixed.

The fixed cost must be paid regardless of whether any of the good is produced. The variable cost will increase when the quantity produced increases.

Deriving the Short-run Total Cost Curve

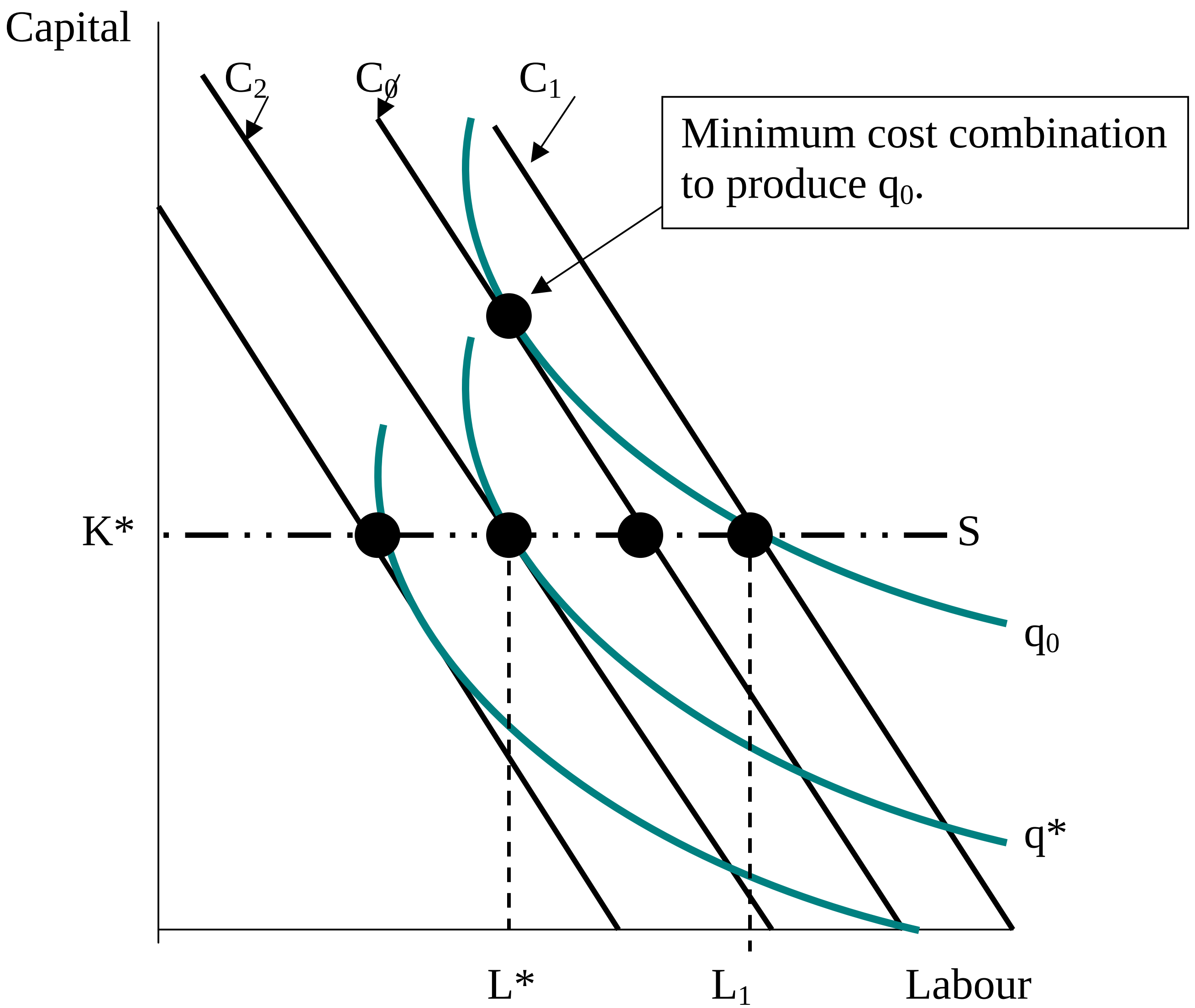
- Refer to the diagram on the next page.

In the short-run, the firm has K* units of capital.

The firm's **expansion path** is the horizontal line 'S' because the amount of capital is fixed.

If the firm wants to produce q_0 units in the short run, it must employ L_1 units of labour and incur a total cost of C_1.
The lowest total cost of producing quantity q_0 is along the isocost line C_0.

However, since the cost C_1 is greater than C_0, it is more expensive to produce q_0 units in the short run when the amount of capital is fixed at K*, than in the long run when capital is variable.



With K* of capital, production of q* units of output with L* units of labour in the cost minimization production solution.

Short run Cost Curve.

FIGURE 10-1

Output as a Function of One Variable Input
This production process shows increasing marginal productivity of the variable input up to $L = 4$, and diminishing marginal productivity thereafter.

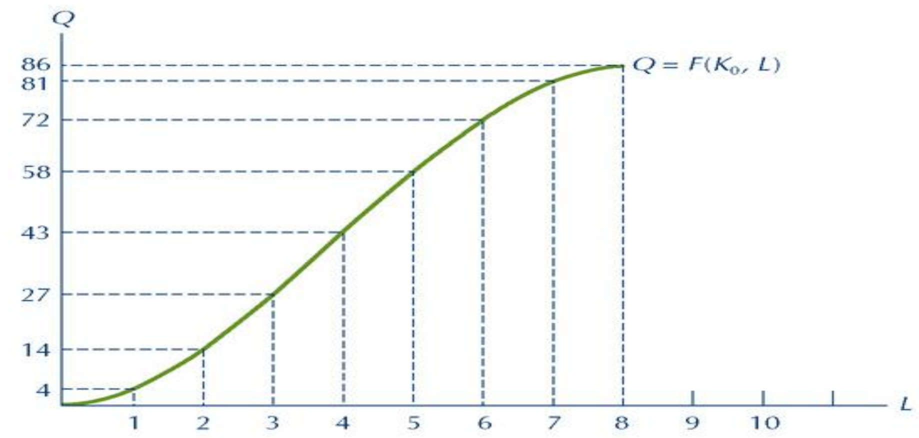
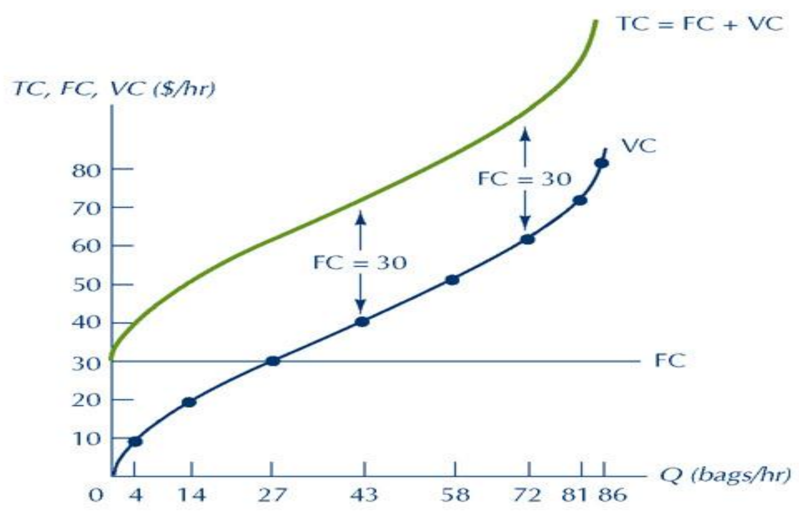


FIGURE 10-2

The Total, Variable, and Fixed Cost Curves
These curves are for the production function for Kelly's Cleaners, shown in Figure 10-1. The variable cost curve passes through the origin, which means that the variable cost of producing zero units of output is equal to zero. The TC curve, which is the sum of the FC and VC curves, is parallel to the VC curve and lies FC = \$30/hour above it. See Table 10-2.



Short run Cost.

