ECON 545 – Fall 2014
Mid-Term Test Solution

Q.1. (d) ; Q.2. (d) ; Q.3. (d) ; Q.4. (b) ;

Q.5. (c) ; Q.6. (c) ; Q.7. (b).

Q.8. (a)

FALSE With a large p-value we would NOT reject $H_0 : \text{plim}[\frac{1}{n}X'\varepsilon] = 0$, & we'd use OLS.

(b) FALSE Although OLS is still weakly consistent even if the errors are heteroskedastic, $E(b_1 b_2) \neq E(b_1)E(b_2) \neq \beta_1\beta_2$. By Slutsky's Theorem, we do have $\text{plim}(b_1 b_2) = \beta_1\beta_2$.

(c) TRUE or FALSE (!) The Wald test applies in a wider range of situations than does the F-test. However, it is valid only asymptotically whereas the F-test is an exact, finite-sample, test.

(d) TRUE

$$Z'e_{IV} = Z'(y - X(Z'X)^{-1}Zy)$$
$$= Z'y - Z'y = 0.$$

So, if one column of Z is a constant, the I.V. residuals will sum to zero.

Q. 9.

(a) Just requires a general discussion of the results.

The F-statistic is testing

$$H_0 : \beta_2 = \beta_3 = \cdots = \beta_8 = 0$$

vs H_A : H_0 false.

The p-value is essentially zero. We'd REJECT H_0 & conclude that we have a significant linear relationship between y & the x's.

(b) $$R^2 = 1 - \frac{e'e}{\Sigma(y_i - \bar{y})^2}$$

$$e'e = 9716547$$

$$s.d.(y) = 242.1072 = \sqrt{\frac{1}{n-1}\sum_i (y_i - \bar{y})^2}$$

So, $$\sum_i (y_i - \bar{y})^2 = (204)(242.1072)^2$$
$$= 11957642.84$$

And so, $$R^2 = \left\{1 - \frac{9716547}{11957642.84}\right\}$$

$$= 0.1874$$

The model explains 18.74% of the sample variation of the dependent variable.

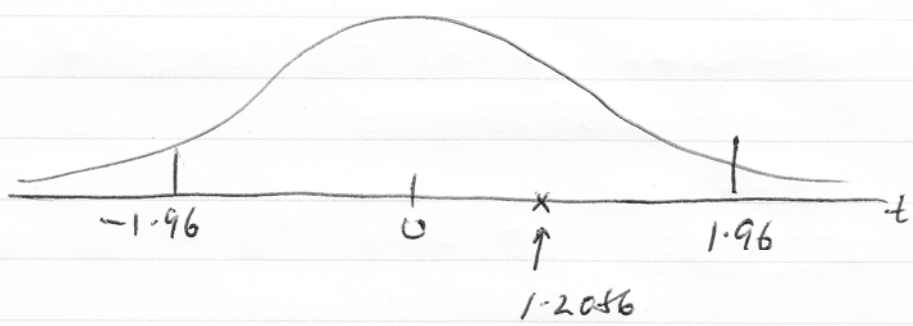
(c) The sample is so large that the Student-t & Standard Normal densities are essentially equal.
The interval is:

$$10.39204 \pm 1.96\,(3.9827) \qquad \$'000/yr$$

or, $[\,2.586,\ 18.198\,]$ $\$'000/yr$.

If constructed such intervals in the same way, many times, using the same sample size, 95% of such intervals would cover the true, unknown, value of β_5. This <u>particular</u> interval may or may not cover β_5. We can't tell.

(d) $t = (17.475 - 10) / 6.1847 = 1.2086$



We would <u>NOT REJECT</u> $H_0: \beta_2 = 10$ against $H_A: \beta_2 \neq 10$.

