Economics 205
UNIVERSITY OF VICTORIA
Managerial Economics
Spring Term, 2014
Assignment 3 Solutions

Due: Thursday, March 6**, 2014**, **3 pm.** (In the boxes marked "ECON 205" near the Economics Department Office.) Show your work!

Question 1: Textbook question page 314, #1. (3 marks)
Given: $Q = 100K^{0.5}L^{0.5}$,C=$1000, w=$30 and r =$40, show how to determine the amount of labour and capital that the firm should use in order to maximize output?

$$Q = 100K^{0.5}L^{0.5}$$
$$C = 1000$$
$$C = wL + rK$$
$$w = 30$$
$$r = 40$$
$$\frac{MP_k}{r} = \frac{MP_L}{w}$$
$$\frac{50L^{0.5}}{40K^{0.5}} = \frac{50K^{0.5}}{30L^{0.5}}$$
$$1500L = 2000K$$
$$K = 0.75L$$
$$Cost\ \ function:$$
$$1000 = (0.75L)40 + 30L$$
$$1000 = 60L$$
$$L = 16.67$$
$$K = 0.75(16.67) = 12.5$$
$$Q = 100(12.5)^{0.5}(16.67)^{0.5} = 1443.52$$

Question 2: Textbook question page 314 , #3. (3 marks)
Given: $Q = 100K^{0.5}L^{0.5}$, w=$50, and r =$40, show how to determine the amount of labour and capital that the firm should use in order to minimize the cost of producing 1118 units of output. What is this minimum cost?

$$Q = 100K^{0.5}L^{0.5}$$

$$w = 50$$
$$r = 40$$
$$\frac{MP_k}{r} = \frac{MP_L}{w}$$
$$\frac{50L^{0.5}}{40K^{0.5}} = \frac{50K^{0.5}}{50L^{0.5}}$$
$$50L = 40K$$
$$K = 1.25L$$
$$\text{Production } \textit{function:}$$
$$1118 = 100(1.25L)^{0.5}L^{0.5} = 1.118L$$
$$1118 = 111.8L$$
$$L = 10$$
$$K = 1.25L = 12.5$$
$$C = 50(10) + 40(12.5) = \$1000.$$

Question 3: Textbook question page 357. #15. (8 marks)

The manager of the Electronic Corporation has estimated the total variable costs and the total fixed cost functions for producing a particular type of camera to be:

$TVC = 60Q-12Q^2 + Q^3$

$TFC = 100$

The corporation sells the camera at the price of \$60 each. An engineering study just published estimated that if the corporation employs newly developed technology, the long-run total cost function would be

$TC= 50 + 20Q + 2w +3r.$

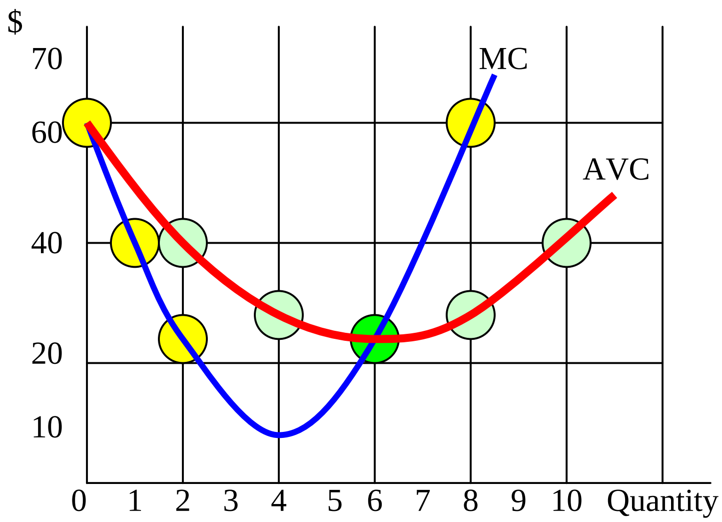
The manager asks you to find
(a) AVC and MC functions, the output level at which the two curves cross, and a plot of them.
(b) the breakeven output of the firm and the output at which the firm maximizes its total profits; and (c) the long-run average cost and long-run marginal cost functions with the new technology if w = \$20 and r =\$10, and plot them.
(d) Should the corporation adopt the new technology? It if did, what would be the profit maximizing level of output if the firm can continue to sell its cameras at the price of \$60 per unit?

The manager asks you to find:

(a) AVC and MC functions, the output level at which the two curves cross, and a plot of them.

