Topic 3:
The Production Function and Costs of the Firm
Part I

1) The Production Function

2) Changing Factors of Production in the ______ and Long runs

3) The Short-Run Production Function
   Total Product
   Average Product
   Marginal Product

4) **The Long-Run Production Function**
   Marginal Rate of __________ Substitution
   Returns to _____
   Cobb-Douglas Production Function

5) **The I_______ Function**

6) **The Long and Short Run Total Cost Functions**

7) **Shifts in the Long-Run Total Cost Function**
   Change in the Price of a Factor
   Technological Change

Objective: To examine how firm and industry ______ *curves* are derived

Introduction: Up to this point we have examined how the market demand function is derived. Next, we will examine the **supply** side of the market. We will explore how firms **minimize _____** and **maximize productive efficiency** in order to produce goods and services. By effectively combining labour and capital, the firm develops a production process with the objective of efficient ________allocation and cost minimization. The firm is assumed to produce a given output at minimum cost.

The Production Function

Definitions:

Factors of Production: are factors used to produce output.

Example: Labour

Capital - machines
-buildings
Land
_______ _________

State of technology: consists of existing k_________ about method of production.

The quantity that a firm can produce with its factors of production depends on the state of technology.

The relationship between factors of production and the output that is created is referred to as the **p__________ function**.

"The production function describes the m_______ quantity of output that can be produced with each combination of _______ of production given the state of technology."

For any product, the production function is a table, a graph or an equation showing the **maximum** _____rate of the product that can be achieved from any specified set of usage rates of inputs.

The production function summarizes the characteristics of existing technology at a given time; it shows the technological constraints that the _____must deal with.

Model Assumption: The quantity produced per period is **'q'**, and the two factors of production are **labour** and **capital**.

Let: 'L' represent the number of ________ or aggregate hours of work of a given quality.

Let: 'K' represent the number of _________ or machine hours, the size of the plant, or the number of plants.

Notation: The Production Function of the Firm: $\mathbf{q}=\boldsymbol{f}\mathbf{(L, K)}$

where f , the function, describes the relationship between the inputs L, K and the output each different combination produces per period.

Changing Factors of Production in the Short and Long Runs

We must make a distinction between the **short** and **long run**.

In the short run, a firm is able to change some of the factors of production, but at least one factor is **f____**.

In the long run, ***all*** factors of production can be v_____

The Short-Run Production Function

Model: consider the simplest case where there is one input whose quantity is fixed and one input whose quantity is variable.

Suppose that the fixed input is the number of machines (capital) and the variable input is l________

- In the short run the firm cannot change the number of machines quickly without incurring a high ____.

- With one fixed input the short-run production function shows how total output changes as the **v_______** factor changes.

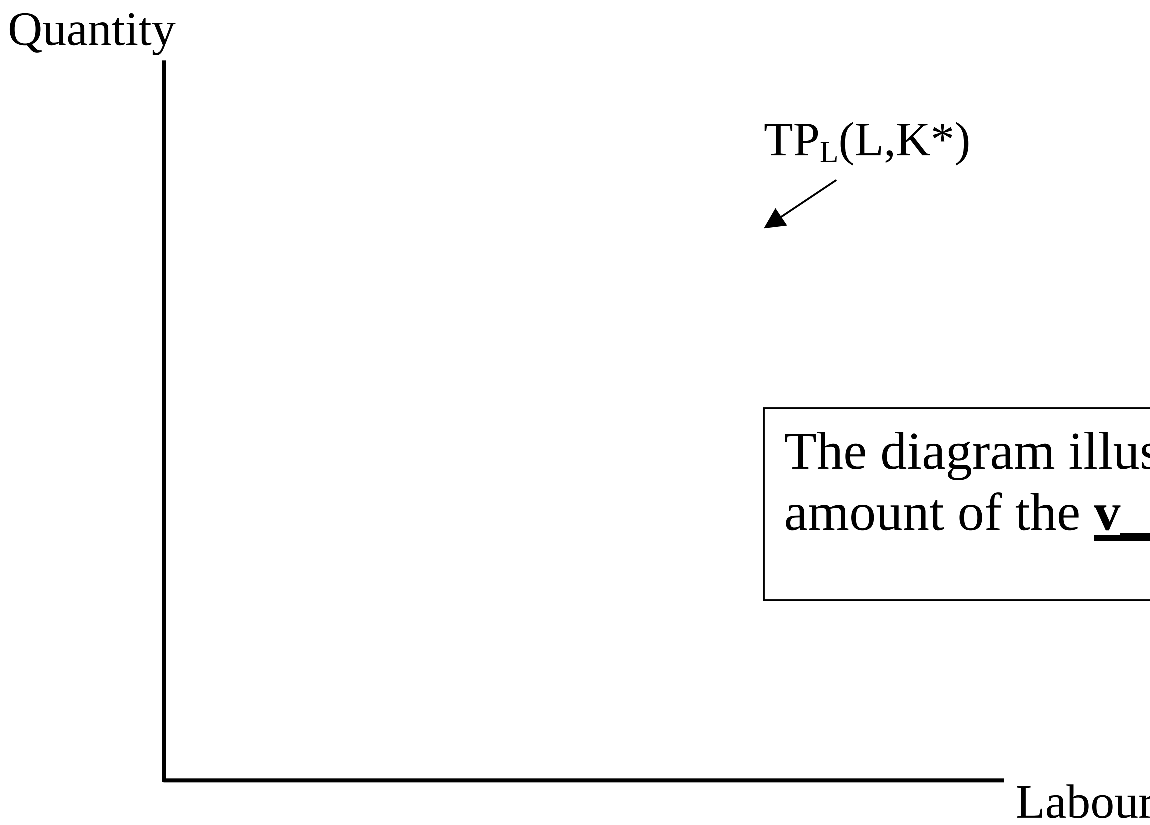
The Total, Average and Marginal Product of Labour

Total Product Function: expresses the relationship between the v_______ input and the total output.

The total product function of labour: TP_L: shows the various amounts of output that is produced when the amount of labour is varied with a given fixed amount of capital.

$$TP_L(L,K^*)=f(L, K^*)$$

The **total product function of capital**: TP_K show the various amounts of output that is produced when the amount of capital is varied with a given f_____amount of labour.



The diagram illustrates what occurs when the amount of the **v________** input increases.

The total product (output) increases when the amount of labour increases, holding the amount of capital fixed at K*.

Quantity increases initially at an <u>increasing rate</u>, but eventually quantity increases at a d_________ rate when more labour is employed.

Algebraically: $$\frac{\partial^2 TP_L(L,K^*)}{\partial L^2} > 0$$ at first;

and then eventually, $$\frac{\partial^2 TP_L(L,K^*)}{\partial L^2} < 0$$ (becomes negative).

At some point, adding more labour units no longer increases o______.

We can derive the average and marginal product function of labour from the total product function.