(e) $s.e = \sqrt{1435.012} = 37.882$

Q. 10 (a) $e_{IV} = y - X b_{IV} = y - X(Z'X)^{-1} Z'y$

$= Wy = W(X\beta + \varepsilon).$

However, $WX = (I - X(Z'X)^{-1}Z')X = (X - X)$
$= 0$

So, $e_{IV} = W\varepsilon.$

(b) $e_{IV}' e_{IV} = (W\varepsilon)'(W\varepsilon) = \varepsilon' W' W \varepsilon$

$= \varepsilon' [I - Z(X'Z)^{-1}X'][I - X(Z'X)^{-1}Z']\varepsilon$

$= \varepsilon'\varepsilon - \varepsilon' Z(X'Z)^{-1}X'\varepsilon - \varepsilon' X(Z'X)^{-1}Z'\varepsilon$

$+ \varepsilon' Z(X'Z)^{-1}X'X(Z'X)^{-1}Z'\varepsilon$

(c) $\text{plim}[\tfrac{1}{n} e_{IV}' e_{IV}] = \text{plim}[\tfrac{1}{n}\varepsilon'\varepsilon] - 2A + B.$

(because $\varepsilon' X(Z'X)^{-1}Z'\varepsilon = \varepsilon' Z(X'Z)^{-1}X'\varepsilon$,
as both are <u>scalars</u>.)

$A = \text{plim}[\tfrac{1}{n}X'\varepsilon]'[\tfrac{1}{n}\text{plim}(Z'X)]^{-1}[\tfrac{1}{n}\text{plim}(Z'\varepsilon)]$

by Slutsky's Theorem.

So, $A = \gamma' Q_{ZX}^{-1} \cdot 0 = 0$

& $B = 0 \cdot Q_{ZX}'^{-1} Q_{XX} Q_{ZX} \cdot 0 = 0$

So, $\text{plim}\left[\frac{1}{n} e_{IV}' e_{IV}\right] = \text{plim}\left[\frac{1}{n}\varepsilon'\varepsilon\right]$

$$= \text{plim}\left[\frac{1}{n}\sum_i \varepsilon_i^2\right]$$

By Slutsky's Theorem, this is $E(\varepsilon_i^2)$, or σ^2 #.

Q-11

(a) $(y - X_1\hat{\beta}_1) = X_2\beta_2 + v$

$$\tilde{\beta}_2 = (X_2'X_2)^{-1}X_2'(y - X_1\hat{\beta}_1)$$
$$= (X_2'X_2)^{-1}X_2'[X_1\beta_1 + X_2\beta_2 + \varepsilon - X_1\hat{\beta}_1]$$

$$E(\tilde{\beta}_2) = (X_2'X_2)^{-1}X_2'X_1\beta_1 + \beta_2 + 0 - (X_2'X_2)^{-1}X_2'X_1E(\hat{\beta}_1)$$

But $\hat{\beta}_1$ is unbiased, so $E(\hat{\beta}_1) = \beta_1$

& So $E(\tilde{\beta}_2) = \beta_2$. (Unbiased, too).

(b) $e = y - Xb = My = M\varepsilon$.

$$\hat{\beta} = [X'M_eX]^{-1}X'M_ey$$

where $M_e = [I - e(e'e)^{-1}e']$

$$X'M_e = X' - X'e(e'e)^{-1}e' = X'$$

So, $\hat{\beta} = (X'X)^{-1}X'y = b$. #

(c) $e'y = e'\hat{y} + e'e = e'Xb + e'e$.

But $X'e = 0$, so $e'y = e'e$

(d) $\hat{\alpha} = [e'M_x e]^{-1} e'M_x y$

where: $M_x = I - X(X'X)^{-1}X'$.

So, $e'M_x = e' - e'X(X'X)^{-1}X' = e'$,

because $X'e = 0$.

So, $\hat{\alpha} = (e'e)^{-1} e'y$

We've just seen that $e'y = e'e$, so

$\hat{\alpha} = (e'e)^{-1} e'e = 1$.

(because $e'e$ is a scalar).