The Isocost Function

The production function summarizes the technological options facing the firm. Unfortunately it is not enough to be just aware of these options when making the output and factor input decision.

- The **price** of a factor of production is extremely important in this decision.

- In order to minimize costs and produce efficiently, the firm must know exactly what its costs will be.

Let ‘w’ be the annual cost of each unit of labour.

Let ‘M’ be the price of capital that never needs to be replaced.

Let ‘i’ be the interest rate.

Let r=iM= the opportunity cost of maintaining one unit of capital.

Notation:

P_K=r = price of capital

P_L=w = price of labour

q=Q= output

The total annual cost of producing the good is:

Total cost = Cost of labour + Cost of Capital

$$\mathbf{C = wL + rK}$$

Rearranging we have an expression for K:

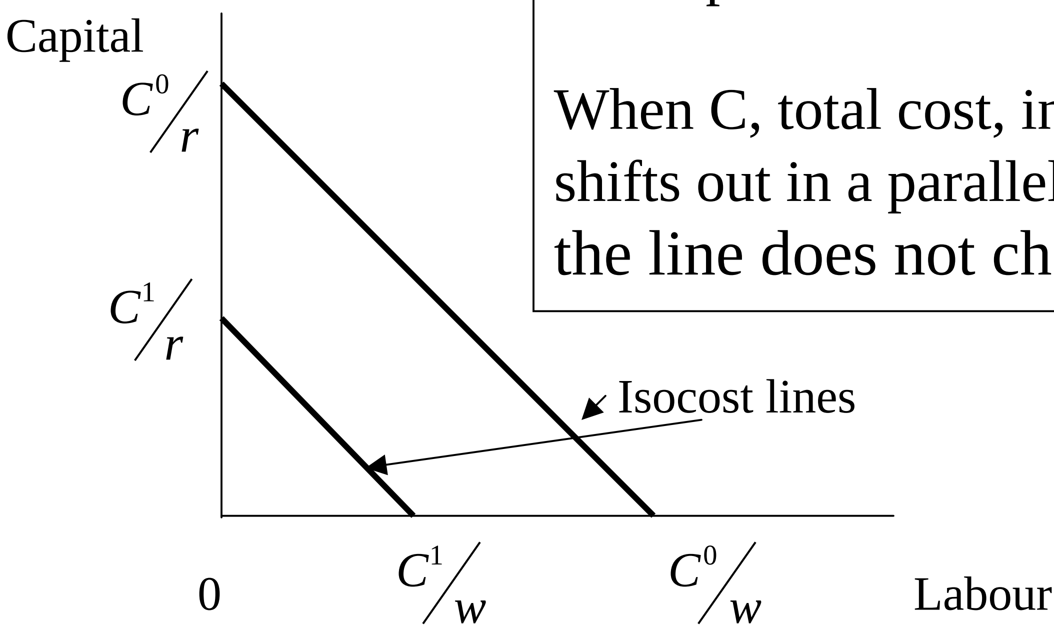
$$K = \frac{C}{r} - \frac{w}{r}L \quad \Leftarrow \text{Isocost Line}$$

The isocost line represents the total cost C as constant for all K-L combinations satisfying the equation.

"An isocost line shows the different combinations of factors of production that can be employed with a given total cost."

For a given cost C, the vertical intercepts of these lines are C/r.
C/r is the amount of capital that can be employed when no labour is used.
The slope of the line is -w/r = the negative of the factor price ratio.

