Large Extra Dimensions and the Hierarchy Problem

The Hierarchy Problem

- At Planck energies ($M_{PL} \approx 10^{19} GeV$) all four forces have the same strength.

-At the Electroweak scale ($M_{EW} \sim 1TeV$) the four forces have different strengths due to symmetry breaking.

-If there is a GUT that describes all four forces, then the symmetry group G must be broken at both M_{PL} and M_{EW}:

$$M_{PL}: \quad G \rightarrow L \times K$$

$$M_{EW}: \quad K \rightarrow SU(3) \times SU(2) \times U(1)$$

Why is one symmetry breaking term at $10^{19} GeV$ and the other at $10^3 GeV$? This extreme difference is called the *Hierarchy Problem*.

- The difference in symmetry breaking scales causes problems in the Higgs mechanism. Each symmetry is broken by a scalar field with a non-zero VEV,

$$< 0|\Phi|0 >= V \sim M_{PL} \qquad < 0|\phi|0 >= v \sim M_{EW}$$

For a simple theory which only contains these two scalars, the Higgs mass is of the form

$$m_h^2 = \lambda v^2 - \beta V^2$$

-To explain W,Z boson masses, $m_h \sim 100GeV$, and so the V^2 term must be removed by renormalization - at all orders.

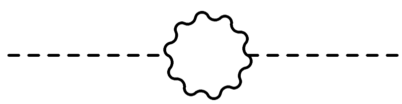
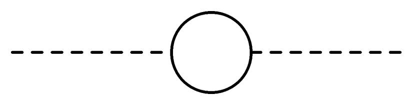
Traditional Solutions

How can the Hierarchy problem be resolved?

More Symmetry - It is possible to create a new weakly broken symmetry which reduces the Higgs mass and electroweak energy scale from the Planck mass. In practice these models are difficult to construct and require more tuning than the hierarchy problem does.

Technicolour - Either the Higgs boson is a composite particle, bound together by a new technicolour force, or the Higgs mechanism is replaced by a more complicated theory. However these theories predict new particles that are not observed. (Susskind 1979, Weinberg 1976)

Supersymmetry - If the SM particles are accompanied by supersymetric partners (ie W^+ is part of a doublet with a spin-1/2 Wino) then the effects of the weak sparticles can alter both the Higgs mass and the electroweak scale. But no sparticles have ever been observed. (Witten 1981)

$iM_h^2 =$  $+$  $+ \ldots$

Quantum Gravity - Gravity has really only been studied at ranges greater than a few millimeters, although from particle physics we know that it is still weak at $\sim 10^{-15}$ mm. But the Planck mass corresponds to distances of 10^{-32} mm! It is possible that the effects of some unknown quantum theory of gravity may strengthen the gravitational coupling at 10-100 TeV.

Extra Dimensions

Another solution is to introduce extra dimensions to the universe (Arkani-Hamed, Dimopoulos and Dvali [hep-ph/9803315]).

Suppose that the universe has 4+n dimensions, and a Planck mass $M_{PL(4+n)}$. Then the potential energy for two masses would be

$$V(r) = \frac{m_1\, m_2}{M_{PL(4+n)}^{n+2}} \frac{1}{r^{n+1}}$$

Above 1cm, gravitational potential energy is observed to have a 1/r dependence, so n=0. But if the extra dimensions are not infinite, and have length R (in general R_i), then at large distances the potential would be

$$V(r) = \frac{m_1\, m_2}{M_{PL(4+n)}^{n+2} R^n} \frac{1}{r}$$

-At large distances a 4+n dimensional universe appears to be 4-dimensional with

$$M_{PL}^2 = M_{PL(4+n)}^{n+2} R^n \left(= M_{PL(4+n)}^{n+2} V\right)$$

If R is large, then $M_{PL(4+n)}$ can be equal to the electroweak energy scale and still produce the observed large value of the Planck mass.

-If $M_{PL} \approx 10^{19} GeV$ and $M_{PL(4+n)}^{n+2} \approx 10^3 GeV$, then

$$R_{n=1} \approx 10^8 km \qquad R_{n=2} \approx 1mm$$

- The theory of extra spatial dimensions has been around since Kaluza(1919) and Klein(1926), but their models required $R \approx 10^{-35} m$

- If all SM fields are confined to a small region in each extra dimensions ($\sim 10^{-15} mm$), then $M_{EW} \approx M_{PL(4+n)}^{n+2}$.

-If the 'weak region' is even smaller (confined to a distance ρ in each extra dimension), then

$$\frac{M_{EW}^2}{M_{PL}^2} = \left(\frac{\rho}{R}\right)^n$$

Brane Models

- The SM fields could also be confined to one or more 4D branes or domain walls.(Randall & Sundrum [hep-th/9905221,hep-th/9906064])

-The most common model (called RS2) also introduces warping of the extra dimensions:

$$ds^2 = e^{-kr_c\phi} g_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\phi^2 \qquad \phi \in [-\pi, \pi]$$

The branes are located at $\phi = \pm\pi$ (our universe) and $\phi = 0$ (a hidden brane).