The **average product function, AP$_L$,** measures output per unit of labour:

$$\text{Average product of labour} = \frac{\text{Total product of labour}}{\text{Number of labour units}}$$

$$AP_L(L, K^*) = \frac{TP_L(L, K^*)}{L}$$

Average product of labour is the measure of ***p*__________** of labour.

The **marginal product** of an input is the addition to total _______ resulting from the addition of the last unit of the input when the amount of other inputs used is held constant.

The **marginal product function of labour**, MP_L, measures the change in quantity due to a change in the labour input, or the ______ of the total product function of labour:

$$\text{Marginal product of labour} = \frac{\Delta \text{ in total product of labour}}{\Delta \text{ in number of labour units}}$$

$$MP_L(L, K^*) = \frac{\Delta TP_L(L, K^*)}{\Delta L} = \frac{\partial TP_L}{\partial L}$$

If the amount of labour is fixed and capital is varied, the marginal product of capital is:

$$\text{Marginal product of capital} = \frac{\Delta \text{ in total product of capital}}{\Delta \text{ in capital}}$$

$$\mathrm{MP_K}(\mathrm{L}^*, \mathrm{K}) = \frac{\Delta \mathrm{TP_K}(\mathrm{L}^*, \mathrm{K})}{\Delta K} = \frac{\partial TP_K}{\partial K}$$

The ______of the total product function of labour determines the shape of the average and marginal product functions.

Average product of labour, AP_L, measures output per unit of labour.

It is the **<u>slope</u>** of a ____drawn from the origin to any point on the TP_L function.

The **<u>average product of labour</u>** for any given level of employment is equal to the <u>slope</u> of a straight line drawn from the origin to the _____ product function at that employment level.

Generally, the AP_L increases at first as labour is increased. I.e. the output per worker increases initially.

Further increases in labour reduce AP_L.

- AP_L declines when employment increases.

There is a distinct relationship between marginal product and average product:

When: MP>AP , AP is _________ng

MP<AP , AP is _________ng

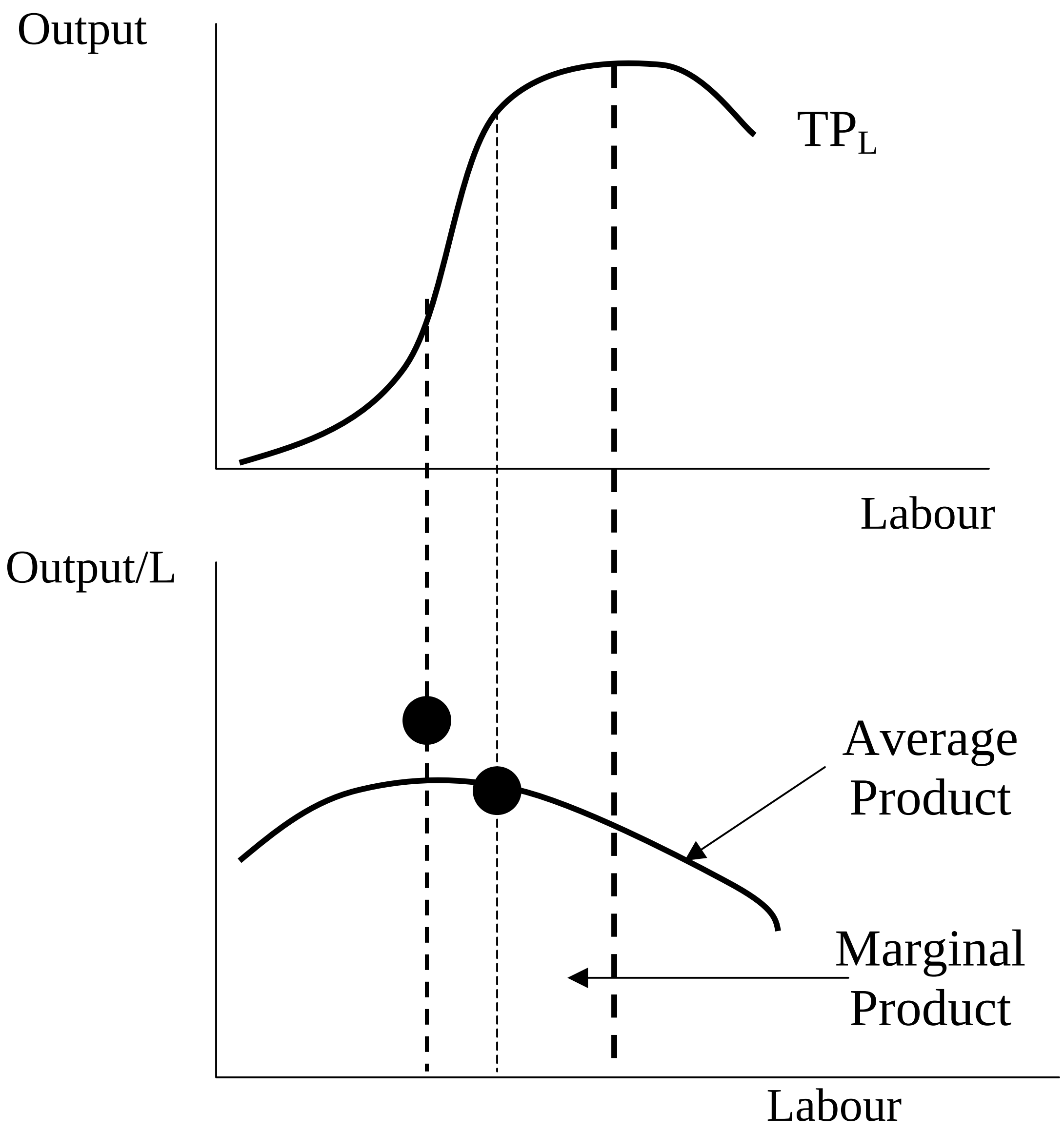
MP=AP , AP is _________ and at a maximum

The **law of diminishing returns** describes the eventual decline in the _________product of the variable factor as the variable factor increases with other factors held constant.

The <u>law of diminishing returns</u> applies only to situations where one _______is increasing and the other factors are fixed.

"The law of diminishing marginal returns: if equal increments of an input are added, and the quantities of other inputs are held constant, the resulting increments of product will decrease beyond some point; that is, the marginal product of the input will diminish.

Note: 1) this law is an empirical generalization
2) it assumes that technology remains fixed
3) it assumes that there is at least one input whose quantity is being held constant.



Output
TP$_L$
Labour
Output/L
Average Product
Marginal Product
Labour

The Long-Run Production Function

- In the long run, all factors of production are _________.

Substitution Among Factors

Similar to the notion of substituting between goods to maintain constant utility along an indifference curve, firms usually can produce the same output quantity by substituting between factors of production.

The important question that needs to be addressed is:

"What _____________ of factors should be used to produce this output?"

This question is difficult to answer because there is more than one way to produce the product.