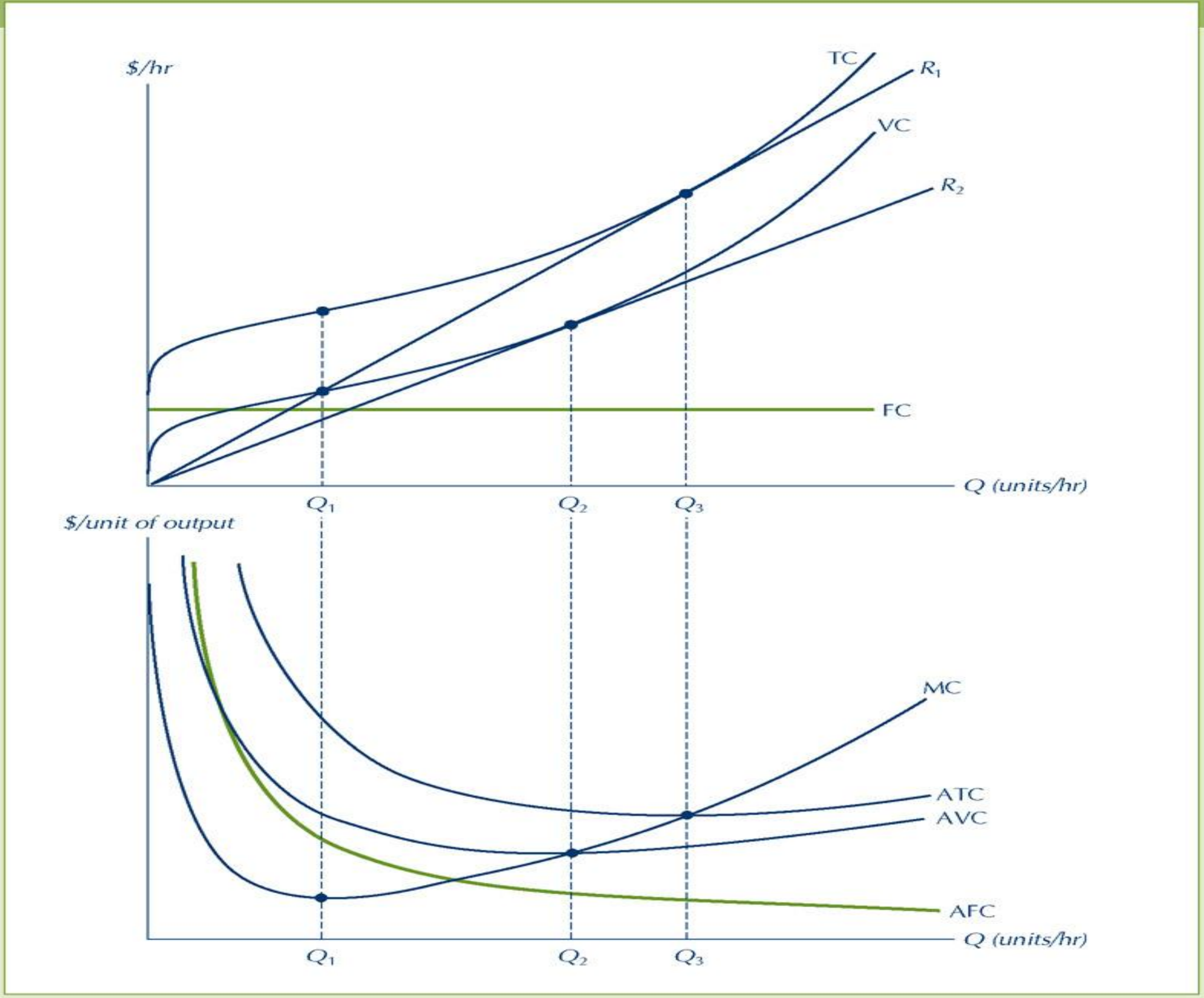
- Other concepts of costs.
  - Average fixed cost (AFC) : is fixed cost divided by the quantity of output.
    - $AFC_{Q1} = FC/Q_1 = rK_0/Q_1$
  - Average variable cost (AVC): is variable cost divided by the quantity of output.
    - $AVC_{Q1} = VC/Q_1 = wL_1/Q_1$
  - Average total cost (ATC): is total cost divided by the quantity of output.
    - $ATC_{Q1} = AFC_{Q1} + AVC_{Q1}$
  - Marginal cost (MC): is the change in total cost that results from producing an additional unit of output.
    - $MC_{Q1} = dTC/dQ$

More on Short run Cost Curve

FIGURE 10-5

The Marginal, Average Total, Average Variable, and Average Fixed Cost Curves

The MC curve intersects the ATC and AVC curves at their respective minimum points. With TC curves having this form, it is always the case that minimum MC occurs to the left of minimum AVC, which is left of minimum ATC.



Short run cost.

- The most important cost in production decision is the marginal cost.
  - Similar to marginal product (C.9), when MC is less than the average cost (either ATC or AVC), the average cost curve must be decreasing with output; and when MC is greater than average cost, average cost must be increasing with output.

The Short-Run Cost Functions of the Firm

Fixed Cost: is a cost that does not change with the quantity of output produced.

A Sunk Cost: a previous expenditure that a firm cannot avoid.

The short run total cost function, $C_S(q)$ represents the total cost of producing each quantity with a given plant size.

The short-run total cost function is the sum of the fixed and variable cost functions:

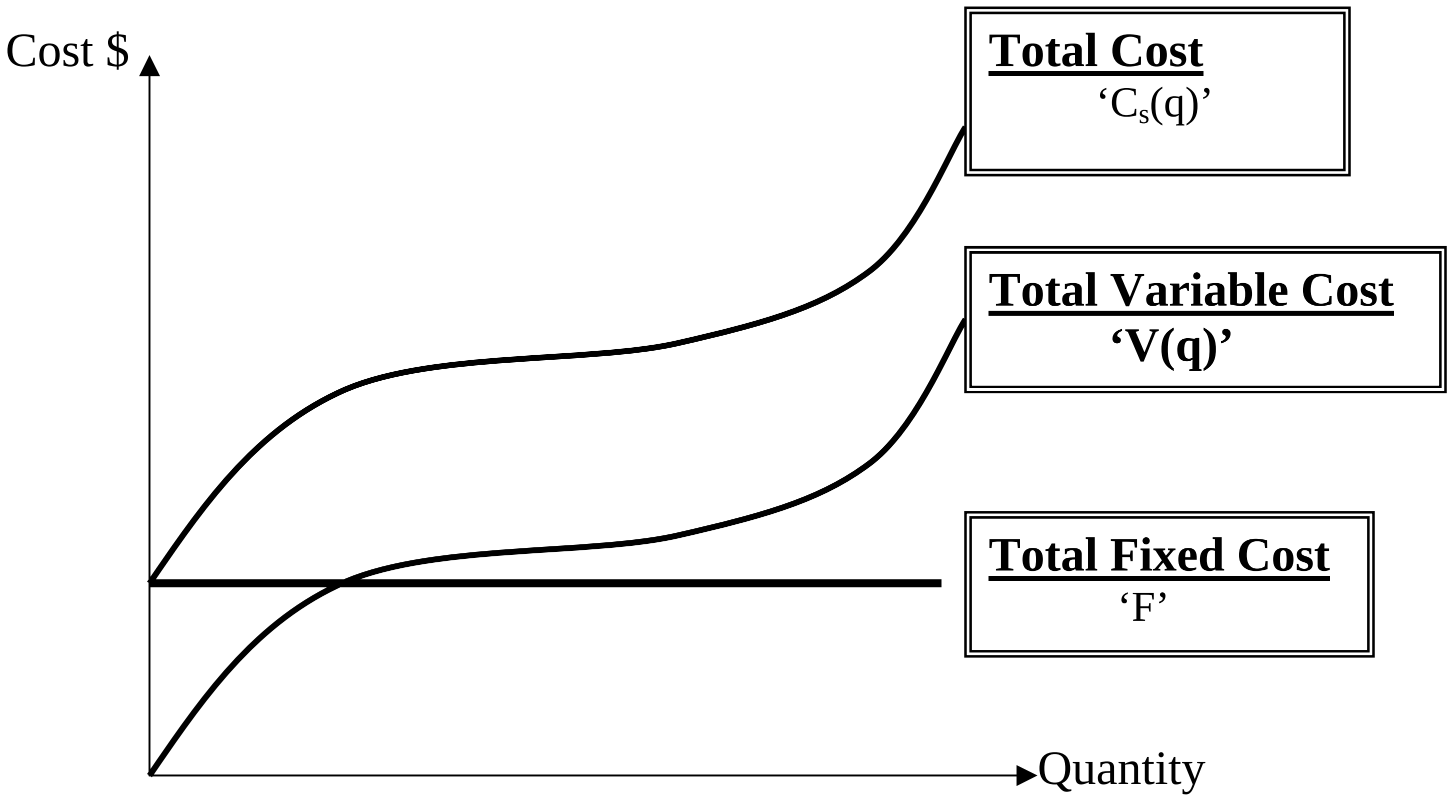
$$\mathbf{C_S(q) = F + V(q)}$$

where: F = fixed cost
V(q) = variable cost (costs that change with output produced.)

The short-run total cost function shows the lowest total cost of producing each quantity when at least one factor is fixed.

<u>Graphing:</u>

To derive the short-run total cost function, we can graph total fixed and total variable costs and then sum them vertically.



Next, we can derive the average costs function and the marginal cost function from these curves.

There are seven cost functions you need to know:

The first three we have already discussed.

1) Short-run total cost: $C_S(q)$
2) Short-run total variable cost: $V(q)$
3) Total fixed cost: F

4) Short-run marginal cost: $MC_S(q)$

$$\boxed{MC_S(q) = \frac{\Delta C_S(q)}{\Delta q} = \frac{\Delta V(q)}{\Delta q}}$$

5) Short-run average cost: $AC_S(q)$

$$\boxed{\mathrm{AC_S(q)} = \frac{\mathrm{C_S(q)}}{\mathrm{q}} = \frac{\mathrm{F}}{\mathrm{q}} + \frac{\mathrm{V(q)}}{\mathrm{q}} = \mathrm{AFC(q)} + \mathrm{AVC(q)}}$$

6) Average variable cost: AVC(q) $\boxed{AVC(q) = \frac{V(q)}{q}}$

7) Averaged fixed cost: AFC(q)

The SAC at any quantity of output is the slope of a straight line drawn from the origin to the point on Cs(q) associated with that output.

