We require: $I = \int_0^\infty (\sigma)^{-(n+1)} \exp\left[-\frac{1}{2\sigma^2} \sum_t (C_t - \beta Y_t)^2 \right] d\sigma.$

Let $z = \frac{1}{\sigma^2} \sum_t (C_t - \beta Y_t)^2.$

So, $dz = -\left[2 \sum_t (C_t - \beta Y_t)^2 / \sigma^3 \right] d\sigma$

or, $d\sigma = -\sigma^3 / \left\{ 2 \sum_t (C_t - \beta Y_t)^2 \right\} dz.$

Also, $\sigma = \left[\sum_t (C_t - \beta Y_t)^2 / z \right)^{1/2}$,

& so $\sigma^{-(n+1)} = \left[\sum_t (C_t - \beta Y_t)^2 / z \right]^{-(n+1)/2}.$

Then, $I = \int_\infty^0 \left(\frac{a}{z} \right)^{-(n+1)/2} e^{-z/2} \left(-\frac{\sigma^3}{2a} \right) dz$,

where $a = \sum_t (C_t - \beta Y_t)^2.$

So, $I = \int_0^\infty (a/z)^{-(n+1)/2} e^{-z/2} \left(\frac{1}{2}\right) \left(\frac{1}{z}\right)^{3/2} a^{3/2} \frac{1}{a} dz$

$= a^{-n/2} \int_0^\infty z^{n/2 - 1} e^{-z/2} dz.$

$\propto a^{-n/2} = \left[\sum_t (C_t - \beta Y_t)^2 \right]^{-n/2}.$