Intermediate Microeconomics Calculus Module

The following syllabus is intended both as a refresher of topics from Math 100 / 102, and to emphasize calculus involving several variables (multivariate calculus). This syllabus will make extensive use of the math review from the Frank-Parker text:

http://www.mcgrawhill.ca/college/olcsupport/frank4/math/

It is assumed that students are already quite familiar with Modules 1 and 2.

This review is designed to be delivering in two 45-minute lectures. In lecture 1 we will cover algebra and some calculus topics, using material mainly from Modules 3, 8 and 10 in the Frank-Parker text. You may use the previous mentioned modules to supplement this syllabus. It is expected that you are already familiar with modules 1 and 2, which are required for the material in module 3. In lecture 2, we will cover multivariate calculus, using Module 12 from Frank-Parker, and Section 7.4 and 7.5 from *Brief Calculus – An Applied Approach* by Larson. You will be tested on this material on January 23rd or 25th, depending on your section.

This syllabus is a work in progress, covering only the material for Lecture 1. This syllabus will be updated by January 16th.

<u>Lecture 1</u>

1 Algebra

Overlooking some basic results from algebra can be very detrimental to a test score. Some frequent mistakes involve exponents, namely whether they should be added (subtracted) or multiplied (divided). For example, if $y = x^2$, then:

$$y^3 = x^{2\times 3} = x^6$$

$$y \times x^7 = x^2 x^7 = x^{2+7} = x^9$$

$$y/_{x^4} = x^2 x^{-4} = x^{2-4} = x^{-2} = 1/_{x^2}$$

You should see the following notes on *using powers or exponents:* http://www.mcgrawhill.ca/college/olcsupport/frank4/math/pdf/mod8.pdf

<u>Exercises:</u>

-Exercise 1 from the above module.

-Example 1 from Section 4.1, Larson.

It is also important to know how to solve a *system of equations*. Perhaps the simplest example is when a demand function is equated to a supply function in order to solve for *price* and *quantity*.

Example:

The direct demand and supply functions are $Q^D = 500 - 3P$ and $Q^S = 50 + 6P$, respectively. If demand = supply, then you should find that $P^* = 50$ and $Q^* = 350$.

You should see http://www.mcgrawhill.ca/college/olcsupport/frank4/math/pdf/mod3.pdf

Exercises: 2 through 8 from the above module.

2 Derivatives

The derivative is an important tool in economics. Understanding and being able to use a derivative will be vital to your success in many economics course.

A derivative expresses a *rate of change*. In the case of a function of one variable, $f(x)$, the derivative can be interpreted as the *slope* of the function in point x. So why should we care about a rate of change? There are many problems in economics that ask, *how much?* The answer is usually found when the benefit of *one more unit* is just equal to the cost of *one more unit* (i.e., where the marginal benefit equals the marginal cost). If we can mathematically express benefits and costs, then we can usually find marginal benefits and costs by taking derivatives.

The following section on derivatives summarizes *Math Module 10:* http://www.mcgrawhill.ca/college/olcsupport/frank4/math/pdf/mod10.pdf

2.1 Notation

There are several notations commonly used when referring to a derivative. Here are a few:

Function:	Alternative notations			
$y = f(x)$	$f'(x)$	$\frac{df}{dx}$	y'	$\frac{dy}{dx}$

2.2 Important functional forms and their derivatives

There are several functional forms which you should readily know the derivatives of. Below is a table reproduced from *Math Module 10* of the Frank-Parker text.

	Function: $y = f(x)$	**Function: Example**	**Derivative:** $dy/dx = y' = f'(x)$	**Derivative: Example**
1.	$y = a$	$y = 4$	$y' = 0$	$y' = 0$
2.	$y = ax$	$y = 12x$	$y' = a$	$y' = 12$
3.	$y = ax^n$	$y = 4x^3$ $y = 3x^4$	$y' = nax^{n-1}$	$y' = 12x^2$ $y' = 12x^3$
4.	$y = f(x) + g(x)$	$y = 4x^3 + 3x$	$y' = f'(x) + g'(x)$	$y' = 12x^2 + 3$
5.	$y = f(x) \times g(x)$	$y = (3x^2)(2+x)$ $= 6x^2 + 3x^3$	$y' = f(x)g'(x) +$ $g(x)f'(x)\ y'$	$y' = (3x^2)(1) +$ $(2+x)(6x)$ $=12x + 9x^2$
6.	$y = \dfrac{f(x)}{g(x)}$	$y = \dfrac{(1+2x)}{(x+4)}$	$y' = [g(x)f'(x) -$ $f(x)g'(x)] / [g(x)]^2$	$y' = \dfrac{7}{(x+4)^2}$
7.	$y = f(u(x))$	$y = (3x+4)^2$, with $u = 3x + 4$, $y = u^2$	$y' = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$	$y' = 2(3x+4)(3)$ $= 18x + 24$
8.	$y = ln\ x$	$y = ln\ x$	$y' = 1/x$	$y' = 1/x$
9.	$y = e^x$	$y = e^x$	$y' = e^x$	$y' = e^x$

These are well known results which can be found in any introductory calculus text. You should practise these results.

