Review:

Question 2: Use the **Bayes'** formula to answer the following: An apple cider company has two production facilities, one in Vancouver and one in Richmond. The same type of cider is made at both factories. The Vancouver factory produces 35% of the company's cider and the Richmond factory produces the remaining 65%. All cider produced from the two factories is sent to a central facility, where it is bottled for sale. After extensive sampling, the quality assurance manager has determined that 12% of the cider produced in Vancouver and 15% of the cider produced in Richmond is unusable due to poor quality.

(i) What is the probability that the cider was produced at the Vancouver factory, given that the cider is of poor quality?

(ii) What is the probability that the cider was produced at the Richmond factory, given that the cider is of good quality?

(i)

$$P(Van|Poor) = \frac{P(Van)P(Poor|Van)}{P(Poor)}$$

$$P(Van|Poor) = \frac{P(Van)P(Poor|Van)}{P(Van)P(Poor|Van) + P(R)P(Poor|R)}$$

$$P(Van|Poor) = \frac{(0.35)(0.12)}{(0.35)(0.12) + (0.65)(0.15)} = \frac{0.042}{0.1395} = 0.301$$

(ii)

$$P(Rich|Good) = \frac{P(R)\,P(Good|Rich)}{P(Good)}$$

$$P(Rich|Good) = \frac{P(Rich)\,P(Good|Rich)}{P(Van)\,P(Good|Van) + P(Rich)\,P(good|Rich)}$$

$$P(Rich|Good) = \frac{(0.65)(0.85)}{(0.35)(0.88) + (0.65)(0.85)} = \frac{0.5528}{0.8605} = 0.6421$$

<u>Question 3:</u>

Consider the joint probability distribution of inflation rates and money growth:

		Inflation Rate % (Y)	
		1	**2**
Money Growth % (X)	**10**	0.20	0.10
	12	0.30	0.40

A) Find E(Money Growth) and E(Inflation rate).

		Inflation Rate(Y)		
		1	**2**	**P(X)**
Money (X)	**10**	0.20	0.10	0.30
	12	0.30	0.40	0.70
P(Y)		0.50	0.50	1.00

E(x) = 10(0.30) + 12(0.70) =3+7.4=11.4

E(Y) = 1(0.50) + 2(0.5) =0.5+1=1.5

B) Find the variance of money growth [Var(X)] and find the variance of the inflation rate [Var(Y)]. ***(2 marks)***

First determine the $E(x^2)$ and $E(y^2)$

$E(x^2) = 10^2(0.30) + 12^2(0.70) = 30+100.8=130.8$

$E(Y^2) = 1^2(0.50) + 2^2(0.5) = 0.5+2=2.5$

Var(Money x)=$E(x^2)-(E(x))^2$ =130.8-129.96=0.84

Var(interest rates Y)=$E(y^2)-(E(y))^2$=2.5-2.25=0.25

C) Find the covariance of the inflation rate and money growth.

First determine the Expected value of money growth * inflation rates or (E(X*Y)):

$=\sum(XY)P(x,y)$

$= 10(1)\ 0.2 + 10(2)(0.1)+12(1)0.3+12(2)(0.4)$

$=2+2+3.6+9.6=17.2$

$$\begin{aligned}Cov(X,Y) &= E(X,Y) - E(x)E(Y)\\ &= 17.2 - 11.4 \times 1.5\\ &= 17.2 - 17.1 = 0.1\end{aligned}$$

D) Find Var(5X-2Y).

$25V(x)+4V(Y)+2(5)(-2)Cov(x,y)$

$=25(0.84) +4(0.25) + 2(5)(-2)(0.10)$
$=21+1-2= 20$

E) Find the correlation between inflation rates and money growth. Interpret the results.

$$\begin{aligned}Cor(X,Y)&=Cov(x,y)/sd(x) \times sd(y)\\ &=(20)/(0.916515139 \times 0.5)\\ &=0.1/0.458257569=0.218\end{aligned}$$

Weak positive correlation.