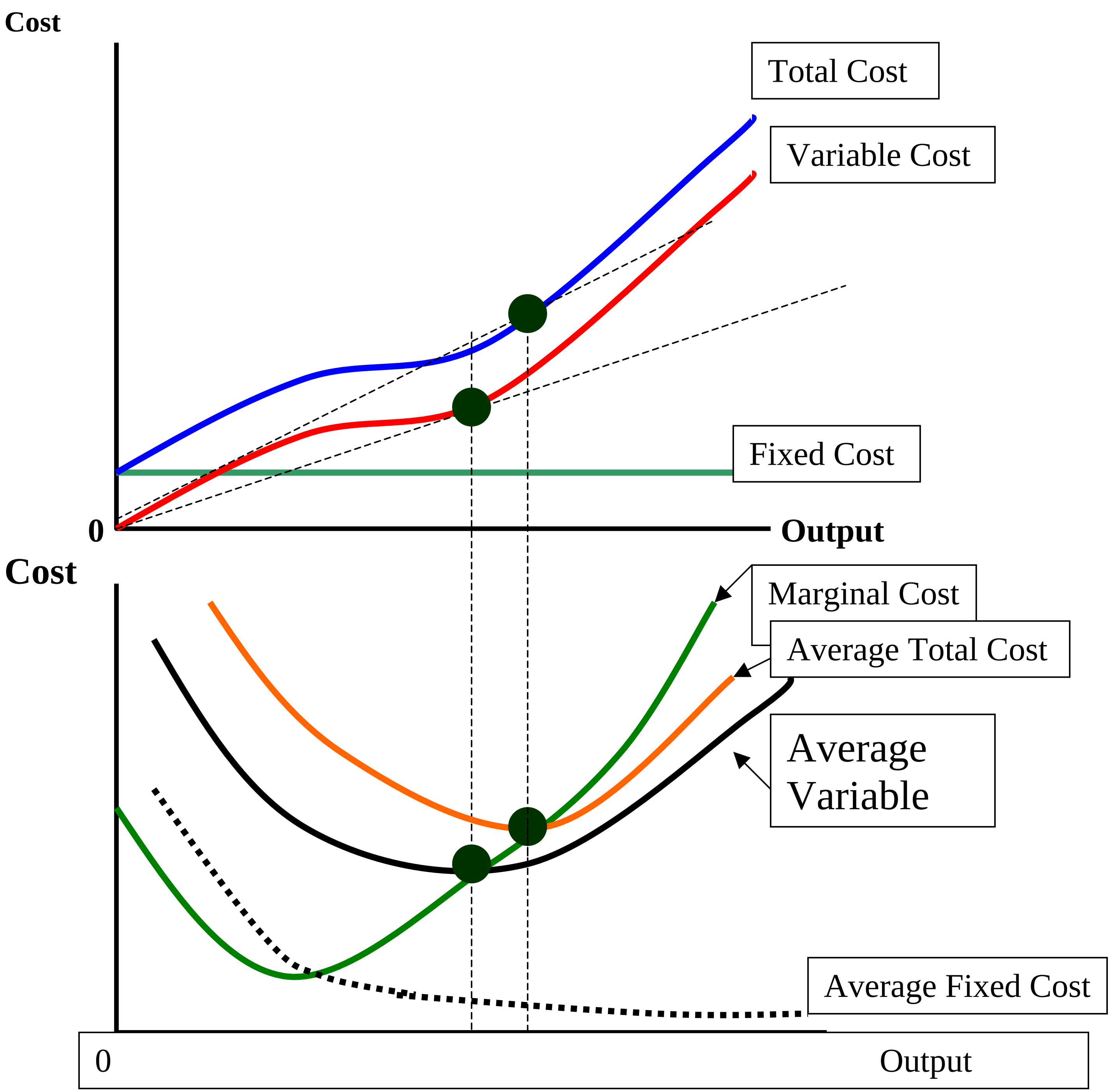
To determine if ACs is increasing or decreasing as quantity produced changes, simply determine how the slopes of successive rays to different points on the total cost curve change.

The ACs reaches a minimum when the slope of the ray from the origin to the total cost curve is tangent.

AVC is the slope of a ray from the origin to the total variable cost function.

When the slope of the ray from the origin to the V(q) curve is tangent, AVC is at a minimum.

The difference between AC and AVC decreases as the quantity increases because AFC=F/q decreases.


Cost
Total Cost
Variable Cost
Fixed Cost
0
Output
Cost
Marginal Cost
Average Total Cost
Average Variable
Average Fixed Cost
0
Output

The Relationship Between Marginal and Average Costs

1) When MC < AC: AC is decreasing.
2) When MC = AC: AC is constant.
3) When MC > AC: AC is increasing.

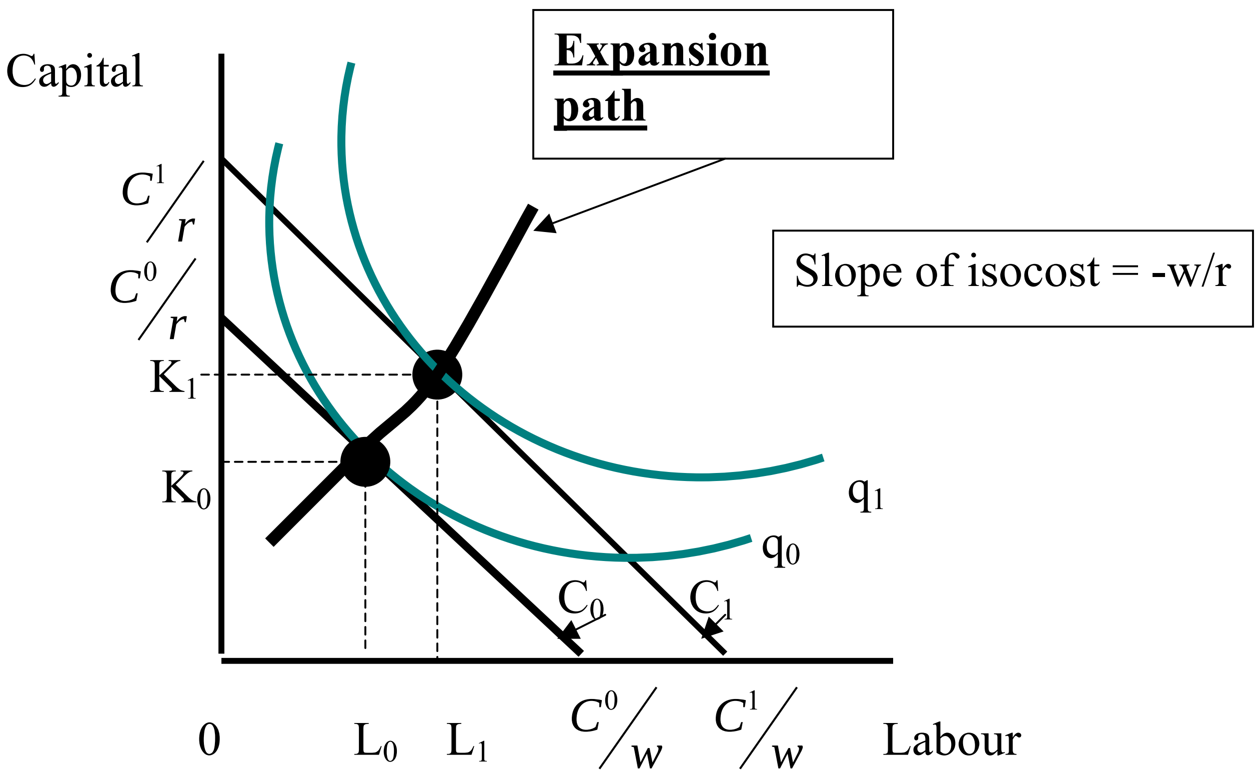
The marginal cost function goes through the minimum points of the average variable cost and short-run average cost functions.

Short run Cost.

- Allocating production between 2 production processes.
  - If have two distinct production processes, allocating inputs so that MC of the two production processes are equal.
    - $MC_A = MC_B$
  - Example: Firm has 2 processes with:
    - $MC1=0.4Q1$ and $MC2=2+0.2Q2$
      - How much should this firm produces with each process if it wants to produce 8 units and 4 units?

The Long-run Total Cost Function

The long-run total cost function represents the lowest total cost of producing a unit of a good when all inputs are variable.



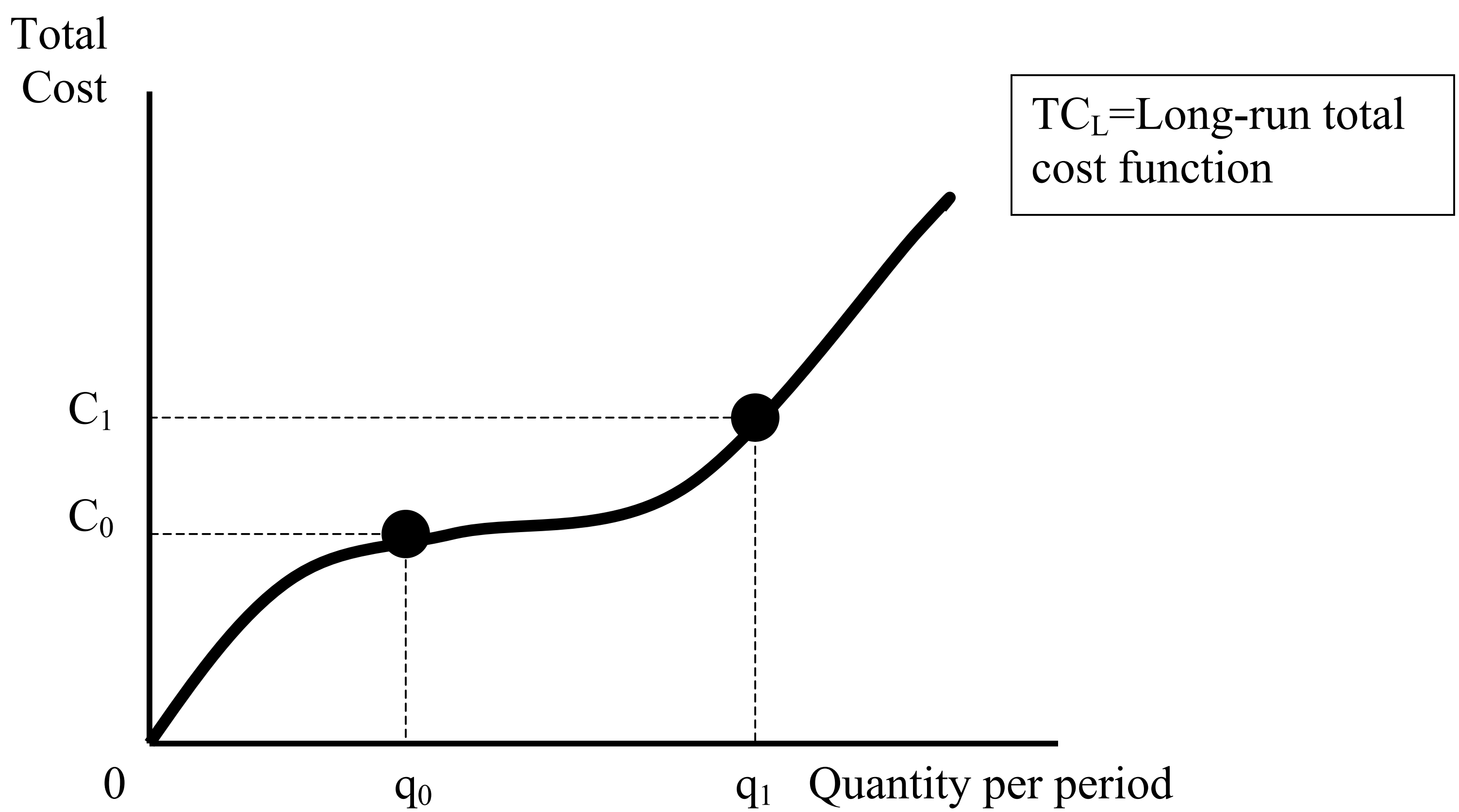
On the diagram, there are two point of tangency of isocost lines with two isoquants.

