Topic 3:Index Numbers and Times Series

Part I: Index Numbers:

- What is an ________ number?
- Constructing Indices
- Choosing between Indices
- Using Price Indices
- The Canadian ______
- Price Indices and the "Cost of Living"

Reference: Chapter 20 and Chapter 17

⇨We need to develop a measure that _______________ the characteristics of large data sets and is useful for period to period comparisons.

⇨An **index number** accomplishes this by aggregating information into a _______ measure that permits easy comparisons.

Definition: An INDEX NUMBER is a number which summarizes pieces of information in a **u______** way, and forms the basis for comparative judgements.

⌘ An index number is a ratio of 2 numbers expressed as a percentage.

Used if we want to keep track of "**overall**" price _______ over time or need to get an **aggregate** measure of output– different items have the own units.

Examples: CPI (Consumer Price Index);
PPI (Producer Price Index)

Basic Considerations for a "good" Index Number:

(i) Include information on **____** items.

(ii) Combine in way which recognizes **r________** importance of each item.

(iii) Need a "**bench mark**" value and period to provide reference point for comparisons. ****Base period required***

Price and Quantity Indices

Price Indices

Two broad types: (and there are several variants of each)

(i) Aggregative Indices

It consists in expressing the aggregate price of all commodities in the current year as a percentage of the aggregate price in the base year.

$$P_{01} = \frac{\sum p_1}{\sum p_0} \times 100$$

P_{01} = Index number of the current year.
p_1 = Total of the current year's price of all commodities.
p_0 = Total of the base year's price of all commodities.

(ii) Relatives Indices

- The current year price is expressed as a price relative of the base year price. These price relatives are then averaged to get the index number. The average used could be arithmetic mean, geometric mean or even median.

$$P_{01} = \frac{\sum\left(\frac{p_1}{p_0} \times 100\right)}{N}$$

Where N is Numbers Of items.

Looking at (i): Aggregative Indices

Example: Bundle of Two Goods

Item	1990			2019	
	Price ($) (P)	Quantity (Q)		Price ($) (P)	Quantity (Q)
Good 1	0.75			1.80	
Good 2	0.50			2.70	

We want to summarize ___________price change from 1990 to 2019.
(Base Year =1990.)

Must be the **same goods** in each ______! (Assumption)

Type of Price Index

Price of the "ith" item in the year of interest.

$$P_{0t}^{S} = \left[\frac{\sum_{i=1}^{n} P_{it}}{\sum_{i=1}^{n} P_{i0}} \right] \times 100$$

The 100 indicates that we have a base value of 100 in the base year.

Base Year

Year of interest

Price of the "ith" item in the base year.

Simple A____________ Index:

Let P_{it}= Price of the i^{th} item at time "t".
(i= 1, 2,....,n; t=0, 1, 2,....,T)

Construction of the **Simple P_____ Index**:

$$P_{0t}^{S} = \left[\frac{\sum_{i=1}^{n} P_{it}}{\sum_{i=1}^{n} P_{i0}} \right] \times 100$$

The simple _____ index from base year '0' to year 't' for items 1 to n.

If t=0, index=____ (Base year value).

Example: See previous data for 1990 and 2019:
Base Year =1990

$$P_{0t}^{S} = \left[\frac{\sum_{i=1}^{n} P_{it}}{\sum_{i=1}^{n} P_{i0}} \right] \times 100$$

$$P_{0=1990,t=2019}^{S} = \left[\frac{1.80 + 2.70}{0.75 + 0.50} \right] \times 100 = 360 \Leftarrow \$/\$ \; Unitless$$

<u>Prices rose, overall, to be 3.6 times higher in 2019 than in 1990.</u>

⇨This type of index ***only*** tells about changes in ______.

⇨Individual ______ falls can be offset by other individual rises.

⇨With the simple index, all goods are given **<u>equal weight</u>**.

So......

Weighted Aggregative Index:

$$P_{0t}^{W} = \left[\frac{\sum_{i=1}^{n} P_{it} W_i}{\sum_{i=1}^{n} P_{i0} W_i} \right] \times 100$$

where W_i=(fixed) weight given to the ith price.

Limitation of these indices*: quality changes:*

Assumption of __________ items

How to ________ the Weights:

- ❖ Quantity of goods purchased. (Which quantity?)
- ❖ Percentage of expenditure on good. ($p_i q_i$)

Both measure "importance", but may *differ* over time.

So, when should they be measured?

Possibilities:

(I) Laspeyres' Index:

$$P_{0t}^{L} = \left[\frac{\sum_{i=1}^{n} p_{it} q_{i0}}{\sum_{i=1}^{n} p_{i0} q_{i0}} \right] \times 100$$

Base period quantities= weights

Disadvantage:

Weights get __________ over time.

Tends to **overstate** changes in _______ for the price index.

(II) Paasche's Index:

$$P_{0t}^{P} = \left[\frac{\sum_{i=1}^{n} p_{it} q_{it}}{\sum_{i=1}^{n} p_{i0} q_{it}} \right] \times 100$$

Current period quantities=______

Disadvantage:

😳 Weights constantly changing, so index change may be partly due to _________ changes. (Tends to understate changes.)

<u>Some Compromises:</u>

<u>(III) Marshall-Edgeworth Index:</u>

$$P_{0t}^{ME} = \left[\frac{\sum_{i=1}^{n} p_{it} \left(\frac{q_{i0} + q_{it}}{2} \right)}{\sum_{i=1}^{n} p_{i0} \left(\frac{q_{i0} + q_{it}}{2} \right)} \right] \times 100$$

Average of base period and current period quantities are the weights.

(IV) Fisher's "Ideal" Index:

$$P_{0t}^{F} = \sqrt{P_{0t}^{L} \times P_{0t}^{P}}$$

(Geometric Mean of the Laspeyres' and Paasche indices)

Example: Calculations from earlier data:

Year	P_{0t}^{S}	P_{0t}^{L}	P_{0t}^{P}	P_{0t}^{ME}	P_{0t}^{F}
1990	100	100	100	100	100
2019	360	458.18	390.1	422.61	422.7

<u>Calculations:</u>(from data on page 2)

If the base year is 1990, with a value of 100, then for t=2019:

<u>Simple Aggregative Index</u>

$$P_{0t}^{S} = \frac{1.80 + 2.70}{0.75 + 0.50} \times 100 = 360$$

<u>Laspeyres':</u>

$$P_{0t}^{L} = \frac{1.80(1) + 2.70(4)}{0.75(1) + 0.50(4)} \times 100 = 458.18$$

Paasche:

$$P_{0t}^{P} = \frac{1.80(2) + 2.70(3)}{0.75(2) + 0.50(3)} \times 100 = 390.1$$

Marshall-Edgeworth:

$$P_{0t}^{ME} = \left[\left[\frac{\sum_{i=1}^{n} p_{it}\left(\frac{q_{i0} + q_{it}}{2}\right)}{\sum_{i=1}^{n} p_{i0}\left(\frac{q_{i0} + q_{it}}{2}\right)} \right] \right] \times 100$$

$$= \left[\frac{1.80\left(\frac{1+2}{2}\right) + 2.70\left(\frac{4+3}{2}\right)}{0.75\left(\frac{1+2}{2}\right) + 0.50\left(\frac{4+3}{2}\right)} \right] \times 100 = 422.61$$

Fisher:

$$P_{0t}^{F} = \sqrt{P_{0t}^{L} \times P_{0t}^{P}} = \sqrt{458 \times 390} = 422.7$$

<u>Changing The Base:</u>

Often one may want to change the <u>base period</u> and /or base value of an index after it has been constructed.

Suppose we have an index with a base value of 100 in 1970. But, we desire to have a base of 100 in 1972:

Year	Old Index		New Index
1970		Adjust	83.33
1971		each	91.67
1972		entry	100
1973		by	108.33
1974		dividing	118.33
⋮	⋮	by	⋮
1994		1.2	246.66

What numerical value multiplied by 120 equals 100, such that the relatives are left unaltered? (Or equivalently 120 divided by what numerical value equals 100?)

$$\frac{120}{X} = 100 \text{ solving for X: } \frac{120}{100} = 1.2$$

or

$$120 \times X = 100 \text{ solving for X: } \frac{100}{120} = 0.833$$

- Base period is ____________by proportional scaling– ***relatives unaltered***.

▲To create the new index, divide each value of the "old" index by 1.2 (or multiply each value of the "old" index by 0.833).

Overall % changes do ______ change.

Quantity Indices: Same idea as price indices.

Reverse the roles of prices and quantities. Prices are now acting as the weights.

Simple Quantity Index:

$$Q_{0t}^{S} = \frac{\sum q_{it}}{\sum q_{i0}} \times 100$$

Laspeyres' Quantity Index:

$$Q_{0t}^{L} = \left[\frac{\sum_{i=1}^{n} q_{it} p_{i0}}{\sum_{i=1}^{n} q_{i0} p_{i0}} \right] \times 100$$

Base period prices = weight

Paasche Quantity Index:

$$Q_{0t}^{P} = \left[\frac{\sum_{i=1}^{n} q_{it} p_{it}}{\sum_{i=1}^{n} q_{i0} p_{it}} \right] \times 100$$

Current period prices are the weights.

Marshall Edgeworth:

$$Q_{0t}^{ME} = \left[\left[\frac{\sum_{i=1}^{n} q_{it} \left(\frac{p_{i0} + p_{it}}{2} \right)}{\sum_{i=1}^{n} q_{i0} \left(\frac{p_{i0} + p_{it}}{2} \right)} \right] \right] \times 100$$

Fisher:

$$Q_{0t}^{F} = \sqrt{Q_{0t}^{L} \times Q_{0t}^{P}}$$

Choosing Between Indices: 3 TESTS

(1) ______Reversal Test:

P_{it}=price of the i[th] good at time t.

For the single good: $$\frac{P_{it}}{P_{i0}} = \frac{1}{\left(P_{i0} / P_{it} \right)}$$

I.e. Information about price change is the same, regardless of _______________**date**.

Example:

Gum: Price in 2000 is $1.15.
Price in 2005 is $1.75.

$$\frac{1.75}{1.15} = \frac{1}{\left(1.15 / 1.75\right)} \Leftrightarrow (1.75)\left(1.15 / 1.75\right) = 1.15$$

$$1.52173913 = 1.52173913$$

● This motivates the idea of the <u>**Time Reversal Test**</u> for indices.

<u>If it passes:</u>

$$(P_{0t} \times P_{t0}) = 1$$

$where:$

P_{0t} is base at 0 to time t and

P_{t0} is base at t to time 0.

Example: Laspeyres' Index:

$$P_{01}^{L} = \left[\left[\frac{\sum_{i=1}^{n} p_{i1}q_{i0}}{\sum_{i=1}^{n} p_{i0}q_{i0}}\right]\right]$$ (assume the base value is 1 not 100)

$$P_{10}^{L} = \left[\left[\frac{\sum_{i=1}^{n} p_{i0}q_{i1}}{\sum_{i=1}^{n} p_{i1}q_{i1}}\right]\right]$$

Test: $\left(P_{01}^{L} \times P_{10}^{L}\right) = 1$

Fails if: $\left(P_{01}^{L} \times P_{10}^{L}\right) \neq 1$

Laspeyres' generally ______ the Time Reversal Test.

Using the data from the previous example:

$$P^L_{1990,2019} = \left[\left[\frac{\sum_{i=1}^{n} p_{i2019} q_{i1990}}{\sum_{i=1}^{n} p_{i1990} q_{ii1990}}\right]\right] = \left[\frac{(1.80)(1)+(2.70)(4)}{(0.75)(1)+(0.50)(4)}\right] = \frac{12.6}{2.75} = 4.5818$$

$$P^L_{2019,1990} = \left[\left[\frac{\sum_{i=1}^{n} p_{i1990} q_{i2019}}{\sum_{i=1}^{n} p_{i2019} q_{i2019}}\right]\right] = \left[\frac{(0.75)(2)+(0.50)(3)}{(1.80)(2)+(2.70)(3)}\right] = \frac{3}{11.7} = 0.2564$$

$$\text{Test:}\left(P^L_{1990,2019} \times P^L_{2019,1990}\right) = (4.5818)(0.2564) = 1.1748 \neq 1$$

Laspeyres' Price index _____ the Time Reversal test.

Generally: Time Reversal Test >1 for ________ and < 1 for _________.

(2) **Factor Reversal Test:**

For a _______ good, price ratio is $\left(\frac{p_1}{p_0}\right)$ and quantity ratio is $\left(\frac{q_1}{q_0}\right)$.

The change in ______________ is the product: $\left(\frac{p_1 q_1}{p_0 q_0}\right) = \left(\frac{p_1}{p_0}\right)\left(\frac{q_1}{q_0}\right)$.

⇨The **Factor Reversal** test for an index checks:

$$\left[P_{01} \times Q_{01}\right] = \left[\frac{\sum p_{i1} q_{i1}}{\sum p_{i0} q_{i0}}\right]$$

$Where\ P_{01}$ = price index

Q_{01} = quantity index

Suppose P_{01} shows a 20% rise and expenditure has risen by 50%, then Q_{01} should show a 25% rise:

$$\left[P_{01} \times Q_{01}\right] = \left[\frac{\sum p_{i1}q_{i1}}{\sum p_{i0}q_{i0}}\right]$$

$$(1.20) \times (1.25) = \_\_\_\_\_\_$$

Both Laspeyres' and Paasche' Indices ____ the Factor Reversal Test.

Example: Using the previous example:

$$P_{01}^{L} = \left[\frac{\sum_{i=1}^{n} p_{i1} q_{i0}}{\sum_{i=1}^{n} p_{i0} q_{i0}} \right] \qquad Q_{01}^{L} = \left[\frac{\sum_{i=1}^{n} q_{i1} p_{i0}}{\sum_{i=1}^{n} q_{i0} p_{i0}} \right]$$

$$\textit{The test}: \quad \left(\mathrm{P}_{01}^{\mathrm{L}} \times Q_{01}^{L} \right) = \left[\frac{\sum_{i=1}^{n} p_{i1} q_{i1}}{\sum_{i=1}^{n} p_{i0} q_{i0}} \right]$$

$$P^{L}_{1990,2019} = \left[\left[\frac{\sum_{i=1}^{n} p_{i2019} q_{i1990}}{\sum_{i=1}^{n} p_{i1990} q_{ii1990}}\right]\right] = \left[\frac{(1.80)(1)+(2.70)(4)}{(0.75)(1)+(0.50)(4)}\right] = \frac{12.6}{2.75} = 4.5818$$

$$Q^{L}_{1990,2019} = \left[\left[\frac{\sum_{i=1}^{n} q_{i2019} p_{i1990}}{\sum_{i=1}^{n} q_{i1990} p_{i1990}}\right]\right] = \left[\frac{(2)(0.75)+(3)(0.5)}{(1)(0.75)+(4)(0.5)}\right] = \frac{3}{2.75} = 1.0909$$

$$LHS \mapsto \left(P^{L}_{1990,2019} \times Q^{L}_{1990,2019}\right) = (4.5818)(1.0909) = 4.998$$

$$\sum p_{it}q_{it} = (1.8)(2) + (2.7)(3) = 11.7$$

$$\sum p_{i0}q_{i0} = (0.75)(1) + (0.5)(4) = 2.75$$

$$RHS \mapsto \frac{\sum p_{it}q_{it}}{\sum p_{i0}q_{i0}} = \frac{11.7}{2.75} = 4.2545$$

Since, $4.998 \neq 4.2545$, ________ test.