This can be illustrated with the aid of **iso_____** analysis.

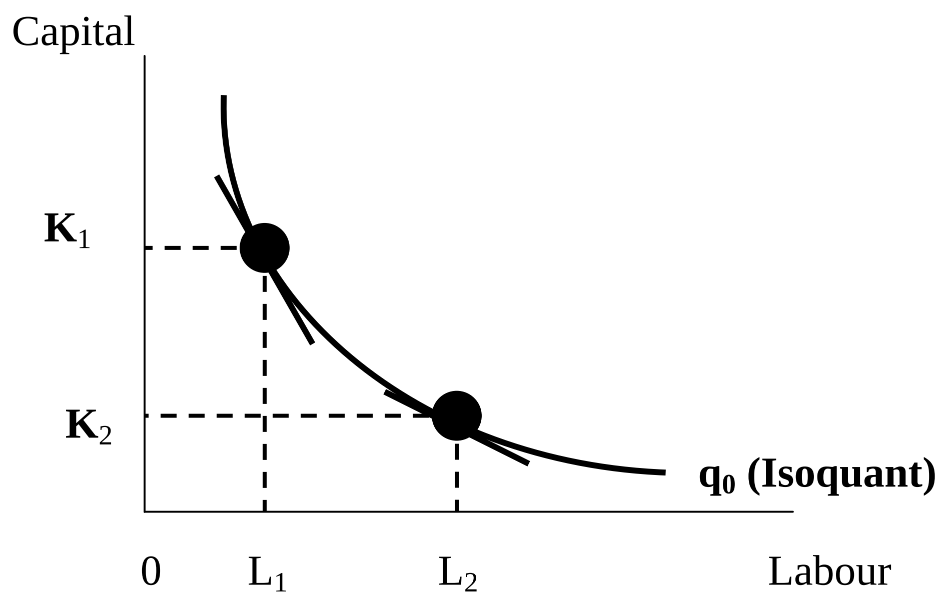
The amount of capital is on the vertical axis and number of labour units is on the horizontal axis.

The curve with an output of 'q_0' is called an isoquant.

- "Iso" means _____.
- "quant" means q_______.

The _______ produced is the **<u>same</u>** along the isoquant. The points along the isoquant q_0 represent the different factor combinations that can produce q_0 units per period.

"An isoquant shows the different combinations of factors of production that can produce a given _________ of output."



The Marginal Rate of Technical Substitution

The marginal rate of technical substitution (MRTS) measures the rate of substitution of one factor for another along an _________.

"The marginal rate of technical substitution is the rate at which a firm can substitute capital and labour for one another such that the output is _________."

$$\mathrm{MRTS}_{\mathrm{KL}} = \left.\frac{\Delta K}{\Delta L}\right|_{q=\text{constant}}$$

where $\Delta K/\Delta L$ is the slope between two point on an isoquant.

Note: An isoquant **cannot** have a _________ slope.

- An increase in one factor of production causes output to increase. Hence this increase in one factor must be offset by a decrease in other factor in order to keep output at the same level.

- As the firm moves along the isoquant from left to right, the ______ increases. The firm substitutes labour for capital, but at a diminishing rate. When this occurs there is a ***diminishing*** marginal rate of technical substitution.

Returns to Scale

In general, the level of a firm's productivity changes as the quantity produced by the firm changes.

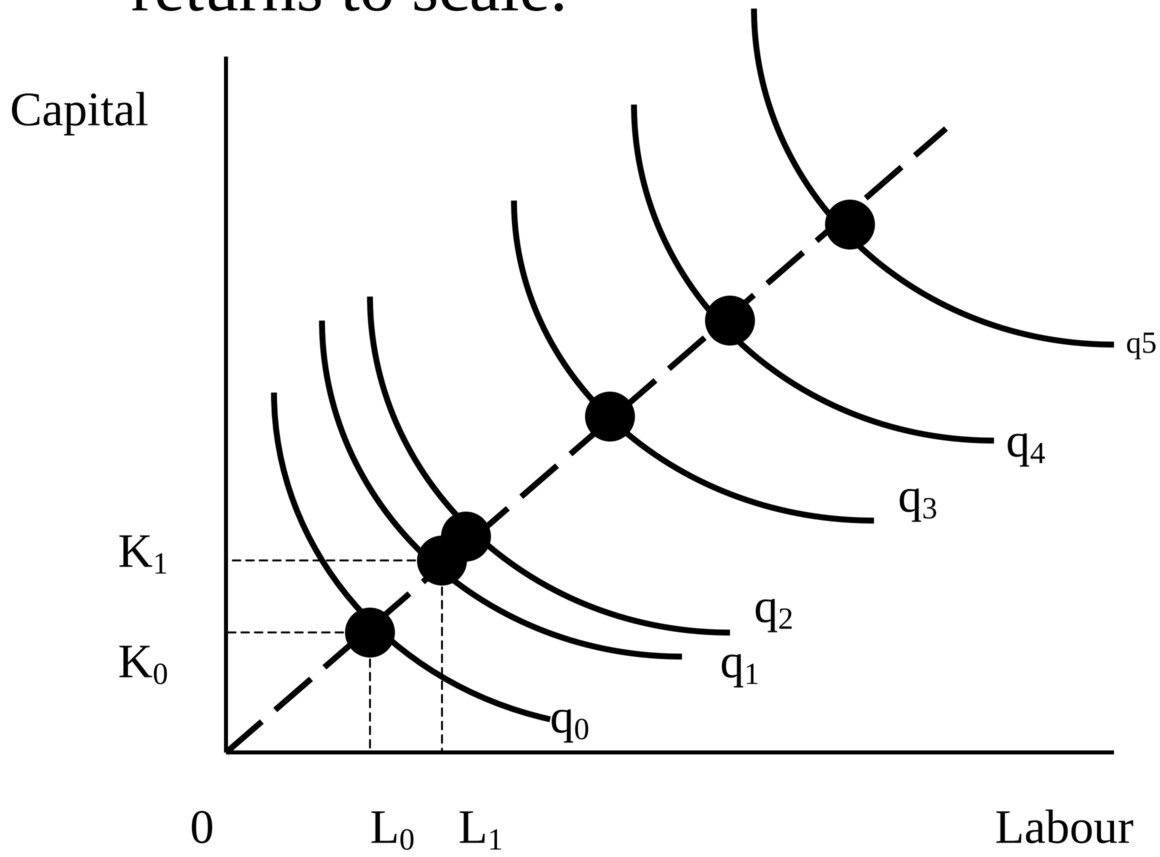
Returns to scale refers to the percentage change in _____ to a percentage change in ______.

Three Cases:

1) When the percentage increase in inputs is smaller than the percentage increase in output, there are__________ returns to scale.

2) When the percentage increase in inputs leads to the ____ percentage increase in output, there are **constant** returns to scale.