If the firm wants to produce q_0 units of the good, it can minimize its total cost by employing L_0 units of labour and K_0 units of capital, such that its minimum cost is C_0.

If the firm wants to produce q_1 units of the good, it can minimize its total cost by employing L_1 units of labour and K_1 units of capital, such that its minimum cost is C_1.

The curve that connects all these points of tangency between an isoquant and an isocost line is referred to as the **expansion path**. Each point relates a quantity with a minimum total cost.

To derive the long-run total cost function, we take the pairs of total cost and quantity from the expansion path.



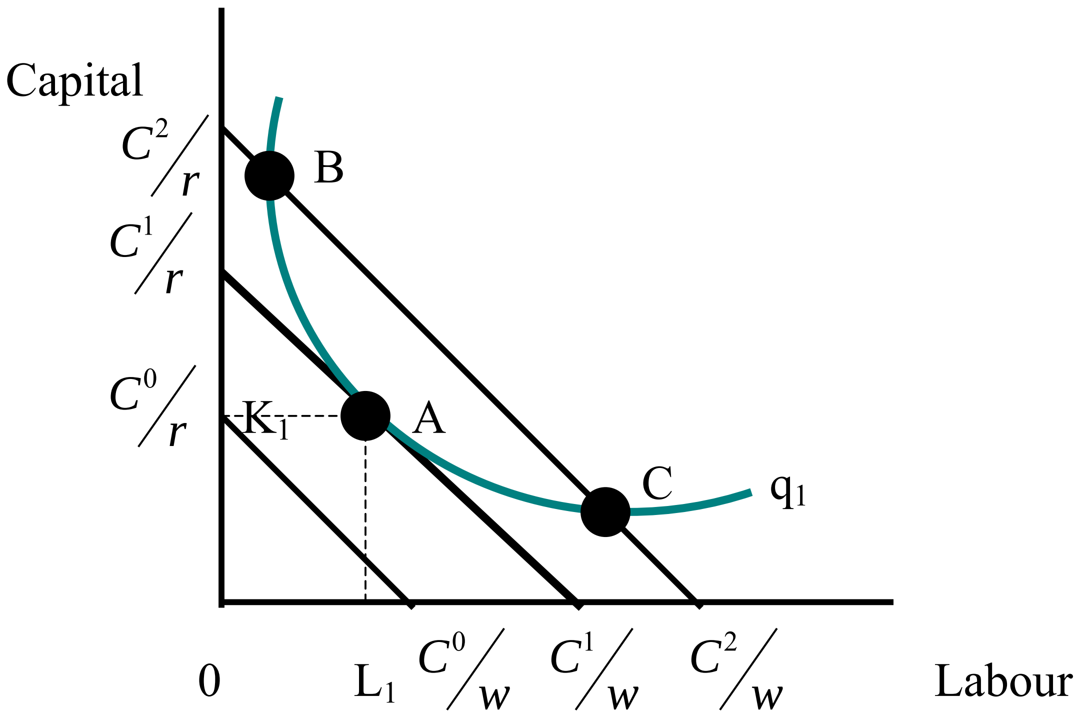
"The long-run total cost function shows the lowest total cost of producing each quantity when all factors of production are variable."

Long run Cost

- The Isocost curve: similar to the budget line in consumer problem, it is the locus of all possible input bundles that can be purchased for a given level of total expenditure C. The slope is –w/r if K and L are the only two inputs with K on the vertical axis.
  - $rK + wL = C \rightarrow K = C/r - (w/r)L$
- The minimum cost for a given level of output is the bundle at which the isocost curve is tangent to the isoquant curve (C9).
  - $MRTS = MPL/MPK = w/r$

The Production Decision

We can now determine which combination of factors produces a given quantity at the lowest total cost.



Long run Cost

- Example:
  - A firm with a production function as

$$Q(K,L) = 2K^{1/2}L^{1/2}$$

    - Derive the optimal input mix as a function of wage and rent.
    - If this firm currently operate optimally with 8 units of labour and 2 units of capital at the total cost of \$16, what are the wage and rent in this market?

There are three isocost lines. The firm wants to produce q_1 units. The cost minimizing point is at A.

- Total cost increases by moving from each isocost line.
- The firm can produce q_1 units along the isoquant q_1.
- The total cost of produces q_1 units is minimized at point A.

If the firm is producing q_1 units at minimum total cost, the slope of the isoquant equals the slope of the isocost line:

$$\boxed{MRTS_{KL} = -\frac{w}{r}}$$ (Minimum cost Condition)

"A firm minimizes the total cost of producing a given quantity by selecting a combination of factors where the slope of the isoquant equals the slope of the isocost line."

And since $\boxed{MRTS_{KL} = -\frac{MP_L}{MP_K}}$,

and $\boxed{MRTS_{KL} = -\frac{w}{r}}$, then $\boxed{-\frac{MP_L}{MP_K} = -\frac{w}{r}}$.

Rearranging $\boxed{\frac{MP_L}{w} = \frac{MP_K}{r}}$ (Minimum Cost Condition)

The firm minimizes the total cost of providing a given quantity if the ratio of the marginal product of a factor to its price is the same for all factors.

"The lowest total cost of producing a given quantity occurs when the ratio of the marginal product of a factors to the last dollar spent on it is equal for all factors of production."

The rate of the MP_L to the price of labour represents the increase in output due to the last dollar spent on labour.

To minimize total cost, the additional output due to the last dollar spent on labour must be equal to the addition output due to the last dollar spent on capital.

If they are not equal, it pays the firm to reallocate its expenditure from one factor to another.

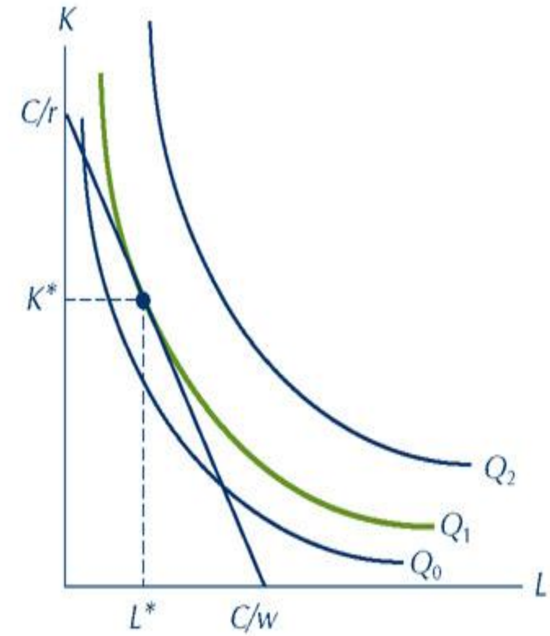
The firm should spend more money on the factor that gives the firm a greater boost in output for the extra dollar spent.

Deriving the cost function

- Solving the minimization problem graphically

FIGURE 10-11

The Maximum Output for a Given Expenditure A firm that is trying to produce the largest possible output for an expenditure of C will select the input combination at which the isocost line for C is tangent to an isoquant.

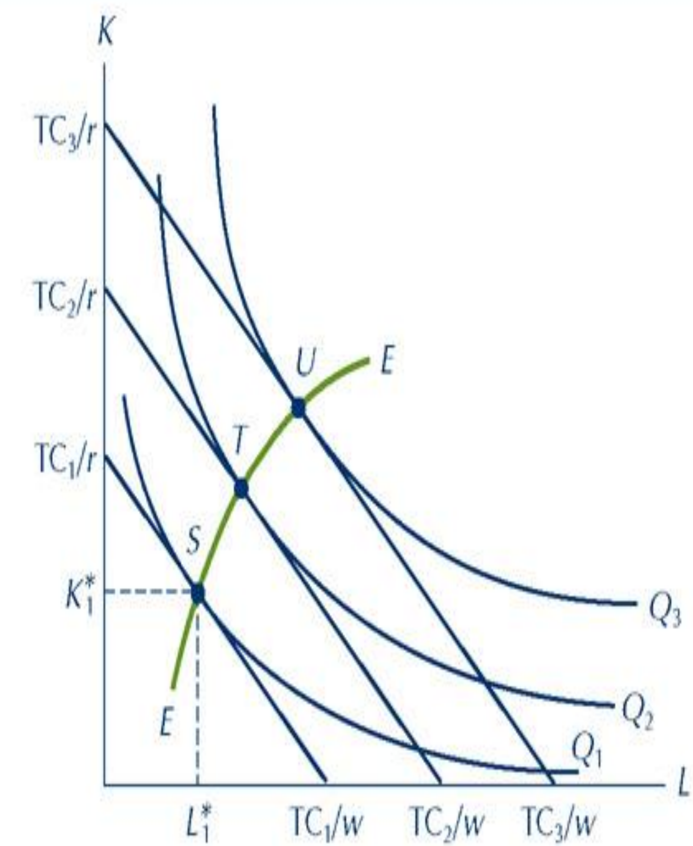


Long run Cost.

