Question: *(5 Marks)*

A shipping company operates a fleet of ships that carry commodities between Vancouver, B.C. and San Diego California. The travel time varies between these two locations. These shipping times are as follows:

$$x = \text{shipping days}$$
$$= \{5,\ 9,\ 7,\ 4,\ 4\}$$

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	TIME
Mean	5.800000
Median	5.000000
Maximum	9.000000
Minimum	4.000000
Std. Dev.	2.167948
Skewness	0.612269
Kurtosis	1.815527
Jarque-Bera	0.604681
Probability	0.739086
Sum	29.00000
Sum Sq. Dev.	18.80000
Observations	5

The manager is interested in knowing the:

(i) median of shipping time
5

(ii) coefficient of variation of shipping time

$$c.v. = \frac{\sigma}{\mu} \times 100$$

$$c.v. = [1.9391 / 5.8] \times 100 = 33.42$$

(iii) 10^{th} percentile of shipping time
4

(iv) 3rd quartile of shipping time
Answer:7

(v) 50th midrange of shipping time
3

Question 3: (4 marks)

The January 2001 and December 2002 price and volumes (actual quantities purchased) in millions of shares for the two high-technology company stocks are shown in the table.

A) Calculate the Fisher quantity index using January 2001 as the base period with a base value of 1. (2 Marks)

	IBM		MICROSOFT	
	Price	Volume (Quantity)	Price	Volume (Quantity)
Jan 2001	112	9.9	61.06	49.7
Dec. 2002	77.50	7.6	51.70	31.6

Source: Standard $ Poor's. NYSE Daily Stock Price Record, Jan 2001, 2002

$$Q_{0t}^{F} = \sqrt{Q^{L} \times Q^{P}} = \sqrt{0.6711 \times 0.6661} = \sqrt{0.44704} = 0.6686$$

Year	Index
2001	1
2002	0.6686

B) Prove that the Laspreyres price index fails the factor reversal test. (2 marks)

TEST:

$$P_{0t}Q_{0t} = \frac{p_{it}q_{it}}{p_{i0}q_{i0}}.$$

$$P_{0t}Q_{0t} = 0.805296 \times 0.671101262 = 0.540435162$$

$$\frac{p_{it}q_{it}}{p_{i0}q_{i0}} = \frac{2222.72}{4143.482} = 0.53643.$$

The price index fails the test.

<u>Question 4:</u> (2 marks) Prove that if two random variables are independent, their covariance equals zero.

If X and Y are independent: P(X,Y)=P(X) * P(Y)

$$Cov(X,Y) = \sum_{x}\sum_{y}\left[(x-\mu_x)(y-\mu_y)\right]P(x,y)$$
$$= \left[\sum_{x}(x-\mu_x)P(x)\right]\left[\sum_{y}(y-\mu_y)P(y)\right]$$
$$= \left[\sum_{x}(x)P(x) - \mu_x\sum_{x}P(x)\right]\left[\sum_{y}(y)P(y) - \mu_y\sum_{y}P(y)\right]$$
$$= \left[E(x) - \mu_x\right]\bullet\left[E(y) - \mu_y\right] = 0$$
$$= (\mu_x - \mu_x = 0)\ (\mu_y - \mu_y = 0)$$

<u>Question 5:</u> Determine the final seasonal indices. ***(2 marks)***

$S_1 = 0.92$
$S_2 = 1.05$
$S_3 = 1.23$
$S_4 = 0.77$
GM=0.978

$S_1* = 0.92 / 0.978 = 0.9407$
$S_2* = 1.05 / 0.978 = 1.0736$
$S_3* = 1.23 / 0.978 = 1.2577$
$S_4* = 0.77 / 0.978 = 0.7873$

<u>Question 6:</u> (2 marks) Use the Bayes' formula to answer the following: A cider company has two production facilities, one in Vancouver and one in Richmond. The same type of cider is made at both factories. The Vancouver factory produces 25% of the company's cider and the Richmond factory produces the remaining 75%. All cider produced from the two factories is sent to a central facility, where it is bottled for sale. After extensive sampling, the quality assurance manager has determined that 2% of the cider produced in Vancouver and 5% of the cider produced in Richmond is unusable due to poor quality.