______________-_____________

⇨passes time-reversal test
⇨fails factor-reversal test

____________’s Ideal

⇨passes time reversal test
⇨passes factor reversal test

<u>(3) Time __________ Sequences:</u>

- So far we have considered only 2 points in time.

▲ **How do we measure ______ changes from period 1 to period 2?**

Either: (1) Form P_{12} directly (using period 1 as the base year)

Or (2) take $\left[P_{02} / P_{01} \right] = P_{12}$ directly.

Should get the same result either way.

Note:

$$P_{02} = \frac{P_2}{P_0}$$

$$P_{01} = \frac{P_1}{P_0}$$

$$\frac{P_2 / P_0}{P_1 / P_0} \text{ rearranging: } \frac{P_2}{P_0} * \frac{P_0}{P_1} = \frac{P_2}{P_1} = P_{12}$$

Suppose: Prices rise by 20% from period 0 to 1.
Prices ______by 10% from period 1 to 2.

Overall rise from period 0 to period 2 is (1.2 × 1.1) =1.32 (32% rise)

Index Values: 100 120 ______
(0) (1) (2)

This motivates the **<u>Circularity Test:</u>**

Index ________ this test if: $\boxed{P_{02} = (P_{01} \times P_{12})}$

For example, consider the Laspeyres':

$$P_{01}^{L} = \left[\frac{\sum_{i=1}^{n} (p_{i1} q_{i0})}{\sum_{i=1}^{n} (p_{i0} q_{i0})} \right]$$

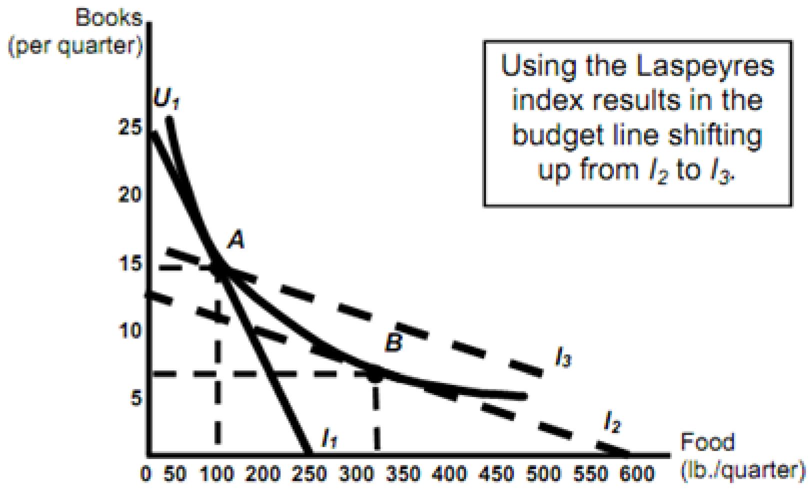
$$P_{12}^{L} = \left[\frac{\sum_{i=1}^{n} (p_{i2} q_{i1})}{\sum_{i=1}^{n} (p_{i1} q_{i1})} \right]$$

$$But: \quad \left(\mathrm{P}_{02}^{\mathrm{L}} \right) = \left[\frac{\sum_{i=1}^{n} (p_{i2} q_{i0})}{\sum_{i=1}^{n} (p_{i0} q_{i0})} \right] \neq \left[P_{01}^{L} \times P_{12}^{L} \right]$$

L_________ index _____ the circularity test.

Is this important? YES!!–the ____ is essentially a Laspreyres'-type of index. ☞ Fails tests and tends to _____**-state** price rises.

▲All indices mentioned so far _____ this test, except for the weighted aggregative index with fixed weights.



We have only considered Aggregative Indices up to this point. Now consider:

<u>(II) Averages of Relatives Indices</u>

A **<u>price relative</u>** is: $\left[P_{it} / P_{i0} \right]$.

▲The assumption is that ___ commodity is used <u>equally</u> as much.

But, an index should take into account the differing _________ of the items used.

⇨Weighted price indices permit the consideration of the <u>relative</u> importance of the commodities in the basket of goods. I.e. use quantities as weights.

A weighted average of *price relatives* index weights the various price relatives in a market basket by the total amount ______ on that commodity.

Weighted Arithmetic Mean of Price Relatives

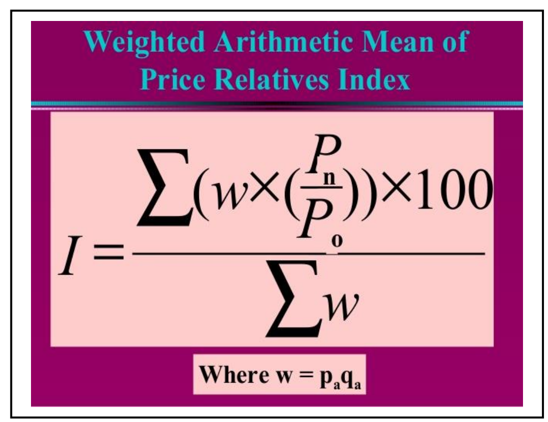
$$P_{0t}^{wm} = \left[\frac{\sum v_i \left({P_{it}}/{P_{i0}} \right)}{\sum v_i} \right] * 100$$

$$P_{0t}^{wm} = \left[\frac{\sum p_{it} q_{i0}}{\sum p_{i0} q_{i0}} \right] = P_{0t}^{L}.$$

Note: if $v_i = \left(p_{i0} q_{i0} \right)$, then $P_{0t}^{wm} = P_{0t}^{L}$

$(i.e. base\ year\ weights - - \exp enditure)$

v_i represents the weight.



Geometric Mean of Price Relatives

$$P_{0t}^{G} = \left[\prod \left(\frac{p_{it}}{p_{i0}} \right) \right]^{1/n} \Leftarrow \text{can also be weighted.}$$

Harmonic "Mean of Price Relatives"

$$P_{0t}^{H} = \left\{ \left[\frac{\sum v_i \left(P_{it} / P_{i0} \right)^{-1}}{\sum v_i} \right] \right\}^{-1}$$

Note: if $v_i = \left(p_{it} q_{it} \right)$, then $P_{0t}^{H} = P_{0t}^{P}$

$(i.e.\ current\ year\ weights -- \text{exp}\,enditure)$

Converting data in index number format: Measuring the level of real national output

When we are measuring the level of national income we often make use of **index numbers** to track what is happening to real GDP. In the table below we see the value of consumer spending and also real GDP expressed in £ billion. I have chosen 1995 as the base year for our index of spending and output. So the data for consumer spending and real GDP has an index value of 100.0 in 1995.

To calculate the index number for consumer spending in 1996 we use the following formula

Index (1996) = (consumer spending (1996) / base year consumer spending) x 100

	Consumer spending	Index of consumer spending	Real GDP	Index of real GDP
	£ billion	1995 = 100	£ billion	1995 = 100
1995 (Base)	512.6	100.0	857.5	100.0
1996	531.9	103.8	880.9	102.7
1997	551.1	107.5	908.7	106.0
1998	572.3	111.6	938.1	109.4
1999	598.8	116.8	966.6	112.7
2000	625.1	121.9	1005.5	117.3
2001	644.9	125.8	1027.9	119.9
2002	667.4	130.2	1048.5	122.3
2003	684.8	133.6	1074.9	125.3
2004	710.2	138.5	1108.9	129.3

Using Price indices: "Price Deflators"

It is common practice to distinguish between **nominal** and _____ values for a series.
I.e. current dollar terms versus __________ dollar terms.

Recall: the real value is obtained by:

$$\text{Real Value} = \frac{\text{Nominal Value}}{\text{Index Number}} \times 100$$

Example: invest money for a year at 7% p.a. interest. Now suppose prices also rise by 2%.

Real Rate of Interest: $\left(\frac{1.07-1.02}{1.02}\right)=\left(\frac{1.07}{1.02}-1\right)\approx 5\%\ \ p.a.$

You can purchase approximately 5% more goods with each dollar than was possible a year ago.

Convert a nominal variable into a "real" variable by **deflating** – _______ "nominal" value by price index to get "______" value.

Price Index used as a price deflator.

Must choose the appropriate price index for the purpose of price deflator.

Real income is commonly referred to as the **purchasing ______ of the money income.**

In comparing incomes, wages, rents, GNP, and personal income per capita of different countries, the use of an appropriate deflator is common practice. Hence, the **real value** is more easily recognized.

$$\text{Real Exports} = \frac{\text{Nominal Exports}}{\text{Export Price Index}} \times 100$$

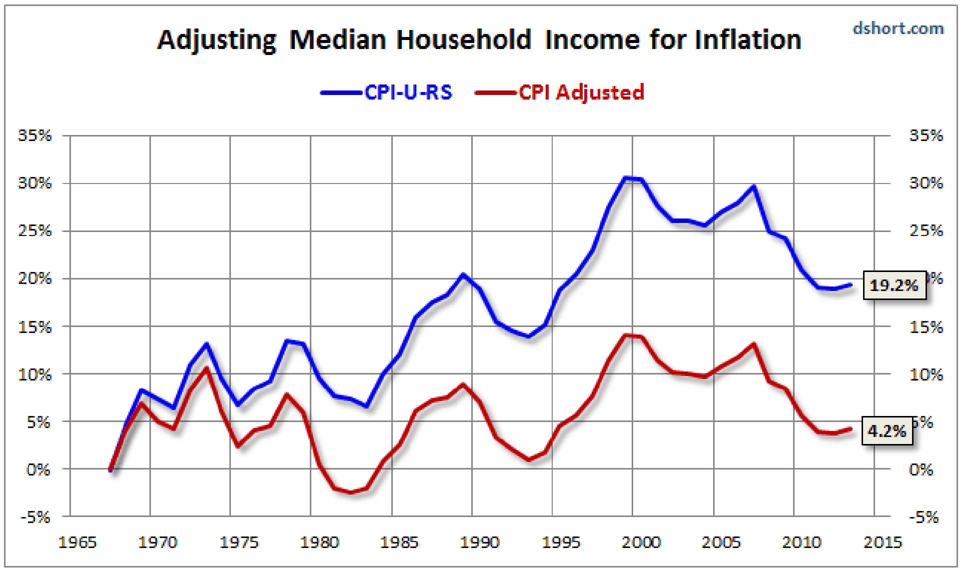
$$\text{Real Consumption} = \frac{\text{Nominal Consumption}}{\text{CPI}} \times 100$$

$$\text{Real Payroll} = \frac{\text{Nominal Value of Payroll}}{\text{Wage Rate Index}} \times 100$$

Obviously, increases/decreases in “nominal” series may be in the same/ opposite direction to those in corresponding “real” series.

Final Interest Rate Data

Year	CPI	Nominal Interest Rate	Inflation Rate	Real Interest Rate
1	100	--	--	--
2	110	15%	10%	5%
3	120	13%	9.1%	3.9%
4	115	8%	-4.2%	12.2%



5.3.3 Splicing techniques

Splicing in this context refers to the combining or joining of more than one method to form a complete time series. Several splicing techniques are available if it is not possible to use the same method or data source in all years. This section describes techniques that can be used to combine methods to minimise the potential inconsistencies in the time series. Each technique can be appropriate in certain situations, as determined by considerations such as data availability and the nature of the methodological modification. Selecting a technique requires an evaluation of the specific circumstances, and a determination of the best option for the particular case. It is *good practice* to perform the splicing using more than one technique before making a final decision and to document why a particular method was chosen. The principal approaches for inventory recalculations are summarised in Table 5.1.

5.3.3.1 OVERLAP

The overlap technique is often used when a new method is introduced but data are not available to apply the new method to the early years in the time series, for example when implementing a higher tier methodology. If the new method cannot be used for all years, it may be possible to develop a time series based on the relationship (or overlap) observed between the two methods during the years when both can be used. Essentially, the time series is constructed by assuming that there is a consistent relationship between the results of the previously used and new method. The emission or removal estimates for those years when the new method cannot be used directly

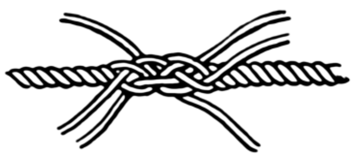
TABLE 5.1
SUMMARY OF SPLICING TECHNIQUES

Approach	Applicability	Comments
Overlap	Data necessary to apply both the previously used and the new method must be available for at least one year, preferably more.	• Most reliable when the overlap between two or more sets of annual estimates can be assessed. • If the trends observed using the previously used and new methods are inconsistent, this approach is not *good practice*.
Surrogate Data	Emission factors, activity data or other estimation parameters used in the new method are strongly correlated with other well-known and more readily available indicative data.	• Multiple indicative data sets (singly or in combination) should be tested in order to determine the most strongly correlated. • Should not be done for long periods.
Interpolation	Data needed for recalculation using the new method are available for intermittent years during the time series.	• Estimates can be linearly interpolated for the periods when the new method cannot be applied. • The method is not applicable in the case of large annual fluctuations.
Trend Extrapolation	Data for the new method are not collected annually and are not available at the beginning or the end of the time series.	• Most reliable if the trend over time is constant. • Should not be used if the trend is changing (in this case, the surrogate method may be more appropriate). • Should not be done for long periods.
Other Techniques	The standard alternatives are not valid when technical conditions are changing throughout the time series (e.g., due to the introduction of mitigation technology).	• Document customised approaches thoroughly. • Compare results with standard techniques.

Linking ("Splicing") Indices

➪ Suppose we have a Laspeyres' price index, P_{0t}^{L} for the period t=1963,...,1973. The weights have become outdated over this time period.

➪ You decide to form new weights in 1973 and construct a separate P_{0t}^{L} series for t=1973,1974,1975,...,1990.



Although these are different indices, you may need a continuous price time series, so combine the two series into one linked series:

Year	Series 1	Series 2		Linked
1963	**100**			
1964	**120**			
⋮	⋮			
1973	**160**	**100**	**(100) × 1.6 =**	
1974		**110**		
⋮		⋮		
1990		**185**	**(185) × 1.6**	

To make the 100 in year 1973 be equal to 160, simply multiply (100)(1.6)=160.