3) When the percentage increase in inputs is ______ than the percentage increase in output, there are **decreasing** returns to scale.



The **returns to scale** can be measured along the ray from the origin.

Here the capital to labour ratio is given.

- There is a relationship between returns to scale and the spacing of the iso_______.

When there are ____________ returns to scale, the isoquants are bunched closer together.

When there are ________ing returns to scale, the isoquants are farther apart.

Returns to Scale and the Cobb-Douglas Production Function

A common production function is the Cobb-Douglas production function.

Algebraically:

$$q = AL^aK^b$$

where A, a and b are constant and greater than zero.

To determine the returns to scale for this function, we could change labour and capital by a factor ‘m’ and then determine if output changes by more than, equal to or less than ‘m’ times.

$$q = A\,(mL)^{a}(mK)^{b}$$

$$q = Am^{a}L^{a}m^{b}K^{b}$$

$$q = m^{a+b}[AL^{a}K^{b}]$$

Since originally output was $q=AL^{a}K^{b}$, we can determine if output will increase by either less than m times if $a+b<1$ (because $m^{a+b}<m$), by exactly m times if $a+b=1$ or by more then m times if $a+b>1$.

The MRTS and MP of Both Factors of Production

The MRTS and marginal ________ of labour and capital are related.

Suppose the firm decides to increase the amount of capital it employs, holding the amount of labour constant. Output will increase by the amount Δq_K because of the increase in capital ΔK.

The increase in output is approximated by

$$\Delta q_K = MP_K \Delta K$$

If the firm holds capital constant and decreases the amount of labour it employs, ______ decreases by Δq_L, where the amount of labour decreases by ΔL.

The decrease in output is approximated by

$$\Delta q_L = MP_L \Delta L \, .$$

Along a given isoquant, output must be ________.

If the firm increases capital by ΔK, labour must decrease by an amount ΔL such that: $\Delta q_K + \Delta q_L$ =__in order to remain on the same isoquant.

Substituting in these expressions, the condition becomes:

$$MP_K \Delta K + MP_L \Delta L = 0$$

Rearranging such that we have an expression for the MRTS in terms of the marginal products of the two factors:

$$MRTS_{KL} \equiv \frac{\Delta K}{\Delta L} = -\frac{- -_L}{- -_K}$$

The MRTS of K for L equals the negative of the ratio of the marginal ________ of L and K.

The Isocost Function

The production function summarizes the t_____________ options facing the firm. Unfortunately it is not enough to be just aware of these options when making the output and factor _____ decision.

- The**_____** of a factor of production is extremely important in this decision.

- In order to minimize costs and produce efficiently, the firm must know **exactly** what its costs will be.

Let ‘w’ be the annual cost of each unit of labour.
Let ‘M’ be the price of capital that never needs to be replaced.
Let ‘i’ be the interest rate.
Let r=iM= the opportunity cost of maintaining one unit of capital.

The total annual cost of producing the good is:

Total cost = Cost of labour + Cost of Capital

C = wL + rK (Total Cost Function)

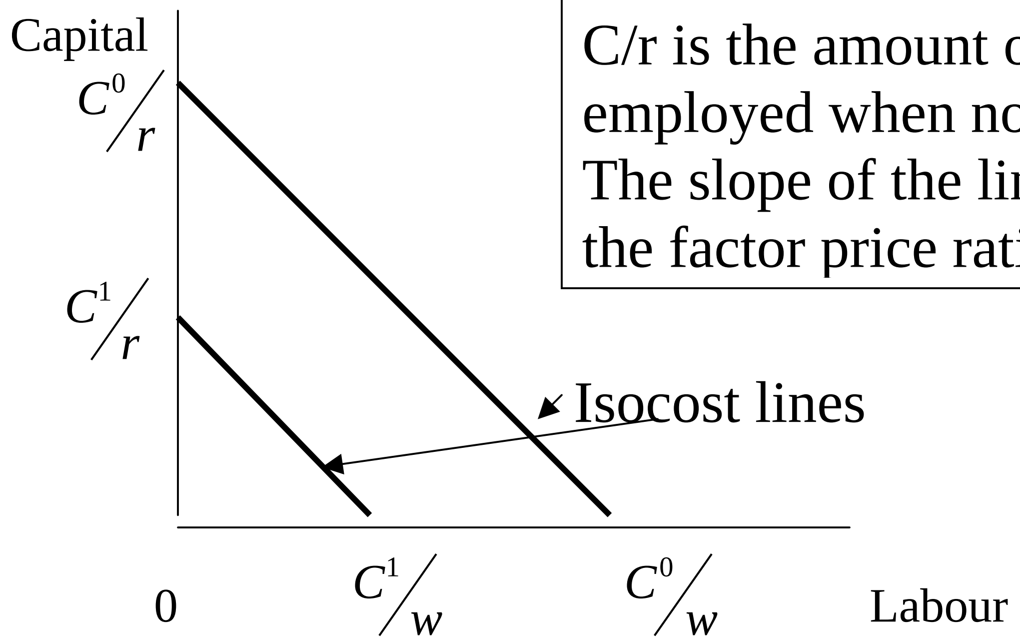
Rearranging we have an expression for K:

$$\boxed{K = \frac{C}{r} - \frac{w}{r}L \quad \Leftarrow \_\_\_\_\_\_\_\_ \text{ Line}}$$

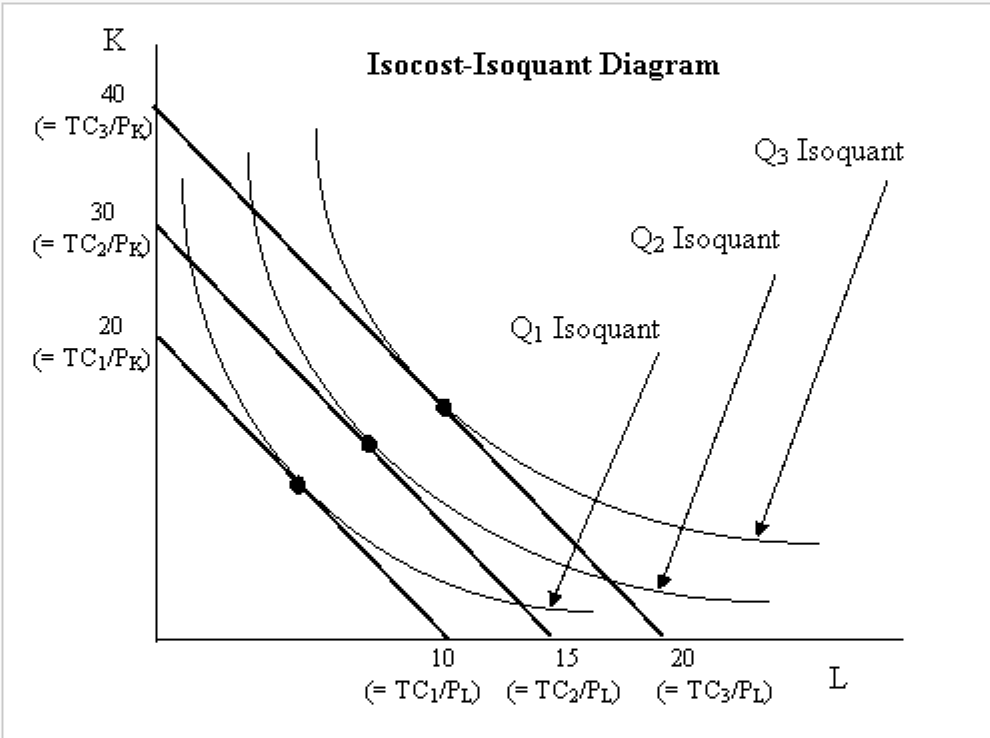
The isocost line represents the total ____ C as constant for all K-L combinations satisfying the equation.

