

## Case Econometric Game 2015

Many countries face the issue of ageing of their population. People live longer and longer, sometimes with increases of life expectancy in the order of two months per year. The implied changes in demography are of a global nature and are not only important in Europe and the US, but also in, for instance, China.

Some background information is given in the book by Haberman and Pitacco. They write:

*“Life expectancy at birth among early humans was likely to be between 20 and 30 years as testified by evidence that has been gleaned from tombstones inscriptions, genealogical records, and skeletal remains. Around 1750, the first national population data began being collected in the Nordic countries. At that time, life expectancy at birth was around 35–40 years in the more developed countries. It then rose to about 40–45 by the mid-1800s. Rapid improvements began at the end of the 19th century, so that, by the middle of the 20th century it was approximately 60–65 years. By the beginning of the 21st century, life expectancy at birth has reached about 70 years. The average life span has thus, roughly tripled over the course of human history. Much of this increase has happened in the past 150 years: the 20th century has been characterized by a huge increase in average longevity compared to all of the previous centuries. Broadly speaking, the average life span increased by 25 years in the 10,000 years before 1850. Another 25-year increase took place between 1850 and 2000. And there is no evidence that improvements in longevity are tending to slow down.*

*The first half of the 20th century saw significant improvement in the mortality of infants and children (and their mothers) resulting from improvements to public health and nutrition that helped to withstand infectious diseases. Since the middle of the 20th century, gains in life expectancy have been due more to medical factors that have reduced mortality among older persons. Reductions in deaths due to the ‘big three’ killers (cardiovascular disease, cancer, and strokes) have gradually taken place, and life expectancy continues to improve.*

*The population of the industrialized world underwent a major mortality transition over the course of the 20th century. In recent decades, the populations of developed countries have grown considerably older, because of two factors – increasing survival to older ages as well as the smaller numbers of births (the so-called ‘baby bust’ which started in the 1970s). In this new demographic context, questions about the future of human longevity have acquired a special significance for public policy and fiscal planning. In particular, social security systems, which in many industrialized countries are organized according to the pay-as-you-go method, are threatened by the ageing of the population due to the baby bust combined with the increase in life expectancy. As a consequence, many nations are discussing adjustments or deeper reforms to address this problem.”*

This case consists of two parts. The first is *modelling* life expectancies and survival probabilities for a country of your choice. The second part considers possible mitigations of *risks* in longevity.

## 1. Modeling survival probabilities

Model survival probabilities, for a country of your choice, using freely available data from the Human Mortality Database (HMDB); see [www.mortality.org](http://www.mortality.org).

Deliverables:

1. A model, duly motivated, for the *joint* dynamics/distribution of individual survivor probabilities of males and females of various ages with a prediction horizon of, at least, 100 years.
2. A model description which is such that your competing teams would be able to implement the model without any ambiguities.
3. A set of 1.000 scenarios of possible developments of survivor probabilities for males and females of all ages and a 100 year prediction horizon.
4. The life expectancy of a newborn male and female as predicted by your model.
5. The life expectancy of a 65 year old male and female in 2025 as predicted by your model.
6. A graph showing the cumulative survival probabilities over age for newborns and 65 year olds, in 2015, 2025, and 2035.

Hints:

1. It is customary to model the male and female subpopulations independently of each other. Any attempt to model these jointly would be appreciated.
2. One issue is the modelling of survival probabilities for very high ages. If desired, you may assume that this probability converges to a constant of, say, 50%.
3. Several models exist in the literature. Useful references may be
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  - m. Lee, R. D., & Miller, T. (2001). Evaluating the Performance of the Lee-Carter Method for Forecasting Mortality. *Demography*, 38(4), 537-549.
  - n. Li, N., & Lee, R. D. (2005). Coherent mortality forecasts for a group of populations: An extension of the Lee-Carter method. *Demography*, 42(3), 575-594.
  - o. Oeppen, J., & Vaupel, J. W. (2002). Broken Limits to Life Expectancy. *Science*, 296(5570), 1029-1031.
  - p. Peters, F., Nusselder, W., & Mackenbach, J. (2012). The Longevity Risk of the Dutch Actuarial Association's Projection Model (Tech. Rep.). Tilburg: Netspar Design paper.
  - q. Plat, R. (2009). On Stochastic Mortality Modelling. *Insurance: Mathematics and Economics*, 45(3), 393-404.
  - r. Preston, S. H. (2007). The Changing Relation between Mortality and Level of Economic Development. *International Journal of Epidemiology*, 36(3), 556-570.
  - s. Renshaw, A. E., & Haberman, S. (2006). A Cohort-based Extension to the Lee-Carter Model for Mortality Reduction Factors. *Insurance: Mathematics and Economics*, 38(3), 556-570.
4. We suggest that, as a benchmark, you first implement the classical Lee-Carter model for your country of choice and study fitted and realized survivor probabilities.