F) If W=6X-3Y, find the expected value and standard deviation of W.
(x=money growth and y=inflation)

$$E(W)=6E(X)-3E(Y)$$

$$E(W)=(6 \times 11.4) - (3 \times 1.5)= 68.4+4.5=72.9$$

$$V(W)=36V(x)+9V(Y)+(2)(6)(-3)Cov(X,Y)$$

$$V(W)= (36 \times .84) +(9 \times .25) +(-36)(.1)$$

$$V(W)=30.24 + 2.25 - 3.6 = 28.89$$

Question 4:

An airline is considering changing from an assigned seating reservation system to one in which fliers would be able to take any seat they wish on a first-come-first-serve basis. The airline believes that 85% of its fliers would like this change if it is accompanied with a reduction in ticket prices. The airline has selected a random sample of 50 customers and determined that 37 like the proposed change. We assume the binomial distribution applies.)

a) If the airline is correct in its assessment of the probability, what is the expected number and standard deviation of the people in a sample of n=50 who will like the change?

$\prod = 0.85$, n=50

$E(x) = n\prod = (50)(0.75)=37.5$

$V(x) = n\prod(1-\prod) = (50)(0.75)(0.25)=9.375$

$Sd(x)=3.06186$

b) What is the probability of finding 32 to 37 customers who like the change if the probability is 0.85 that a customer will like the change? (3 marks)
[i.e. $P(32 \le x \le 37)$]

$$P(32 \le x \le 37) = 0.0001 + 0005 + .0013 + .0033 + .0079 + .0169 = 0.03$$

Question 5

The p.d.f. for X is: $f(x) = \begin{cases} \frac{1}{8} & \text{for } -2 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$.

A) Is this a proper p.d.f.? Prove that it is or is not. ***Use integration to solve.***

Total area is =1 if it's proper.

F(x)=[1/8]X $\leftarrow$ Evaluate between -2 and 6.

(1/8)[6-(-2)]=(1/8)(8)=1

B) What is the expected value of the random variable x? ***Use integration to solve.***

$$E(x) = \int_{-2}^{6} xf(x)dx = \int_{-2}^{6} x\left(\frac{1}{8}\right)dx$$

$$\int_{-2}^{6}\left[\frac{1}{8}x\right]dx = \frac{1}{8}x^2\left(\frac{1}{2}\right)$$

$$= \frac{1}{16}x^2$$

evaluating between -2 and 6:

$$\left[\frac{1}{16}\right]36 - \left[\frac{1}{16}\right]4 = 32/164 = 2$$

C) What is the probability of x being larger than 3? (P(X>3)) Use integration to determine the probability and show your work!

$$P(x>3)=\int_{3}^{6} f(x)dx=\int_{3}^{6}\left(\frac{1}{8}\right)dx$$

$$=\frac{1}{8}x\Big]_{3}^{6}$$

evaluating between 3 and 6:

$$\left[\frac{1}{8}\right]6-\left[\frac{1}{8}\right]3=\left[\frac{3}{8}\right]$$

D) What is the probability of x being less than 5? (P(X<5)) Use integration to determine the probability and show your work!

$$P(x<5)=\int_{-2}^{5} f(x)dx=\int_{-2}^{5}\left(\frac{1}{8}\right)dx$$

$$=\frac{1}{8}x\Big]_{-2}^{5}$$

evaluating between -2 and 5:

$$\left[\frac{1}{8}\right]5-\left[\frac{1}{8}\right](-2)=\left[\frac{5}{8}+\frac{2}{8}\right]=\frac{7}{8}$$

<u>Question 6:</u> A study concluded the time it took the average employee to get to work each day is normally distributed with a mean equal to 19 minutes and a standard deviation equal to 5 minutes. One employee indicated that he could get to work in 10.5 minutes per day.

(i) Find the probability that an employee could travel to work in 10.5 or more minutes per day.

$$Z = \left[\frac{x-\mu}{\sigma}\right] = \frac{10.5-19}{5} = -1.7$$

P(X>10.5)=1-P(Z<-1.7)==P(Z<1.7)=0.9554

(ii) Find the probability that an employee could travel to work in 21 or less minutes per day.

$$Z = \left[\frac{x-\mu}{\sigma}\right] = \frac{21-19}{5} = 0.4$$

P(X<21)=P(Z<0.4)=0.6554

(iii) Find the probability that an employee could travel to work between 10 and 25 minutes per day.

$$P\left[\frac{x_1-\mu}{\sigma} \le Z \le \frac{x_2-\mu}{\sigma}\right] = P\left[\frac{10-19}{5} \le Z \le \frac{25-19}{5}\right]$$

