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THE ESTIMATION OF ALLOCATION MODELS WITH AUTOCORRELATED DISTURBANCES

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The estimation of singular allocation models, with errors which are spatially autocorrelated or exhibit a serially correlated component, is considered. The restrictions implied for the error process parameters are identical to those for autoregressive or moving average errors.

1. Introduction

The problem of estimating allocation models is well known [e.g. Barten (1969), Powell (1969)]. Such models comprise a system of m equations in which, at each sample point, the dependent variables sum to a linear combination of (some of) the regressors. For example, the system may explain budget or factor shares and each equation may include an intercept,

$$y_j = \Pi x_j + v_j; \quad j = 1, \ldots, n, \tag{1}$$

where y_j and v_j are $(m \times 1)$, Π is $(m \times k)$ and x_j is $(k \times 1)$ with first element unity, and

$$\iota' y_j = 1; \quad j = 1, \ldots, n, \tag{2}$$

where ι is $(m \times 1)$ with all elements unity. Such systems are singular, as (1) and (2) imply

$$\iota' \Pi = (1, 0, \ldots, 0) \tag{3}$$

and

$$\iota' v_j = 0; \quad j = 1, \ldots, n. \tag{4}$$

Berndt and Savin (1975) discuss the estimation of such systems when the disturbances follow a vector autoregressive or moving-average process, as migh be appropriate with time-series data. They show that (2) implies restrictions on the columns of the parameter matrices associated with these error processes. If these restrictions are ignored then the usual 'solution' to the problem of singularity of the disturbance covariance matrix no longer applies – the maximum-likelihood estimates are *not* invariant to the equation deleted from the system.

In this paper we consider some less common error processes and show that *precisely the same* restrictions must be imposed on the parameters of these processes as for the AR and MA cases, if the maximum-likehood estimates are to be invariant to the equation deleted.

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2. Spatial autocorrelation

When cross-section data have a natural ordering it may be appropriate to consider the spatial autoregressive process suggested by Cliff and Ord (1973) and studied by King and Evans (1985). In vector form, this is

$$
\begin{aligned}
v_j &= Rv_{j-1} + Rv_{j+1} + \epsilon_j; \quad j = 2, \ldots, n-1, \\
v_1 &= Rv_2 + \epsilon_1, \\
v_n &= Rv_{n-1} + \epsilon_n,
\end{aligned} \tag{5}
$$

where $\epsilon_j \sim \mathrm{IN}(0, \Omega)$, and R is $(m \times m)$. An example of an allocation model for which (5) may be appropriate is a system of Engel curves where the data are categorised by income group [Giles and Hampton (1975)] or by region [Hampton and Giles (1987)].

If the data satisfy (2) then (4) and (5) imply

$$\iota' R(v_{j-1} + v_{j+1}) + \iota'\epsilon_j = 0, \tag{6}$$

and

$$\iota'(v_{j-1} + v_{j+1}) = 0 \tag{7}$$

for $j = 2, \ldots, n-1$. Now, ϵ_j is independent of v_{j-1} and v_{j+1}, so from (6),

$$\iota'\epsilon_j = 0, \tag{8}$$

and

$$\iota' R(v_{j-1} + v_{j+1}) = 0, \tag{9}$$

so that (7) and (9) imply

$$\iota' R = c\iota', \tag{10}$$

where c is a scalar constant. It is easily verified from (4) and (5) that (8) and (10) hold for $j = 1, n$. From (8), $\Omega\iota = 0$: the system is singular.

King and Evans (1985) also consider a spatial moving average process, which in vector form is

$$
\begin{aligned}
v_j &= \epsilon_j + R\epsilon_{j-1} + R\epsilon_{j+1}; \quad j = 2, \ldots, n-1, \\
v_1 &= \epsilon_1 + R\epsilon_2, \\
v_n &= \epsilon_n + R\epsilon_{n-1}.
\end{aligned} \tag{11}
$$

Applying the same argument as for the autoregressive case, when the data 'add up', we again find that R must satisfy (10) and that Ω has rank $(m-1)$.

These results are *identical* to that of Berndt and Savin (1975, pp. 938–939). Maximum-likelihood estimation of single equations with spatially autocorrelated errors is discussed by King and Evans

(1985). The estimator may be extended for the system (1) and (2), and the arguments of Berndt and Savin (1975, pp. 939–940) can be used to establish the invariance of the estimates to the equation deleted to overcome the singularity of Ω, *provided* that (10) is imposed.

3. Serially correlated error components

As a further example of the way in which the model's error process may be generalized, suppose that v_j comprises two components, one serially independent and the other autoregressive:

$$v_j = u_j + \eta_j; \quad j = 1, \dots, n, \tag{12}$$

where u_j and η_j are independent, $\eta_j \sim \mathrm{IN}(0, \Sigma)$ and

$$u_j = Ru_{j-1} + \epsilon_j; \quad j = 2, \dots, n, \tag{13}$$

with $\epsilon_j \sim \mathrm{IN}(0, \Omega)$. Revankar (1980) and King (1986) give algorithms for the maximum-likelihood estimation of single equations with a serially correlated error component. Their approaches can be extended to systems such as (1). A general discussion of sets of equations with error components is given by Srivastava and Giles (1987, pp. 266–274).

Following the arugment of the last section, if the data satisfy (2) then v_j satisfies (4), so by the independence of the u_j and η_j, and of u_j and ϵ_j,

$$\iota' u_{j-1} = \iota' R u_{j-1} = 0 \tag{14}$$

and

$$\iota' \epsilon_j = 0.$$

It follows that R again satisfies (10) and $\Omega\iota = 0$. The conclusion of section 2 applies here: if the maximum-likelihood estimator of Π and R is to be invariant to the equation dropped from (1) to deal with the singularity of Ω, each column of R must be restricted to sum to the same constant value.

4. Conclusion

We have shown that the restrictions associated with the parameters of a singular allocation model with autoregressive or moving average errors, also apply *exactly* for several other relevant error processes. The error component considered can be expressed as a *restricted* ARMA (1, 1) model, but our results do not apply when the errors follow an unrestricted ARMA scheme, as may be verified by applying the arguments used in this paper. While the discussion is in terms of first-order processes, it is easily extended to higher-order schemes.

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