Cosmology in Models with Large Extra Dimensions

History

- Serious research into the effects of 4+d dimensions began in 1921/1926 with the works of Kaluza & Klein.

- Originally one extra dimension was added in an attempt to unify Electromagnetism with General Relativity

- Although succesful at reproducing Maxwell's equations from Einstein's equations, extra dimensional models were abandoned because:

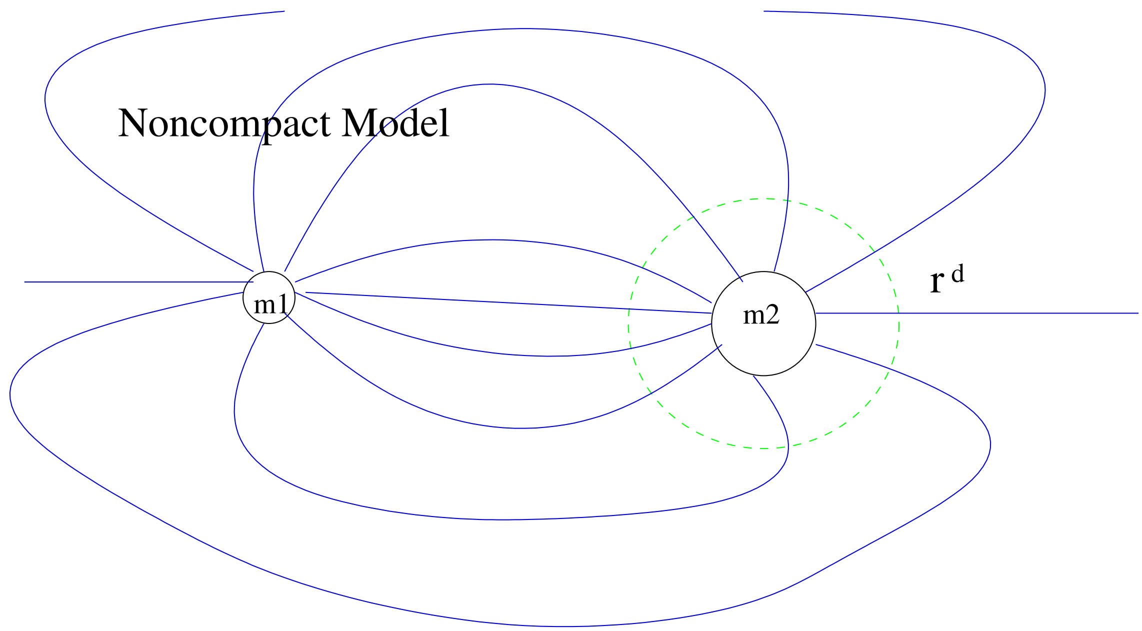
(1) To reproduce EM correctly required the extra dimension to be compactified to a length $R \sim 10^{-35}m$

(2) The models predict new short range interactions which are not observed. (There exists an infinite number of massive photons)

(3) Every model included a new scalar field which was not observed. (Now referred to as the radion)

(4) KK theories were classical theories introduced during the golden age of quantum mechanics, and were largely ignored.

Extra Dimensional Models

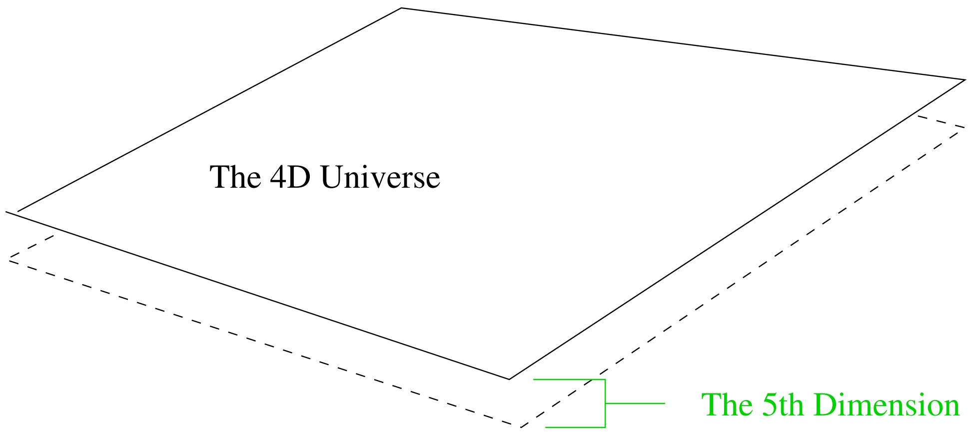


- If all dimensions are infinite and nonwarped, then the gravitational potential should be