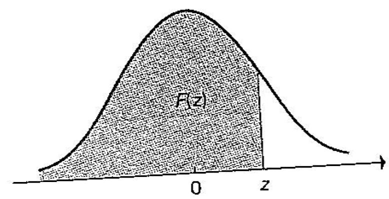
$$= P[-1.8 \le Z \le 1.2]$$

$$= 0.8849 - (1-0.9641) = 0.8849 - 0.0359 = 0.849$$

84.9%

APPENDIX TABLES

Table 1 Cumulative Distribution Function of the Standard Normal Distribution



z	F(z)	z	F(z)	z	F(z)	z	F(z)	z	F(z)	z	F(z)
.00	.5000										
.01	.5040	.31	.6217	.61	.7291	.91	.8186	1.21	.8869	1.51	.9345
.02	.5080	.32	.6255	.62	.7324	.92	.8212	1.22	.8888	1.52	.9357
.03	.5120	.33	.6293	.63	.7357	.93	.8238	1.23	.8907	1.53	.9370
.04	.5160	.34	.6331	.64	.7389	.94	.8264	1.24	.8925	1.54	.9382
.05	.5199	.35	.6368	.65	.7422	.95	.8289	1.25	.8944	1.55	.9394
.06	.5239	.36	.6406	.66	.7454	.96	.8315	1.26	.8962	1.56	.9406
.07	.5279	.37	.6443	.67	.7486	.97	.8340	1.27	.8980	1.57	.9418
.08	.5319	.38	.6480	.68	.7517	.98	.8365	1.28	.8997	1.58	.9429
.09	.5359	.39	.6517	.69	.7549	.99	.8389	1.29	.9015	1.59	.9441
.10	.5398	.40	.6554	.70	.7580	1.00	.8413	1.30	.9032	1.60	.9452
.11	.5438	.41	.6591	.71	.7611	1.01	.8438	1.31	.9049	1.61	.9463
.12	.5478	.42	.6628	.72	.7642	1.02	.8461	1.32	.9066	1.62	.9474
.13	.5517	.43	.6664	.73	.7673	1.03	.8485	1.33	.9082	1.63	.9484
.14	.5557	.44	.6700	.74	.7704	1.04	.8508	1.34	.9099	1.64	.9495
.15	.5596	.45	.6736	.75	.7734	1.05	.8531	1.35	.9115	1.65	.9505
.16	.5636	.46	.6772	.76	.7764	1.06	.8554	1.36	.9131	1.66	.9515
.17	.5675	.47	.6803	.77	.7794	1.07	.8577	1.37	.9147	1.67	.9525
.18	.5714	.48	.6844	.78	.7823	1.08	.8599	1.38	.9162	1.68	.9535
.19	.5753	.49	.6879	.79	.7852	1.09	.8621	1.39	.9177	1.69	.9545
.20	.5793	.50	.6915	.80	.7881	1.10	.8643	1.40	.9192	1.70	.9554
.21	.5832	.51	.6950	.81	.7910	1.11	.8665	1.41	.9207	1.71	.9564
.22	.5871	.52	.6985	.82	.7939	1.12	.8686	1.42	.9222	1.72	.9573
.23	.5910	.53	.7019	.83	.7967	1.13	.8708	1.43	.9236	1.73	.9582
.24	.5948	.54	.7054	.84	.7995	1.14	.8729	1.44	.9251	1.74	.9591
.25	.5987	.55	.7088	.85	.8023	1.15	.8749	1.45	.9265	1.75	.9599
.26	.6026	.56	.7123	.86	.8051	1.16	.8770	1.46	.9279	1.76	.9608
.27	.6064	.57	.7157	.87	.8078	1.17	.8790	1.47	.9292	1.77	.9616
.28	.6103	.58	.7190	.88	.8106	1.18	.8810	1.48	.9306	1.78	.9625
.29	.6141	.59	.7224	.89	.8133	1.19	.8830	1.49	.9319	1.79	.9633
.30	.6179	.60	.7257	.90	.8159	1.20	.8849	1.50	.9332	1.80	.9641