"An isocost line shows the different combinations of _______ of production that can be employed with a given total cost."

For a given cost C, the vertical intercepts of these lines are C/r.
C/r is the amount of capital that can be employed when no labour is used.
The slope of the line is -w/r = the negative of the factor price ratio.



When C, total cost, increases, the isocost line shifts ___ in a parallel fashion, but the _____ of the line does not change.



Isocost v. Isoquant Graph

In economics an **isocost** line shows all combinations of inputs which cost the same total amount.[1][2] Although similar to the budget constraint in consumer theory, the use of the isocost line pertains to cost-minimization in production, as opposed to utility-maximization. For the two production inputs labour and capital, with fixed unit costs of the inputs, the equation of the isocost line is

$$rK + wL = C$$

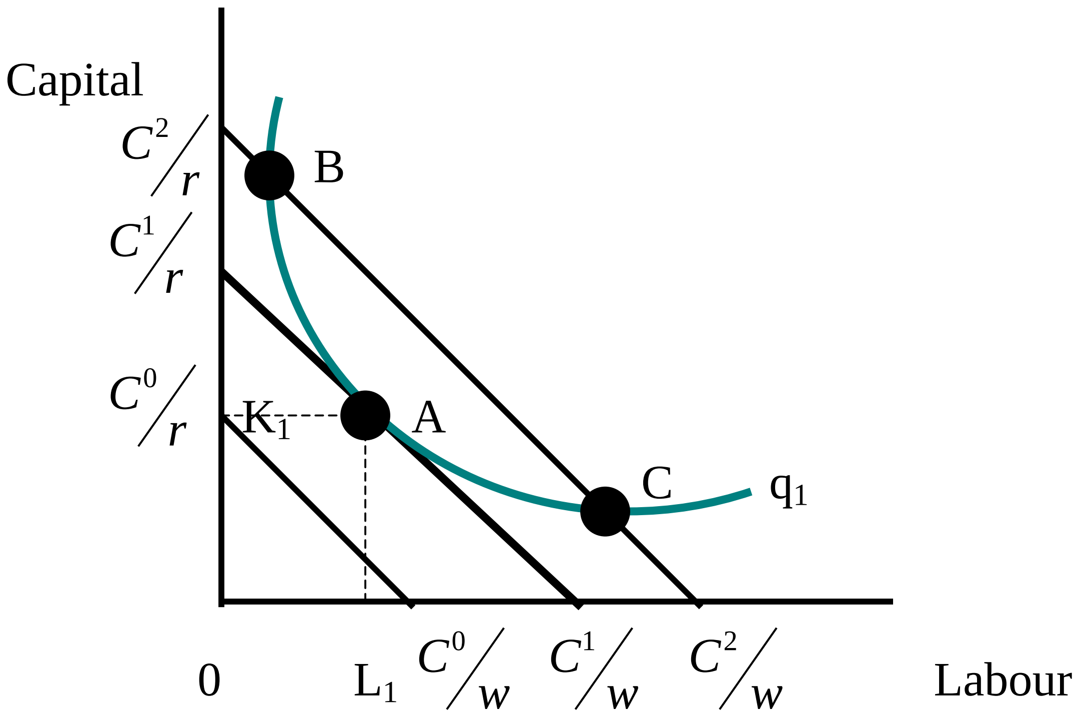
where w represents the wage rate of labour, r represents the rental rate of capital, K is the amount of capital used, L is the amount of labour used, and C is the total cost of acquiring those quantities of the two inputs.

The absolute value of the slope of the isocost line, with capital plotted vertically and labour plotted horizontally, equals the ratio of unit costs of labour and capital. The slope is:

$$-w/r.$$

The Production Decision

We can now determine which combination of factors produces a given quantity at the lowest total cost.



There are three isocost lines. The firm wants to produce q_1 units. The cost minimizing point is at A.

- Total cost increases by moving from each isocost line.
- The firm can produce q_1 units along the isoquant q_1.
- The total ____ of produces q_1 units is minimized at point A.

If the firm is producing q_1 units at ________total cost, the slope of the isoquant equals the slope of the isocost line:

$$MRTS_{KL} = -\frac{\_}{\_}$$

(Minimum cost Condition)

"A firm minimizes the total cost of producing a given quantity by selecting a combination of factors where the ___ of the isoquant equals the ______ of the isocost line."

And since $\boxed{MRTS_{KL} = -\frac{\_\,\_L}{\_\,\_K}}$,

and $\boxed{MRTS_{KL} = -\frac{\_}{\_}}$, then $\boxed{-\frac{MP_L}{MP_K} = -\frac{w}{r}}$.

Rearranging $\boxed{\frac{MP_L}{w} = \frac{MP_K}{r}}$ (Minimum Cost Condition)

The firm minimizes the total cost of providing a given quantity if the ratio of the marginal product of a factor to its ______ is the same for all factors.

"The lowest total cost of producing a given quantity occurs when the ratio of the marginal product of a factors to the last dollar spent on it is equal for all factors of production."

The rate of the MP_L to the price of labour represents the increase in output due to the last dollar spent on labour.

To minimize total cost, the additional output due to the last dollar spent on labour must be equal to the addition output due to the last dollar spent on capital.

If they are not equal, it pays the firm to re_________ its expenditure from one factor to another.

The firm should spend more money on the factor that gives the firm a greater boost in output for the extra dollar spent.