- **Relative prices are preserved over time.**

We could link with any other base year:

Year	Series 1		Series 2	Linked
1963	100	÷ 1.6		
1964	120	÷ 1.6		
	⋮	⋮		
1973	160	(160) ÷ 1.6 =	100	
1974			110	
			⋮	
1990			185	

In each case, the linked series suggests prices are **1.96 times** higher in 1990 than in 1963.

$$\frac{185-62.5}{62.5}=1.96$$

$$\frac{296-100}{100}=1.96$$

Constructing the CPI

- The CPI is based on "price relatives" for each item in each location. Then they are aggregated to get regional indices and indices for groups. Eventually, you get to an "All groups" index for Canada.

- The construction of the CPI involves the "***weighted average of price ___________***" approach.

The weights are (base period) _____________, so this is the L_________' Index.

- Base year updated regularly -- every 2 years.
- Weights revised to reflect changing expenditure patterns.

(______ is the latest.)

An Analysis of the 2019 Consumer Price Index Basket Update, Based on 2017 Expenditures

By: **Taylor Mitchell**

Release date: February 27, 2019

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On this page

Introduction

The Consumer Price Index (CPI) is one of the most widely-known, quoted and utilized economic indicators in Canada and is of interest to a wide range of users. It can be used to compare, through time, the cost of a fixed basket of goods and services purchased by consumers. The CPI is used for economic analysis and provides insight into overall economic conditions.

> Private and public pension programs, income tax deductions, and some government social payments are adjusted using the CPI. The index is used as a deflator of various economic aggregates to obtain estimates at constant prices. The CPI is also a tool for setting and monitoring economic policy. For example, the Bank of Canada uses the CPI and special aggregates of the CPI for this purpose.

As a Laspeyres-type [1] price index, the CPI basket quantities are those of the reference period of the basket weights, which are used to estimate quantities consumed for upper level aggregation. The larger the basket weight of a given aggregate in the CPI basket, the more a price change in that aggregate will impact the all-items CPI.

Basket weights are derived primarily from household expenditures reported in Statistics Canada's Survey of Household Spending (SHS) [2] and are updated every two years. The January 2019 CPI marks the introduction of updated basket weights in the calculation of the index. As of its release on February 27 2019, the basket weights used in the aggregation of the CPI were updated based on consumer spending patterns from the 2017 SHS, replacing those from the 2015 SHS. In addition to updates to the classification structure of the basket, these changes enhance the quality of the CPI.

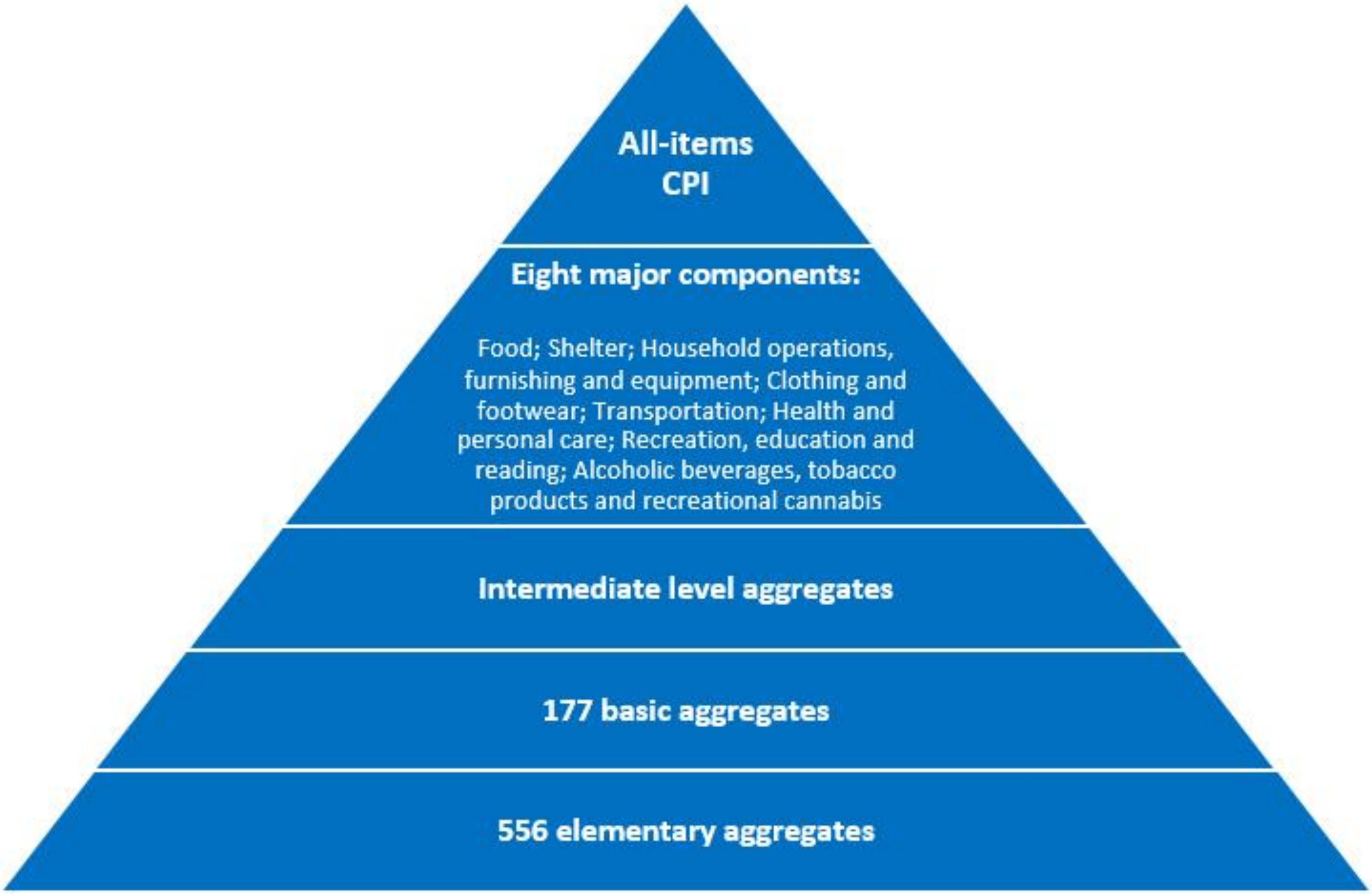
Composition of the CPI Basket

The Consumer Price Index is a weighted average of the price changes of a fixed basket of goods and services, based on the expenditures of a target population [3] in a certain reference period. Each good or service in the basket is representative of consumer spending patterns, and price movements are assigned a basket share that is proportional to the consumption expenditure for which they account. For example, 26.92% of total consumer expenditures in scope of the CPI accounted for shelter-related goods and services. This corresponds with the 26.92% weight assigned to shelter in the 2017 CPI basket. The weights of goods and services are fixed during the life of a given basket and play an important role in determining the impact of a given item's price change on the CPI. For instance, Canadians on average spend a much larger share of their total expenditures on gasoline than on milk. As a result, a 10% price increase in gasoline prices will have a greater impact on the all-items CPI than an equivalent increase in the price of milk [4].

The Consumer Price Index classification of goods and services is organized according to a top-down hierarchical structure (see diagram below). At the top of the structure is the all-items CPI, which contains eight major components. Below the eight major components are intermediate level aggregates, such as owned accommodation and operation of passenger vehicles, which, along with the major components, provide insight into the sources of monthly and annual price change. There are 177 basic aggregates. [5] These basic aggregates are typically the result of aggregating one or more elementary aggregates, many of which are unpublished.

Elementary aggregates are added or deleted from the basket as consumption patterns change over time. The aggregate for DVD rentals, [6] for instance, was deleted from the basket as they became less popular with consumers and subsequently commanded a lower share of overall expenditures. At the elementary aggregate level, the classification includes a sample of items that are chosen to characterize all products in that class. Representative products are chosen with emphasis on items that are widely available and known to be among the most popular with consumers, ensuring that the items selected are representative of the purchases consumers actually make. The number of representative products assigned to an elementary aggregate can vary based on the basket weight of the aggregate, as well as the price variability and heterogeneity of products in that class. For instance, when pricing certain dry grocery products, representative products typically include both brand-name and store-brand items. At the same time, there is only one representative product priced under the bananas aggregate.

Classification Structure of the Consumer Price Index



The Consumer Price Index classification is organized according to a top-down hierarchical structure, depicted in a pyramid chart with five levels.

At the first level, or the top of the pyramid, is the "All-items Consumer Price Index".

Below at the second level of the pyramid are the eight major components which are:

- Food;
- Shelter;
- Household operations, furnishings and equipment;
- Clothing and footwear;
- Transportation;
- Health and personal care;
- Recreation, education and reading;
- Alcohol beverages, tobacco products and recreational cannabis.

At the third level of the pyramid there are "Intermediate level aggregations".

At the fourth level of the pyramid there are "177 basic aggregates".

At the fifth and lowest level of the pyramid there are "556 elementary aggregates".

Importance of Updating the Consumer Price Index Basket Weights

If the fixed-quantity basket of goods and services was kept unchanged for an extensive period of time, it would gradually lose accuracy and relevance as a reflection of consumer spending. This is partly due to the nature of consumption patterns, which have a tendency to evolve in response to shifts in relative prices. For example, if the price of chicken increased between basket updates, consumers may opt away from chicken and substitute other meats. A Laspeyres-type price index cannot reflect this expenditure change until the basket weights are updated. This can lead to an overstatement of the importance of changes in the price of chicken in the index and a subsequent upward bias in the CPI. Typically, the longer a fixed set of basket weights is used, the greater this upward bias.

Consumer spending patterns are also influenced by factors such as variations in the level and distribution of household income, demographics (such as an aging population), evolving habits and technological advances. New products and services are introduced to the market and existing ones may be modified or become obsolete. As a result, the basket needs to be revised periodically to reflect changes in consumers' spending patterns. For example, the significant increase of the basket share for Internet access services from 0.53% in 2005 to 1.06% in 2017 reflects the growing importance of the Internet in the daily lives of Canadians. [7] In the same time period, the basket share of cigarettes fell from 1.27% to 0.82% as consumer spending patterns shifted away from cigarettes. [8]

In addition to the review of the expenditure weights, a basket update is also an opportunity to review and update other aspects of the indices. This includes changing the CPI classification to make it more representative of consumer spending and the products and services available for purchase. It is also an opportunity to review and update the sample of prices collected, review price index estimation methodologies, and update documentation and dissemination products, although these activities are not limited to basket updates.

Table 1
Selected product classes added or deleted from the 2017 CPI basket

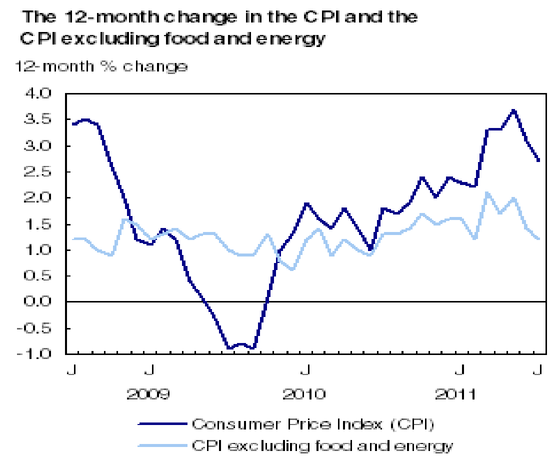
Product class	Parent index	Type of aggregate	Added	Deleted
Broadband and other Internet access services	Internet access services	Elementary	..	√
Area rugs and mats	Household textiles	Basic	..	√
Commuter trains	City bus and subway transportation	Elementary	√	..
Ride sharing	Taxi and other local and commuter transportation	Elementary	√	..
Medicinal cannabis	Medicinal and pharmaceutical products	Basic	√	..
Blank CDs and DVDs	Purchase of digital media	Elementary	..	√
Portable drives	Purchase of digital media	Elementary	..	√
Other traveller accommodation	Traveller accommodation	Elementary	√	..
Audio streaming services	Audio and video subscription services	Elementary	√	..
Cannabis	Recreational cannabis	Elementary	√	..

The index reference period or index base period is the period in which the index is set to equal 100. For the CPI, the index base period is usually a calendar year expressed as "index year=100". The current index base period for the CPI remains 2002=100 in the 2017 basket.

Note:

Laspeyres: weighted aggregative index, with base period ____________ as weights.

CPI: weighted average of price relatives, with base period _______________ as weights.



ALL-ITEMS, CPI

1. Food
2. Shelter
3. Household operations and furnishings
4. Clothing and footwear
5. Transportation
6. Health and personal care
7. Recreation, education and reading
8. Alcoholic beverages and tobacco products

The Consumer Price Index

The CPI measures changes in consumer prices facing the Canadian consumer.

It is derived by comparing the cost of a fixed basket of commodities purchased by consumers over time.

The “basket” is assumed to contain commodities of ***unchanging*** quantity and ________ and hence is believed to reflect only “pure price movements.”

Between 1913 and1926 both the Department of Labour and the Dominion Bureau of Statistics (Statistics Canada) each produced separate CPI's. The present CPI series descended from these sources.

Some component indexes in the CPI begin with the date they were introduced into the CPI.

The CPI is widely used as an indicator of the **changes in the rate of__________.** Hence, consumers can monitor changes in their personal income.

The CPI is generally employed in four ways:

1) Evaluate and escalate the ___________power over time of:
   -wages -rents
   -leases -child or spousal support allowance
   -private and public pension programs (Old Age Security and Can. Pension)
   -personal income tax deductions
   -government social payments

2) Used to _______ current dollar estimates to obtain constant dollar estimates.

3) Setting and evaluating economic policies
   - Bank of Canada's monetary policies
   - Assessment of public policy– food prices, etc.