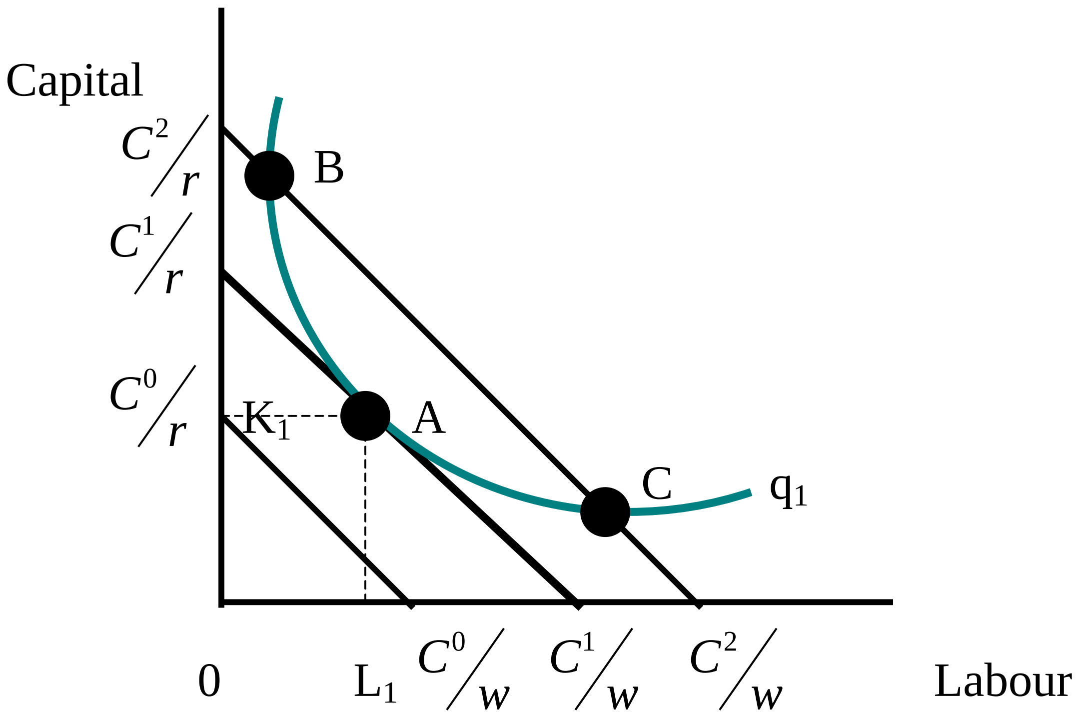
When C, total cost, increases, the isocost line shifts out in a parallel fashion, but the slope of the line does not change.



When C, total cost, increases, the isocost line shifts out in a parallel fashion, but the slope of the line does not change.

The Production Decision

We can now determine which combination of factors produces a given quantity at the lowest total cost.



There are three isocost lines. The firm wants to produce q_1 units. The cost minimizing point is at A.

- Total cost increases by moving from each isocost line.
- The firm can produce q_1 units along the isoquant q_1.
- The total cost of produces q_1 units is minimized at point A.

If the firm is producing q_1 units at minimum total cost, the slope of the isoquant equals the slope of the isocost line:

$$\boxed{MRTS_{KL} = -\frac{w}{r}}$$ (Minimum cost Condition)

"A firm minimizes the total cost of producing a given quantity by selecting a combination of factors where the slope of the isoquant equals the slope of the isocost line."

And since $MRTS_{KL} = -\frac{MP_L}{MP_K}$,

and $MRTS_{KL} = -\frac{w}{r}$, then $-\frac{MP_L}{MP_K} = -\frac{w}{r}$.

Rearranging $\frac{MP_L}{w} = \frac{MP_K}{r}$ (Minimum Cost Condition)

The firm minimizes the total cost of providing a given quantity if the ratio of the marginal product of a factor to its price is the same for all factors.

“The lowest total cost of producing a given quantity occurs when the ratio of the marginal product of a factors to the last dollar spent on it is equal for all factors of production.”

The rate of the MP_L to the price of labour represents the increase in output due to the last dollar spent on labour.

To minimize total cost, the additional output due to the last dollar spent on labour must be equal to the addition output due to the last dollar spent on capital.

If they are not equal, it pays the firm to reallocate its expenditure from one factor to another.