$AVC=60-12Q+Q^2$
$MC=60-24Q+3Q^2$

AVC=MC
$60-12Q+Q^2 = 60-24Q+3Q^2$
$Q(-12+24)=2Q^2$
$12Q-2Q^2=0$
$Q(12-2Q)=0$
Q=6 and Q=0
AVC=60-72+36=24=MC



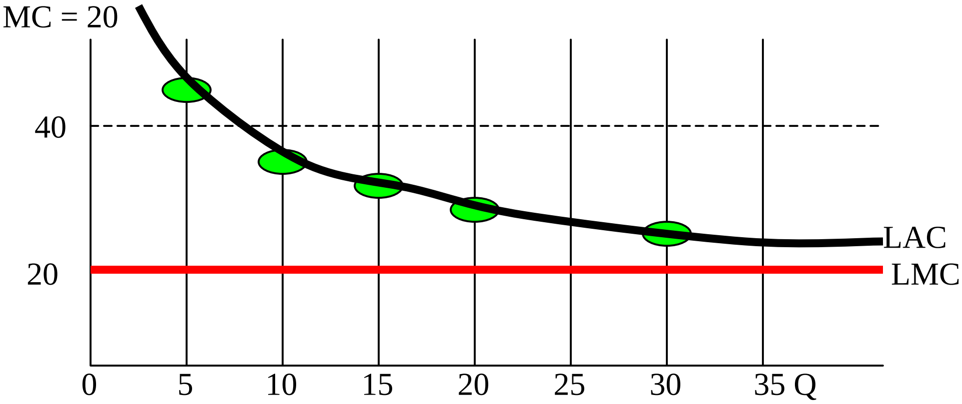
(b) the breakeven output of the firm and the output at which the firm maximizes its total profits;

MR = MC
$MC=60-24Q+3Q^2$

TR= (P)(Q)
From Part A, we found MC=AVC =P=$60
TR= 60Q
MR= 60
$60 = 60-24Q+3Q^2$
$24Q=3Q^2$
24= 3Q
Q=8 Firm Maximizes profits at Q=8.

(c) the long-run average cost and long-run marginal cost functions with the new technology if w = $20 and r =$10, and plot them.

TC= 50 + 20Q + 2w +3r
TC= 50 + 20Q + 40 + 30 = 120 + 20Q
ATC = $120Q^{-1}$ +20
MC = 20



(d) Should the corporation adopt the new technology? It if did, what would be the profit maximizing level of output if the firm can continue to sell its cameras at the price of $60 per unit?

TC_{new} = 120 + 20Q
TC_{old} = 100+60Q-12Q^2 + Q^3

The corporation should adopt the new technology; otherwise it will be unable to compete with other firms that do at any level of output. The LAC curve of the firm would continue to fall and get closer and closer to LMC =$20 as the firm expands output. Since the firm can sell cameras at $60, its profits will increase as output increases until it may capture the entire market for cameras.
MC=20
MR = 60Q
20 = 60Q
Q=20/60?
LAC= LMC

LAC=120/Q+20
Slope of LAC = -120/Q^2=0
Q= infinity

Question 4: The Speedy Company is a manufacturer race timers. The operations manager has determined that the firm's output (Q) is related to how engineers (E) and technicians (T) are combined in the production process: $Q = -5.68 - 0.32E - 0.42T + 6.35\sqrt{E} + 8.52\sqrt{T} + 0.34\sqrt{ET}$.

If the wage of an engineer is $36,000 and the wage of a technician is $24,000, and the total amount the firm spends on both engineers and technicians is $6 million, determine the expressions that must be satisfied simultaneously to obtain the optimal values of E and T.
(3 marks)

My answers are in thousands of $.

To maximize output, the company must choose a combination of engineers and technicians such that : $\frac{MP_E}{P_E} = \frac{MP_T}{P_T}$.

$$MP_E = \frac{\partial Q}{\partial E} = -0.32 + 6.35 \times 0.5E^{-0.5} + 0.34 \times 0.5E^{-0.5}T^{0.5}$$

$$MP_E = -0.32 + 3.175E^{-0.5} + 0.17E^{-0.5}T^{0.5}$$

$$MP_T = \frac{\partial Q}{\partial T} = -0.42 + 8.52 \times 0.5T^{-0.5} + 0.34 \times 0.5T^{-0.5}E^{0.5}$$

$$MP_T = -0.42 + 4.26T^{-0.5} + 0.17T^{-0.5}E^{0.5}$$

$\frac{MP_E}{P_E} = \frac{MP_T}{P_T}$:

First Equation:

$$\frac{-0.32 + 3.175E^{-0.5} + 0.17E^{-0.5}T^{0.5}}{36} = \frac{-0.42 + 4.26T^{-0.5} + 0.17T^{-0.5}E^{0.5}}{24}$$