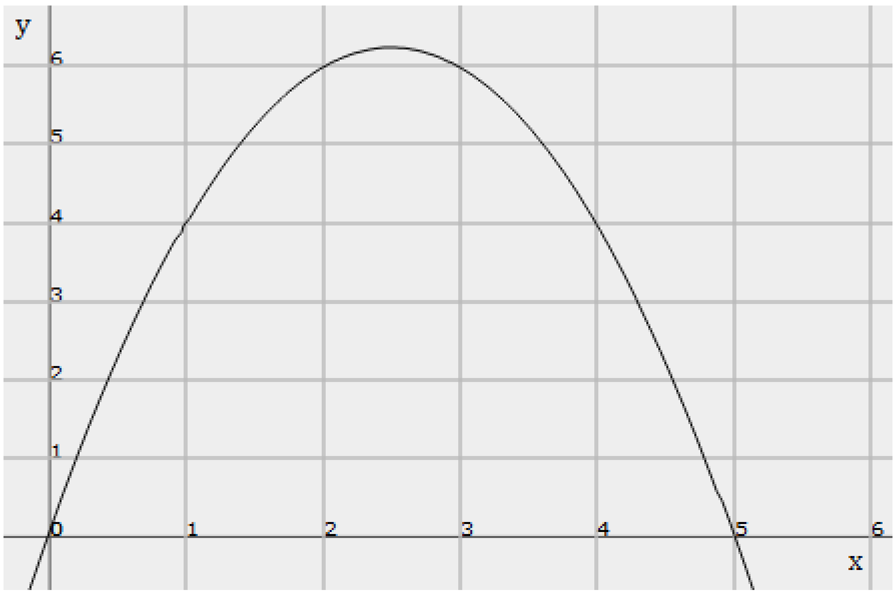
<u>Exercises:</u> examples 1 – 9, Section 2.2; examples 1, 2, 4, 8, Section 2.3; examples 1, 2, 4, Section 2.4; Larson.

2.3 An application of the derivative – finding maxima and minima

A derivative can be used to find the maximum or minimum of a function through the idea of a *first order (necessary) condition*. A first order necessary condition is a logical argument which states that: in order for x to achieve a maximum or minimum of a function $f(x)$, it is necessary that the first derivative of that function be zero at x. Recall that the first derivative is the slope of the function.

<u>Example:</u>

Consider the function $y = 5x - x^2$. Find the maximum of this function.



From the above graph, it looks like *y* is at a maximum when *x* = 2.5. Let's verify this guess.

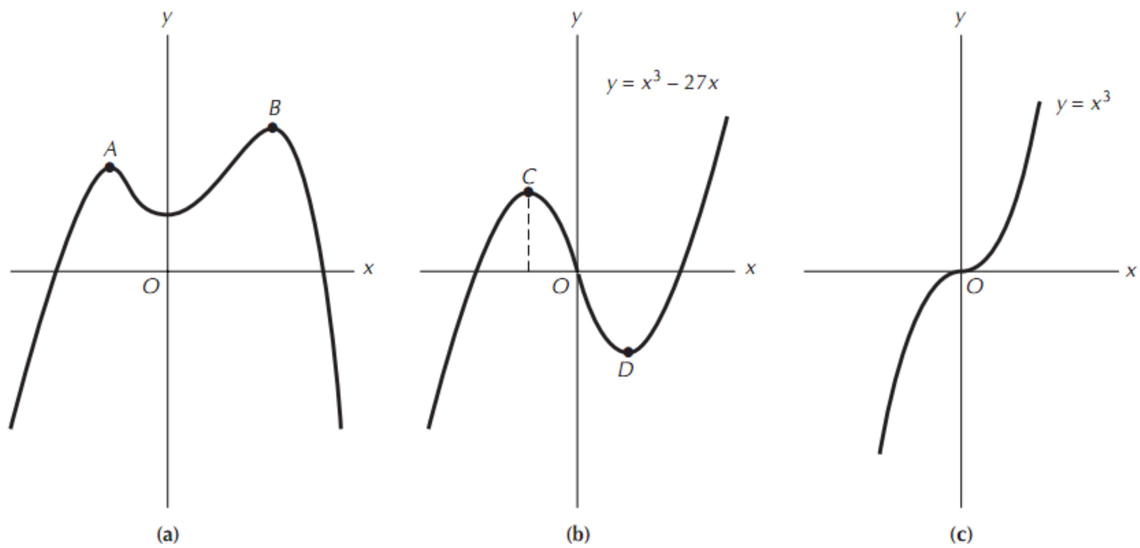
At a maximum of *y*, *x* must be at a value such that $\frac{dy}{dx}$ equals zero. Here,

$$\frac{dy}{dx} = 5 - 2x.$$

Setting this derivative equal to zero and solving for *x* gives *x* = 2.5.

The following figure reproduced from *Math Module 10* illustrates the importance of a *second order condition*, which makes use of the second derivative of the function, denoted $f''(x)$ or $\frac{d^2y}{dx^2}$. The second derivative of $f(x)$ is simply found by taking the derivative of $f'(x)$. The interpretation of a second derivative is a little less intuitive, however, as it is the rate of change of the rate of change. In the graph below, the second derivative will be negative at points A, B and C. It will be positive at point D. It will be zero at the origin in figure (c). At all of these points, the first derivative at *x* will be equal to zero. In order to be certain as to whether you have maximized or minimized a function, the second derivative should be checked. To reiterate, at a maximum, the second derivative will be negative, since the first derivative is decreasing, or becoming more negative.

FIGURE M.10-3



Example:

Continuing from the previous example, we see that $\frac{d^2y}{dx^2} = -2$, which is negative for all values of *x*. This means that the function is concave, and has only one *global* maximum. Hence, we have maximized, not minimized.

Exercises: 1 and 3 from the above module.

References

Microeconomics and Behaviour Math Review, Robert Frank and Ian Parker, 4th Canadian Edition. URL: http://www.mcgrawhill.ca/college/olcsupport/frank4/math/

Brief Calculus – An Applied Approach, Ron Larson, 8th Edition.