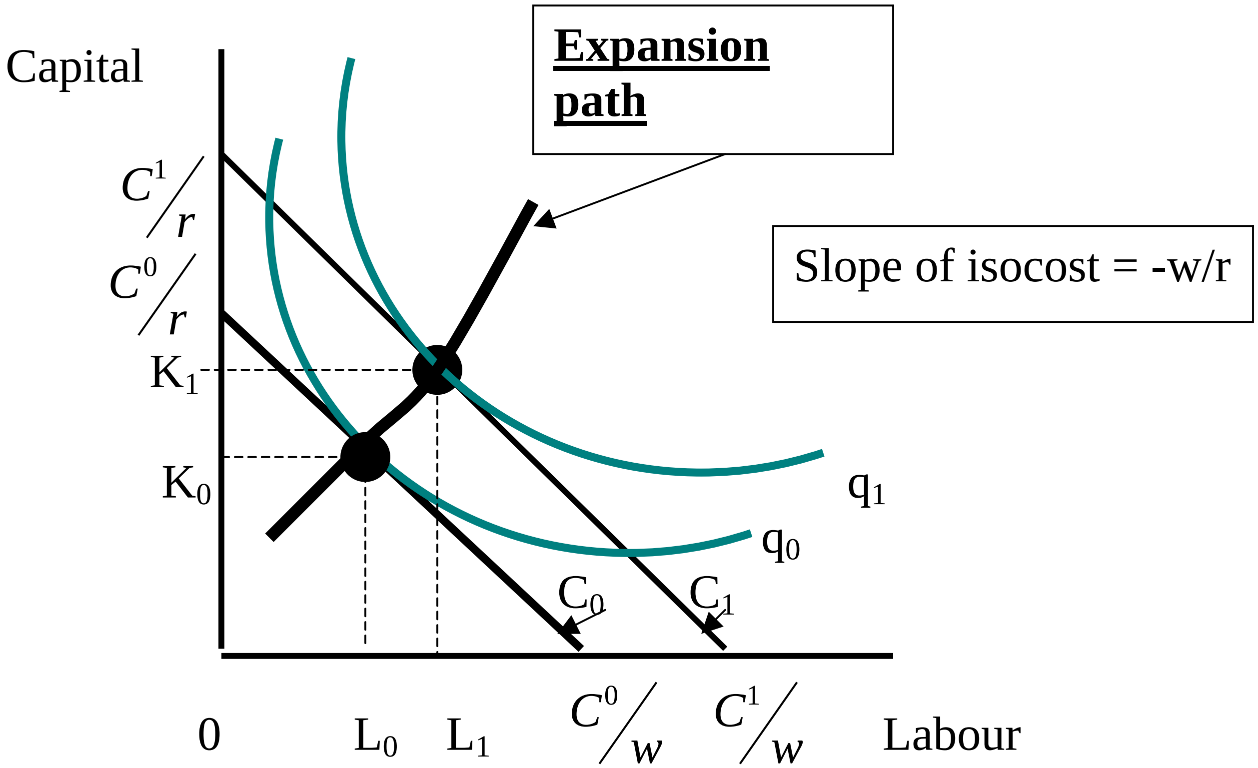
The firm should spend more money on the factor that gives the firm a greater boost in output for the extra dollar spent.

The Long-Run and Short-Run Total Cost Functions

We will first examine the relationship between total cost and quantity produced when all factors of production can be varied and then when one factor is fixed and the other factor is variable.

The Long-run Total Cost Function

The long-run total cost function represents the lowest total cost of producing a unit of a good when all inputs are variable.



