Q.8. (a) \[ \text{FALSE} \] with a large p-value we would not reject \( H_0: \lim \frac{1}{n} X' E_1 S = 0 \), we'd use C.S.

Q.8. (b) \[ \text{FALSE} \] although C.S is still weakly consistent even if the errors are heteroskedastic, \( E(\beta_1, \beta_2) \neq E(\beta_1) E(\beta_2) = \beta_1 \beta_2 \). By Slutsky's Theorem, we do have \( \lim \frac{1}{n} (\beta_1, \beta_2) = (\beta_1, \beta_2) \).

Q.8. (c) \[ \text{TRUE or FALSE} \] (\#) The Wald test applies in a wider range of situations than does the F-test. However, it is valid only asymptotically whereas the F-test is an exact, finite-sample test.

Q.8. (d1) \[ \text{TRUE} \] \[ 2' \hat{e}_{IV} = 2' l_y - X (2' X)^{-1} 2' y \]
\[ = 2' y - 2' y = 0. \]

So, if the column of \( Z \) is a constant, the I.V. residuals will sum to zero.
Q. 9

(a) Just requires a general discussion of the results.

The F-statistic is testing

\[ H_0 : \beta_1 = \beta_3 = \ldots = \beta_8 = 0 \]

vs

\[ H_A : \text{Ho false.} \]

The p-value is essentially zero. We'll reject \( H_0 \) and conclude that we have a significant linear relationship between \( y \) and the \( x \)'s.

(b) \[ R^2 = 1 - \frac{e'e}{\sum (y_i - \bar{y})^2} \]

\[ e'e = 9716547 \]

s.d. (\( y \)) = 242.1072 = \[ \sqrt{\frac{1}{n-1} \sum (y_i - \bar{y})^2} \]

So, \[ \sum (y_i - \bar{y})^2 = (204)(242.1072)^2 \]

\[ = 11957642.84 \]

And so, \[ R^2 = \left[ 1 - \frac{9716547}{11957642.84} \right] \]

\[ = 0.1874 \]

The model explains 18.74% of the sample variation of the dependent variable.
(c) The sample is so large that the Student-$t$ and Standard Normal densities are essentially equal.
   The interval is:

   \[ 10.39204 \pm 1.96 (3.9827) \quad \$000/\text{yr} \]

   or, \[ \{ 2.556, 18.198 \} \quad \$1000/\text{yr}. \]

   If constructed such intervals in the same way, many times, using the same sample size, 95\% of such intervals would cover the true unknown value of $\beta_5$. This particular interval may or may not cover $\beta_5$. We can't tell.

(\text{a}) \quad t = \frac{(17.475 - 10)}{6.1847} = 1.2056

\[
\begin{align*}
-1.96 & \quad \downarrow & \quad 0 & \quad \uparrow & \quad 1.96 \\
1.2056 & \\
\text{We would NOT REJECT } H_0: \beta_2 = 10 \\
\text{against } H_A: \beta_2 \neq 10. 
\end{align*}
\]

(\text{e}) \quad s.e. = \sqrt{1435.012} = 37.882
\( a_{10} \) \( e_{1V} = y - x b_{1V} = y - x (z'x)^{-1} z'y \)

\( = wy = W(\beta + \epsilon) \).

However, \( W x = (I - x (z'x)^{-1} z') x = (x - x) = 0 \).

So, \( e_{1V} = W \epsilon \).

\( b \) \( e_{1V}' e_{1V} = (W \epsilon)' (W \epsilon) = \epsilon' W' W \epsilon = \epsilon' [I - x (z'x)^{-1} z'] [I - x (z'x)^{-1} z'] \epsilon = 3' \epsilon + 3' \epsilon x' (x'z/z'x)^{-1} \epsilon' z' + 3' (z'x)^{-1} z' x (z'x)^{-1} \epsilon \)

\( c \) \( \text{plim} \left[ \frac{1}{n} e_{1V}' e_{1V} \right] = \text{plim} \left[ \frac{1}{n} \epsilon' \epsilon \right] = 2A + B \) (because \( \epsilon' x (z'x)^{-1} z' \epsilon = \epsilon' z (z'x)^{-1} \epsilon' \) as both are scalars).

\( A = \text{plim} \left[ \frac{1}{n} x' \epsilon \right] = \frac{1}{n} \text{plim} [z'x]^{-1} \frac{1}{n} \text{plim} (z' \epsilon) \)

by Slutsky's Theorem.

So, \( A = 0 \cdot Q_{xx}^{-1} \cdot 0 = 0 \)

\( B = 0 \cdot Q_{xx}^{-1} \cdot Q_{xx} \cdot 0 = 0 \).
So, \( \text{plim} \left[ \frac{1}{n} e_{iv} e_{iv} \right] = \text{plim} \left[ \frac{1}{n} \varepsilon \varepsilon \right] \\
= \text{plim} \left[ \frac{1}{n} \sum_{i=1}^{n} \varepsilon_{i}^2 \right] \\
By Slutsky\'s Theorem, this is \( E(\varepsilon^2) \), or \( \sigma^2 \). \\

\[ (a) \]
\[ (y - X_1 \hat{\beta}_1) = X_2 \beta_2 + \nu \]
\[ \tilde{\beta}_2 = (X_2'X_2)^{-1} X_2' (y - X_1 \hat{\beta}_1) \]
\[ = (X_2'X_2)^{-1} X_2' [X_1 \beta_1 + X_2 \beta_2 + \varepsilon - X_1 \hat{\beta}_1] \]
\[ E(\tilde{\beta}_2) = (X_2'X_2)^{-1} X_2' X_1 \beta_1 + \beta_2 + 0 - (X_2'X_2)^{-1} X_2' X_1 E(\hat{\beta}_1) \]
But \( \hat{\beta}_1 \) is unbiased, so \( E(\hat{\beta}_1) = \beta_1 \)
\( \therefore \) \( E(\tilde{\beta}_2) = \beta_2 \). (Unbiased, too)

\[ (b) \]
\[ e = y - X\beta = My = ME. \]
\[ \hat{\beta} = [X'M_e X]^{-1} X'M_e y \]
where \( M_e = [I - e(e'e)^{-1}e'] \)
\[ X'M_e = X' - X'e (e'e)^{-1}e' = X' \]
So, \( \hat{\beta} = (X'X)^{-1} X'y = b \).

\[ (c) \]
\[ e'y = e'y + e'e = e'Xb + e'e. \]
But \( X'e = 0 \), so \( e'y = e'e \)
(d) \[ \hat{z} = [e'M_x e]^{-1} e'M_x y \]

where: \[ M_x = I - X(x'X)^{-1}x' \]

So, \[ e'M_x = e' - e'X(x'X)^{-1}x' = e' \]

because \[ x'e = 0. \]

So, \[ \hat{z} = (e'e)^{-1} e'y \]

We've just seen that \[ e'y = e'e \], so

\[ \hat{z} = (e'e)^{-1} e'e = 1. \]

(because \( e'e \) is a scalar).