"ECONOMIC GOODNESS-OF-FIT

vs.

STATISTICAL GOODNESS-OF-FIT"

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(Seminar Overheads)

Introduction

- **Many economic models derived by considering optimizing behaviour of agents.**

- **Can this be used to evaluate the "quality" of our empirical estimates of such relationships?**

- **Provide some background motivation based on work by Varian.**

- **Define "Economic Goodness-of-Fit" in context of demand systems.**

- **Provide some illustrative empirical results.**

- **Conclusions & directions for further research.**

Background

- **Basic ideas date back at least to Samuelson (1938, 1947), Afriat (1967, 1972).**
- **More recent work by Varian (1982a, 1982b, 1984, 1985).**
- **Generally we test an economic model by "fitting" it to some data, for a particular parametric functional form, and testing to see if the estimated parameters satisfy the restrictions implied by the economic theory.**

- **Why may this be less than satisfactory?**

  Most theories don't really need a parametric framework.
  Why use "statistical significance" to judge the "economic significance" of the results?

Three Examples

1. Profit-Maximization:

(p^t, y^t) ; $t = 1, 2, \ldots\ldots\ldots, T$

Require $p^t y^t \geq p^t y^s$; $\forall\ t, s$

(*Weak Axiom of Profit Maximization* – Varian, 1984)

2. Cost-Minimization:

(w^t, x^t, y^t) ; $t = 1, 2, \ldots\ldots\ldots, T$

Require $w^t x^t \leq w^t x^s$; $\forall\ y^s \geq y^t$

(*Weak Axiom of Cost Minimization* – Varian, 1982a)

3. Utility-Maximization:

(p^t, x^t) ; $t=1,2,3, \ldots\ldots, T$

Revealed preference:

$x^t R x^s$ ***iff*** $\exists$ $x^r, \ldots\ldots, x^u$ **satisfying**

$p^t x^t \geq p^t x^r, \ldots\ldots\ldots, p^u x^u \geq p^u x^s$

Require: $x^t R x^s \Rightarrow p^s x^s \leq p^s x^t$

(*Axiom of Revealed Preferences* – Varian, 1982b)

Why a Statistical Approach?

- **Varian (1982a,b; 1984) provides non-parametric methods for examining whether the above inequalities are satisfied, empirically.**

- **Non-Parametric methods are unduly "sharp" - no "error term".**

- **So standard (parametric) inferential procedures tend to be used to examine economic hypotheses. Does "statistical significance" equate with "economic significance"?**

- **Don't ask: "Does optimization hold exactly?" Instead, ask: "Is optimization a reasonable way to describe this behaviour?"**

- **"Nearly Optimizing Behaviour" is good enough.**

Non-Parametric Economic Goodness-of-Fit

- **Ask: "How large are the violations of the theoretically required inequality?"**

- ***Example* (profit-maximization):**
  Suppose we observe a pair of observations, *s* and *t*, such that $p^t y^t < p^t y^s$.

- **A reasonable measure of this violation of the WAPM would be:**

$$r^{ts} = [(p^t y^s - p^t y^t) / p^t y^t] = (p^t y^s / p^t y^t) - 1.$$

- **These r^{ts} values are essentially "economic residuals". They could be listed, or their average or maximum could be reported.**

- **The distribution (pattern) of these residuals would also be informative (time-series or cross-section).**

Parametric Goodness-of-Fit

- **Use Consumer behaviour as an example.**

  (Extend Varian, 1990.)

- **Parametric utility function,** $u(x, \beta)$

- **"Money metric utility function",**

$$m(p, x, \beta) = \min.\ py$$

  $s.t. \quad u(y, \beta) \geq u(x, \beta).$

- **An "index of the degree of violation of utility-maximizing behaviour" is:**

$$i^t = m(p^t, x^t, \beta) / (p^t x^t).$$

  N.B.: (1 - i^t) is "wasted expenditure".

- **Can use this as an index of Goodness-of-Fit, and also as the basis for estimating** β**.**

- **In the latter case, need to decide on a loss function:**

  e.g., **min.** $\Sigma (\log(i^t))^2$.

Empirical Example: Cobb-Douglas Utility

- $u(x_1, x_2, x_3) = x_1^{a1}\, x_2^{a2}\, x_3^{a3}$; $a_1 + a_2 + a_3 = 1$

- **The demand functions are:** $x_i = [(a_i\, e) / p_i]$, **where**

  e **= total expenditure.**

- m **= amount of money at prices** (p_1, p_2, p_3) **needed to choose an optimal bundle that has same utililty as** (x_1, x_2, x_3).