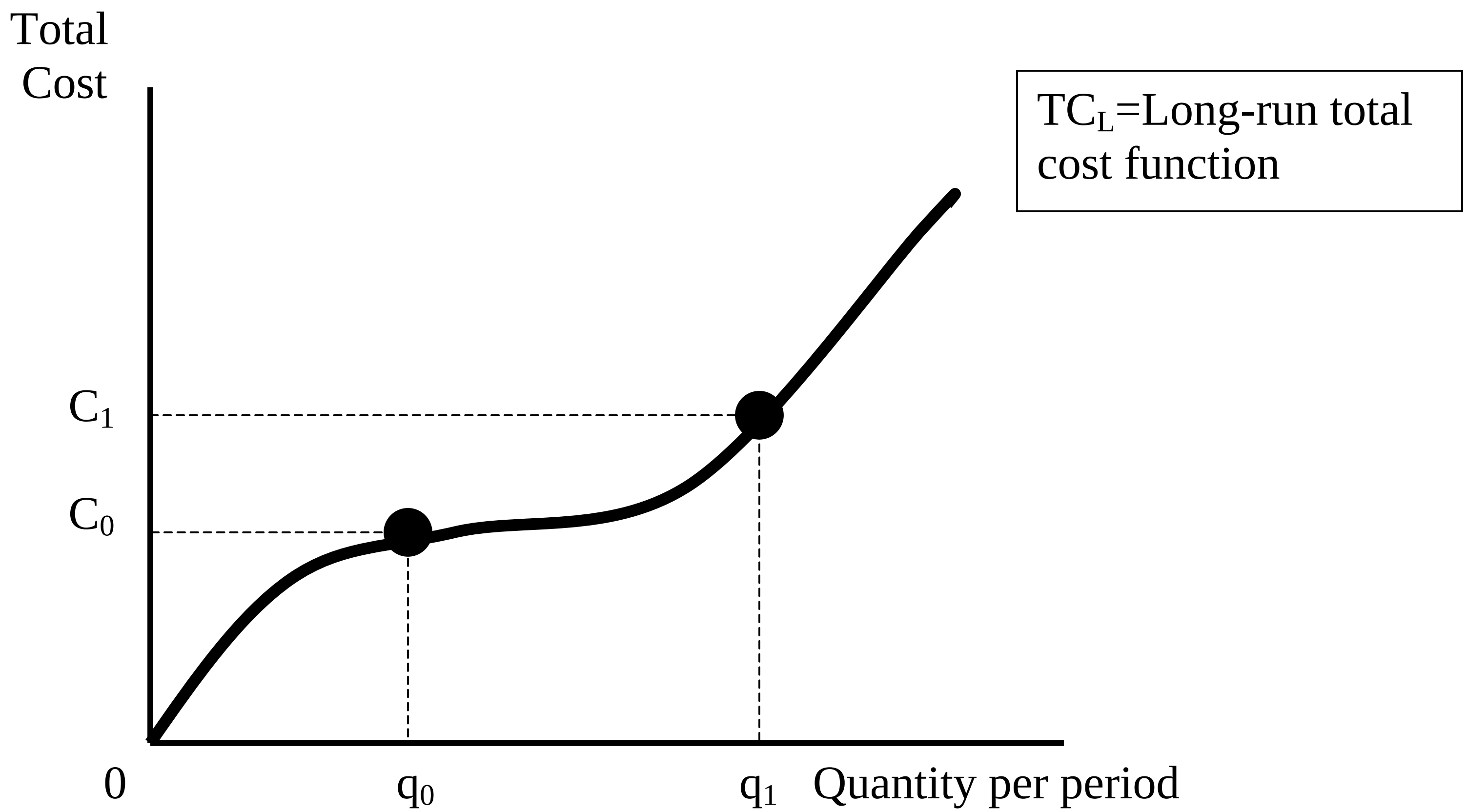
On the diagram, there are two point of tangency of isocost lines with two isoquants.

If the firm wants to produce q_0 units of the good, it can minimize its total cost by employing L_0 units of labour and K_0 units of capital, such that its minimum cost is C_0.

If the firm wants to produce q_1 units of the good, it can minimize its total cost by employing L_1 units of labour and K_1 units of capital, such that its minimum cost is C_1.

The curve that connects all these points of tangency between an isoquant and an isocost line is referred to as the **expansion path**. Each point relates a quantity with a minimum total cost.

To derive the long-run total cost function, we take the pairs of total cost and quantity from the expansion path.



"The long-run total cost function shows the lowest total cost of producing each quantity when all factors of production are variable."

The Short-Run Total Cost Function

In the short run, one factor is fixed. Hence, the cost of this fixed factor does not change as quantity produced changes.

The short-run total cost function consists of two components:
Short-run total cost = total fixed cost + total variable cost

$$TC_S = TC_S(q) = F + V(q)$$

The short-run total cost function shows the lowest total cost of producing each quantity when one factor is fixed.

The fixed cost must be paid regardless of whether any of the good is produced. The variable cost will increase when the quantity produced increases.