Table 1 Cumulative Distribution Function of the Standard Normal Distribution Continued

z	F(z)	z	F(z)	z	F(z)	z	F(z)	z	F(z)	z	F(z)
1.81	.9649	2.21	.9864	2.61	.9955	3.01	.9987	3.41	.9997	3.81	.9999
1.82	.9656	2.22	.9868	2.62	.9956	3.02	.9987	3.42	.9997	3.82	.9999
1.83	.9664	2.23	.9871	2.63	.9957	3.03	.9988	3.43	.9997	3.83	.9999
1.84	.9671	2.24	.9875	2.64	.9959	3.04	.9988	3.44	.9997	3.84	.9999
1.85	.9678	2.25	.9878	2.65	.9960	3.05	.9989	3.45	.9997	3.85	.9999
1.86	.9686	2.26	.9881	2.66	.9961	3.06	.9989	3.46	.9997	3.86	.9999
1.87	.9693	2.27	.9884	2.67	.9962	3.07	.9989	3.47	.9997	3.87	.9999
1.88	.9699	2.28	.9887	2.68	.9963	3.08	.9990	3.48	.9997	3.88	.9999
1.89	.9706	2.29	.9890	2.69	.9964	3.09	.9990	3.49	.9998	3.89	1.0000
1.90	.9713	2.30	.9893	2.70	.9965	3.10	.9990	3.50	.9998	3.90	1.0000
1.91	.9719	2.31	.9896	2.71	.9966	3.11	.9991	3.51	.9998	3.91	1.0000
1.92	.9726	2.32	.9898	2.72	.9967	3.12	.9991	3.52	.9998	3.92	1.0000
1.93	.9732	2.33	.9901	2.73	.9968	3.13	.9991	3.53	.9998	3.93	1.0000
1.94	.9738	2.34	.9904	2.74	.9969	3.14	.9992	3.54	.9998	3.94	1.0000
1.95	.9744	2.35	.9906	2.75	.9970	3.15	.9992	3.55	.9998	3.95	1.0000
1.96	.9750	2.36	.9909	2.76	.9971	3.16	.9992	3.56	.9998	3.96	1.0000
1.97	.9756	2.37	.9911	2.77	.9972	3.17	.9992	3.57	.9998	3.97	1.0000
1.98	.9761	2.38	.9913	2.78	.9973	3.18	.9993	3.58	.9998	3.98	1.0000
1.99	.9767	2.39	.9916	2.79	.9974	3.19	.9993	3.59	.9998	3.99	1.0000
2.00	.9772	2.40	.9918	2.80	.9974	3.20	.9993	3.60	.9998		
2.01	.9778	2.41	.9920	2.81	.9975	3.21	.9993	3.61	.9998		
2.02	.9783	2.42	.9922	2.82	.9976	3.22	.9994	3.62	.9999		
2.03	.9788	2.43	.9925	2.83	.9977	3.23	.9994	3.63	.9999		
2.04	.9793	2.44	.9927	2.84	.9977	3.24	.9994	3.64	.9999		
2.05	.9798	2.45	.9929	2.85	.9978	3.25	.9994	3.65	.9999		
2.06	.9803	2.46	.9931	2.86	.9979	3.26	.9994	3.66	.9999		
2.07	.9808	2.47	.9932	2.87	.9979	3.27	.9995	3.67	.9999		
2.08	.9812	2.48	.9934	2.88	.9980	3.28	.9995	3.68	.9999		
2.09	.9817	2.49	.9936	2.89	.9981	3.29	.9995	3.69	.9999		
2.10	.9821	2.50	.9938	2.90	.9981	3.30	.9995	3.70	.9999		
2.11	.9826	2.51	.9940	2.91	.9982	3.31	.9995	3.71	.9999		
2.12	.9830	2.52	.9941	2.92	.9982	3.32	.9996	3.72	.9999		
2.13	.9834	2.53	.9943	2.93	.9983	3.33	.9996	3.73	.9999		
2.14	.9838	2.54	.9945	2.94	.9984	3.34	.9996	3.74	.9999		
2.15	.9842	2.55	.9946	2.95	.9984	3.35	.9996	3.75	.9999		
2.16	.9846	2.56	.9948	2.96	.9985	3.36	.9996	3.76	.9999		
2.17	.9850	2.57	.9949	2.97	.9985	3.37	.9996	3.77	.9999		
2.18	.9854	2.58	.9951	2.98	.9986	3.38	.9996	3.78	.9999		
2.19	.9857	2.59	.9952	2.99	.9986	3.39	.9997	3.79	.9999		
2.20	.9861	2.60	.9953	3.00	.9986	3.40	.9997	3.80	.9999		

Rules of Expectation and Variance

Rules of “Expectation”: $E(x) = \sum_x xP(x)$

The following are some helpful rules to assist us in calculations using expectation operator:

1) The E(c)=c, where c is a constant.

$$E(c) = \sum (c) = \sum c \times P(c) = c \times 1 = c.$$

2) V(c)=0; The variance of a constant is zero.