$$V(r) = \frac{m_1 m_2}{M_*^{d+2}} \frac{1}{r^{d+1}}$$

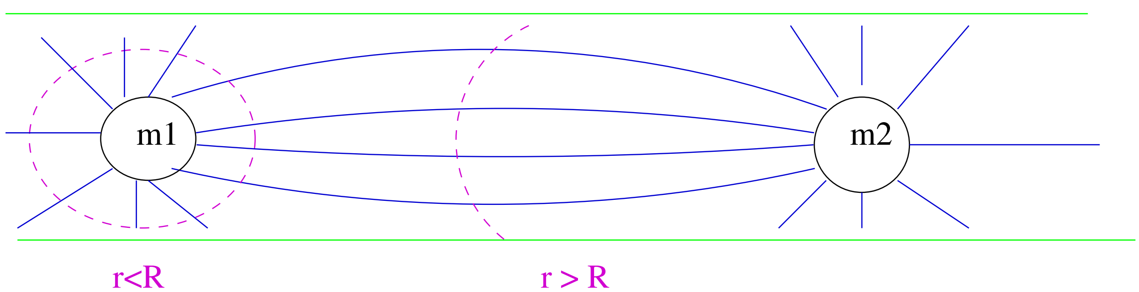
- At large distance it has a $1/r$ dependence so $d = 0$.

-Thus it has been traditionally assumed that the extra dimensions are compactified. (This can be avoided with extreme warping)



- If the compactified manifold is T^d (a d-dimensional torus), with all dimensions having length R, the potential is

$$V(r) = \frac{m_1 m_2}{M_*^{d+2} R^d}\,\frac{1}{r} = \frac{m_1 m_2}{M_P^2}\,\frac{1}{r} \qquad\qquad r > R$$



Large Dimensions

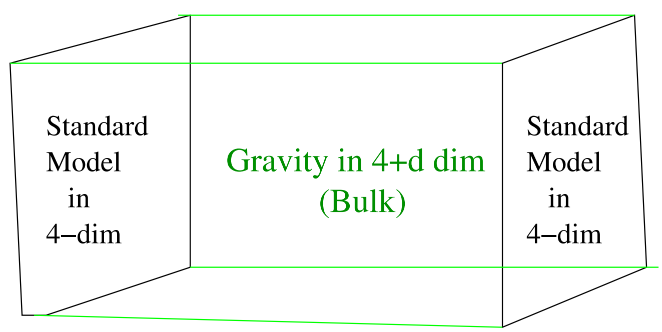
- Recently it has been proven that the dimensions can be large. (Arkani-Hamed, Dimopoulos, & Dvali, hep-ph/9803315& hep-ph/9807344)

- Such models are especially interesting since they can also solve the hierarchy problem.

-In 4D models, the characteristic energies of the Standard model are $\sim 1TeV$ while gravity has characteristic energy $M_P \sim 10^{16}TeV$.

- In these new theories the SM is confined to a 4D region, but gravity is a (4+d) dimensional theory with fundamental energy M_*

$$M_*^{d+2} = M_P^2/R^d$$



- The first restriction is that gravity appears four-dimensional for $r \gtrsim 0.1mm$. Below these distances gravity has not been measured accurately.

$$V \propto \left(\begin{array}{ll} \frac{1}{M_*^{d+2} R^d} \frac{1}{r} & r > R \\ \\ \frac{1}{M_*^{d+2}} \frac{1}{r^{d+1}} & r < R \end{array} \right.$$