- Long run output expansion path.

FIGURE 10-15

The Long-Run Output Expansion Path
With fixed input prices *r* and *w*, bundles *S*, *T*, *U*, and others along the locus *EE* represent the least costly ways of producing the corresponding levels of output.



Long run Cost

- Example:
  - A firm with a production function as

$$Q(K,L) = 2K^{1/2}L^{1/2}$$

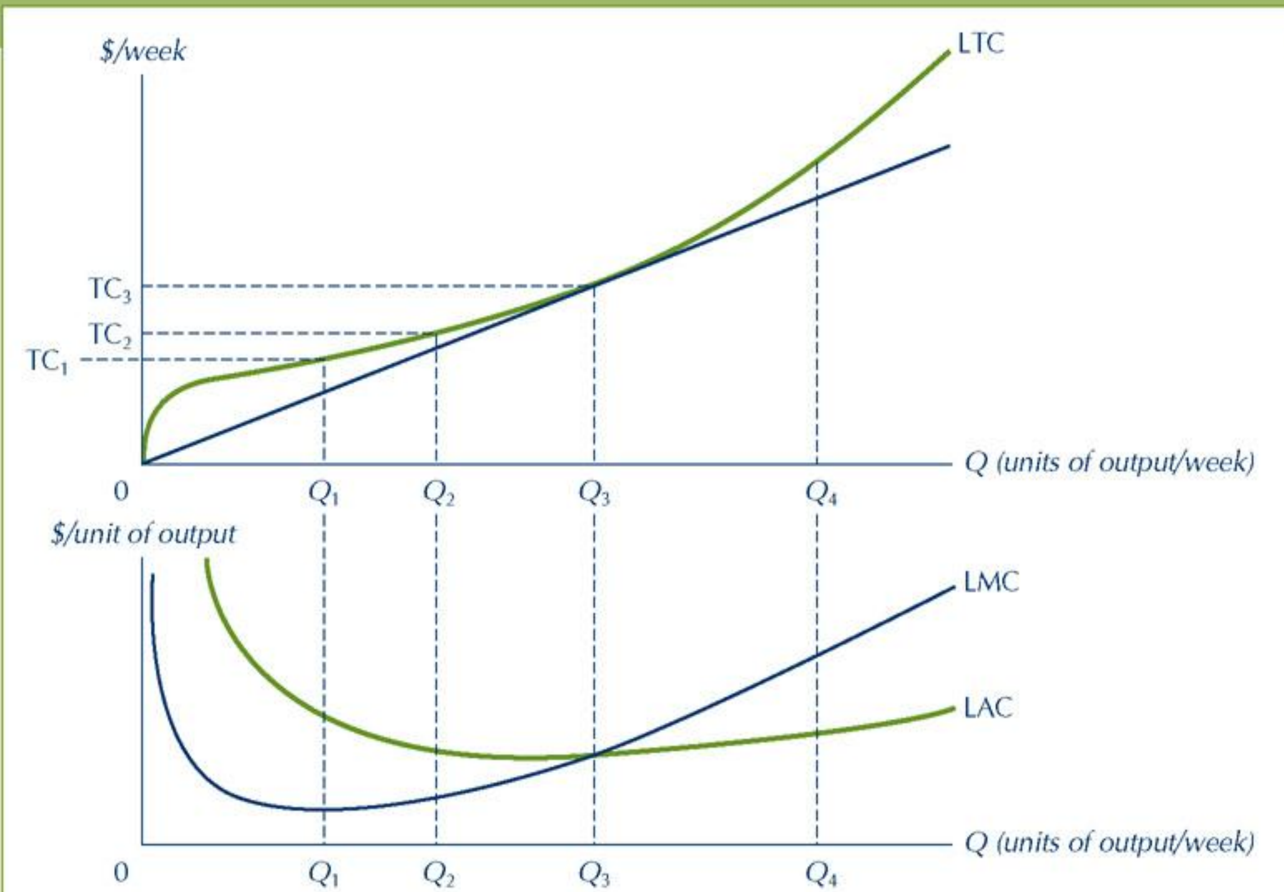
- Derive the optimal input mix as a function of wage and rent.
- If this firm currently operate optimally with 8 units of labour and 2 units of capital at the total cost of $16, what are the wage and rent in this market?

Long run Cost

- Graph of long run total cost, long run average cost and long run marginal cost. Fixed cost?

FIGURE 10-16

The Long-Run Total, Average, and Marginal Cost Curves
In the long run, the firm always has the option of ceasing operations and ridding itself of all its inputs. This means that the long-run total cost curve (top panel) will always pass through the origin. The long-run average and long-run marginal cost curves (bottom panel) are derived from the long-run total cost curves in a manner completely analogous to the short-run case.



Long run Cost

- Steps to get the long run cost from the production function.
  - Use the equilibrium relationship: MRTS = w/r to get the relationship between K and L.
  - Substitute K or L out of the production function and isolate L or K in term of Q.
  - Substitute K or L into the expenditure/cost function to get the long run total cost function.

Long run Cost

- Example:
  - Derive the total cost function for the following production function:

$$Q(K, L) = K^{1/2} L^{1/2}$$

$$Q(K, L) = K^{1/2} L$$

$$Q(K, L) = K^{1/3} L^{1/3}$$

FIGURE 10-17

The LTC, LMC, and LAC Curves with Constant Returns to Scale
(*a*) With constant returns, long-run total cost is strictly proportional to output. (*b*) Long-run marginal cost is constant and equal to long-run average cost.

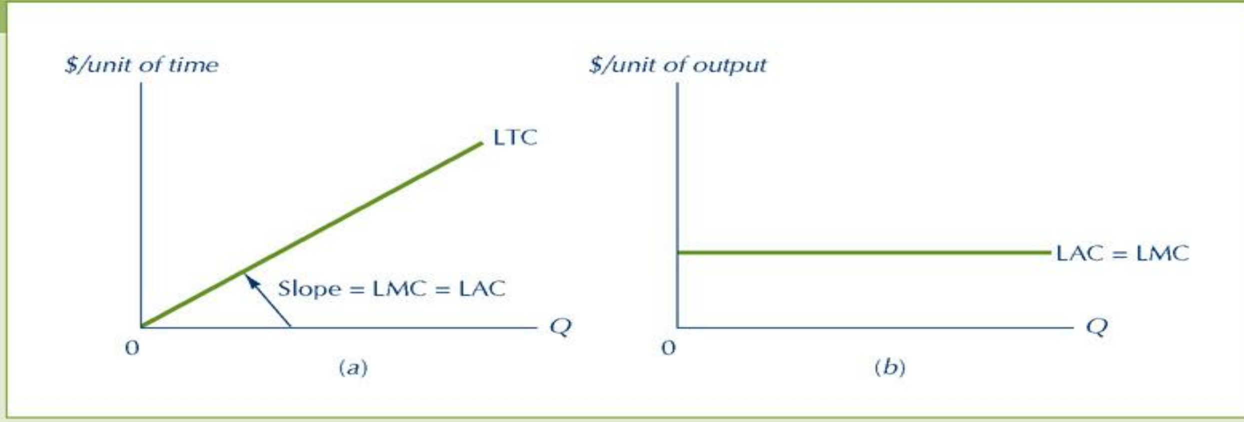


FIGURE 10-19

The LTC, LAC, and LMC Curves for a Production Process with Increasing Returns to Scale
With increasing returns, the large-scale firm has lower average and marginal costs than the smaller-scale firm.