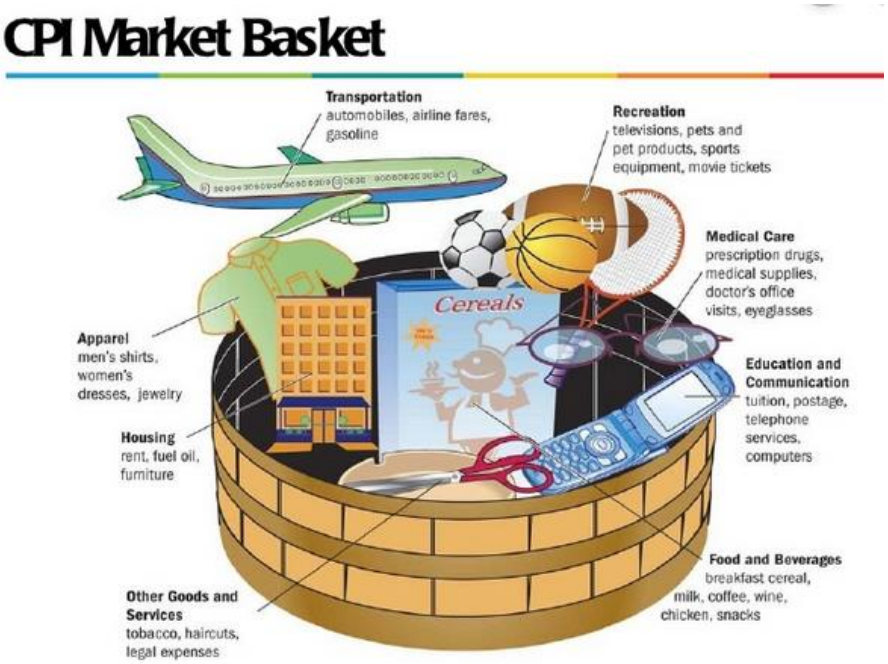
4) Economic analysis and research by economists
   - explore effects of ___________
   - causes of inflation
   - regional disparities in ______ movements

CPI is based upon prices (including ______) paid by consumers in private retail outlets, government stores, offices and other consumer service establishments.

CPI considers a constant "basket" of goods over time – prices in cities greater than or equal to 30,000.

"Price movements of the goods and services represented in the CPI are weighted according to the relative importance of commodities in the total expenditure of consumers."

CPI basket weights are collected from multiple types of surveys relating to a specific year. The weights are currently based on _____consumer expenditure data. The current time base of the index is _____ = 100.



Sampling

This is a sample survey.

The CPI price sample is obtained from a selection of geographical areas, representative goods and services, and types and locations of retail outlets, to estimate price changes experienced by Canadians. The timing of price collection during the month is predefined. The extent and quality of the sample is constrained by budgets and the information available on which to base the sampling process.

Regular reviews and updates ensure that price changes calculated with samples of products are representative of the price change for the entire class of products. New products are introduced into the pricing samples based on information about their market shares, market and product trends and the expert judgment of product officers. Particular efforts are made to introduce new products into samples for product categories with relatively high rates of change. Outlet sample reviews and updates are performed periodically for many CPI basic classes.

There are approximately 700 goods and services identified to represent the price movement in 177 basic goods and services classes. Sample goods and services are chosen on the basis of representativeness and expected continuous availability.

The geographical distribution of the sample varies by product. The most geographically dispersed price samples are for goods and services where prices are likely to be heavily influenced by local market conditions (e.g., locally determined prices such as rents, water charges, local transit fares, and property taxes). In contrast, prices, such as car registration fees or postage fees, are collected from provincial or national agencies.

The selection of outlets is based on market intelligence and is designed primarily to include retail outlets with high sales revenues. Almost all prices are collected from retail outlets or from local, regional or provincial agencies.

Note: The CPI does not measure changes in the "cost of living" – the latter is the change in income needed to keep a consumer as "well off" – depends on individual ___________.

The CPI is not a ***cost-of-living index***, though people frequently call it this. In theory, the objective behind a cost-of-living index is to measure price changes experienced by consumers in maintaining a constant standard of living. The idea is that consumers would normally switch between products as the price relationship of goods changes. If, for example, consumers get the same satisfaction from drinking tea as they do from drinking coffee, then it is possible to substitute tea for coffee if the price of tea falls relative to the price of coffee. The cheaper of the interchangeable products may be chosen.

We could compute a cost-of-living index for an individual if we had complete information about that person's taste and consuming habits. To do this for a large number of people, let alone the total population of Canada, is impossible. For this reason, regularly published price indexes are based on the ***fixed basket*** concept rather than the cost-of-living concept.

Highlights of the Canadian CPI

(i) Relates to prices in ______ centres.

(ii) Based on prices of over 300 items.

(iii) 8- "Groups" of goods:

- ________
- Shelter
- Household Operations and Furnishings
- Clothing and Footwear
- Transportation
- Health and Personal Care
- Recreation, Education and Reading
- Tobacco and Alcoholic beverages

● There are separate indices plus an "All Groups" category.

(iv) Index computed monthly –regional indices also available.

(v) Weight given to each individual price change is based on _____________as % of total household expenditure. The primary sources of expenditure data on consumer goods and services are from the Survey of Household Spending and the Food Expenditure Survey.

(vi) Prices of each group/service sampled at several outlets: 110,000 price quotations each month. The All-items index at the Canada level is based on an annual sample of over 500,000 prices quotes.

(vii) Some prices quoted twice/month; some once/quarter; some twice/year; some annual. EG. Food: 2 / month; Haircuts: 1 / 3 months; Car insurance: bi-annual

(viii) The extent and _________of the sample is constrained by budgets and the information available on which to base the sampling prices.

Cost of Living

It is reasonable to measure the change in the **cost of living** of an individual between two periods as ***the change in their money income which will be necessary for the individual to maintain his or her ________standard of living*** – no more or no less.

It must be pointed out that if all prices change in the same proportion, that proportion will measure the change in the cost of living and there will be no problem of measurement.

☆However, all prices do not change in the same ____________.

Suppose in period 0 an individual spends his income by purchasing q_0 of various commodities at price p_0. Assuming he saves nothing, his total income equals his total expenditure: $\sum p_0 q_0$.

➢ <u>In period 1, prices change to p_1</u>. How much income does he need in period 1 to make him as well off as he was in period 0?

If the quantities of goods which he would need to buy to leave him exactly as well off as he was in period 0 are $\bar{q}_1$, then with an income of $\sum p_1 \bar{q}_1$ he would be <u>exactly</u> as well off.

It follows that the change in the cost of maintaining his original (period 0) standard of living will be given by

$$C_{01}^{0} = \frac{\sum p_1 \bar{q}_1}{\sum p_0 q_0}.$$

The superscript (0) indicates that the change in his cost of living is being measured in terms of his period 0 standard of living.

Unfortunately we do not know the $\bar{q}_1$'s because they are not the __________he actually buys in period 1, but only what he would need to buy to be as well off as before.

➢ **<u>Consequently, we do not know the aggregate</u>** $\sum p_1\bar{q}_1$.

However, we do know the aggregate $\sum p_1 q_0$. This is the amount of income he would require in period 1 to enable him to purchase the <u>quantities</u> he purchased in period 0.

It can be shown that $\sum p_1 q_0$ will be __________than $\sum p_1\bar{q}_1$, provided his tastes have ___________ unchanged.

If he did have the income $\sum p_1 q_0$ in period 1, he would not buy the same _________as he bought in period 0, (the q_0 quantities) because he would take advantage of the changes in relative prices and would alter his allocation of ________, buying relatively more of those goods whose prices had fallen relatively more or risen relatively less.

The fact that he would do this in preference to buying the q_0 quantities indicates that an income of $\sum p_1 q_0$ would make him **better off** than the original income of $\sum p_0 q_0$ and, hence better off than an income of $\sum p_1 \bar{q}_1$ which is its equivalent.

- We have accordingly, $\sum p_1 q_0 > \sum p_1 \bar{q}_1$.

This assumption that his ______ must have remained unchanged is vital because if they have changed, we cannot conclude that the quantities he would buy in period 1 with an income of $\sum p_1 q_0$ are preferred to those he actually did buy in period 0.

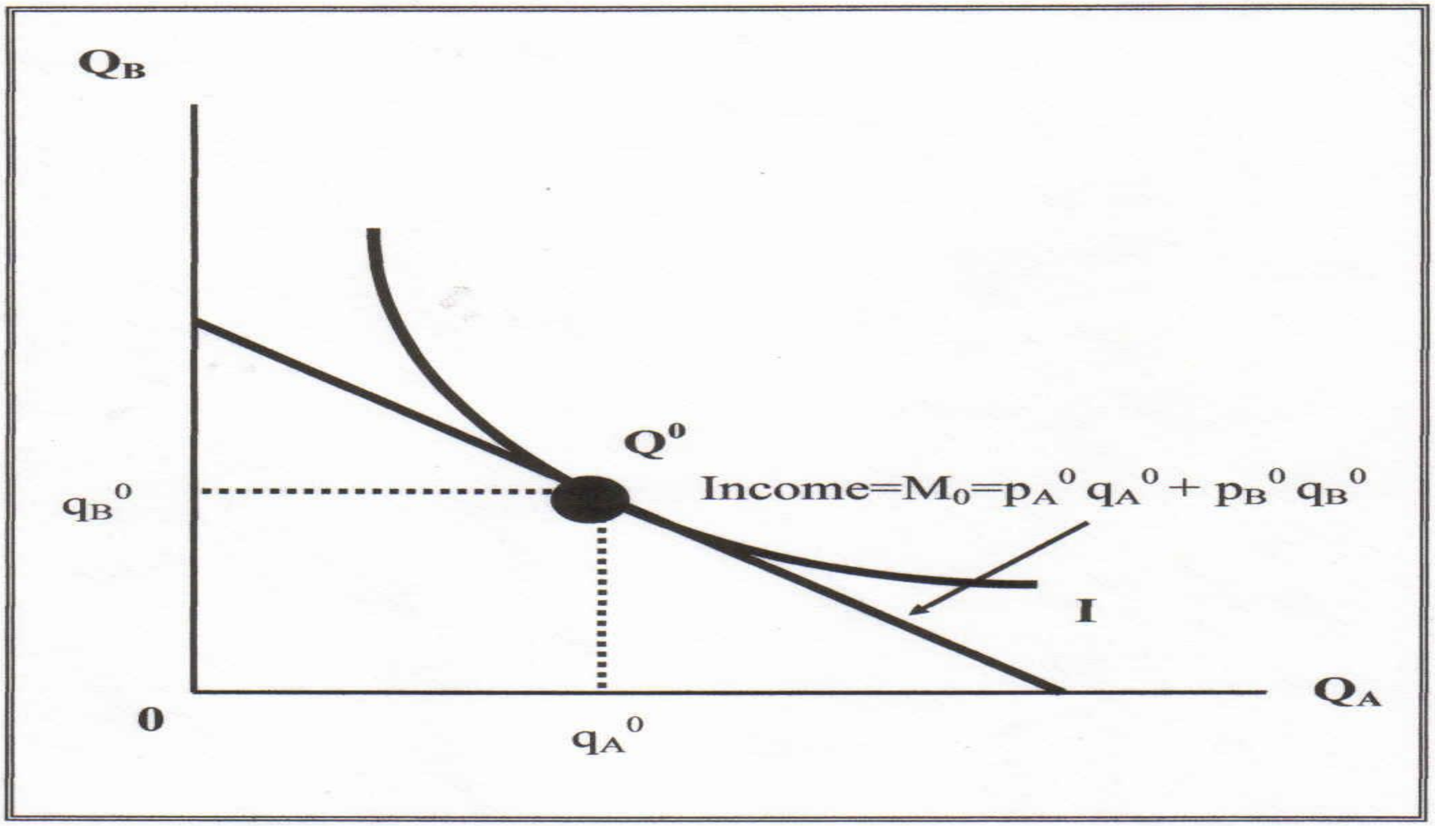
This can be illustrated, by means of **indifference curves**, in the case of an individual spending his income on two goods.

We call the two goods A and B. Let the price of A in period 0 be p_A^0 and of B be p_B^0. If the individual's money income in period 0 is M_0, we shall have:

$$M_0 = p_A^0 q_A^0 + p_B^0 q_B^0$$

where q_A^0 and q_B^0 are the quantities of A and B respectively which he can purchase under these circumstances.

2 Good Example: Time Period 0



Any combination of good A and good B can be purchased along or below the budget line.
Assume fixed preferences and one individual.

$$\text{Slope} = -\frac{P_A^0}{P_B^0}$$

On the diagram, q_A^0 is measured on the X-axis and q_B^0 on the Y-axis. The individual can take up any position on the line – his budget line, M_0. It has a slope given by the price of ***A*** relative to that of ***B***.

➔ Slope: $\left(-\frac{p_A^0}{p_B^0}\right)$

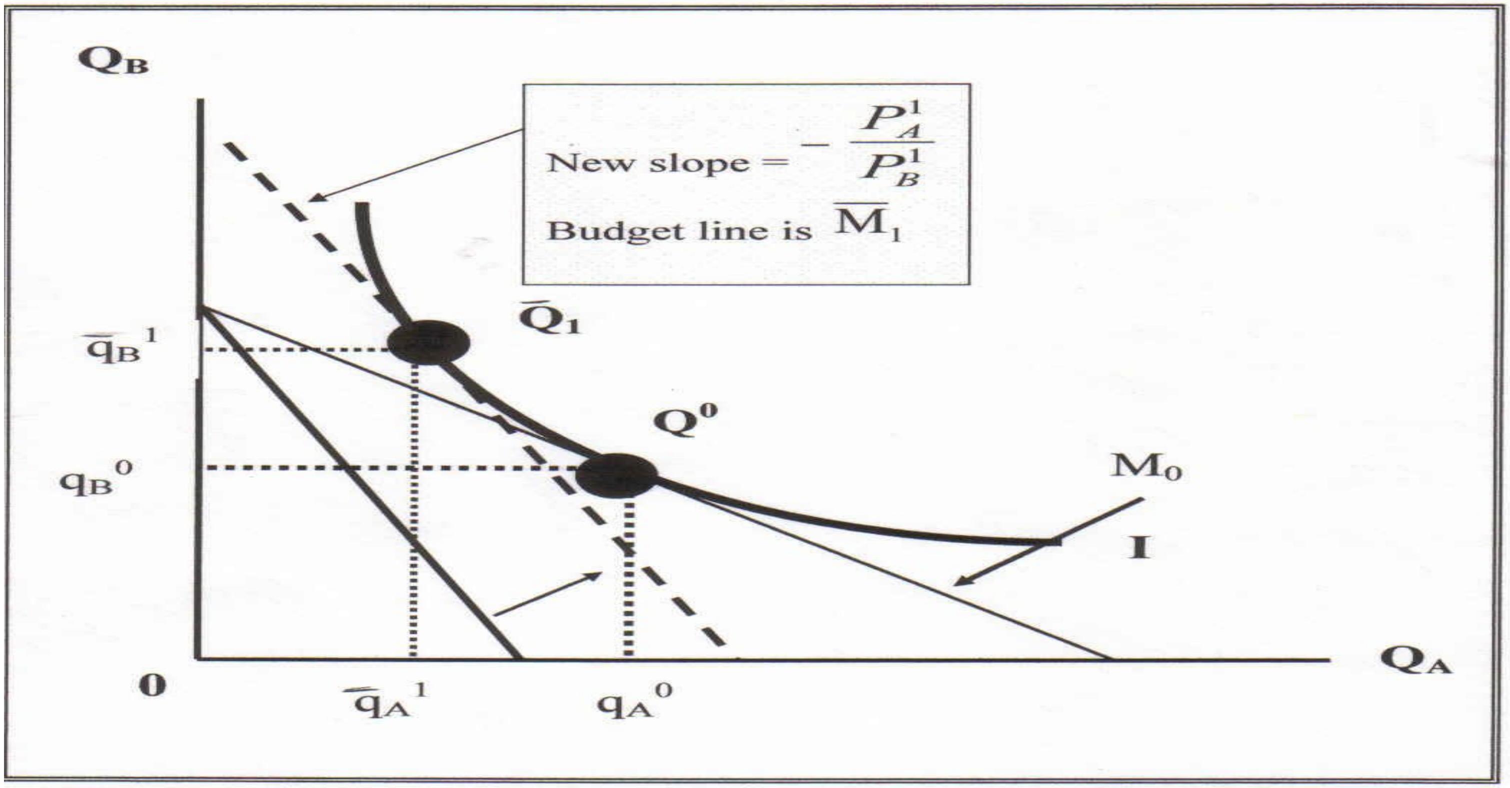
The individual will move to the point Q_0, where quantities of A and B, q_A^0 and q_B^0, meet on the budget line, at which point he reaches his highest indifference curve. The individual will purchase q_A^0 and q_B^0 in period 0.