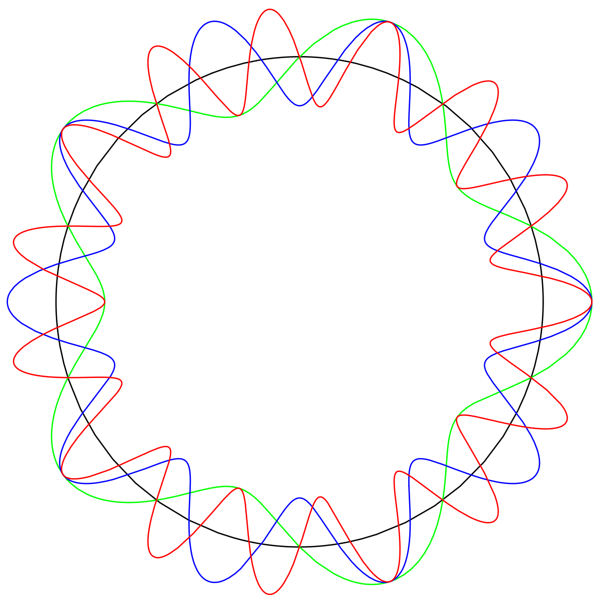
- If d=1, then for $M_* \sim 1TeV$ the extra dimension must be of the same scale as the solar system, and so is forbidden.

- For d=2, $R \lesssim 1mm$ and so cannot be excluded with current gravity experiments.

-The latest results of Adelberger & Heckel give $R < 0.1mm$, though results for r $\simeq$ 0.01mm are expected early next year.

- For d=3, $R \lesssim 10^{-5}mm$, which is well below the range of any current experiment.

Kaluza-Klein Fields



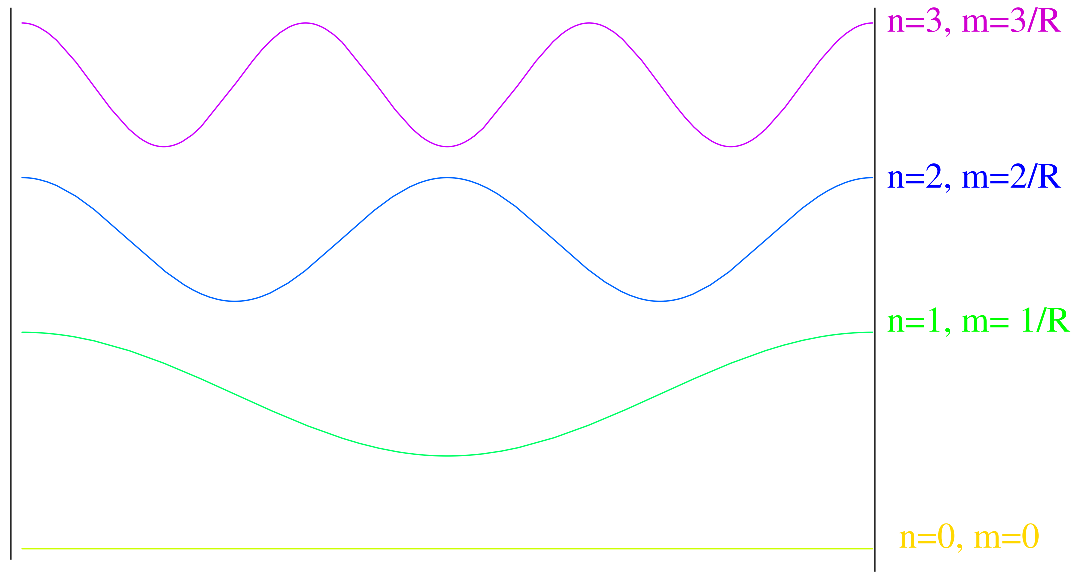
Any field defined in the extra dimension must have integer modes.

-As a result, any wavefunction will have discrete values of momentum in the extra dimension.

- If the metric for the 4+d manifold is

$$ds^2 = (\eta_{\mu\nu}(1+\kappa_d(\phi_{ij}))+\kappa_d h_{\mu\nu})dx^\mu dx^\nu - \kappa_d(2\phi_{ij}dy^i + A_{\mu j}dx^\mu)dy^j$$

then Einstein's equations predict that ϕ_{ij} act like 4D scalar fields (called radions) and $A_{\mu j}$ act as 4D vector fields.



