Econometric Research and Special Studies Department

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ABSTRACT

The theory of distributed lags is that any cause produces a supposed effect only after some lag in time, and that this effect is not felt all at once, but is distributed over a number of points in time. Irving Fisher initiated this theory and provided an empirical methodology in the 1920’s. This article provides a small overview.

Key words: distributed lags.

JEL Codes: B31, C22.

SAMENVATTING

De theorie van de zogeheten ‘distributed lags’ houdt niet alleen in dat elke oorzaak na enige tijd bepaalde effecten met zich meebrengt, maar ook dat deze effecten zich op verschillende tijdstippen openbaren. Irving Fisher gaf in de jaren twintig van deze eeuw de aanzet tot deze theorie en leverde een eerste empirische invulling. Dit artikel geeft hiervan een kort overzicht.

Trefwoorden: distributed lags.

JEL Codes: B31, C22.

1 INTRODUCTION

Imagine that today the oil price will increase with 50%, and accordingly, petrol companies will immediately increase their gasoline rates with a similar percentage. Common sense suggests that the demand for gasoline will drop as an effect of this price change. With respect to the timing of this drop in demand, it seems not very likely that the effect is completely realised tomorrow. Most consumers are not able to adapt their behaviour immediately. The ability to react is limited by factors as the geographical distribution of residences and workplaces, the existing stock of vehicles and the existing supply of alternative transport systems. For example, a car commuter may find it hard to change his mode of transportation. First he needs to look for new modes of transportation, and when he reaches the conclusion that public transport will be a better option, he might wait until his car needs to be replaced. Moreover, some people are able to react faster than others are. For example, some people might immediately decide to use their bicycles for shopping purposes. In general, the effect of a price change will have a delayed effect on the demand for the good. The total effect of the price change is not felt at one particular point in time, but will be distributed over time. We, i.e. economists and econometricians, say that the effect is modelled as a distribution of lags, or more popularly, the effect has a distributed lag.

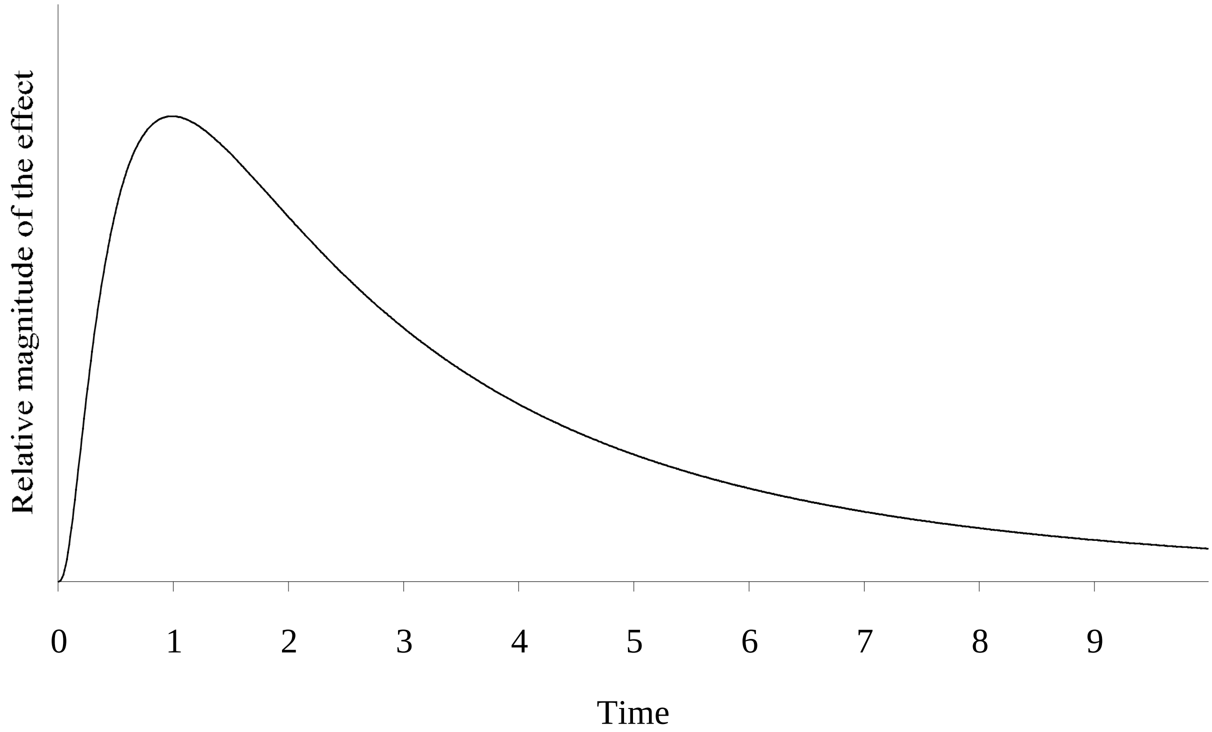
In his ‘Theory of Interest’ (1930), Irving Fisher was already aware of the fact that certain effects will be delayed over time. He initiated the theory of distributed lags incidentally in the course of his studies of price-trade relations in the 1920’s. In a short note, published in the Bulletin de l’Institut International de Statistique in 1937, Fisher elaborated on this theory in a rather elegant way. In this article, we would like to focus our attention on this note.