Second equation:

$$6000 = EP_E + TP_T$$

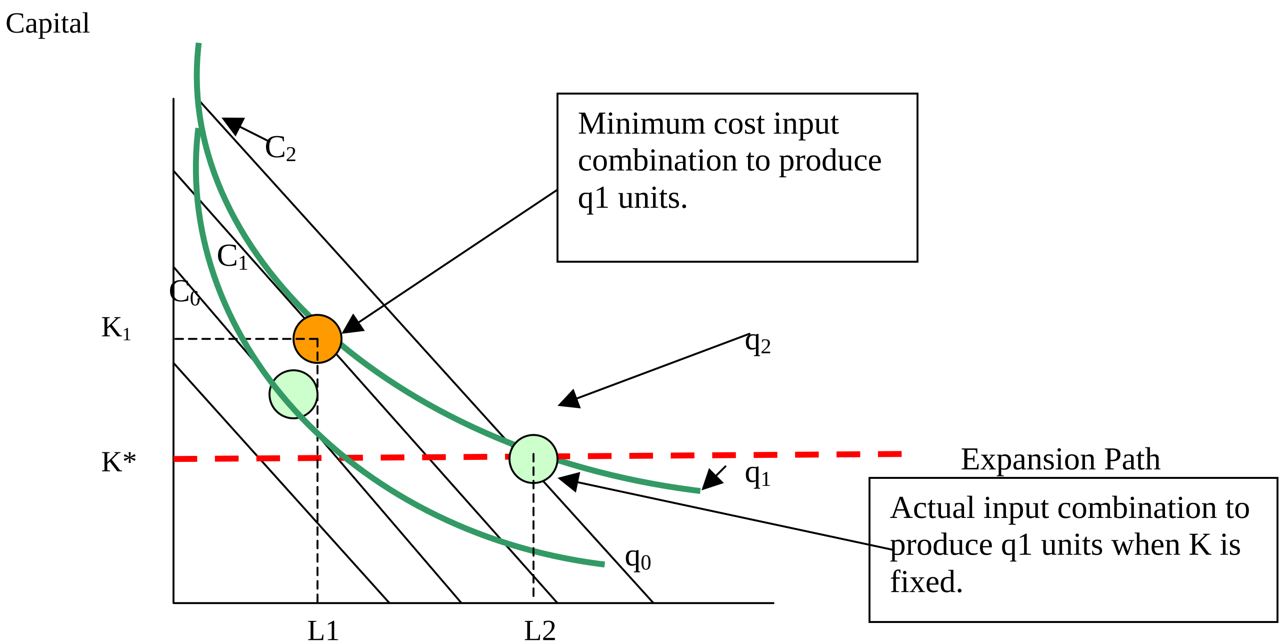
$$6000 = 36E + 24T$$

$$T = 250 - 1.5E$$

Question 5 (5 Marks)

Explain the why the short-run minimum cost of producing a certain output may differ from the long-run minimum cost. Illustrate your explanation with a diagram.

In the short-run, at least one factor of production is assumed fixed. When this happens and the firm must produce a specific quantity of output, the firm may not be able to use the input combination that attains minimum cost relative to the long-run situation. In the long run, all factors of production are variable. Hence, a firm will produce a specific output where the isocost line is tangent to the isoquant (the specific quantity that must be produced). In the short-run, this may not be possible for a specific quantity.



If capital is fixed at K* units, and the firm must produce q1 units, L2 units of labour will be used with this fixed amount of capital to produce q1 units of output. The total cost is C2. If the firm were able to use any combination of K and L, it would use L1 and K1 units of labour and capital and lower its cost to C1. This would be the long-run solution. Total cost is C1.

Question 6: (3 marks)
For each production function determine the:

(A) Determine the marginal product of labour.
(B) Determine the marginal product of capital.
(C) Marginal rate of technical substitution

i) $Q = 9L^{0.3}K^{0.7}$
ii) $Q = 3L^{0.8}K^{0.2}$

i)
$MP_L=2.7L^{-.7}K^{0.7}$
$MP_K=6.3L^{0.3}K^{-0.3}$

$$MRTS = \frac{MP_L}{MP_K} = \frac{2.7L^{-0.7}K^{0.7}}{6.3L^{0.3}K^{-0.3}} = 0.42\frac{K}{L}$$

MRTS: $$= \frac{aAL^{a-1}K^b}{bAL^a K^{b-1}} = \frac{aL^{a-1-a}K^{b-b+1}}{b} = \frac{aK}{bL}$$

ii)

$MP_L = 2.4L^{-.2}K^{0.2}$

$MP_K = 0.6L^{0.8}K^{-0.8}$

$$MRTS = \frac{MP_L}{MP_K} = \frac{2.4L^{-0.2}K^{0.2}}{0.6L^{0.8}K^{-0.8}} = 4\frac{K}{L}$$

$$= \frac{aAL^{a-1}K^b}{bAL^a K^{b-1}} = \frac{aL^{a-1-a}K^{b-b+1}}{b} = \frac{aK}{bL}$$