Furthermore,the KK fields can be decomposed into an infinite set of modes:

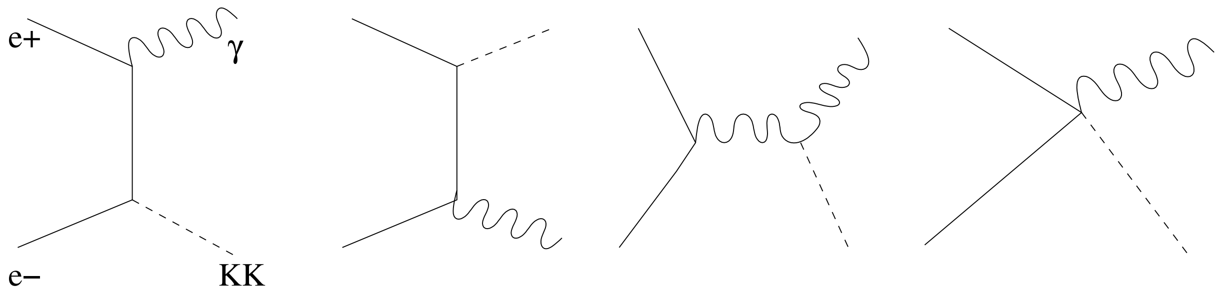
$$\phi = \sum_n \phi^{(n)}\ exp\left(i\frac{2\pi n y}{R}\right)$$

- The orginal fields $\phi_{ij}, ...$ do not have a well defined mass, though the individual modes do:

$$\partial_\mu \partial^\mu \phi = \partial_y \partial^y \phi = m_n^2 \phi \qquad m_n^2 = \frac{4\pi^2 n^2}{R^2}$$

- The zero modes may already be observed in the SM, and the next generation should be produced once energies in the range $\sim 1/R$ are reached.

- The interactions of KK modes with the Standard Model have been studied and reviewed in numerous papers (ie. Han,Lykken, & Zhang (hep-ph/9811350))



-KK modes could be detected as missing energy in the decays of quarkonium ($q\overline{q} \rightarrow \gamma + KK$) or collision experiments ($e^+e^- \rightarrow \gamma + KK$).

- At a given energy E, there are $N \sim (ER)^d$ modes which can be produced.

- Unfortunately the cross section for each is

$$\sigma \propto \frac{\alpha}{M_P^2} \sim 10^{-40} GeV^{-2}$$

- The total cross section for production KK modes is

$$\sigma_{tot} \propto \frac{\alpha E^d}{M_*^{d+2}}$$

Effects of KK Modes

- Any KK modes produced in the early universe or in astrophysical processes will cause observable changes:

(1) Photons produced by the decay of thermal KK modes will increase the energy of the gamma ray background.

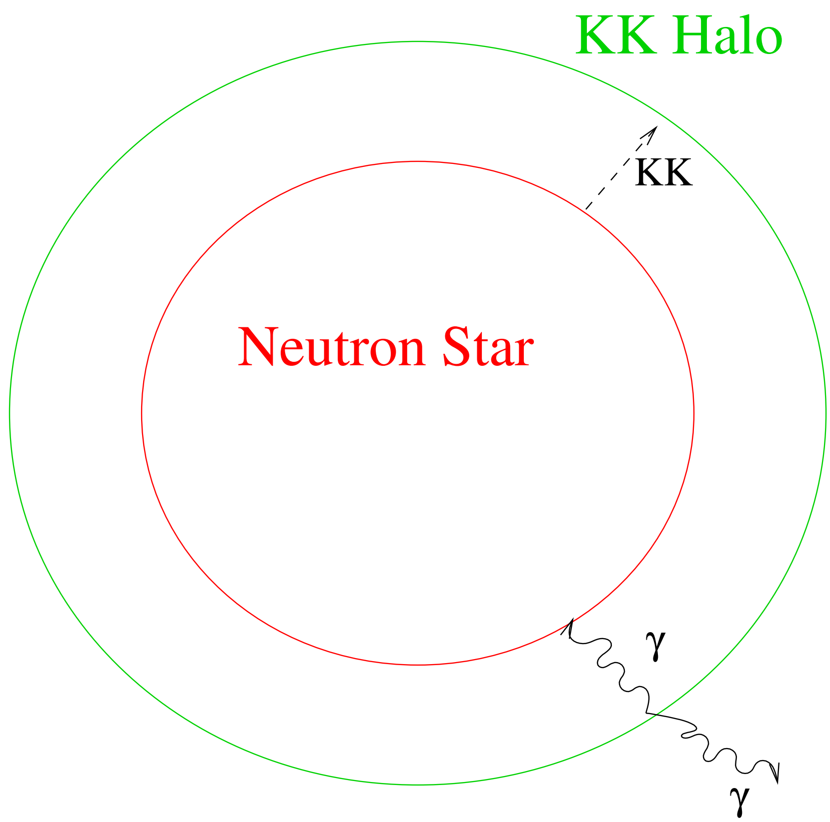
- The gamma ray background has been measured by several experiments, such as EGRET and COMPTEL, and gives a flux of