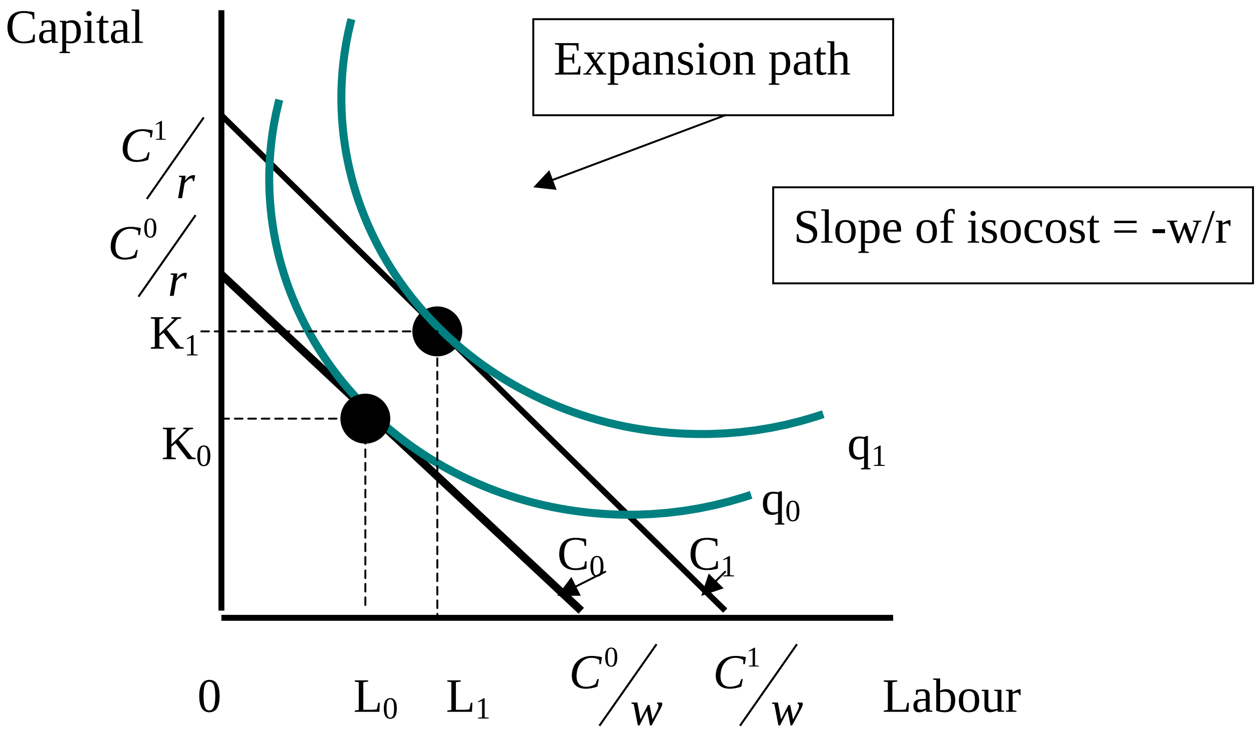
The condition that the MRTS be equal to *w/r* can be given the following intuitive interpretation. We know that the MRTS is equal to the ratio of the marginal products of the two inputs. So the condition that the MRTS be equal to the input cost ratio is equivalent to the condition that the marginal product per dollar is equal for the two inputs. This condition makes sense: at a particular input combination, if an extra dollar spent on input 1 yields more output than an extra dollar spent on input 2, then more of input 1 should be used and less of input 2, and so that input combination cannot be optimal. Only if a dollar spent on each input is equally productive is the input bundle optimal.

The Long-Run and Short-Run Total Cost Functions

We will first examine the relationship between total cost and quantity produced when all factors of production can be ________and then when one factor is fixed and the other factor is variable.

The Long-run Total Cost Function

The long-run total cost function represents the lowest total cost of producing a unit of a good when all inputs are v________.



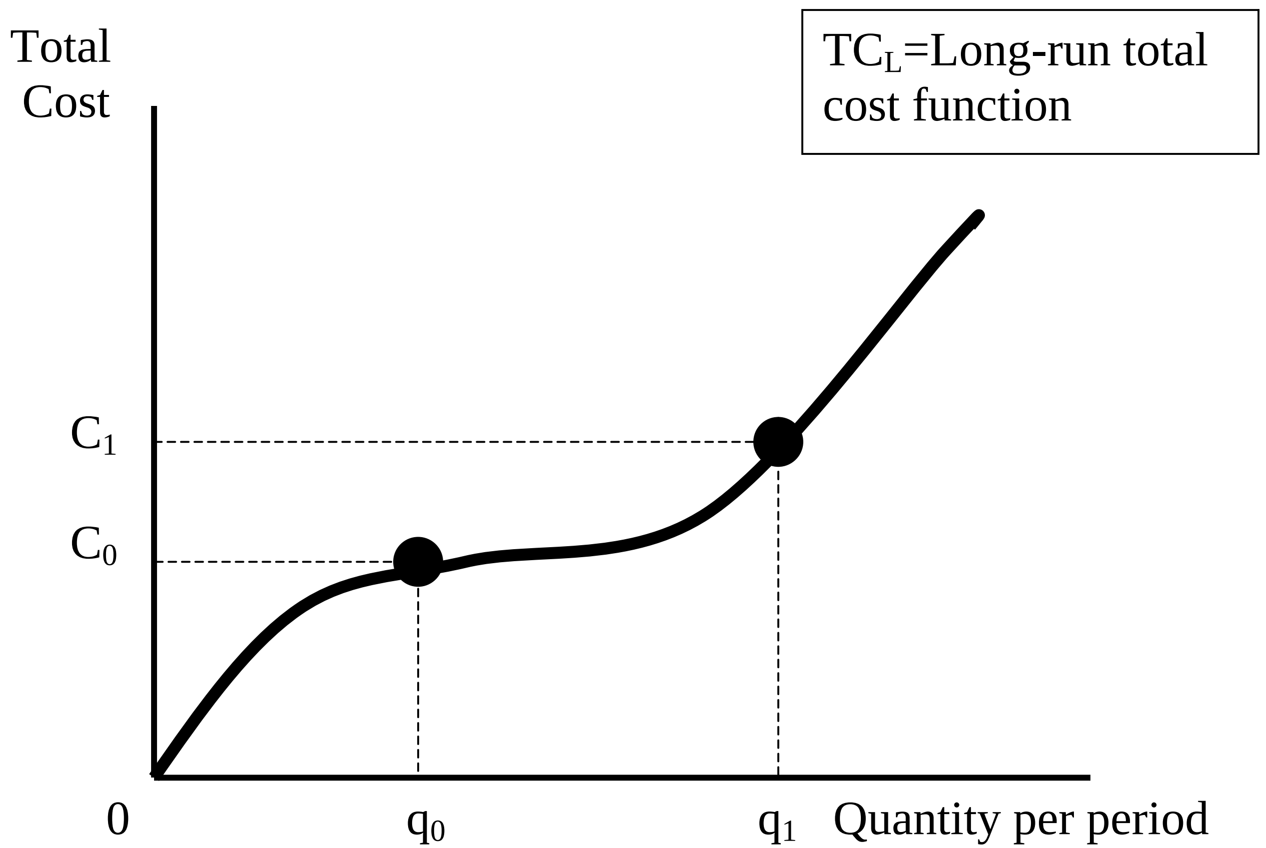
On the diagram, there are two point of tangency of isocost lines with two isoquants.