- **So,** $(x_1^{a1}\, x_2^{a2}\, x_3^{a3})$

  $= [(a_1\, m) / p_1]^{a1}[(a_2\, m) / p_2]^{a2}[(a_3\, m) / p_3]^{a3}$

  $m = (a_1)^{-a1}\, (a_2)^{-a2}\, (a_3)^{-a3}\, (p_1x_1)^{a1}\, (p_2x_2)^{a2}\, (p_3x_3)^{a3}$

  $log(m) = -a_1log(a_1) - a_2log(a_2) - a_3log(a_3) +$

  $a_1log(p_1x_1) + a_2log(p_2x_2) + a_3log(p_3x_3)$

- **Estimate:**

  $log(e^t) = -a_1log(a_1) - a_2log(a_2) - a_3log(a_3) +$

  $a_1log(p^{t1}x^{t1}) + a_2log(p^{t2}x^{t2}) + a_3log(p^{t3}x^{t3}) + \varepsilon^t$

Application: Citibank Data

U.S. consumption data, 1947 - 1987, Non-Durables, Durables & Services

1. Cobb-Douglas Utility:

	SURE	Money Metric Estimator
a_1	**0.129**	**0.150**
	(*80.76*)	**(*13.33*)**
a_2	**0.358**	**0.473**
	(*60.94*)	**(*39.88*)**
a_3	**0.513**	**0.377**
	(*72.88*)	**(*26.56*)**
	Average Wasted Expenditure	
	0.052	**0.019**
$\log \lvert\Omega\rvert$	**17.6**	**-7.1**

2. Klein-Rubin Utility (LES Model):

	SURE	Money Metric Estimator
β_1	**0.111**	**0.084**
	(*51.84*)	(*14.75*)
β_2	**0.298**	**0.254**
	(*48.16*)	(*22.89*)
β_3	**0.591**	**0.662**
	(*28.54*)	(*18.56*)
γ_1	**-29.457**	**-164.160**
	(*-2.62*)	(*-5.78*)
γ_2	**-65.514**	**-519.120**
	(*-2.19*)	(*-5.48*)
γ_3	**-431.639**	**-2185.800**
	(*-4.30*)	(*-4.90*)
	Average Wasted Expenditure	
	0.685	**0.005**
$\log \lvert\Omega\rvert$	**1824.2**	**10.0**

Application: Chen Dongling's Data

- **Household expenditure in Hong Kong, Israel, Singapore, Malta, Mexico, Puerto Rico, Taiwan, Ecuador, Colombia, Korea, Thailand, Sri Lanka, Zimbabwe.**

- **Expenditure on Food, Beverages, Clothing, Housing, Durables, Medicine, Transport, Recreation, Other.**

- **Sample varies - mid 1960's to mid 1980's.**

- **Have looked at Cobb-Douglas and LES models.**

- **Illustrate with C-D results for Singapore.**

SINGAPORE

(Cobb-Douglas Model)

SURE

	Coefficient	*Std. Error*	*t-Ratio*
a1	0.55376	0.51861E-02	106.78
a2	0.93374E-01	0.13315E-02	70.128
a3	0.63934E-01	0.93584E-03	68.316
a4	0.61285E-01	0.17810E-02	34.411
a5	0.43446E-01	0.10659E-02	40.760
a6	0.20880E-01	0.13215E-02	15.800
a7	0.90675E-01	0.50432E-02	17.980
a8	0.44847E-01	0.19846E-02	22.598
a9	0.27795E-01	0.17020E-02	16.331

Money Metric Estimation

	Coefficient	*Std. Error*	*t-Ratio*
a1	**0.56112**	**0.12312E-01**	**45.575**
a2	**0.93402E-01**	**0.81649E-02**	**11.439**
a3	**0.66873E-01**	**0.69614E-02**	**9.6064**
a4	**0.62492E-01**	**0.65889E-02**	**9.4845**
a5	**0.42553E-01**	**0.56502E-02**	**7.5313**
a6	**0.20485E-01**	**0.38966E-02**	**5.2572**
a7	**0.84076E-01**	**0.61502E-02**	**13.670**
a8	**0.45357E-01**	**0.55100E-02**	**8.2317**
a9	**0.23648E-01**	**0.47283E-02**	**5.0014**

Year	*Fit*
1963	0.1717019E-01
1964	0.1789235E-01
1965	0.1626308E-01
1966	0.1456958E-01
1967	0.1193746E-01
1968	0.1236099E-01
1969	0.1205157E-01
1970	0.1019978E-01
1971	0.1398006E-01
1972	0.1371392E-01
1973	0.1095797E-01
1974	0.1651435E-01
1975	0.2131675E-01
1976	0.1448834E-01
1977	0.1462583E-01
1978	0.1012950E-01
1979	0.4689684E-02
1980	0.1626083E-01
1981	0.2710507E-01
1982	0.2111993E-01
1983	0.1706796E-01
1984	0.3207137E-01

Mean	*Std. Dev.*	*Min.*	*Max.*
0.0157	0.0059	0.0047	0.0321

Concluding Remarks

- **Frequently, our models are derived as the solution to an optimization problem.**

- **In such cases, we can use the concept of "Economic Goodness-of-Fit" to augment, or replace, conventional "Statistical Goodness-of-Fit" measures.**

- **A model which "fits" the data in the usual sense need not necesarily exhibit good "economic fit".**

- **Both parametric & non-parametric methods can be used to measure "economic fit". The latter also provide a different basis for determining the parameter values themselves.**

- **Can generalize to other problems, & loss functions.**