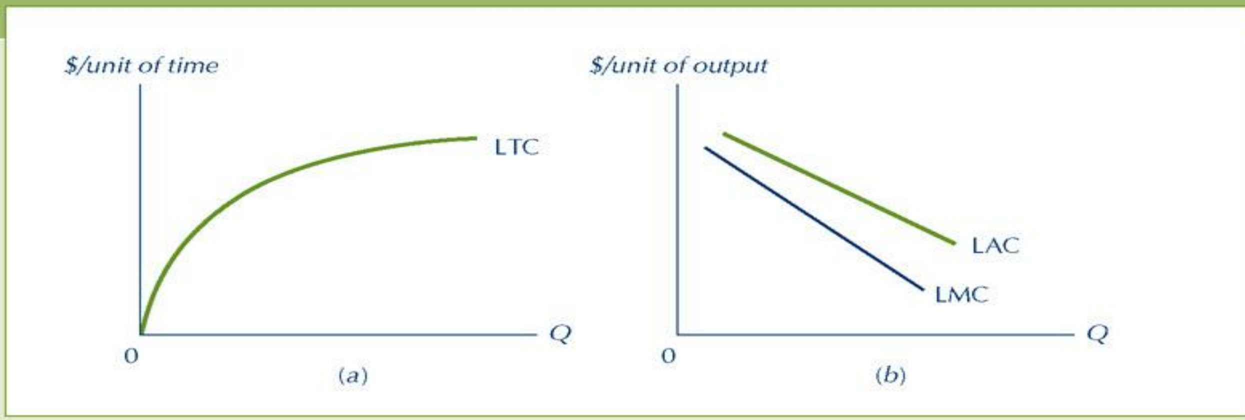
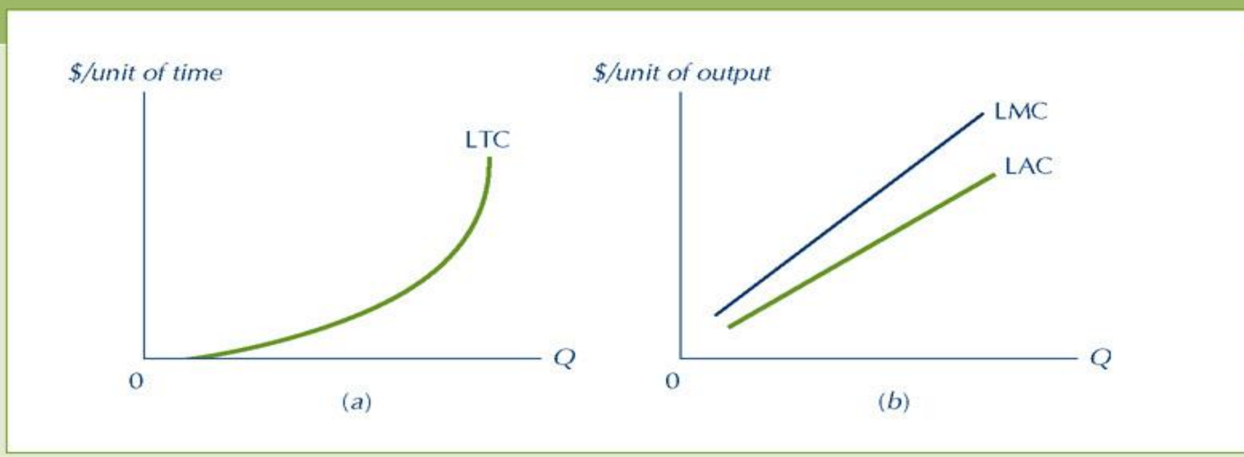


FIGURE 10-18

The LTC, LAC, and LMC Curves for a Production Process with Decreasing Returns to Scale
Under decreasing returns, output grows less than in proportion to the growth in inputs, which means that total cost grows more than in proportion to growth in output.



Example: Cobb-Douglas

- Cost minimization problem
- Let $q=2K^{1/2}L^{1/2}$ (Production function)
- *let* $w_L=2$ and $w_K=4$ (capital and labor costs)

$$\min_{K,L} 4K + 2L$$

- such that

$$q = 2K^{1/2}L^{1/2}$$

- Step (a):

$$\frac{MP_L}{w_L} = \frac{MP_K}{w_K} \Leftrightarrow L = 2K$$

- Step (b):

$$2K^{1/2}L^{1/2} - q = 0$$

- Step (c):

$$2K^{1/2}(2K)^{1/2} - q = 0 \Rightarrow 2\sqrt{2}K - q = 0$$

$$\Rightarrow K^* = \frac{1}{2\sqrt{2}}q \Rightarrow L^* = 2K^* = \frac{1}{\sqrt{2}}q$$

Cobb-Douglas (continued)

- So the cost function will be:

$$C(q) = w_L L^*(q) + w_K K^*(q)$$
$$= 2\frac{1}{\sqrt{2}}q + 4\frac{1}{2\sqrt{2}}q = 2\sqrt{2}q$$

Example 2: the *Leontief technology*

- q = min(1/6L,K)
- Solution cost minimization problem is now obtained by common sense, not by mathematical recipe
- L and K must be on the efficient locus: 1/6L=K (why?)
- So q= 1/6 & L=K and it follows that: L*(q) = 6q and K*(q)=q
- Cost function
- $C(q) = W_L L^*(q) + w_L K^*(q)$ = wL6q +wKq = (wL6 +wK)q
- For example, if wL=wK=1, then C(q)=7q

Notes Page 40:

<u>Appendix 10.1</u>

A firm produces output with the production function

$$Q = \sqrt{K}\sqrt{L} = K^{1/2}L^{1/2}$$

where K and L denote its capital and labour inputs, respectively. If the price of labour is $1/unit and the price of capital is $4/unit, what quantities of capital and labour should it employ if its goal is to produce 2 units of outputs?

Wage=w=1
Rent=r=4
Output=Q=2

$$Q = \sqrt{K}\sqrt{L} = K^{1/2} L^{1/2}$$

$$\frac{\partial Q}{\partial L} = MP_L = \frac{1}{2} K^{1/2} L^{-1/2}$$

$$\frac{\partial Q}{\partial K} = MP_K = \frac{1}{2} K^{-1/2} L^{1/2}$$

$$MRTS = \frac{MP_L}{MP_K} = \frac{\frac{1}{2} K^{1/2} L^{-1/2}}{\frac{1}{2} K^{-1/2} L^{1/2}} = \frac{K}{L} = \frac{w}{r}$$

$$\frac{K}{L} = \frac{1}{4} \Rightarrow K = \frac{L}{4}$$

$$Q = 2 = K^{1/2} L^{1/2}$$

$$2 = \left(\frac{L}{4}\right)^{1/2} L^{1/2} = \left(\frac{1}{4}\right)^{1/2} L = \frac{1}{2} L$$

$$2 = \frac{1}{2} L$$

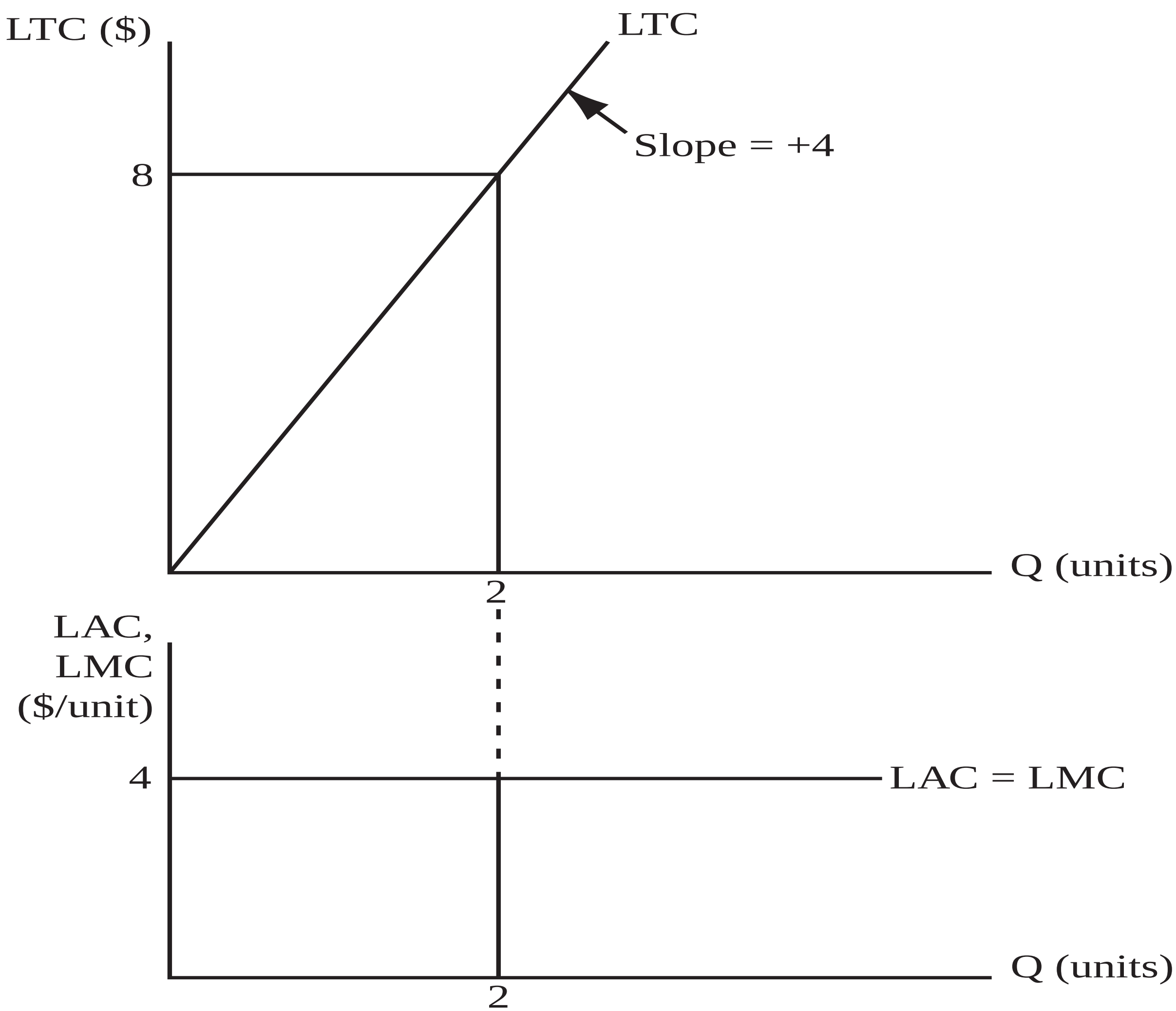
$$4 = L$$

K=4/4=1

2. Sketch the LTC, LAC and LMC.

TC=Kr+Lw=(1)(4)+(4)(1)=8

In Problem 1, producing 2 units of output required 1 unit of capital (at $4/unit) and 4 units of labour (at $1/unit), so that TC = wL + rK = 1(4) + 4(1) = $8, giving ATC = TC/Q = 8/2 = *$4/unit*. Since the sum of the exponents of K and L is ½ + ½ = 1, the production function displays *constant* returns to scale. These characteristics are reflected in the following cost curve diagrams.