If the firm wants to produce q_0 units of the good, it can minimize its total cost by employing L_0 units of labour and K_0 units of capital, such that its minimum cost is C_0.

If the firm wants to produce q_1 units of the good, it can minimize its total ____ by employing L_1 units of labour and K_1 units of capital, such that its minimum cost is C_1.

The curve that connects all these points of tangency between an isoquant and an isocost line is referred to as the e________ path. Each point relates a quantity with a minimum total ___.

To derive the long-run total cost function, we take the pairs of total cost and quantity from the expansion path.



"The long-run total cost function shows the lowest total ____ of producing each quantity when all factors of production are ________."

<u>Returns to Scale and the Shape of the Long-Run Average Cost Function</u>

Notice in the last the example the long-run total cost curve increased initially at a ___________ rate as the quantity increased.

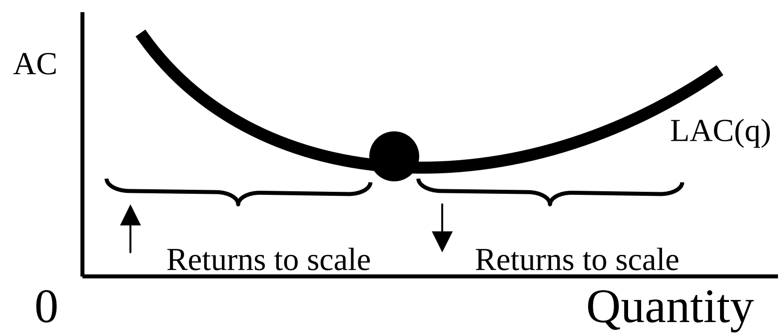
This indicates that the firm is operating in a region of the production function where there are **<u>increasing</u>** returns to scale.

That is, the increase in long-run total cost is less than the increase in output as production increases.

At some point as quantity of output becomes even larger, the long-run total cost increases at an increasing rate, and the firm experiences **decreasing** returns to scale.

The **Long-run Average Cost**: is the long-run total cost divided by quantity:

$$ATC_L(q) = \frac{TC_L(q)}{q}$$



When there are increasing returns to scale, the percentage increase in total cost is ____ than the percentage increase in quantity:

Long-run avenge cost will be ________ as output produced increases.

However, after some point, the long-run average cost reaches a minimum where there are constant returns to scale and then decreasing returns to scale.

Formally, a production function $F(K, L)$ is defined to have:

- constant returns to scale if (for any constant *a* greater than 0) $F(aK, aL) = aF(K, L)$
- increasing returns to scale if (for any constant *a* greater than 1) $F(aK, aL) > aF(K, L)$,
- decreasing returns to scale if (for any constant *a* between 0 and 1) $F(aK, aL) < aF(K, L)$

where K and L are factors of production, capital and labor, respectively.

Formal example

The Short-Run Total Cost Function

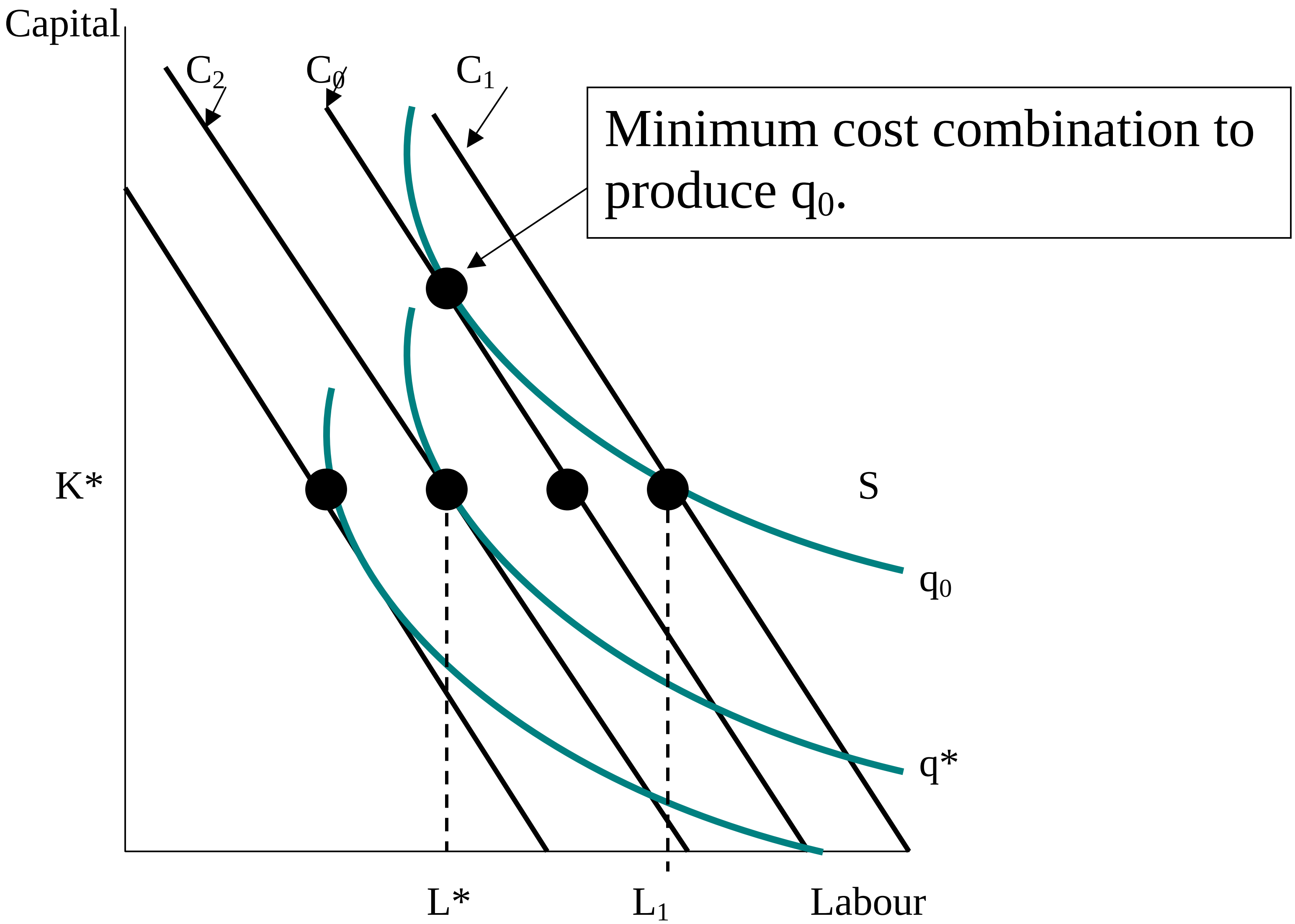
In the short run, one factor is fixed. Hence, the ____ of this fixed factor does not change as _________ produced changes.