Section 2 describes Fisher’s model for the distribution of lags. In section 3 his approach is fit in the econometric theory of distributed lags. In section 4 his estimation procedure is compared with the current approach to estimate unknown parameters. Section 5 concludes.

2 DISTRIBUTED LAGS

Fisher’s theory was that any cause produces a supposed effect only after some lag in time, and that this effect is not felt all at once, but is distributed over a number of points in time. He hypothesised that the best general form for the lag distribution is presumably the lognormal distribution. In essence, this distribution satisfies his idea that the effect will quickly reach its peak after a very short period, and then slowly taper off. Figure 1 clarifies this idea.

Figure 1 Lognormal lag distribution.



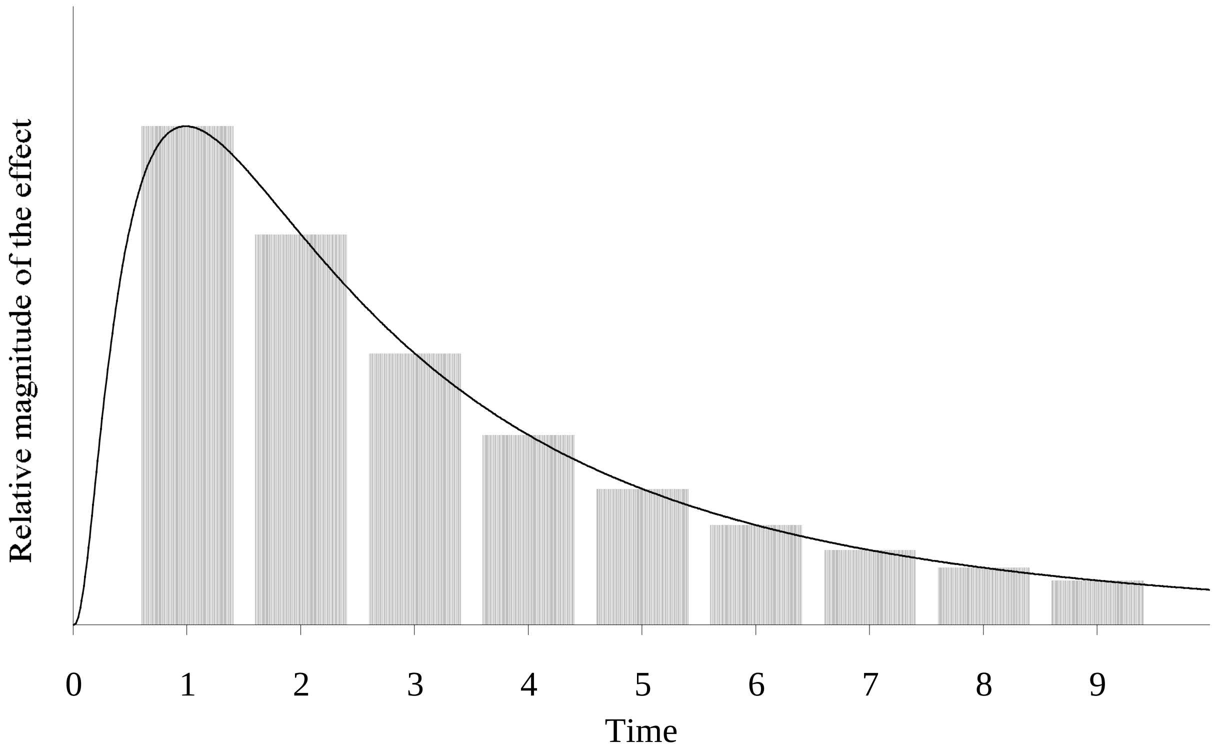
Suppose that at time 0, there is any cause, e.g. a 50% increase in the oil price. Then, Figure 1 shows the relative magnitude of the total effect at any time t>0. The total effect of this price change is represented by the total surface below the figure. The distribution is described by two parameters. The first parameter is the length of time from the cause to the mode of the distribution. The second parameter represents the dispersion of the distribution. One can construct statistics that summarise the information in this lag distribution. For example, the *median lag* is the time t* such that 50% of the total effect is realised. These statistics provide the researcher insight in the timing of the transmission process from cause to effect.