$$\frac{dn}{dE} = 2.3 \times 10^{-3} (E/MeV)^{-2.07} MeV^{-1} cm^{-2} s^{-1} ster^{-1}$$

- Assuming that the entire flux is due to KK decays provides a bound on the abundance of long lived modes with $\tau_{KK} > 10^{12} s$ (or $M_{KK} \lesssim 10 GeV$) (Hannestad (hep-ph/0102290))

- Upper limits on the abundance of KK modes leads to lower bounds on M_* and upper bounds on R:

	$d = 2$	$d = 3$	$d = 4$	$d = 5$	$d = 6$
M_*	167 TeV	21.7 TeV	4.75 TeV	1.55 TeV	< 1 TeV†
R	2.2×10^{-5} mm	2.5×10^{-8} mm	1.1×10^{-9}mm	1.7×10^{-10} mm	$> 2.9 \times 10^{-11}$ mm



(2) Neutron stars produce KK modes, which in turn will decay into e^+e^- and to gamma rays.

- Observations of the flux from neutron stars limits the size of M_*. Future gamma ray telescopes such as GLAST may observe the KK graviton signatures.

- Decays in KK halo that surrounds the neutron star will also provide a heat source that will alter the cooling of the star. (Raffelt & Hannestead, hep-ph/0110067)

- This excess heat can be measured and used to restrict M_* and R.

- The strongest bounds are from the excess heat in the young star PSR J0953+0755:

	$d=2$	$d=3$	$d=4$	$d=5$	$d=6$
M_*	3930 TeV	146 TeV	16.1 TeV	3.4 TeV	1 TeV
R	4.1×10^{-8} mm	1.1×10^{-9}mm	1.7×10^{-10}mm	5.7×10^{-11}mm	2.9×10^{-11} mm

(3) The KK mode decays will also destroy primordial light elements, which alters the nuclei abundances.

- This provides a bound on the abundance of KK modes with $\tau_{KK} < 10^{12}s$, which provides another bound on M_*. (Allahverdi, Bird, Groot Nibbelink & Pospelov,hep-ph/0305010)

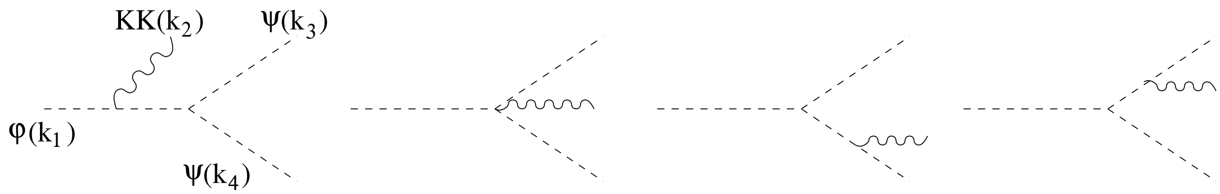
- This bound is strongest in the range $\tau_{KK} > 10^8 s$, which corresponds to