Suppose, in period 1 prices begin to change. (Price of A increases.)

How much income will the individual now need to be as well off as he was before?

The individual will need ____________to enable him to take up a position **just** on the indifference curve. We now draw the line $\overline{M}_1$ with a slope given by $\left(-\frac{p_A^1}{p_B^1}\right)$ such that it just touches the indifference curve.

2 Good Example: Time Period 1
Price of Good A increases.



New Budget line: $\overline{M}_1 = P_A^1 q_A^1 + P_B^1 q_B^1$

The individual will need income $\overline{M}_1$ to consume at the same level of satisfaction as period 0.

$C_{01}^0 = \overline{M}_1 / M_0 =$ change in the cost of living.

With an

With an _______ corresponding to this line, he would take up the position $\overline{Q}_1$, purchasing $\overline{q}_A^1$ of A and $\overline{q}_B^1$ of B.

This would require income: $\overline{M}_1 = p_A^1 \overline{q}_A^1 + p_B^1 \overline{q}_B^1$, as against the old income of $M_0 = p_A^0 q_A^0 + p_B^0 q_B^0$.

■ But the individual is indifferent between the positions Q_0 and $\overline{Q}_1$, so that the ratio $\overline{M}_1 / M_0$ measures the change in his ____income necessary to make him as well off in period 1 as he was in period 0.

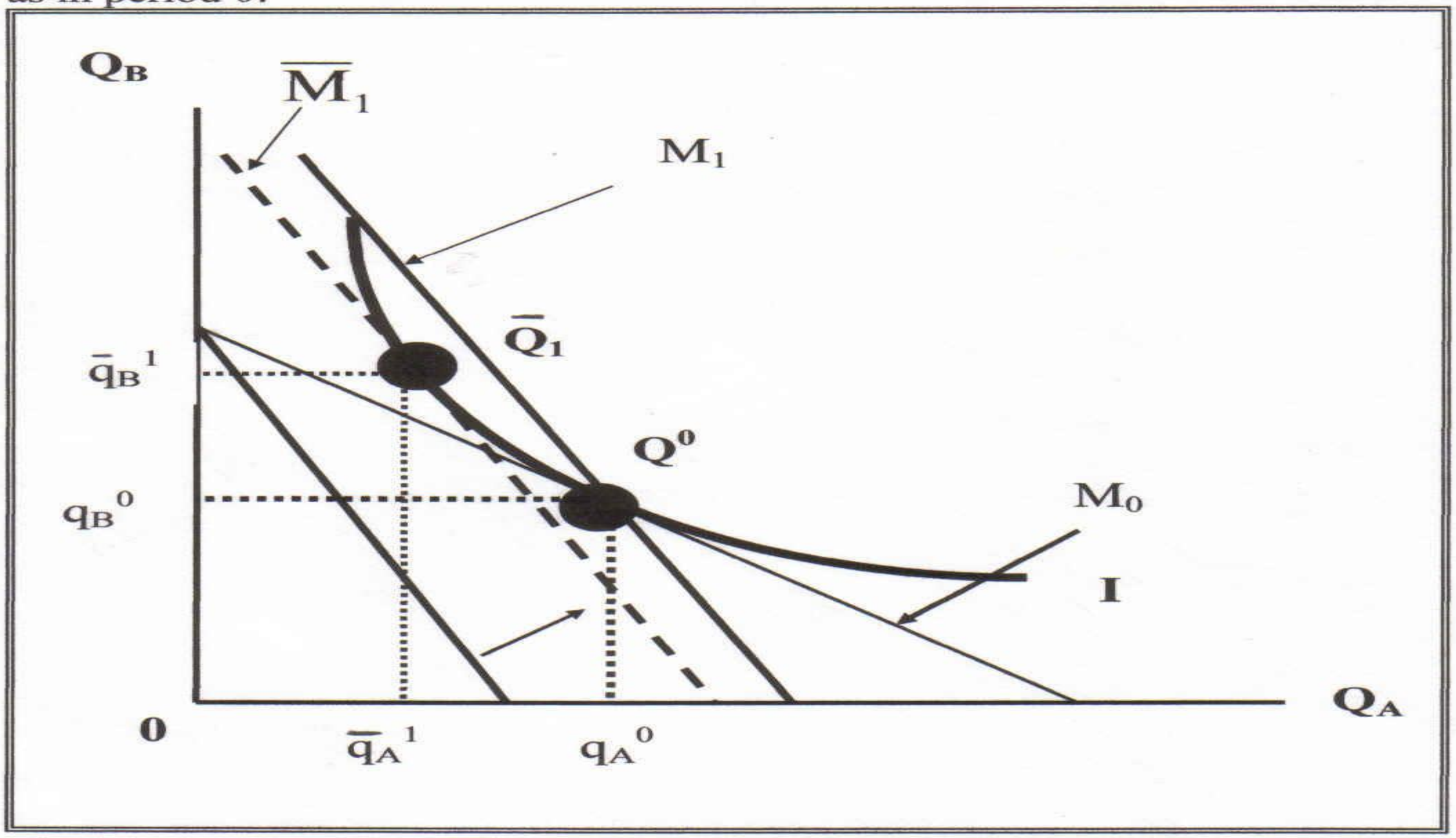
Suppose that the individual is given ________ in period 1 sufficient to enable him to purchase the same quantities as he did in period 0.

This ________must be sufficient to make the budget line for the new prices pass through Q_0 and must be equal to $M_1 = p_A^1 q_A^0 + p_B^1 q_B^0$.

This is shown as the broken line in the diagram, and it must lie parallel to but to the right of $\overline{M}_1$, so that:

$$M_1 = p_A^1 q_A^0 + p_B^1 q_B^0 > \overline{M}_1 = p_A^1 \overline{q}_A^1 + p_B^1 \overline{q}_B^1 .$$

If given enough income to purchase the same quantities in period 1 as in period 0:



$$M_1 > \overline{M}_1$$

M_1 → Laspreyres' index uses base period quantities as the weights. (Hence the tendency for this index to overestimate a rise in price level.)

Price Indices and the "Cost of Living"

- ***Can a price index, such as the ___, measure a change in the "cost of living"?***

⇨Individuals are affected differently by ____ changes.

Definition: A change in the **cost of living** is the change in ($) income needed to _________ ***original*** 'standard of living.'

➡This depends on individual **p___________.**

⇨Let $\bar{q}_{i1}$ be the quantity of good i that I need in period "1" to maintain the standard of living I enjoyed in period "0."

Cost of Living Index is:

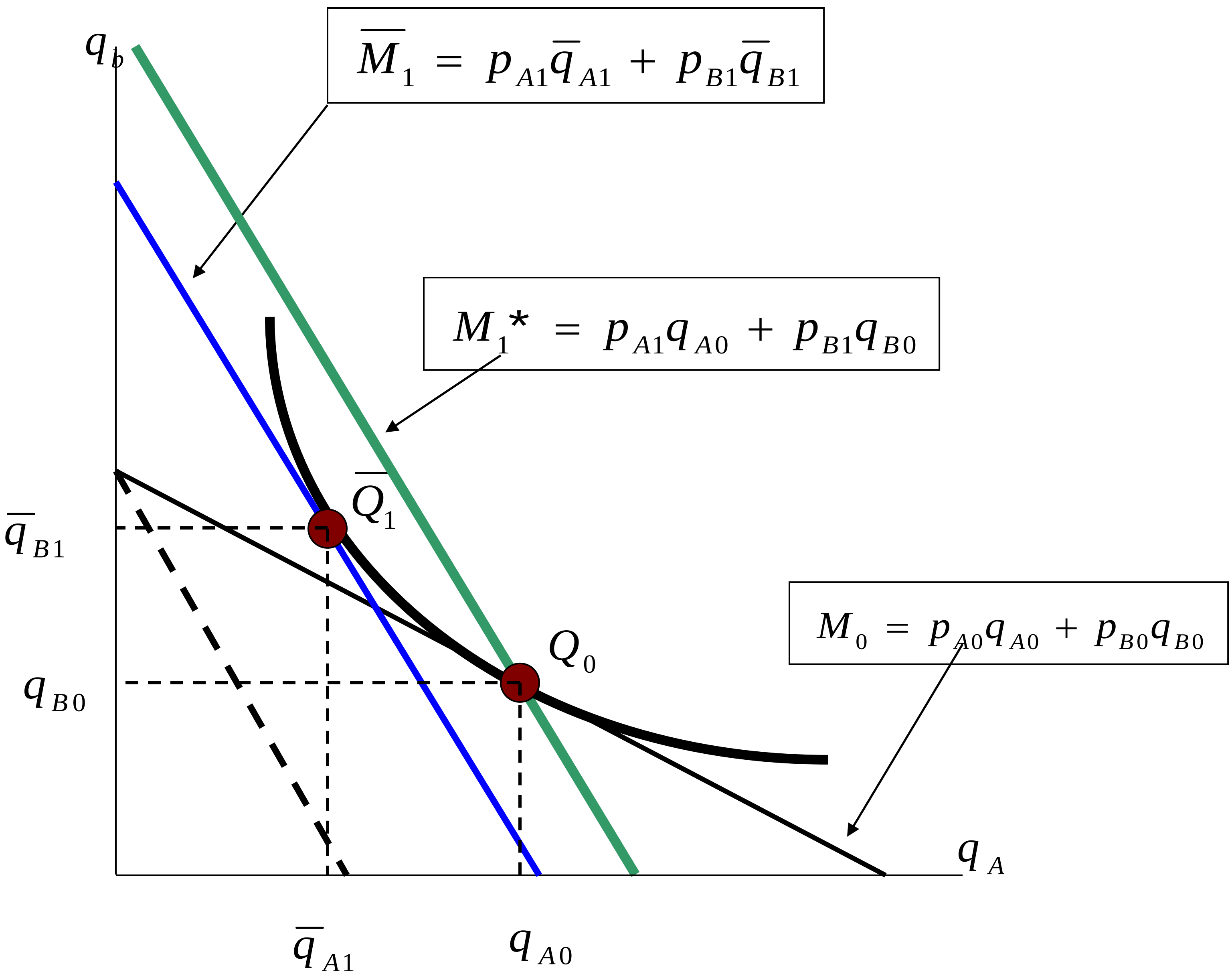
$$C_{01}^{0} = \left[\frac{\sum p_{i1}\bar{q}_{i1}}{\sum p_{i0}q_{i0}} \right]$$

Compare C_{01}^{0} with price indices we have used.

Assume:

(1) One ___________

(2) _______preferences


q_b
$\overline{M}_1 = p_{A1}\overline{q}_{A1} + p_{B1}\overline{q}_{B1}$
$M_1{*} = p_{A1}q_{A0} + p_{B1}q_{B0}$
$\overline{Q}_1$
$\overline{q}_{B1}$
Q_0
$M_0 = p_{A0}q_{A0} + p_{B0}q_{B0}$
q_{B0}
q_A
$\overline{q}_{A1}$
q_{A0}

The price of A ___________:

Relative prices change: "A" is now more expensive.

Budget line shifts ⇨slope increases

Consumer is indifferent between: $\left\{ \begin{array}{c} \left(q_{A0}, q_{B0}\right) \\ \left(\bar{q}_{A1}, \bar{q}_{B1}\right) \end{array} \right\}$

So, $C_{01}^{0} = \left[\dfrac{\sum p_{i1}\bar{q}_{i1}}{\sum p_{i0}q_{i0}} \right] = \dfrac{\overline{M}_{1}}{M_{0}}.$

To purchase the original bundle, consumer needs income of:

$(p_{A1}q_{A0} + p_{B1}q_{B0})$ (the green line). $M_1{}^*$

Note: $\sum p_{i1}q_{i0} > \sum p_{i1}\bar{q}_{i1}$

$(M_1{}^*)$ $(\bar{M}_1)$

If we divide _____ sides by :

$\sum p_{i0}q_{i0} \rightarrow$ base period expendiure :

$$\left[\frac{\sum p_{i1} q_{i0}}{\sum p_{i0} q_{i0}}\right] > \left[\frac{\sum p_{i1} \bar{q}_{i1}}{\sum p_{i0} q_{i0}}\right]$$

or

$$P_{01}^{L} \quad > \quad C_{01}^{0}$$

Laspeyres' Price Index ___________rises in true cost-of-living, and **understates** falls in the ____cost-of-living.