Deriving the Short-run Total Cost Curve

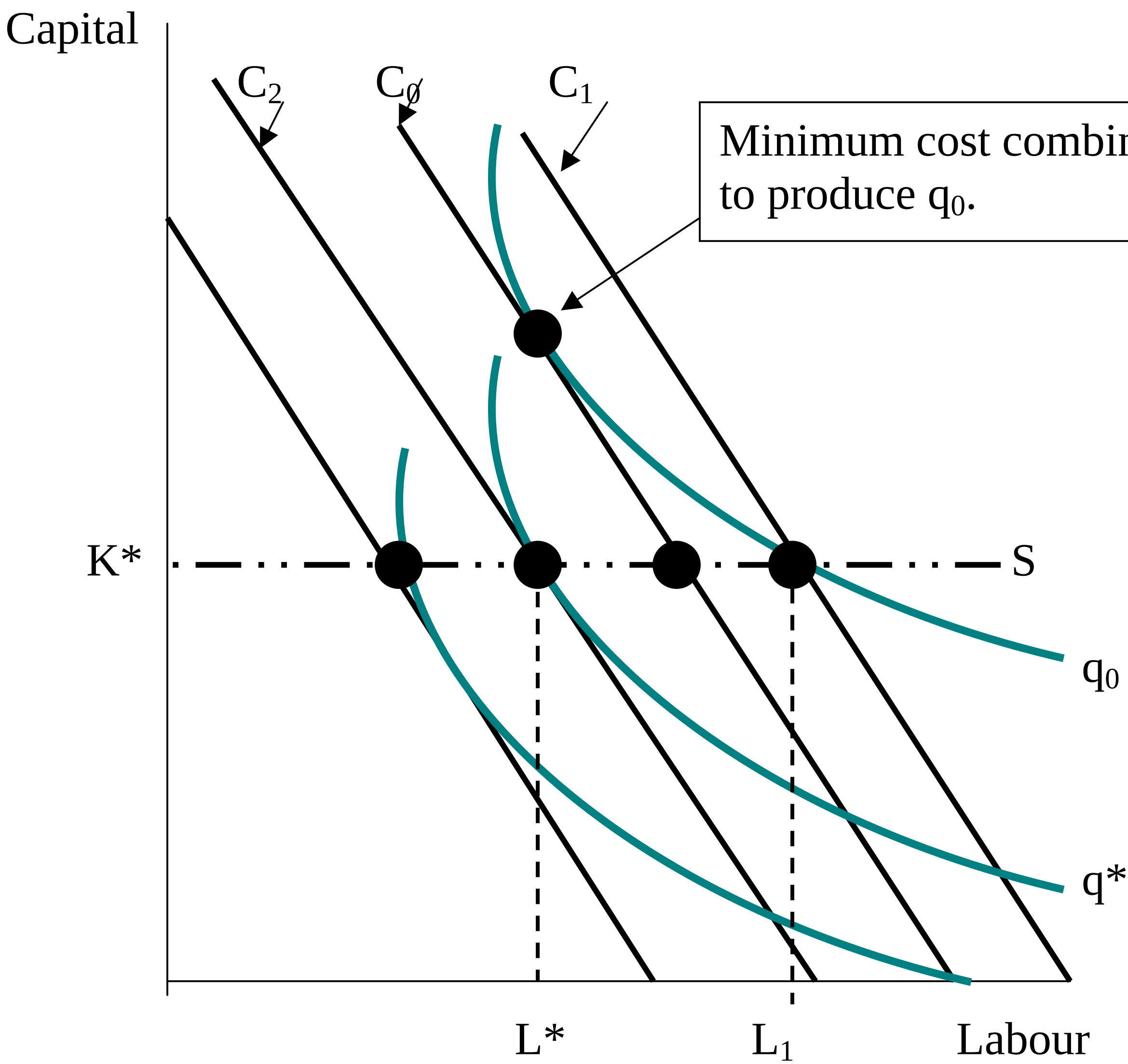
- Refer to the diagram on the next page.

In the short-run, the firm has K* units of capital.

The firm's **expansion path** is the horizontal line 'S' because the amount of capital is fixed.

If the firm wants to produce q_0 units in the short run, it must employ L_1 units of labour and incur a total cost of C_1.
The lowest total cost of producing quantity q_0 is along the isocost line C_0.

However, since the cost C_1 is greater than C_0, it is more expensive to produce q_0 units in the short run when the amount of capital is fixed at K*, than in the long run when capital is variable.



With K* of capital, production of q* units of output with L* units of labour in the cost minimization production solution.

The Short-Run Cost Functions of the Firm

Fixed Cost: is a cost that does not change with the quantity of output produced.

A Sunk Cost: a previous expenditure that a firm cannot avoid.

The short run total cost function, $C_S(q)$ represents the total cost of producing each quantity with a given plant size.

The short-run total cost function is the sum of the fixed and variable cost functions:

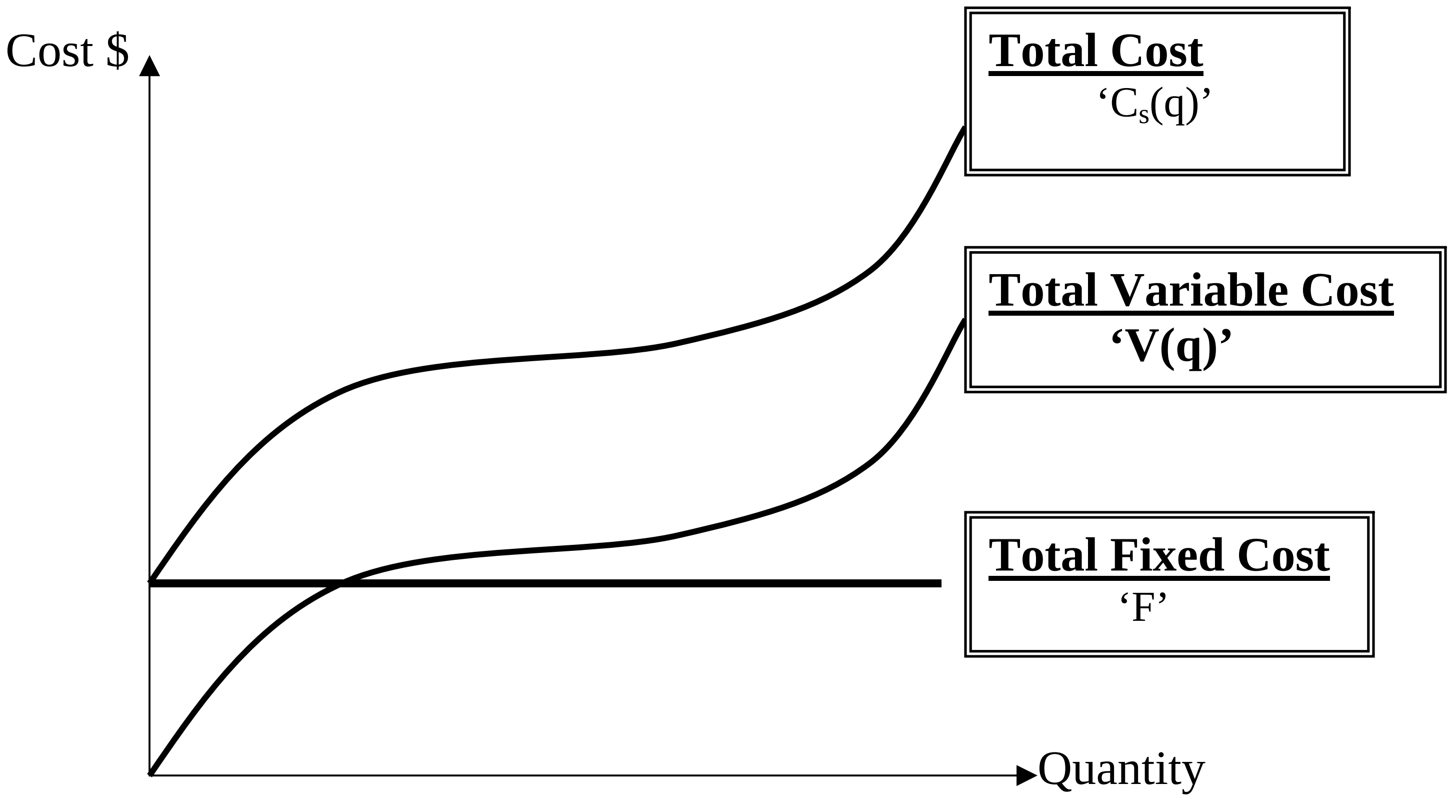
$$\mathbf{C_S(q) = F + V(q)}$$

where: F = fixed cost
V(q) = variable cost (costs that change with output produced.)

The short-run total cost function shows the lowest total cost of producing each quantity when at least one factor is fixed.

Graphing:

To derive the short-run total cost function, we can graph total fixed and total variable costs and then sum them vertically.



Next, we can derive the average costs function and the marginal cost function from these curves.

There are seven cost functions you need to know:

The first three we have already discussed.

1) Short-run total cost: $C_S(q)$

2) Short-run total variable cost: $V(q)$

3) Total fixed cost: F

4) Short-run marginal cost: $MC_S(q)$

$$MC_S(q) = \frac{\Delta C_S(q)}{\Delta q} = \frac{\Delta V(q)}{\Delta q}$$

5) Short-run average cost: $AC_S(q)$

$$AC_S(q) = \frac{C_S(q)}{q} = \frac{F}{q} + \frac{V(q)}{q} = AFC(q) + AVC(q)$$

6) Average variable cost: AVC(q) $AVC(q) = \frac{V(q)}{q}$

7) Averaged fixed cost: AFC(q)

The SAC at any quantity of output is the slope of a straight line drawn from the origin to the point on Cs(q) associated with that output.