$$m_{KK}^{gr} \leq 80GeV$$

$$m_{KK}^{rad} \leq 200,\ 210,\ 225,\ 240,\ 250Gev \quad d=2,3,4,5,6$$

- Observational limits on nuclei abundances leads to the restriction on the KK energy density (Cyburt,Ellis,Fields & Olive, astro-ph/0211258)

$$\frac{\sum n_{KK} m_{KK}}{s} \leq 2 * 10^{-12} GeV$$



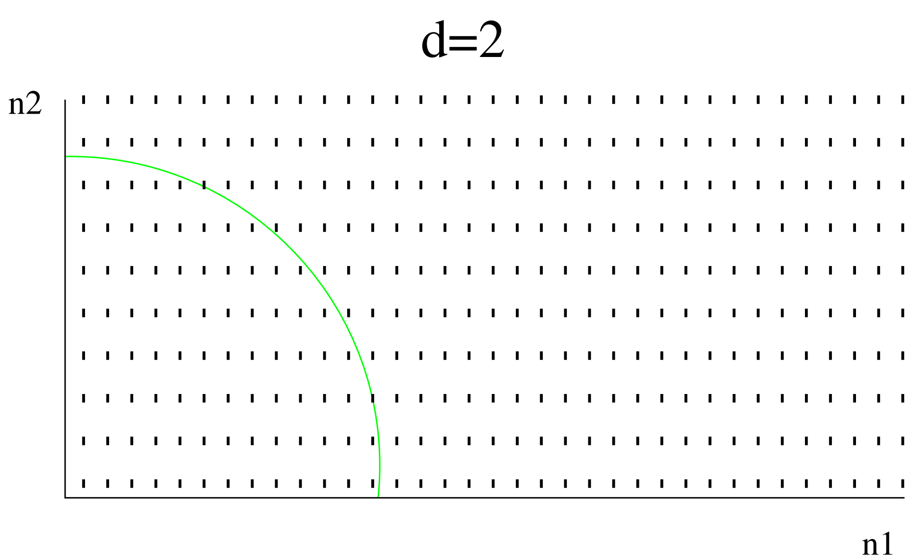
KK Abundance

- The density of KK modes can be calculated, though the decay rate depends on the specific inflaton model used.

- Most of the model dependence can be removed by using the ratio of the number density of *a single KK mode* and the number density of the inflaton

$$\frac{n^i_{KK}}{n_\phi} \approx \frac{\Gamma(\phi \to KK + 2S)}{\Gamma(\phi \to 2S)} \equiv C^i(m_{KK}) \frac{m_\phi^2}{M_P^2}$$

- The total number density of KK modes at a given time is the sum over all modes which are lighter than the critical masses.



- If the number of modes is large enough, (or if the dimensions are large), integration over a d-dimensional sphere can be used instead:

$$\sum_i \frac{m_{KK} n^i_{KK}}{n_\phi} \simeq \frac{m_\phi^2}{M_P^2 x^{d+1}} \int dy\ S_d\ y^d C^i \equiv B_i(x) \frac{m_\phi^2}{M_P^2} \qquad x = \frac{m_{max}}{m_\phi}$$

where $S_d = 2\pi^{d/2}/\Gamma(d/2)$.

- At temperature T, the entropy released from the inflaton decay satisfies

$$\frac{n_\phi}{s} \simeq \frac{3T}{m_\phi}$$

and so the bounds on the energy density gives the bound

$$\frac{3Tm_\phi}{M_*}\left(B_{rad}\left(\frac{m^{rad}_{max}}{M_*}\right)^{d+1} + B_{gr}\left(\frac{m^{gr}_{max}}{M_*}\right)^{d+1}\right) \leq 2 \times 10^{-12}$$