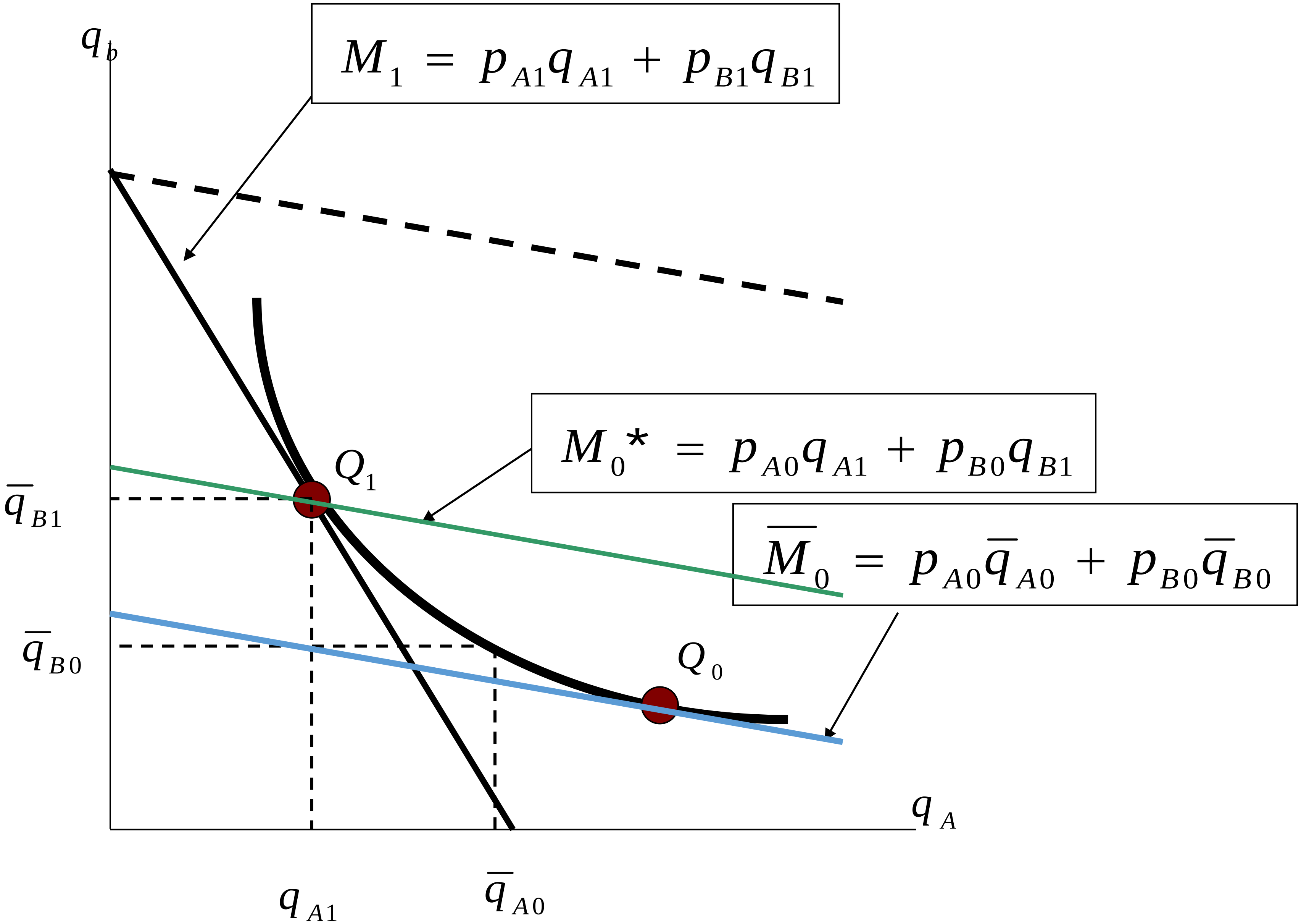
Can also define an alternative indicator of the change in cost of living:

"the individual's income in period "1" as a ratio of what would have been needed in period "0", at period "0" prices to be as well off":

$$C_{01}^{1} = \left[\frac{\sum p_{i1} q_{i1}}{\sum p_{i0} \bar{q}_{i0}} \right] = \frac{M_1}{\bar{M}_0}$$

where $\bar{q}_{i0}$ = quantity of good *i* in period "0" needed to maintain "________" experienced in period "1".

- *the superscript on C.O.L. index refers to the utility base period.*


$M_1 = p_{A1}q_{A1} + p_{B1}q_{B1}$
$M_0^* = p_{A0}q_{A1} + p_{B0}q_{B1}$
$\overline{M}_0 = p_{A0}\overline{q}_{A0} + p_{B0}\overline{q}_{B0}$
q_b
Q_1
Q_0
$\overline{q}_{B1}$
$\overline{q}_{B0}$
q_A
q_{A1}
$\overline{q}_{A0}$

Start at the black line: $M_1 = p_{A1}q_{A1} + p_{B1}q_{B1}$

Suppose _______ is lower is the previous time period:

$$M_1 = (p_{A1}q_{A1} + p_{B1}q_{B1})$$
$$\bar{M}_0 = (p_{A0}\bar{q}_{A0} + p_{B0}\bar{q}_{B0})$$

$$Now: (p_{A0}q_{A1} + p_{B0}q_{B1}) > (p_{A0}\bar{q}_{A0} + p_{B0}\bar{q}_{B0})$$

green line blue line

M_0^* $\bar{M}_0$

or

$$\sum p_{i0}q_{i1} > \sum p_{i0}\bar{q}_{i0}$$

If we divide by $\sum p_{i1}q_{i1}$:

$$\left[\frac{\sum p_{i0}q_{i1}}{\sum p_{i1}q_{i1}}\right] > \left[\frac{\sum p_{i0}\overline{q}_{i0}}{\sum p_{i1}q_{i1}}\right] \leftarrow \text{ invert both sides \&}$$

reverse the inequality sign

$$\text{Paasche price index} \rightarrow \left[\frac{\sum p_{i1}q_{i1}}{\sum p_{i0}q_{i1}}\right] < \left[\frac{\sum p_{i1}q_{i1}}{\sum p_{i0}\overline{q}_{i0}}\right] \leftarrow C_{01}^{1}$$

understates rises & overstates falls in true C.O.L. index.

$$\boxed{P_{01}^{P} < C_{01}^{1}}$$

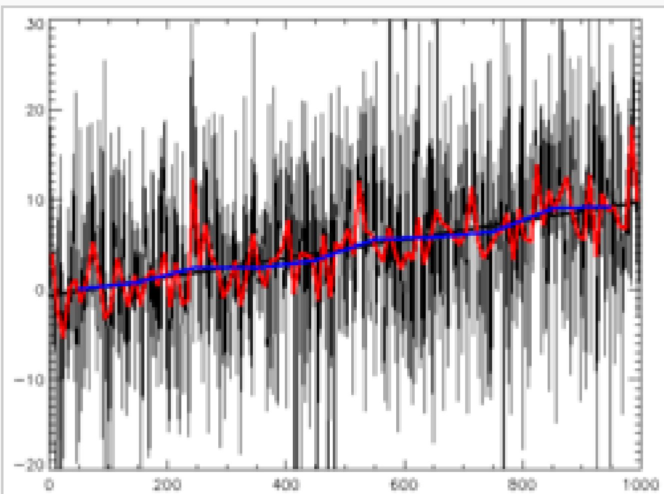
A Paasche _____index **understates** ______and overstates falls in the true cost of living index!

Note:

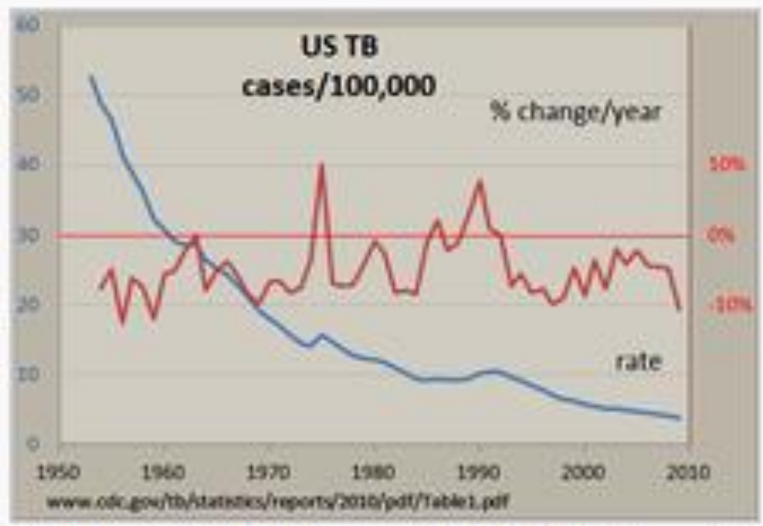
- 2 different C.O.L. indices.
- Each index involves "**hypothetical**" ________
- Cannot tell extent of under/over-statement.
- Each individual has _____ **preferences**.
- A price index takes **no** account of preferences – cannot measure changes in true C.O.L. (Or of welfare").

Topic 3 – Part II
Analysis of Economic Time Series Data

Recording observations of a variable that is a function of _____, results in a set of numbers called a **time series**.



Time series: random data plus trend, with best-fit line and different smoothings



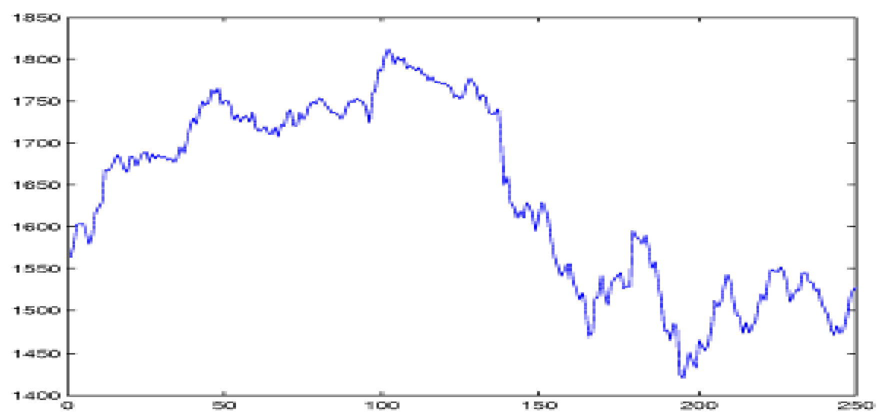
Tuberculosis incidence US 1953-2009

<u>Time Series Data</u>: a sequence of data values, gather over time – usually at some _____ interval.

Since data have a natural order, we can analyse a time series of data in terms of **<u>4</u>** basic components:

(i) Trend: Long term "direction" of the series, over many years.

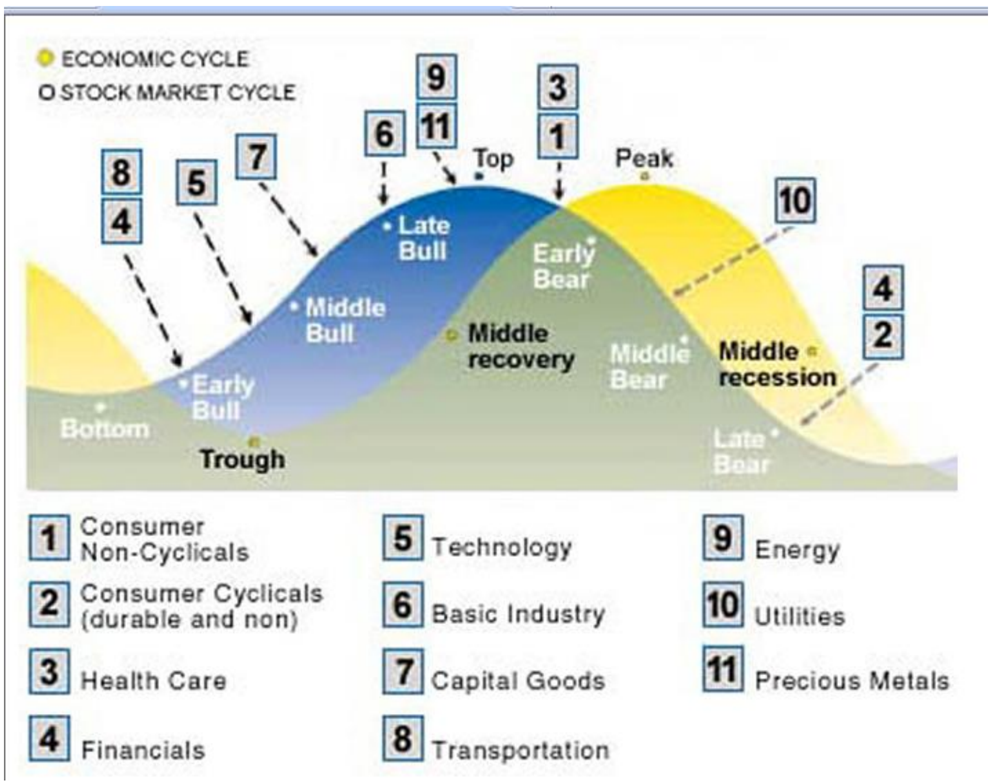
⇨Direction and _____ of the series is important to us.



<u>(ii) Cycle:</u> Represents a pattern repeated over ____periods of different length, usually longer than a year.

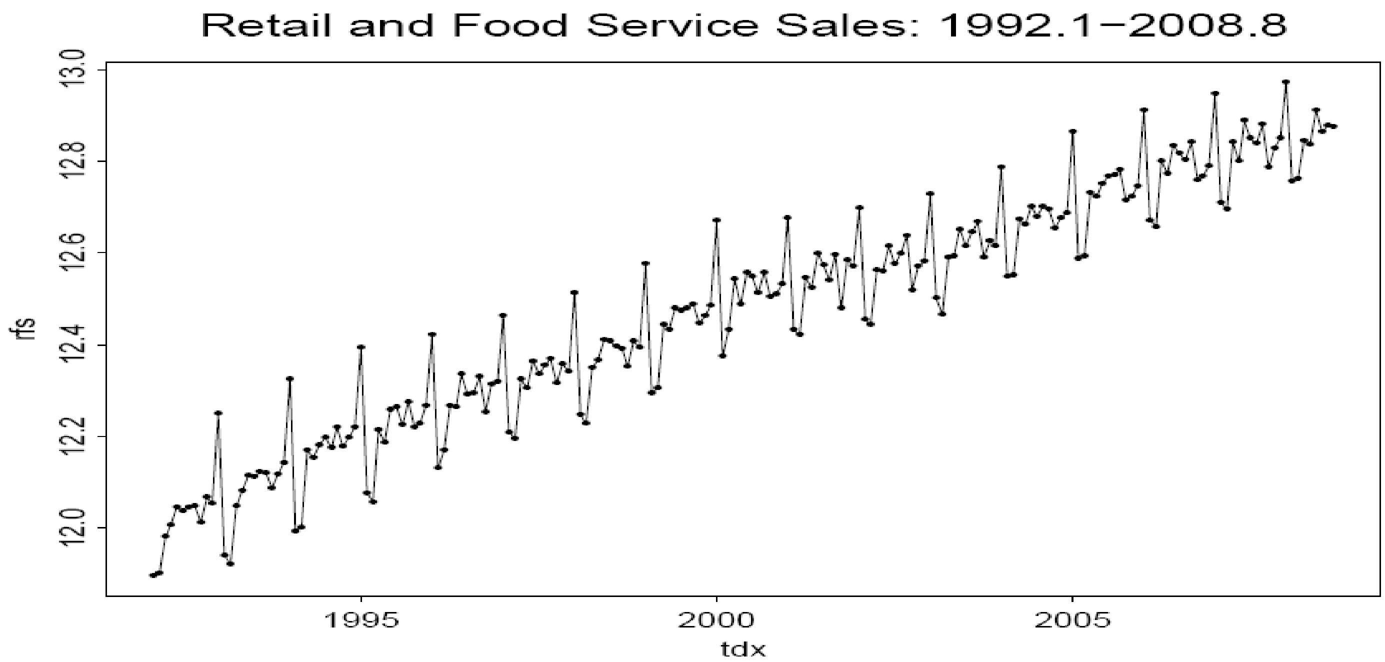
⇨"Wave-like" pattern – (3-5 duration)

⇨eg. Business ______.