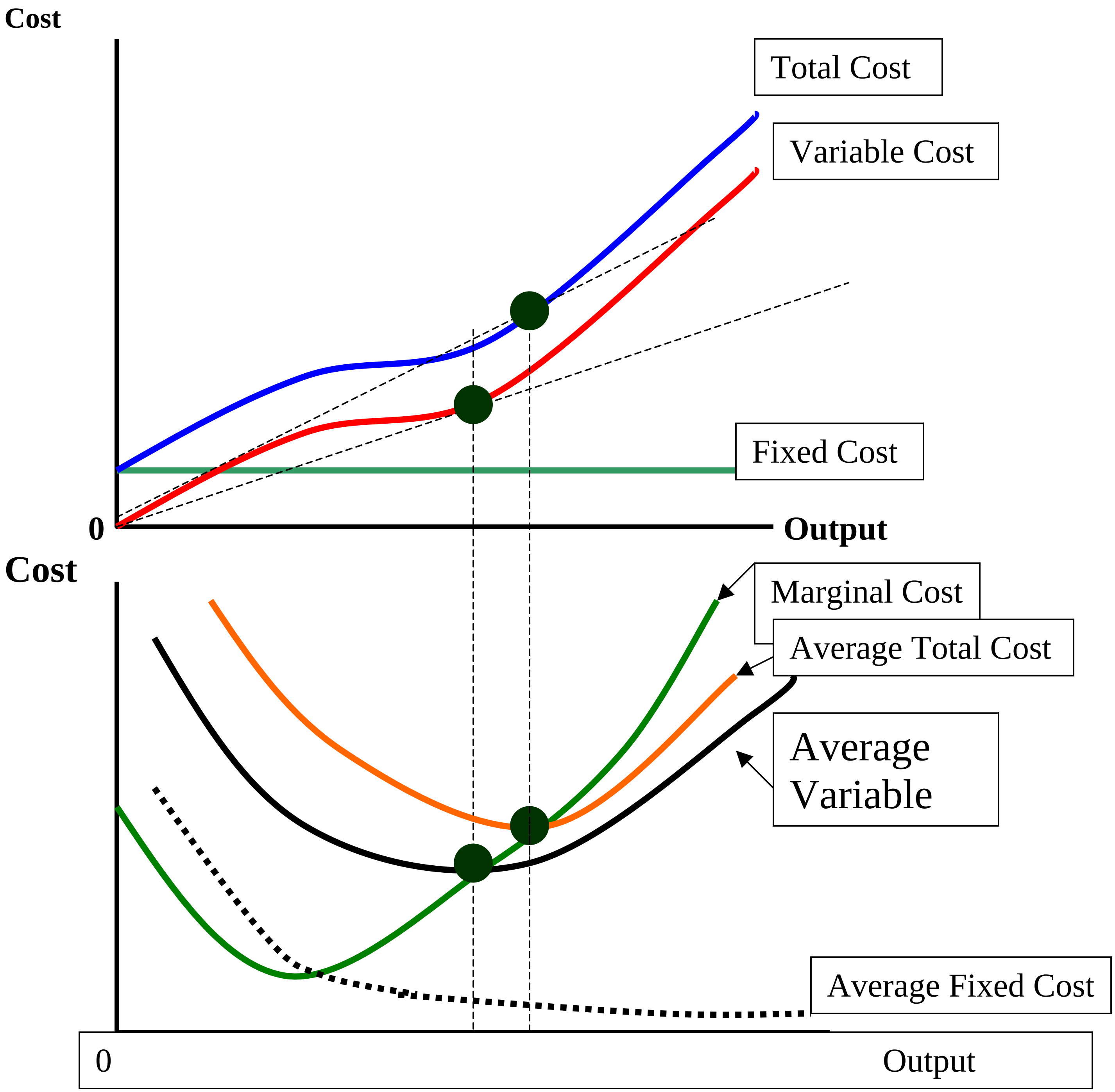
To determine if ACs is increasing or decreasing as quantity produced changes, simply determine how the slopes of successive rays to different points on the total cost curve change.

The ACs reaches a minimum when the slope of the ray from the origin to the total cost curve is tangent.

AVC is the slope of a ray from the origin to the total variable cost function.

When the slope of the ray from the origin to the $V(q)$ curve is tangent, AVC is at a minimum.