Results

Solving this equation for M_* gives the lower bounds:

m_ϕ	$d=2$	$d=3$	$d=4$	$d=5$	$d=6$
1 TeV	35 TeV	13 TeV	7.1 TeV	4.5 TeV	2.8 TeV
2 TeV	47 TeV	17 TeV	9.1 TeV	5.7 TeV	3.4 TeV
M_*	220 TeV	42 TeV	15 TeV	7.9 TeV	4.0 TeV

or an upper bound on R,

m_ϕ	$d=2$	$d=3$	$d=4$	$d=5$	$d=6$
1 TeV	$0.52\mu m$	$6.0\times10^{-5}\mu m$	$5.9\times10^{-7}\mu m$	$3.9\times10^{-8}\mu m$	$7.4\times10^{-9}\mu m$
2 TeV	$0.29\mu m$	$3.8\times10^{-5}\mu m$	$4.1\times10^{-7}\mu m$	$2.8\times10^{-8}\mu m$	$5.7\times10^{-9}\mu m$
M_*	$0.013\mu m$	$8.5\times10^{-6}\mu m$	$1.9\times10^{-7}\mu m$	$1.8\times10^{-8}\mu m$	$4.6\times10^{-9}\mu m$

- It is clear that the bounds from BBN are stronger for d=4,5, or 6. For d=2 and d=3, the astrophysical constraints are better.

- One concern over this derivation is that it assumes the existence of a light scalar in the SM. While valid for heavy inflatons, it fails for $m_\phi \lesssim 400GeV$.

- Fortunately this procedure works with massive scalars, although the integrals are very difficult to calculate numerically.

- The calculation can also use light (or massless) fermions to provide bounds on M_* if the inflaton is light.

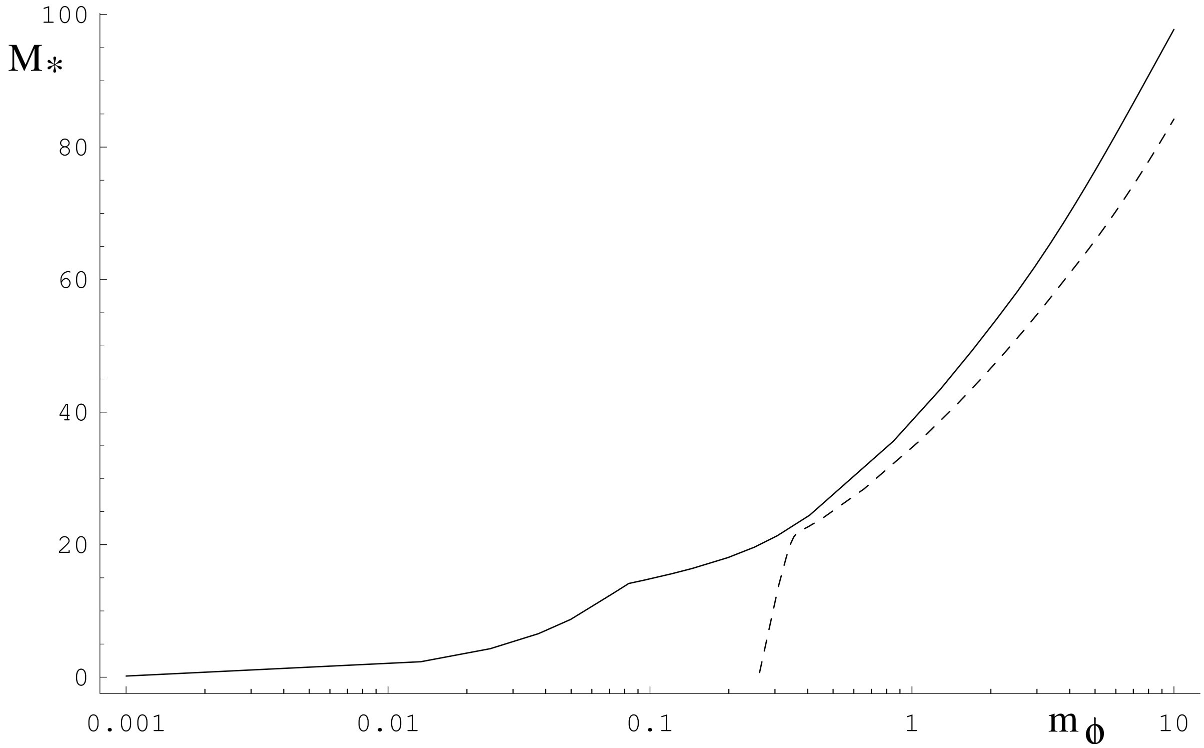


Figure 1: Lower bounds on M_* with d=2, from the decay of inflatons to massive scalars (dashed) and fermions (solid).