Contrary to other economists from his generation, Fisher had a sincere interest in the possibilities to establish an empirical basis for his theory. There are two problems that need to be solved before doing such an analysis. First, real world data are not available on a continuous basis, but are published at discrete intervals. This aspect requires a discrete approximation of the hypothesised lag distribution. A possible approximation is shown in Figure 2.

The second problem was more difficult to overcome in those days. Fisher must have realised that it would be difficult to perform an empirical validation for his conjecture, because he had no computational assistance at his disposal that would have allowed him today to estimate any complex non-linear relationship. Nevertheless, the paper shows Fisher's practical approach for solving problems. He notes that there is an intermediate method that 'saves some nine-tenths of the labor required by the logarithmically normal distribution and yields just as high correlations'.

This short-cut method postulates that the effect will be the greatest at the next time period, and then taper off by equal decrements for each successive time unit. The only parameter in this model is the length of time that the cause will have effect. It may not come to a surprise that Fisher proposed this lag distribution. The discrete approximation of the lognormal lag distribution, as shown in Figure 2, resembles his short-cut method.

Figure 2 Lognormal lag distribution, including discrete approximation.



A formal approach can illustrate his short-cut method. Let us denote the length of time that the cause will have any effect by n*. Then the cause x_t in period t will have effects in the periods t+1, t+2, … , t+n*. The size of the effect will decrease in equal from decrements, from n* in period t+1 to 1 in period t+n*. In other words, the computed effect y_t* in period t is caused by x_{t-1}, x_{t-2}, …, x_{t-n*} according to the following relationship:

$$y_t{*} = \sum_{j=1}^{n*} (n{*}+1-j)x_{t-j}.$$

In the appendix to this article it is shown how Fisher rewrote this relationship to a more compact expression. The relationship depends on the parameter n*. The big question, however, is what is the 'right' choice of n*? Fisher argues that it is the one that makes the computed series y_t* as 'close' to the actual series y_t. Before turning to his concept of closeness, let us first take a look at the current state-of-the-art econometrics on distributed lags.

3 ECONOMETRIC THEORY

How does Fisher's paper fit into today's theory of econometrics? The theory of distributed lags is an important branch within econometrics (see e.g. Dhrymes, 1971; Johnston, 1984; Judge, *et al.*, 1985; Greene, 1993). In general, the underlying data generating process is formalised by

$$y_t = \sum_{j=0}^{\infty} \beta_j x_{t-j} + \varepsilon_t,$$

where the independent variable x_{t-j} represents the cause in period t-j, β_j is the weight that is associated with this cause, ε_t is an independent white noise error term in period t, and y_t is the realised effect in period t. Within this class of models, one can distinguish *finite distributed lags* ($\exists n^*$ such that $\beta_j=0$ for all $j>n^*$) and *infinite distributed lags* ($\beta_j>0$ for all $j>0$).

Fisher's general approach fits into the theory of infinite distributed lags, but his short-cut method falls within the theory of finite distributed lags. His short-cut method is formally called the *arithmetic distributed lag* distribution, and can be described by

$$\beta_j = \begin{cases} (n^*+1-j)\alpha & j = 1,2,\ldots,n^*, \\ 0 & otherwise. \end{cases}$$

Essentially, Fisher's short-cut method requires one parameter to specify the lag distribution, namely the parameter n^* that specifies the number of lags included. In a regression context, there will also be a second parameter α that relates to the size of the effect.