- The action for this model is of the form

$$S = \int d^4x \, d\phi \, e^{-2kr_c\phi} r_c \left[M^2 R^{(5)} + \delta(\phi - \pi) L_{SM} + \delta(\phi) L_{HB} \right]$$

$$= \int d^4x \left[\frac{e^{-2kr_c\pi} - 1}{2k} M^2 (R^{(4)} + (\partial_\mu r_c)^2) + e^{-2kr_c\pi} L_{SM} + L_{HB} \right]$$

which when compared to the Einstein-Hilbert and Standard Model actions:

$$S = \int d^x [M_{PL}^2 R^{(4)} + L'_{SM}]$$

indicates that

$$M_{PL}^2 = \frac{M^2(1 - e^{-2kr_c\pi})}{2k}$$

$$M_{EW}^2 = M^2 e^{-2kr_c\pi}$$

$$L'_{SM} = e^{-2kr_c\pi} L_{SM} + L_{HB}$$

-For $kr_c \approx 12$, the ratio $M_{EW}/M_{PL} \approx 10^{-16}$ as observed. (Which gives $< r_c > \approx 10^{-32} m$)

It is tempting to write a three-brane model, with one SM generation on each brane,

$$L_{eff} = e^{-k_1 R} L_1 + e^{-kR_2} L_2 + e^{-kR_3} L_3$$

since it would create a mass spectrum and explain the generations in SM. But it also predicts a charge spectrum which is not observed.

Experimental Evidence

Short Range Gravity - In a model with N compact dimensions of 'length' R, the gravitational potential should have a $1/r$ dependence for $r > R$, and $1/r^{1+N}$ for $r < R$. As yet there are no accurate measurements of gravity for ranges less than $\approx 1mm$.

Excited States - In the ADD model there is an threshold energy at which particles are able to escape in hyperspace. If at some energy particles start to disappear, it may indicate that they have entered hyperspace.

Massive Weak Bosons - It is possible to predict the existence of the weak bosons extra dimensions, but these theories predict the existence of a series of more massive weak bosons. In the simplest model (Uniform T^{D-4} compactification)

$$M^2 = M_0^2 + \frac{\overline{n} \cdot \overline{n}}{R^2}$$

Experiments indicate the next generation of bosons have masses $> 600GeV$ or $R < 10^{-17}cm$. (These models also do not solve the Hierarchy problem)

Radion Interactions - The g_{55} component of a 5D metric obeys the Klein-Gordon equation, and has the same properties as a spin-0 boson. The radion also interacts with the particles in the Standard Model, and should be observable in experiments. However the mass and interactions of the radion are unknown. (Some papers have indicated that the radion may be the Higgs)

The Future

Stability - Extra dimensions are not stable. A perturbation in the distance between branes will cause the extra dimension to collapse to a point or expand to infinity. In terms of the radion, if a single radion exists it can decay to N radions and each of those decays.

One solution is to add a 5D scalar field to the theory, with interactions on the branes (Goldberger & Wise, 2000). Then radion-scalar oscillations can stabilize the extra dimensions. (This has been proven for massive scalars, and the massless and tachyonic cases are in progress)

Which Model? - At present we have no idea how many extra dimensions exist, how they are compacted, or if they are warped. Each extra dimension can have any number of branes (or other defects) with any number of fields and interactions.

Conclusions

- The effects of the extra dimensions on gravity should be observable in the next generation of experiments.

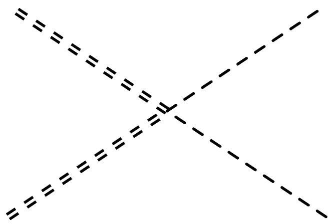
- The same effects that produce the hierarchy problem should produce a scalar field (and possibly a vector field) which can be observed in experiments.

- The Hierarchy problem can be resolved using extra dimensions. The observed ratio M_{EW}/M_{PL} is the result of making 4D observations of a higher-dimensional universe.

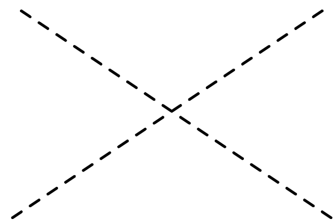
$$L_{SB} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}(\partial_\mu \Phi)^2 + \frac{\lambda}{4!}\phi^4 - \frac{\beta}{4}\phi^2\Phi^2 + \frac{\Lambda}{4!}\Phi^4$$

$$\frac{\delta L}{\delta \phi} = 0 = \frac{\lambda}{3}\phi^3 - \beta\phi\Phi^2 \to v^2 = \frac{3\beta V^2}{\lambda}$$

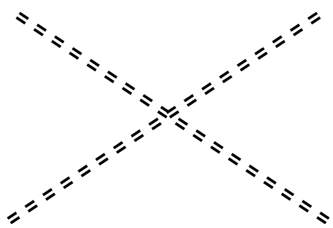
The Feynman rules are



$= -i\beta$

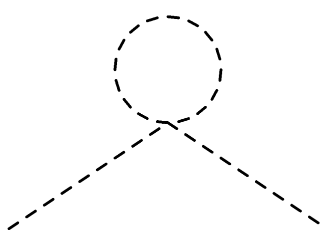


$= i\lambda$



$= i\Lambda$

$iM_h^2 =$



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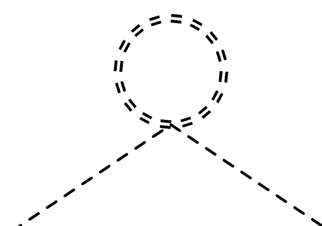

$$= i(\lambda v^2 - \beta V^2) = i(2\beta V^2)$$