(i) What is the probability that the cider was produced at the Vancouver factory, given that the cider is of poor quality? *(1 mark)*

(ii) What is the probability that the cider was produced at the Richmond factory, given that the cider is of good quality? *(1 mark)*

(i)

$$P(Van|Poor) = \frac{P(Van)P(Poor|Van)}{P(Poor)}$$

$$P(Van|Poor) = \frac{P(Van)P(Poor|Van)}{P(Van)P(Poor|Van) + P(R)P(Poor|R)}$$

$$P(Van|Poor) = \frac{(0.25)(0.02)}{(0.25)(0.02)+(0.75)(0.05)} = \frac{0.005}{0.0425} = 0.1176$$

(ii)

$$P(Rich|Good) = \frac{P(R)P(Good|Rich)}{P(Good)}$$

$$P(Rich|Good) = \frac{P(Rich)P(Good|Rich)}{P(Van)P(Good|Van) + P(Rich)P(good|Rich)}$$

$$P(Rich|Good) = \frac{(0.75)(0.95)}{(0.25)(0.98)+(0.75)(0.95)} = \frac{0.7125}{0.9575} = 0.7441$$

Question 7: ***(6 marks)***

Consider the joint probability distribution of inflation rates and money growth:

		Inflation Rate % (Y)	
		2	**4**
Money Growth % (X)	**2**	0.20	0.10
	3	0.30	0.40

A) Find E(Money Growth) and E(Inflation rate). ***(1 mark)***

E(x) =2.7
E(Y) = 3

B) Find the variance of money growth [Var(X)] and find the variance of the inflation rate [Var(Y)]. ***(1 mark)***

Var(Money x)=0.21

Var(inflation rates Y)=1

C) Find the covariance of the inflation rate and money growth. ***(1 mark)***

Cov(X,Y)=0.1

D) Find Var(5X-2Y) = Var [(5 *Money)- (2*inflation rate)]. ***(1 mark)***

7.25

E) Find the correlation between inflation rates and money growth. Interpret the results. ***(1 mark)***

0.218

<u>Weak positive correlation.</u>

F) If W=6X-3Y, find the expected value and standard deviation of W. (1 mark) (x=money growth and y=inflation)

E(w)=7.2
V(W)=12.96
SD=3.6

<u>Question 8:</u> ***(2 marks)***

An airline is considering changing from an assigned seating reservation system to one in which fliers would be able to take any seat they wish on a first-come-first-serve basis. The airline believes that 45% of its fliers would like this change if it was accompanied with a reduction in ticket prices. The airline has selected a random sample of 100 customers and determined that 37 like the proposed change. We assume the binomial distribution applies.)

a) If the airline is correct in its assessment of the probability, what is the expected number and standard deviation of the people in a sample of n=100 who will like the change? (1 mark)

$\prod = 0.45$, n=100

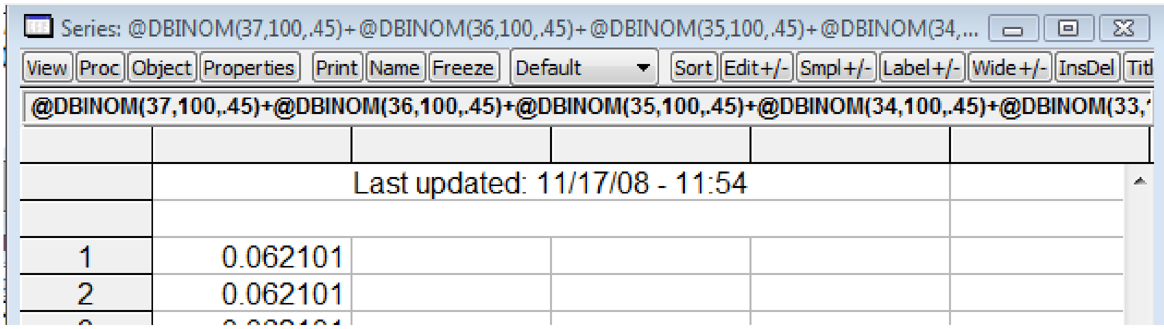
E(x) = 45

V(x) =24.75

Sd(x)=4.9749

b) What is the probability of finding 32 to 37 customers who like the change if the probability is 0.45 that a customer will like the change? (1 mark)
[i.e. $P(32 \le x \le 37)$]

$P(32 \le x \le 37) = 0.0025 + 0043 + .0069 + .0106 + .0157 + .0222 = 0.0622$



Question 9 (4 marks)

The p.d.f. for X is: $f(x) = \begin{cases} \frac{1}{10} & \text{for } -2 \le x \le 8 \\ 0 & \text{otherwise} \end{cases}$.

A) Is this a proper p.d.f.? Prove that it is or is not. ***Use integration to solve. (1 mark)***
1

B) What is the expected value of the random variable x? ***Use integration to solve.(2 marks)***

3

C) What is the probability of x being larger than 5? P(X>5) Use integration to determine the probability and show your work! (1 mark)

3/10

<u>Question 10</u>: (3 marks) A study concluded the time it took the average employee to drive to work each day is normally distributed with a mean equal to 11 minutes and a standard deviation equal to 5 minutes. One employee indicated that he could get to work in 10.5 minutes per day.

(i) Find the probability that an employee could travel to work in 10.5 or more minutes per day.

0.5398

(ii) Find the probability that an employee could travel to work in 15 or less

minutes per day.

$P(X<15)=0.7881$

(iii) Find the probability that an employee could travel to work between 5 and 13 minutes per day.

54.23%

End of Exam