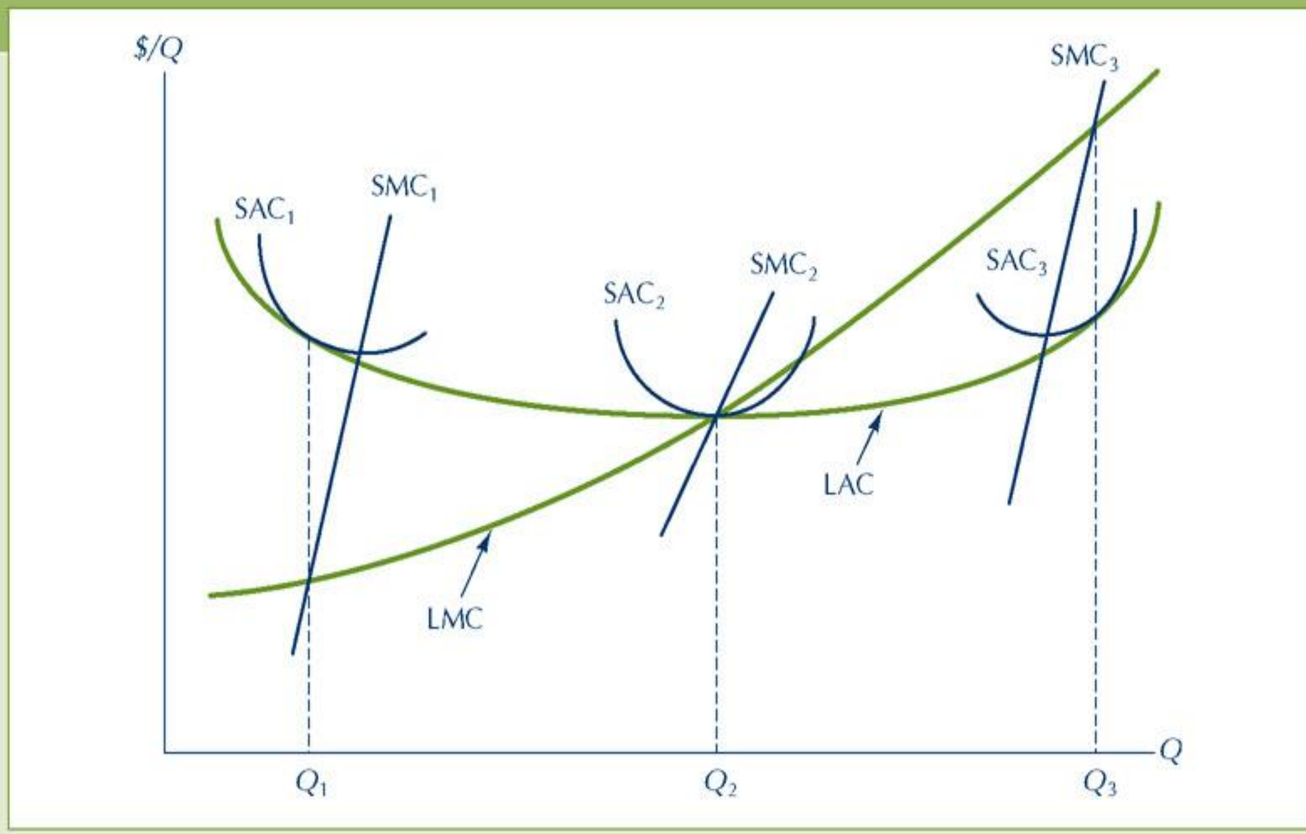

LTC ($)
LTC
Slope = +4
8
Q (units)
2
LAC,
LMC
($/unit)
4
LAC = LMC
Q (units)
2

Long run Cost Curve

- Relationship between LAC and SAC curves.

FIGURE 10-22

The Family of Cost Curves Associated with a U-Shaped LAC
The LAC curve is the "outer envelope" of the SAC curves. LMC = SMC at the *Q* value for which the SAC is tangent to the LAC. At the minimum point on the LAC, LMC = SMC = SAC = LAC. All marginal cost curves, short run and long run, intersect their corresponding average cost curves at their minimum points.



Deriving the LAC Function from the SAC Function

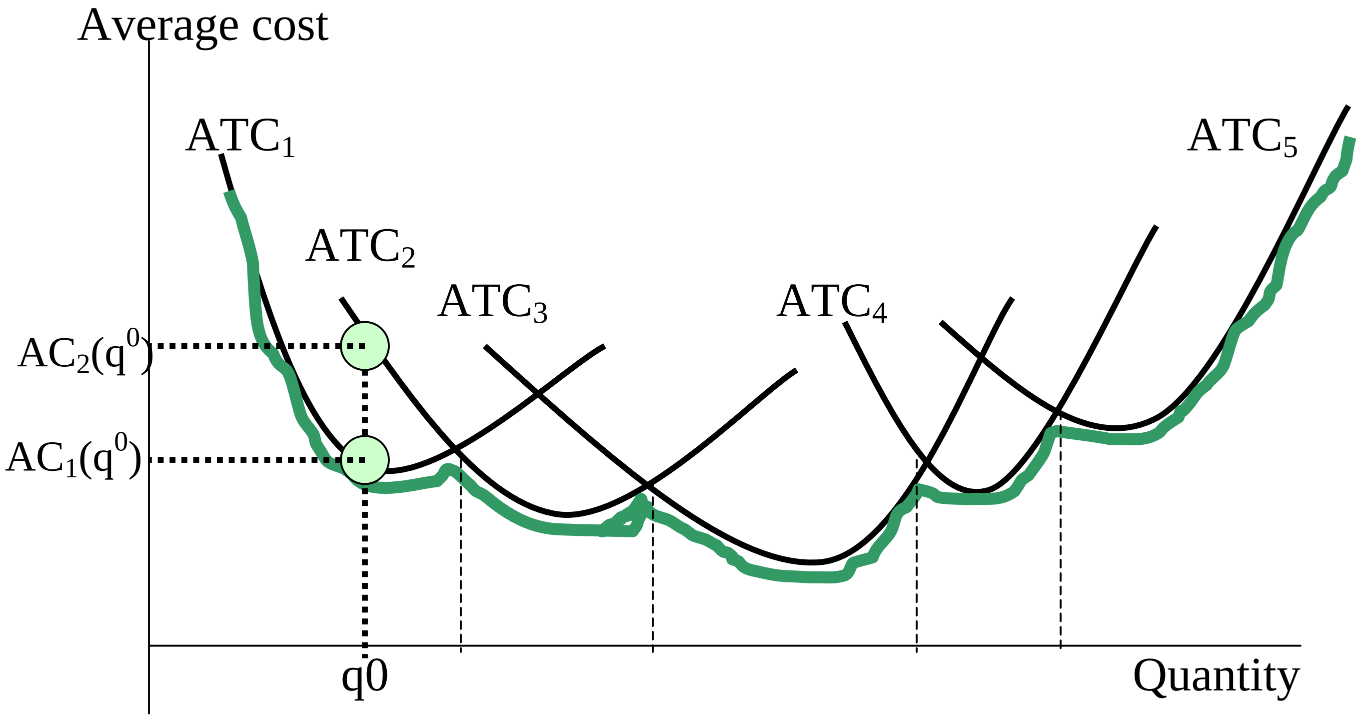
In the long run a firm operates by choosing a plant size and the amount of labour that produces a quantity of product at the lowest possible total cost.

The question we will address is: “How does the firm determine its size to attain the lowest long-run total and average costs?”

1) The Long-Run Average Cost Function with a Limited Choice of Plants:

Suppose we have 5 plants of 5 different sizes.
Let plant 1 represent the smallest plant.

Let plant 5 represent the largest plant.
Let plants 2-4 represent plants of a size in between small and large.



The graph consist of five short-run average cost functions representing the five firms' ACs.

If the firm decides to produce q^0 units, the average cost of producing q^0 with the first plant is lower than the second plant.

The firm should build a small firm if it chooses to produce q^0.

To find the long-run average cost from the firm's short-run average cost functions, for each quantity produced, simply move up vertically until you reach the first short-run average cost function.

The plant with ***that*** average cost function produces that quantity at the lowest average and total costs in the long run.

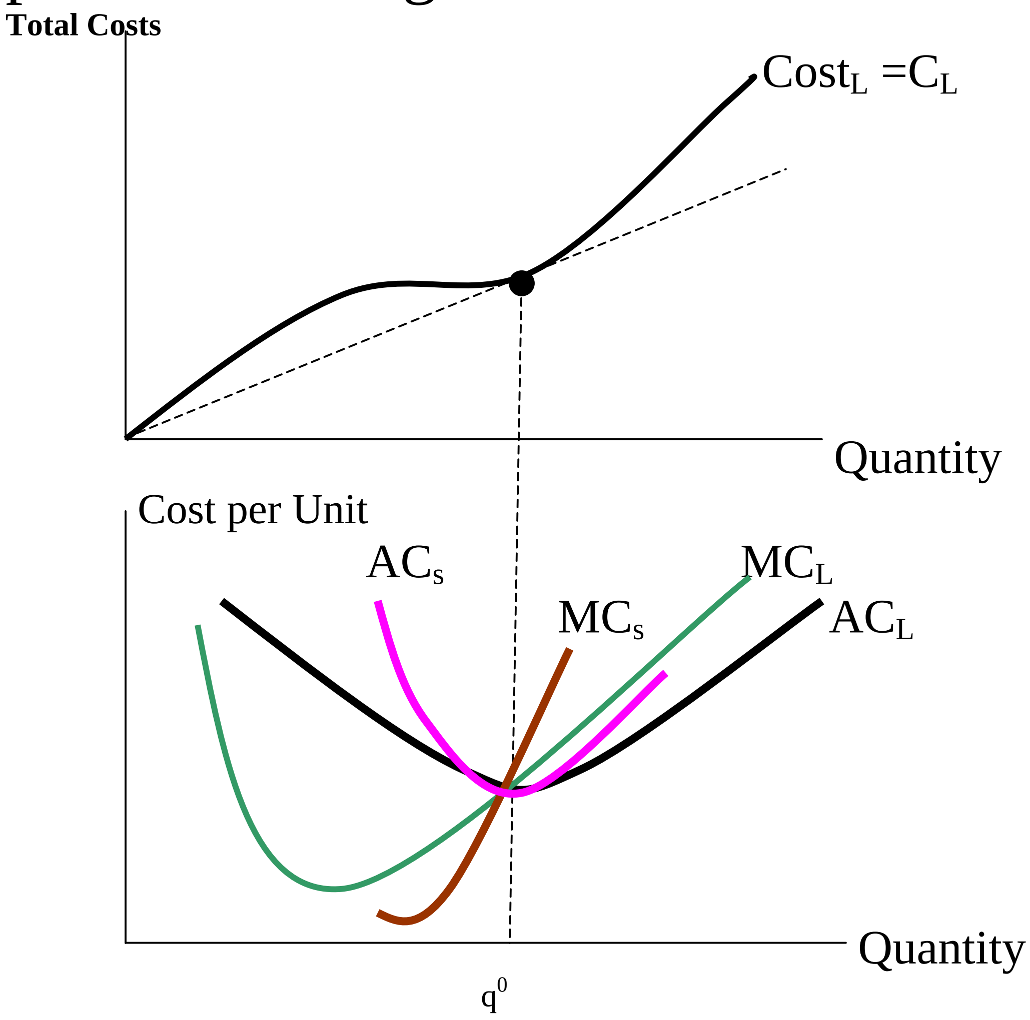
The long run average cost function becomes the *scalloped* average cost function.