Recall: $V(x)=E(x-\mu)^2$;

$$V(c)=E(c-\mu)^2=E(c-c)^2=E(0)^2=0$$

3) E(aX)=aE(X): Multiply X by a constant:

$$E(ax) = \sum (ax)P(x) = a\sum xP(x) = aE(x)$$

4) E(x+b)=E(x)+b: Add a constant

$$E(x+b)=\sum(x+b)=\sum(x+b)P(x)$$
$$=\sum xP(x)+\sum bP(x)$$
$$=E(x)+b\sum P(x)=E(x)+b$$

$$Since,\quad \sum P(x)=1$$

5) $V(ax)=a^2V(x)$:

Recall $V(x)=E(x-\mu)^2$;

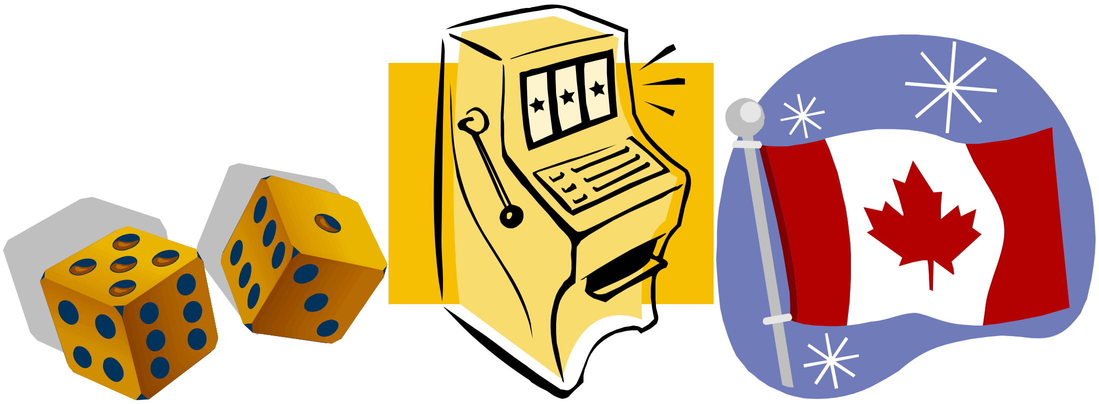
$V(ax)=E(ax-E(ax))^2$
$=E(ax-aE(x))^2$
$=E(a(x-E(x)))^2$
$=E(a^2(x-E(x))^2)$
$V(ax)=a^2E(x-E(x))^2=a^2V(x)$

6) V(x+b)=V(x)

$V(x+b)=E((x+b)-E(x+b))^2$
$=E(x+b-E(x)-b)^2$
$=E(x-E(x))^2=V(x)$

$E(ax \pm b) = aE(x) + b$

$V(ax \pm b) = a^2V(x)$



<u>Example</u>:

Suppose I want \$500 US for my trip to Las Vegas.

⇨What about the results in \$ CANADIAN?

Suppose C\$1 =U.S. \$0.92 (U.S. \$1 =C\$1.084)

$E(ax)=aE(x)$; $a = 1.084$

⇨So, expected value C\$ (500 × 1.084)= C\$542.00

****

Suppose the average hotel room is $209 (US) with a standard deviation of $9 (US).

What is the expected value and standard deviation in Canadian dollar?

C$ $(209 \times 1.084)=$ C$226.56

$V(x)=81$

$V(ax)= a^2\ V(x);\ a= 1.084$

$V(1.084\times 81)=1.084^2 V(x)$

$= 1.084^2 \times 81$

$=1.175056\times 81=95.1795$

Std. deviation $(ax)=|a|$ S.D. (X)

$=$C$$(1.084\times 9)=C9.756