- For heavy inflatons, it is clear that the bounds on M_* do not depend dramatically on which SM particles are used.

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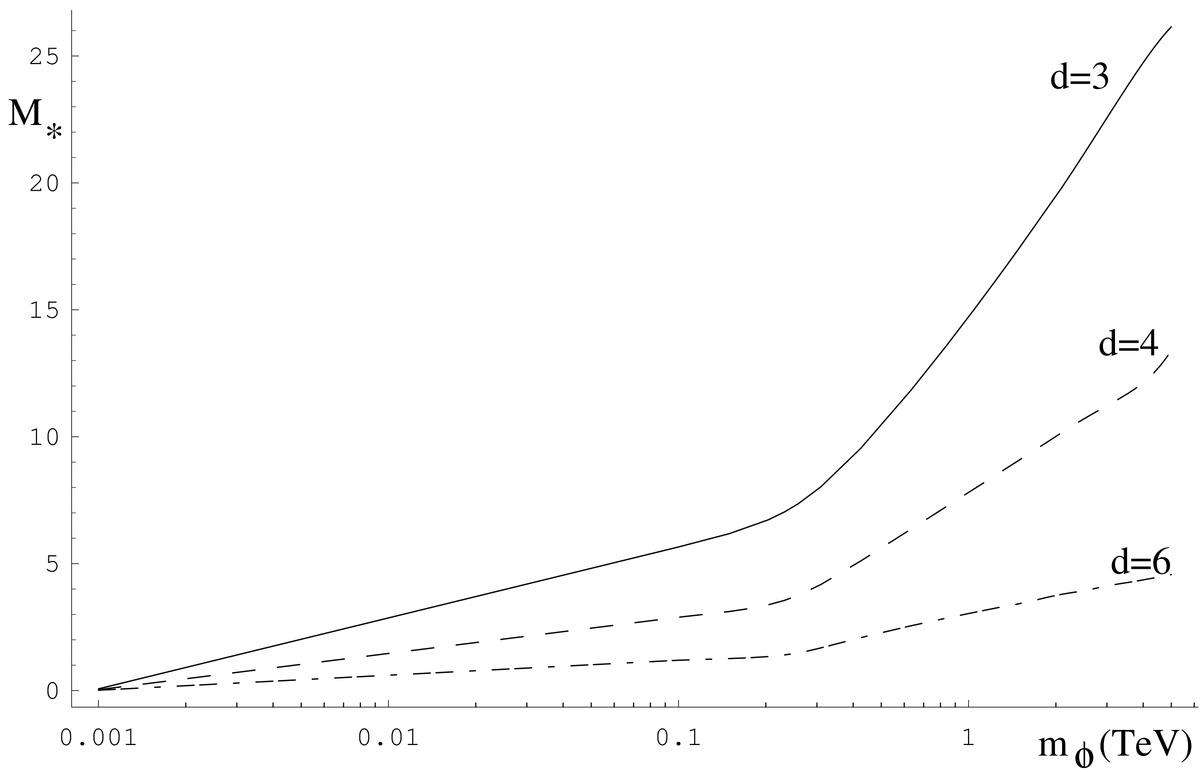

Figure 2: Lower bounds on M_* from fermion decays

Conclusions

Bounds from	$d=2$	$d=3$	$d=4$	$d=5$	$d=6$
gamma rays	167 TeV	21.7 TeV	4.75 TeV	1.55 TeV	<1 TeV†
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m_ϕ	$d=2$	$d=3$	$d=4$	$d=5$	$d=6$
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- It is possible to solve the hierarchy problem using large extra dimensions, but the size (and geometry) of these dimensions is unknown.

- The KK modes in the early universe should reduce the abundance of nuclei, change the properties of hot, dense astrophysical objects, and increase the observed gamma ray background. Any model that includes KK modes is restricted by these observations.

- We have shown that in 4,5, or 6 extra dimensions the constraints imposed by nucleosynthesis are the strongest.