4 ESTIMATION

The statistical underpinning of his work was rather ad-hoc. Fisher writes that the 'best' distribution of lags is the one that maximises the correlation between the actual and the computed series, i.e.

$$n^* = \arg\max\{n^* \mid corr(y, y^*)\}.$$

To find this estimate, Fisher provided a heuristic approach in case one already knows the lag length that makes the correlation between the lagged x_t and y_t at its maximum. A good guess for the value of n^* would be three to four times the numerical value of the aforementioned lag length.

According to Fisher, the absolute value of the correlation coefficient between the actual and the computed series also indicates the model's performance. In one of his first applications of the distributed lag method, his support for the supposition that price changes have a distributed effect on interest rates, relies on the absolute value of this correlation coefficient. In 'The Theory of Interest' (1930), he claims that the high correlation coefficient between the actual and computed series shows that '…the theory .. conforms closely to reality..' (p. 425).

The statistical analysis dates from an age where the econometric school was in its infancy. Today's approach for estimating the parameters of the distributed lag distribution is the least squares technique. Under relatively weak conditions, it can be shown that the least squares estimator is a consistent estimator for the unknown β_j's. There is however a statistical problem when using a general approach to include a large number of lags of the independent variable. When the independent variable is relatively stable and moving around its mean, the series of lags may be nearly linearly dependent. This so-called multicollinearity problem leads to very imprecise estimators for the true parameters.

A solution to the multicollinearity problem might be to use the arithmetic distributed lag approach as proposed by Fisher. Fisher's specification has the advantage of being parsimonious. Nevertheless, Fisher's lag scheme is generally regarded as unduly restrictive (Dhrymes, 1971). More general lag schemes have been proposed in the empirical literature, e.g. the Almon distributed lag. Moreover, Koyck (1954) argues that there exists a comparable assumption regarding the lag structure, that 'saves about another 50 or 60% compared with Fisher's short-cut method'. Koyck's assumption of proportionately decreasing effects allows for an estimation procedure that requires only one calculation.

5 CONCLUSION

In modern econometric textbooks the idea that any cause will have a delayed distributed effect, is well-established. Multicollinearity is a serious problem when one performs unrestricted estimation of the parameters of the lag distribution. This motivates the search for simplifying lag schemes. Textbooks still refer to Fisher's 'Note on a Short-Cut Method for Calculating Distributed Lags' in the Bulletin de l'Institut International de Statistique. Fisher's contribution unintentionally provided the first parsimonious device that is able to solve the multicollinearity problem.

One might criticise Fisher's paper for its statistical underpinnings. Indeed, the econometrics used in this paper is a dated practice. Nevertheless, as one of the founders of the Econometric Society, Fisher should be credited for his attempt to provide an empirical validation for his theoretical models.

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APPENDIX

Fisher showed that the computed series {y_t*} satisfy the following relationship:

$$y_t* = (n*+1)S_{t-1}^{(0)} - S_{t-1}^{(1)} + S_{t-n*-2}^{(1)},$$

where

$$S_t^{(0)} = \sum_{j=1}^{t} x_j, \text{ and } S_t^{(1)} = \sum_{j=1}^{t} S_j^{(0)}, t = 1,2,...$$

This can easily be proved:

$$y_t* = \sum_{j=1}^{n*}(n*+1-j)x_{t-j} = (n*+1)S_{t-1}^{(0)} - (n*+1)S_{t-n*-1}^{(0)} - \sum_{j=1}^{n*}\sum_{i=1}^{j} x_{t-j}$$

$$= (n*+1)S_{t-1}^{(0)} - (n*+1)S_{t-n*-1}^{(0)} - \sum_{i=1}^{n*}\sum_{j=i}^{n*} x_{t-j} = (n*+1)S_{t-1}^{(0)} - (n*+1)S_{t-n*-1}^{(0)} - \sum_{i=1}^{n*}(S_{t-i}^{(0)} - S_{t-n*-1}^{(0)})$$

$$= (n*+1)S_{t-1}^{(0)} - (n*+1)S_{t-n*-1}^{(0)} - (S_{t-1}^{(1)} - S_{t-n*-1}^{(1)} - n*S_{t-n*-1}^{(0)}) = (n*+1)S_{t-1}^{(0)} - S_{t-1}^{(1)} - S_{t-n*-2}^{(0)}.$$