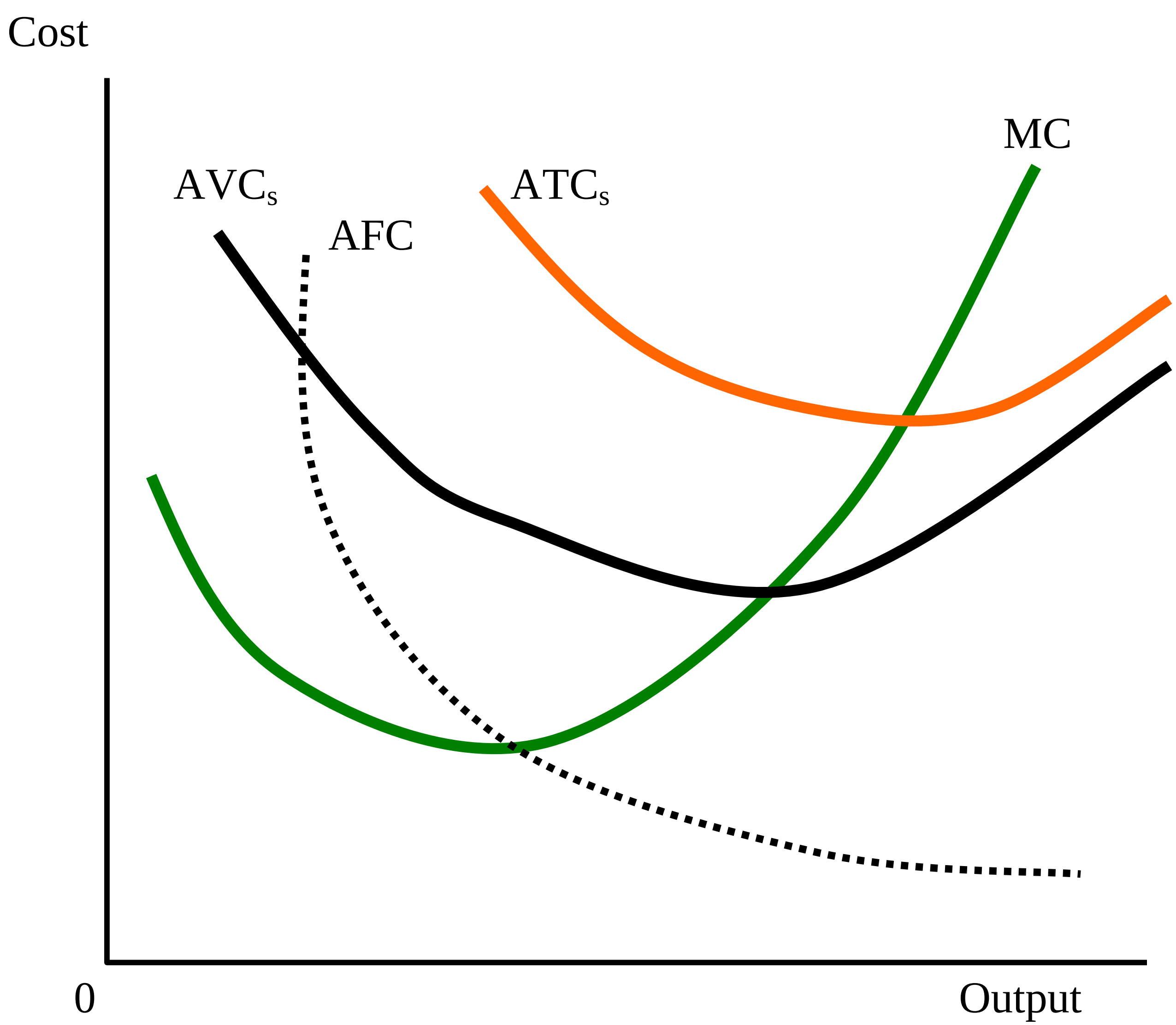
The difference between AC and AVC decreases as the quantity increases because $AFC=F/q$ decreases.


Cost
Total Cost
Variable Cost
Fixed Cost
0
Output
Cost
Marginal Cost
Average Total Cost
Average
Variable
Average Fixed Cost
0
Output

The Relationship Between Marginal and Average Costs

1) When MC < AC: AC is decreasing.
2) When MC = AC: AC is constant.
3) When MC > AC: AC is increasing.

The marginal cost function goes through the minimum points of the average variable cost and short-run average cost functions.



Short-Run Marginal Cost and Short Run Average Cost