(iii) Seasonal: represents fluctuations that repeat themselves within a ______period of one year.

⇨Same pattern each ______.



(iv) Irregular: fluctuation that is unpredictable, or takes place by ________or randomly.

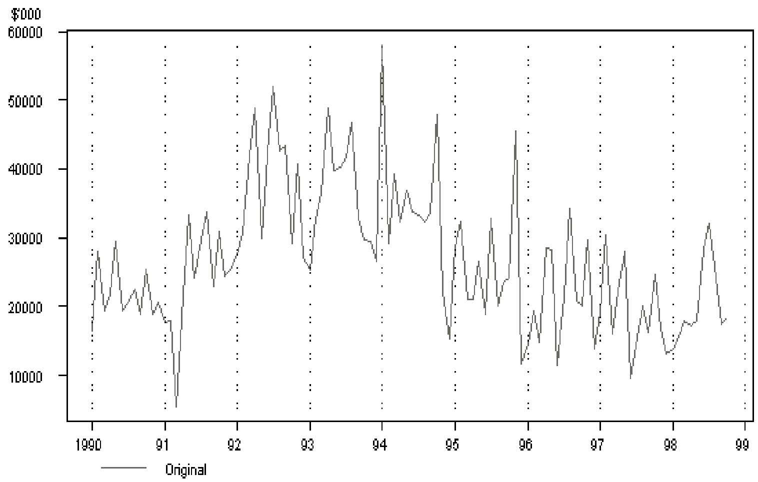
⇨Random "______" in the data.

⇨Non-systematic.

WHAT IS AN IRREGULAR?

The irregular component (sometimes also known as the residual) is what remains after the seasonal and trend components of a time series have been estimated and removed. It results from short term fluctuations in the series which are neither systematic nor predictable. In a highly irregular series, these fluctuations can dominate movements, which will mask the trend and seasonality. The following graph is of a highly irregular time series:

Figure 2: Monthly Value of Building Approvals, Australian Capital Territory (ACT)



The Time Series Model:

It is convenient to "model" a ____ series using these components.

Time series models fall into 2 major categories:

(A) __________: Y=T+C+S+I ⇦ assume components are independent of one another.

(B) ____________: Y=T×C×S×I ⇦ 4 components are related to each other, yet result from different basic causes.

**(Multiplicative model is more important and connection between the two.)

Focus first on the **<u>decomposition of *Y* (the time series) into components</u>**. Why?

(I) Components are useful in own right.

(II) ____ in forecasting of ***Y***.

❑ For the purpose of estimating each of the components of a _______series, models normally treat S_i, C_i, and I_i as **<u>deviations from the trend</u>**.

❑ T____ is usually estimated in units of ***Y***, and the other components are measured in ______form, with values <u>greater than 100</u> indicating a deviation **<u>above</u>** the trend value and values <u>below100</u> indicating a movement **<u>below</u>** the trend.

I.e. Represent cycle and seasonal as fraction of trend:

> 100% ⇨ above trend; < 100%⇨ below trend

The **<u>Steps of Decomposition</u>** of a time series into its components for a **<u>multiplicative model</u>**:

(1) Isolate ___________component by the ratio-to-moving average method.

⇨Determine seasonal _____ with a base value of 100.

(2) Remove ***seasonality*** from Y, leaving (T×C×I).

⇨ Divide the values of Y by the __________index S, and multiply by 100 to obtain:

$$100\left(\frac{Y}{S}\right) = T \times C \times I$$

(3) Determine _______component, and remove, leaving (C×I)

⇨divide T×C×I by the _______value $\hat{Y}$ to obtain (C×I).

(4) "Smooth" remainder to eliminate irregular component
⇨leaves ________.

❑ Once all of the components of a time series are identified, forecasts of the value of the time series can be made by first estimating the value of the trend component at that point in time in the _______, and then modifying this trend value by an adjustment that takes into account S and C components.

Seasonality

We are interested in the seasonal component and in eliminating it from ***Y*** (i.e. Seasonally adjusting the data).

Basic procedure involves **"smoothing"** the _____using moving averages:

$$Y_t = \{\ 8 \quad 2 \quad 6 \quad 1 \quad 10 \quad 4 \quad 12 \quad 2 \quad 8\}$$

2 Period MA: $\frac{8+2}{2}=5 \quad \frac{2+6}{2}=4 \quad \frac{6+1}{2}=3.5 \quad \frac{1+10}{2}=5.5 \quad 7 \quad 8 \quad 7 \quad 5$

⇨More terms in average ➡ smoother
⇨Choose terms to span exactly one year.
⇨Quarterly data ➡ 4 terms; ⇨Monthly data ➡ 12 terms

Need to take account of "_______" of data.

Example: How To determine A Seasonally Adjusted Series (Multiplicative Model)

Year	Quarter	Y_{it}	4-Qtr. M.A.	Centred M.A. (Tit*Cit)	Ratio: $S_i \times I_t = \frac{Y_{it}}{(T_{it} \times C_{it})}$
1994	Q1	40			
	Q2	60	45.0		
	Q3	20	47.5		20/46.25=0.432
	Q4	60	50.0		1.231
1995	Q1	50	52.5		0.976
	Q2	70	57.5		1.273
	Q3	30	55.0		0.533
	Q4	80	55.0		1.455
1996	Q1	40	50.0		0.762
	Q2	70	52.5		1.366
	Q3	10		----	----
	Q4	90		----	----

Obtain Seasonal Indices: (Average)

$$S_1 = (0.976 \times 0.762)^{1/2} \times 100 = 86.2\%$$

$$S_2 = (1.273 \times 1.366)^{1/2} \times 100 = 131.9\%$$

$$S_3 = (0.432 \times 0.533)^{1/2} \times 100 = 48.0\%$$

$$S_4 = (1.231 \times 1.455)^{1/2} \times 100 = 133.8\%$$

<u>Obtain the geometric mean of S_i's</u>:

$$\overline{S}_i^{\,GM} = (86.2 \times 131.9 \times 48 \times 133.8)^{1/4} = 92.44 \neq 100\%$$

Since the geometric mean does not equal **<u>100%</u>**, we must divide <u>each</u> **<u>S_i</u>** by 92.44:

$$S_1^* = \frac{86.2}{92.44} \times 100 = \_\_\_\_\_\_$$

$$S_2^* = \frac{131.9}{92.44} \times 100 = \_\_\_\_\_\_$$

$$S_3^* = \frac{48}{92.44} \times 100 = 51.9$$

$$S_4^* = \frac{133.8}{92.44} \times 100 = 144.7$$

$$\overline{S}_i^{GM*} = (93.2 \times 142.7 \times 51.9 \times 144.7)^{1/4} \cong 100\%$$

$$(GM = 99.97)$$

Apply same **<u>S_i's</u>** to each year. To seasonally adjust the data for a multiplicative model:

$$Y_{it}^{S} = \left[\left(\frac{Y_{it}}{S_i^*} \right) \times 100 \right]:$$

$Y_{it} = T_{it} \times C_{it} \times \mathbf{S_{it}} \times I_{it}$

$$Y_{it}^{S} = \frac{(Y_{it})}{(S_i^*)} * 100$$

Year	Quarter	Y_{it}	S_i^*	Seasonally Adjusted Series
1994	Q1	40	93.2	42.918
	Q2	60	142.7	42.046
	Q3	20	51.9	38.536
	Q4	60	144.7	41.465
1995	Q1	50	93.2	53.648
	Q2	70	142.7	49.054
	Q3	30	51.9	57.803
	Q4	80	144.7	55.287
1996	Q1	40	93.2	42.918
	Q2	70	142.7	49.054
	Q3	10	51.7	19.268
	Q4	90	144.7	62.198

$$\left(\frac{40}{93.2}\right)\times 100 = 42.918$$

Differences Between Additive and Multiplicative Models

Multiplicative Model	Additive Model
1) Seasonal factors should average to 1 or 100%: $GM = \left(\Pi s_i\right)^{1/i} = 1$ Example: $GM = (s_1 \times s_2 \times s_3 \times s_4)^{1/4}$	1) Seasonal factors should average to 0: $\sum \frac{s_i}{i} = 0$ Example: $\bar{x} = \frac{s_1 + s_2 + s_3 + s_4}{4} = 0$
2) When adjusting for seasonality: ⇨Divide by index: $y^s = \frac{y}{s_i^*}$	2) When adjusting for seasonality: ⇨Always subtract: $y^s = y - s_i^*$

Seasonality and Additive Model:

There are only **two** differences:

(1) Always ________instead of divide: $Y_{it}^{S} = (Y_{it} - S_{i}^{*})$

(2) Seasonal factors should average to ______ (not 100).

$$\frac{S_1 + S_2 + S_3 + S_4}{4} = 0$$

Example: Seasonality Topic 3

I have obtained quarterly data from **Cansim** for the period 2005-2009 and using these data I have obtained the following preliminary seasonal indices:

S1 = 2

S2 =-10

S3 = 12

S4 = -3

A) Have I assumed an additive model or a multiplicative model in applying the ratio-to-moving-average method?

This is an additive model because there are negative index numbers. The simple average of these index numbers must equal zero.

B) Amend the four figures so they can be used to seasonally adjust the data.

[2+(-10)+12+(-3)]/4 =1/4=0.25

Adjusted value:

S1 = 2-0.25 =**1.75**

S2 =-10-0.25 = -10.25

S3 = 12- 0.25 =11.75

S4 = -3-0.25 =-3.25

Average of the four index numbers is zero.

C) Suppose that the following data relate to the actual values for 2009.

2009 Q1 = 500

2009 Q2 = 455

2009 Q3 = 520

2009 Q4 = 475

Obtain the seasonally adjusted values for 2009.

Adjusted Values

2009 Q1 = 500	500 -1.75= 498.25
2009 Q2 = 455	455 -(-10.25)= 465.25
2009 Q3 = 520	520 - 11.75= 508.25
2009 Q4 = 475	475 - (-3.25)= 478.25

Question 2:

Determine the two final seasonal indices if the preliminary indices are the following:

$S_1=42$
$S_2=-36$

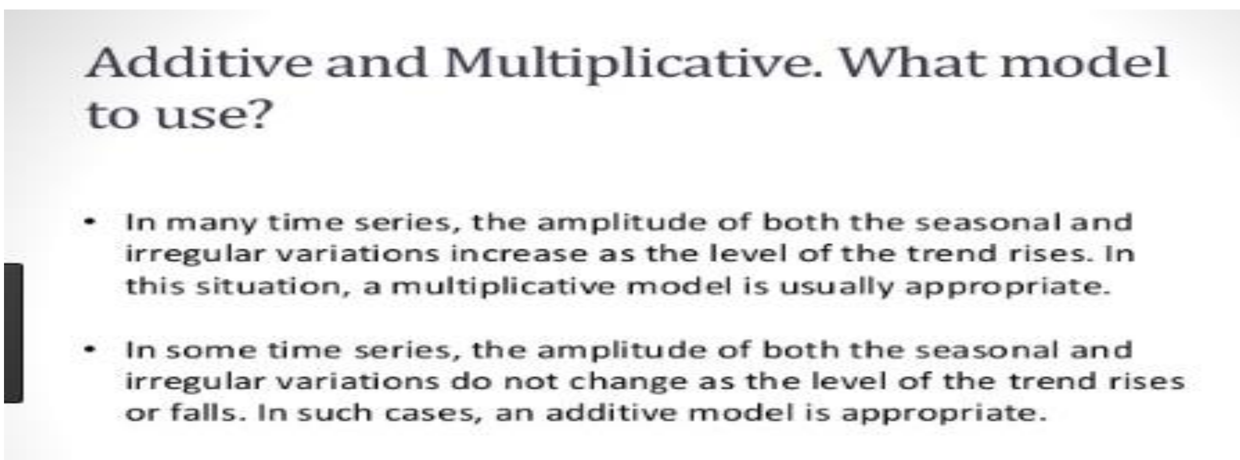
Answer:

Multiplicative or Additive?

Multiplicative: assume ______proportion of Y_{it} is seasonal.

Example: retail trade turnover (December Versus September)
(generally reasonable for trended data.)

Additive: assumes ________amount of Y_{it} is seasonal.
(May be appropriate if no trend; real terms.)



Refinements to Ratio-to-Moving-Average" Method

(1) Trading _____ Adjustments

- Different months have different number of _______days.
- May differ year after year.

(2) Outliers:

May need to "trim" data to ______distorting seasonal factors.

⇨Example: introduction of G.S.T.

(3) "Evolving" S__________ Pattern

- S_i^*'s usually allowed to change cyclically over years.

(4) End Point Problems

Calculation of S_i^*'s may be sensitive to Y_{it} ______at end (s) of series.

<u>The Trend –T</u>

An important step in analyzing a _____series is an estimation of the trend component, T.

One possibility is to use a "moving average" to smooth out ($T_t \times C_t$) to leave trend.

⇨Not favoured because the length of M.A. (and choice of "weights") arbitrary. Can be ____________.