The short-run total cost function consists of two components:
Short-run total cost = total fixed cost + total variable cost

$$TC_S = TC_S(q) = F + V(q)$$

- The short-run total cost function shows the ______ total cost of producing each quantity when one factor is fixed.
- The ______ cost must be paid regardless of whether any of the good is produced.
- The variable cost will increase when the quantity produced increases.

Deriving the Short-run Total Cost Curve



- Refer to the diagram on the former page.

In the short-run, the firm has K* units of capital.

The firm's **<u>expansion path</u>** is the horizontal line 'S' because the amount of capital is <u>f____</u>.

If the firm wants to produce q_0 units in the short run, it must employ L_1 units of labour and incur a total cost of C_1.

The lowest total cost of producing quantity q_0 is along the isocost line C_0.

- However, since the cost C_1 is greater than C_0, it is more __________ to produce q_0 units in the short run when the amount of capital is fixed at K*, than in the long run when capital is variable.

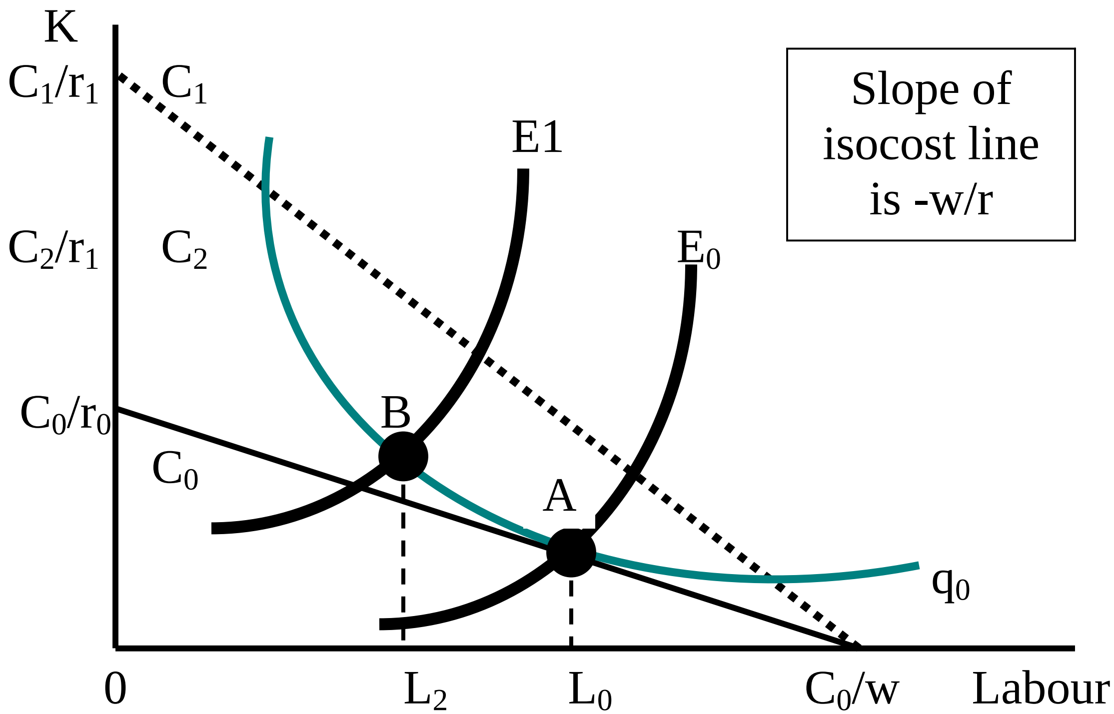
With K* of capital, production of q* units of output with L* units of labour, is the **cost minimization production solution**.

Shifts in the Long-Run Total Cost Function

The position of the long-run cost function will shift if:

1) the prices of the ________ change
2) there is a change in ___________

1) **A Change in the ______ of A Factor**



Initially the expansion path is E_0. The firm produces q_0 units at the lowest total cost C_0 by employing L_0 units of labour and using K_0 units of capital.

At point A, the slope of the isocost line equals the slope of the isoquant.

Suppose the price of capital __________.

- The isocost line becomes C_1. The cost of a unit of capital decreases _________ to the cost of a unit of labour.

After the price reduction of capital, annual cost falls and becomes $r_1 = iM_1$. (New price of capital)

$$-\frac{w}{r_1} = \text{new slope of isocost lines.}$$

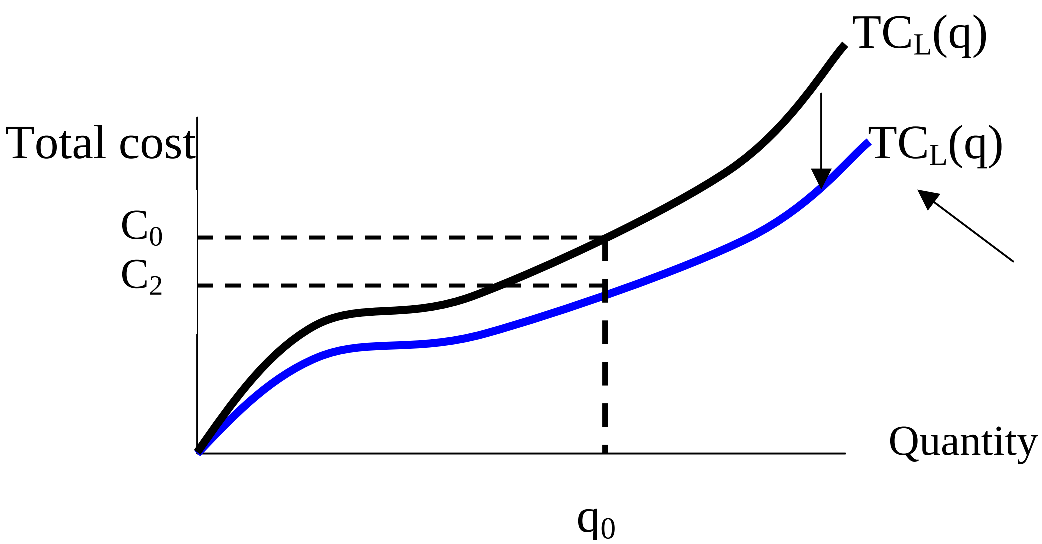
All the firm's isocost lines now have the same slope as isocost C_1.

The firm's new __________ path is E_1, derived by connecting all points of tangency between each new isocost line and each isoquant.

If the firm produces the same output q_0, it will now use more capital (K_2 units) and ____ labour (L_2 units).

Because this combination occurs on the isocost line C_2, below C_1, the total cost associated with producing the output q_0 will be lower after the price of capital falls.

We can now derive the total long-run cost of production of each quantity from the new expansion path E_1.



Notice that the new long run total cost function lies ______ the original total long-run cost function.

2) **Technological Change**

The long-run total cost function shifts _________ when a new technology allows the firm to produce a larger quantity with any given combination of factors.

The technological change allows the firm to produce the same quantity of output at a lower cost.