2) <u>**The Long-Run Average Cost Function With A Continuum of Plants Sizes:**</u>

Now suppose the firm is no longer limited to five plant sizes. It can build any size plant it chooses.

We can derive the ***long run total cost*** function by connecting all the points that identify the <u>minimum total cost of producing each quantity</u>.

That is, each plant produces a certain quantity at a minimum point. By connecting each of these minimum points, we can derive a curve representing these minimum cost-quantity points: the long-run total cost curve:



Let $C_L(q)$=the long run total cost function, (representing the minimum total cost of producing each quantity) in the top diagram.

We can derive the long-run AC and marginal cost function from the long run total cost function.

The long run AC function is the ‘U’ shaped curve labelled AC_L.
The long run MC function is labelled MC_L.

When there is a continuum of plant sizes, each plant has the lowest average cost for producing a unique quantity.

Only one plant can produce a specific quantity at the lowest possible cost. That point will be along the long-run average cost curve. For that plant, there is only one point where the SAC will touch the LAC.

To derive the LAC, simply find the plant with the lowest SAC. By connecting all these points, we form the LAC function.

The long run average cost function shows the lowest average cost of producing each quantity.

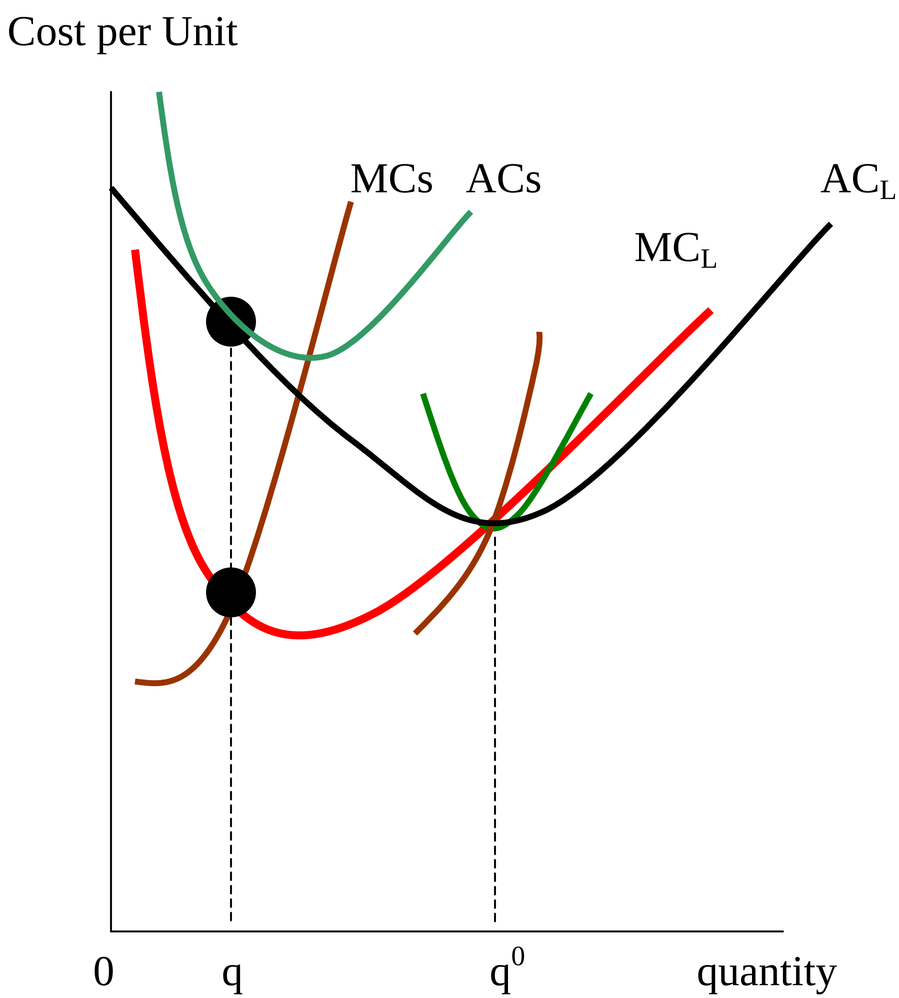
The long run marginal cost function shows the incremental cost of producing another unit.

The long-run MC will equal the long run AC when LAC is at a minimum.

Note:

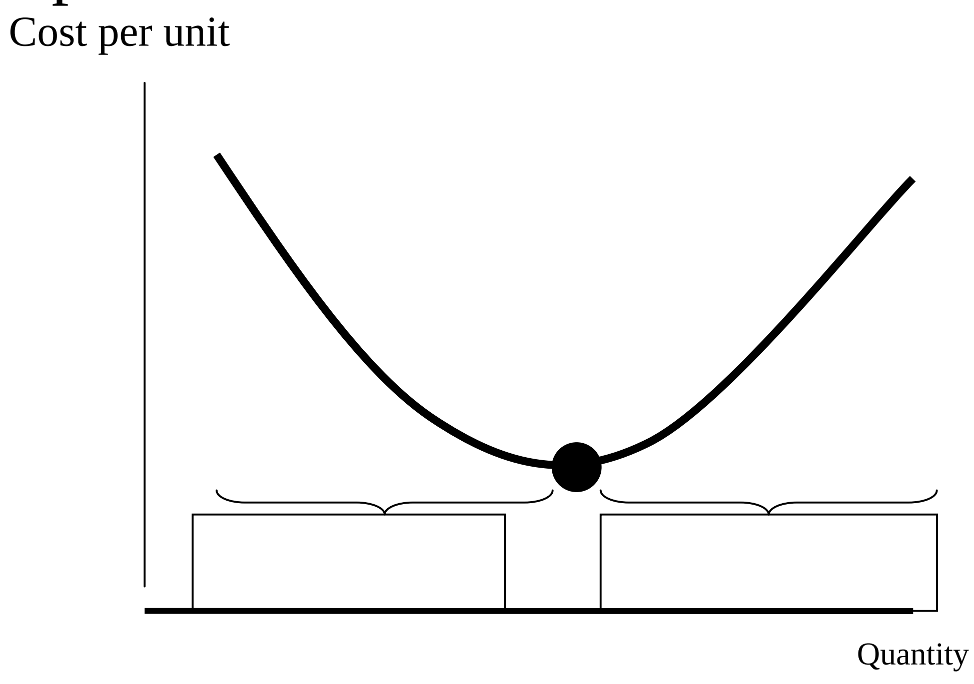
Let q^0= the quantity of output at the **minimum** point along the LAC function. Then:

1) If $q < q^0$, $AC_L = AC_S > MC_L = MC_S$
2) If $q = q^0$, $AC_L = AC_S = MC_L = MC_S$
3) If $q > q^0$, $AC_L = AC_S < MC_L = MC_S$


Cost per Unit
MCs
ACs
AC_L
MC_L
0
q
q0
quantity

The Number of Firms and the Long-Run Cost Function

The number of firms that operate in a particular industry depends on the ***shape*** of the long-run average cost function. The shape places a limit on the number of firms that can efficiently operate in the market and achieve **minimum cost** of production.



When the LAC decreases, the firm is experiencing ***internal economies of scale.*** I.e. as output increases by increasing inputs, the average cost is falling.

Eventually, the firm will hit a minimum and any other increases in output will increase average cost. Here the firm will experience ***internal diseconomies of scale***.

Diseconomies of Scale and Industries with Many Firms

There are ***indicators*** whether there are internal diseconomies of scale.
Many firms, where even a large firm holds only a small market share, is an indicator that diseconomies of scale limit the size of any one firm.

Economies of Scale and Industries With Few Firms

In some industries there are persistent economies of scale.

This is because:

1) Indivisibility of a factor of production: there may be a minimum level of output that the plant size cannot be scaled back to.
2) Volume and area connection: total cost increases less proportionately with output.
3) Specialization within the firm: large firms can separate tasks within the firm to operate more efficiently.

Natural Monopoly: a single firm can produce a given output at the lowest total cost.

This occurs when a firm experiences **continual internal economies of scale.**

The total cost of producing a given quantity increases when there is more than one firm.
Example: B.C. Hydro