☺The <u>preferred</u> approach is to identify a time series' trend, and then "model" the ______ components:

⇨{Increasing / Decreasing}

⇨{Linear / Non-linear}

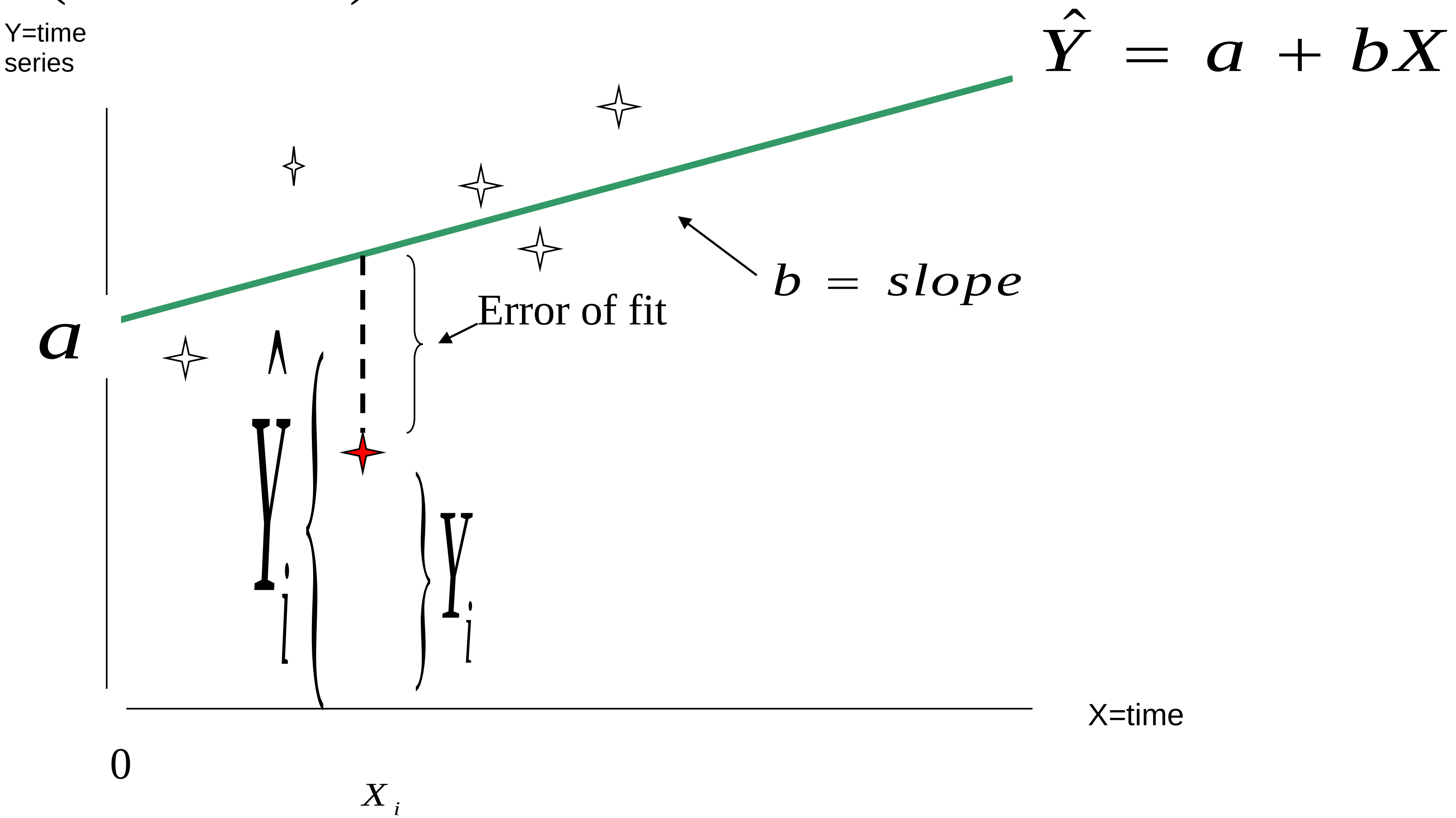
One principle we can use to fit a _______line to de-seasonalized ____is the **Least Squares** principle.

Let: Y_i=i^{th} observation on seasonally adjusted series.
X_i=i^{th} observation on "time."

Example: X = 1, 2, 3, 4, ...
X= 100, 110, 120, 130,...
X=-3, -2, -1, 0, 1, 2, 3, ...

Each observation for data gives us an (X_i, Y_i) point (plot Y_i against X_i).
Try to fit a line through points so it is "_____" to as many data points as possible.

(Use all data.)


Y=time
series
$\hat{Y} = a + bX$
$b = slope$
Error of fit
a
$\hat{Y}_i$
Y_i
X=time
0
X_i

Relationship is $Y_i = \alpha + \beta X_i$

(Not exact due to other components.)

Fitted line is $\boxed{\hat{Y}_i = a + bX_i}$

Error of fit= $\left(Y_i - \hat{Y}_i\right)$ (+/-) ← *Sums to Zero!!*

Place the line so as to <u>minimize</u>: $\boxed{\sum_{i=1}^{T} \left(Y_i - \hat{Y}_i\right)^2}$

(Choose ɑ and b).

That is: $$\underset{a,b}{Min}\left[\sum_{i=1}^{T}\left(Y_i - a - bX_i\right)^2\right]$$

Solution:

$$a = \overline{Y} - b\overline{X}$$

$$b = \frac{\left[\sum_{i=1}^{T}(X_i - \overline{X})(Y_i - \overline{Y})\right]}{\sum_{i=1}^{T}(X_i - \overline{X})^2} = \left[\frac{\left(\sum_{i=1}^{T}X_iY_i\right) - T\overline{X}\,\overline{Y}}{\left(\sum_{i=1}^{T}X_i^2\right) - T\overline{X}^2}\right]$$

where T=# of observations.

Example: T=7

Y_i	10	12	12	13	16	17	16
X_i							

$$\overline{X} = 0$$

$$\overline{Y} = 13.7143$$

$$\sum X_i^2 = 28$$

$$\sum X_i Y_i = 32$$

$$b = \frac{32 - 0}{28 - 0} = 1.1429$$

$$a = 13.7143 - (1.1429)(0) = 13.7143$$

$$\hat{Y}_i = 13.7143 + 1.1429 X_i$$

X_i	Y_i	$\hat{Y}_i$	$(Y_i - \hat{Y}_i)$
	10	10.2856	-0.2856
	12	11.4285	0.5715
	12	12.5714	-0.5714
	13	13.7143	-0.7143
	16	14.8572	1.1428
	17	16.0001	0.9999
	16	17.1430	-1.1430

Some Properties of Least Squares

(1) The predicted ____passes through the means of X and Y, $(\overline{X}, \overline{Y})$:

$$\hat{Y}_i = a + bX_i = a + b\overline{X}$$

but,

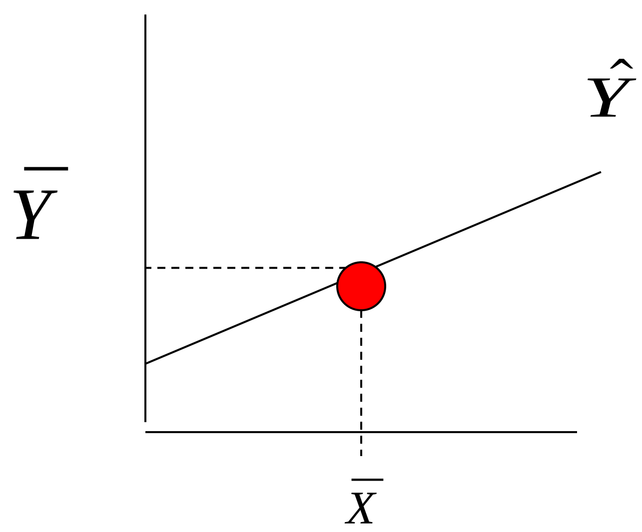
$$a = \overline{Y} - b\overline{X}$$

so,

$$\hat{Y}_i = \left(\overline{Y} - b\overline{X}\right) + b\overline{X} = \overline{Y}$$

when

$$X_i = \overline{X}.$$



(2) Scaling each X_i by a ________ leaves $\hat{Y}_i$ unchanged.

Begin with:

$$\hat{Y}_i = a + bX_i$$

Form:

$$X_i^* = CX_i.$$

Then:

$$b^* = \frac{\left[\sum (X_i^* - \overline{X}^*)(Y_i - \overline{Y})\right]}{\left[\sum (X_i^* - \overline{X}^*)^2\right]},$$

where:

$$\overline{X}^* = C\overline{X};$$

So:

$$b^* = \frac{\left[\sum (CX_i - C\overline{X})(Y_i - \overline{Y})\right]}{\left[\sum (CX_i - C\overline{X})^2\right]} = \frac{1}{C}b$$

Similarly:

$$a^* = \overline{Y} - b^*\overline{X}^*$$

$$= \overline{Y} - \left(\frac{1}{C}\right) b\left(C(\overline{X})\right)$$

$$= \overline{Y} - b\overline{X} = a$$

So:

$$\hat{Y}_i^* = a^* + b^* X_i^*$$

$$= a + \left(\frac{1}{C}\right) b\left(C(X_i)\right) = a + bX_i$$

$$= \hat{Y}_i \Leftarrow \text{ same fitted line.}$$

(3) Adding a __________ to each Xi leaves $\hat{Y}_i$ **unchanged.**

Begin with:

$$\hat{Y}_i = a + bX_i$$

Then form:

$$X_i^* = (X_i + k); \text{ for some 'k'.}$$

$$b^* = \frac{\left[\sum (X_i^* - \overline{X}^*)(Y_i - \overline{Y})\right]}{\left[\sum (X_i^* - \overline{X}^*)^2\right]}$$

where:

$$\overline{X}^* = (\overline{X} + k)$$

So,

$$b^* = \frac{\left[\sum (X_i + k - \overline{X} - k)(Y_i - \overline{Y})\right]}{\left[\sum (X_i + k - \overline{X} - k)^2\right]}$$

$$b^* = \frac{\left[\sum (X_i - \overline{X})(Y_i - \overline{Y})\right]}{\left[\sum (X_i - \overline{X})^2\right]} = b$$

$$a^* = \overline{Y}^* - b^*\overline{X}^*$$
$$= \overline{Y} - b\overline{X}^*$$
$$= \overline{Y} - b(\overline{X} + k)$$
$$= (\overline{Y} - b\overline{X}) - bk$$
$$= (a - bk)$$

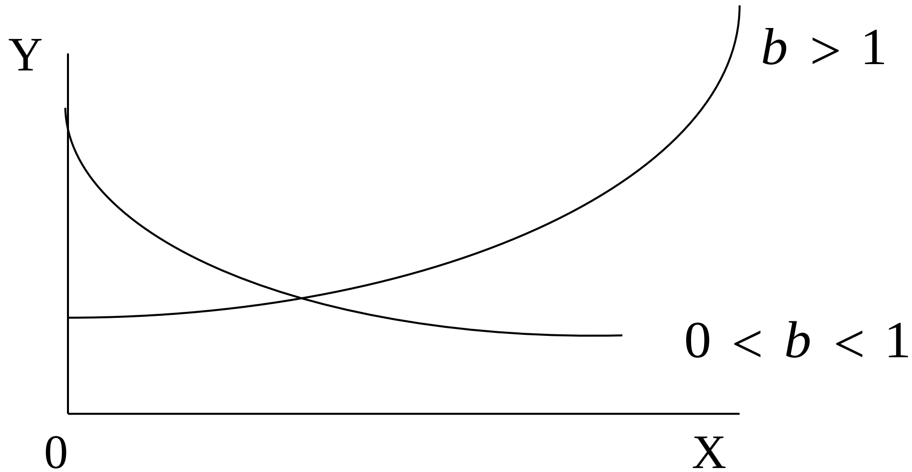
We are free to choose the X_i values just as we wish – the fitted line is unaffected by this choice.

So:

$$\hat{Y}_i^* = a^* + b^* X_i^*$$
$$= (a - bk) + b(X_i + k)$$
$$= a - bk + bX_i + bk$$
$$= (a + bX_i) = \hat{Y}_i \Leftarrow \text{ same fitted line.}$$

Often, a linear trend is clearly not appropriate. There are several _______________possibilities.

Consider an Exponential Trend:



$$\hat{Y}_i = ab^{Xi}$$

To make this trend _______in parameters, take the natural log:

$$\ln \hat{Y}_i = \ln a + X_i \ln b$$

or

$$\hat{Y}_i^* = a^* + b^* X_i \Leftarrow \text{ linear in parameters}$$

Obtain a* and b* by least squares, using Y_i^* $(= \ln Y_i)$ and X_i data.

Then, the fitted trend line is:

$$\text{Exp}(\hat{Y}_i^*) = \text{Exp}\left[\ln(\hat{Y}_i)\right] = \hat{Y}_i$$

Example: T=7

X_i	**Yi**	$Y_i^* \quad (= \ln Y_i)$
	1	0
	3	1.0958
	4	1.3863
	6	1.7918
	9	2.1972
	8	2.0794
	12	2.4849

$$\overline{X} = 0$$

$$\overline{Y}^{*} = 1.5769$$

$$\sum X_i^2 = 28; \qquad \sum X_i Y_i^{*} = 10.2272$$

$$b^{*} = \frac{\left[\sum X_i Y_i^{*} - T\overline{X}\,\overline{Y}\right]}{\sum X_i^{2} - T\overline{X}^{2}} = \frac{10.2272}{28} = 0.36526$$

$$a^{*} = \overline{Y}^{*} - b^{*}\overline{X} = 1.5769$$

So,

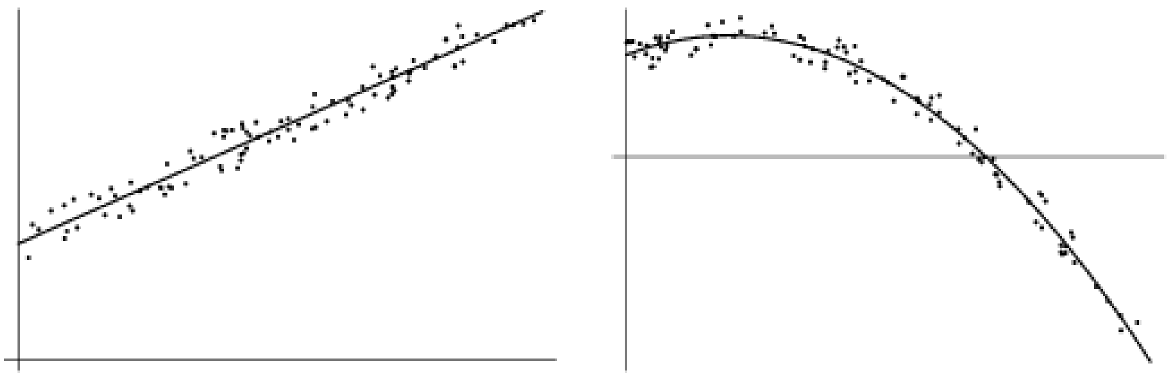
$$\hat{Y}_i^* = 1.5769 + 0.36526 X_i$$

or

$$\hat{Y}_i = \mathrm{Exp}\overbrace{(1.5769 + 0.36526 X_i)}^{\hat{Y}_i^*}$$

$$\hat{Y}_i = \underset{a}{(4.8399)}\underset{b}{(1.4409)}^{X_i}$$

Xi	**Yi**	$\hat{Y}_i$	$(Y_i - \hat{Y}_i)$
	1	1.6178	-0.6178
	3	2.3311	0.6689
	4	3.3589	0.6411
	6	***4.8399***=a	1.1601
	9	6.9738	2.0262
	8	10.0489	-2.0486
	12	14.4790	-2.4790



For nonlinear least squares fitting to a number of unknown parameters, linear least squares fitting may be applied iteratively to a linearized form of the function until convergence is achieved. However, it is often also possible to linearize a nonlinear function at the outset and still use linear methods for determining fit parameters without resorting to iterative procedures. This approach does commonly violate the implicit assumption that the distribution of errors is normal, but often still gives acceptable results using normal equations, a pseudoinverse, etc. Depending on the type of fit and initial parameters chosen, the nonlinear fit may have good or poor convergence properties. If uncertainties (in the most general case, error ellipses) are given for the points, points can be weighted differently in order to give the high-quality points more weight.