Department of Economics **University of Victoria**

ECON 546: Themes in Econometrics
Term Test, February 2016

Instructor:	David Giles
Instructions:	Answer **ALL QUESTIONS**, & put all answers in the booklet provided.
Time Allowed:	75 minutes (Total marks = 75 – *i.e., one mark per minute*.)
Number of Pages:	FOUR (*A separate set of statistical tables & a formula sheet are also provided*.)

Question 1:

Write brief notes (and/or diagrams) to explain what we mean by each of the following:

(a) The Lindeberg-Lévy Central Limit Theorem.

(b) A Mini-Max estimator.

(c) An Asymptotically Efficient estimator.

Total: 18 Marks

Question 2:

A Pearson Type III distribution has three parameters. When one of these parameters is set to 0.5, the density function for a random variable, Y, that follows this distribution is:

$$p(y) = \frac{1}{\beta\sqrt{\pi}}\left(\frac{y-\alpha}{\beta}\right)^{-1/2} \exp\{-(y-\alpha)/\beta\} \quad ; \quad y > \alpha$$

and the associated characteristic function can be shown to be:

$$\phi_y(t) = \exp\{i\alpha t\}(1 - i\beta t)^{-1/2}.$$

(a) Suppose we construct a new random variable, $X = Y_1 + Y_2$, where Y_1 and Y_2 are independent. What is the characteristic function for X?

3 marks

(b) Use this characteristic function to prove that $E(X) = 2\alpha + \beta$.

8 marks

(c) Now, suppose that we know that $\alpha = 0$. If we have n independent sample values, show that the MLE of β is $\tilde{\beta} = 2\bar{y} = \frac{2}{n}\sum_{i=1}^{n} y_i$. (*Don't forget the second-order condition*.)

8 marks

(d) What is the MLE for $E(X)$ in part (b), under the conditions stated in part (c)?

1 mark

Total: 20 Marks

Question 3:

The density for a random variable, *Y*, that follows a Rayleigh distribution is:

$$p(y \mid \theta) = (y/\theta^2)\exp\{-y^2/(2\theta^2)\} \quad ; \quad y > 0 \; ; \; \theta > 0 .$$

and the k^{th} moment about the origin is $\mu_k' = \theta^k 2^{k/2}\Gamma[1+(k/2)]$. Here, the Gamma function satisfies the recurrence relationship, $\Gamma(x+1) = x\Gamma(x)$; $\Gamma(1) = 1$ and $\Gamma(1/2) = \sqrt{\pi}$.

(a) If we have *n* independent sample values, show that the MLE of θ is $\tilde{\theta} = \sqrt{(1/2n)\sum_i y_i^2}$.

7 marks

(b) Show that the mean of *Y* is $\theta\sqrt{\pi/2}$ and the variance of *Y* is $\theta^2(4-\pi)/2$.What are the MLE's for the mean, variance *and standard deviation* of *Y*, and what desirable properties will these estimator have?

7 marks

(c) Derive the Likelihood Ratio Test statistic for testing H_0: $\theta = 1$ against H_1: $\theta \neq 1$.

5 marks

(d) Suppose that *n* = 100 and $\sum_{i=1}^{n} y_i^2 = 180$. Apply the LRT. What assumptions have you made? Is your conclusion sensitive to your choice of significance level?

5 marks

Total: 24 Marks

Question 4:

This question relates to the estimation of a particular "Tobit" model, which explains hours worked in terms of several explanatory variables. The data for the dependent variable are truncated from below – we don't observe the characteristics of people who work zero hours. In addition, the underlying distribution is non-Normal.

(a) What does **OUTPUT** suggest about the success (or otherwise) of this estimation?

4 marks

(b) What do you conclude from **OUTPUT 2**?

3 marks

(c) Use the results in **OUTPUT 1** and **OUTPUT 3** to test the same hypothesis as in **OUTPUT 2**, but using a different test.

4 marks

(d) What do you conclude from **OUTPUT 4**?

2 marks

Total: 13 Marks

OUTPUT 1

Dependent Variable: HOURS
Method: ML - Censored Extreme Value (Newton-Raphson / Marquardt steps)
Date: 02/17/16 Time: 13:45
Sample: 1 753
Included observations: 753
Left censoring (value) at zero
Convergence achieved after 8 iterations
Coefficient covariance computed using the Huber-White method

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	1719.574	486.2058	3.536720	0.0004
FAMINC	0.206550	0.021072	9.802242	0.0000
NWIFEINC	-213.6402	21.96998	-9.724189	0.0000
EDUC	-24.52296	21.26279	-1.153327	0.2488
EXPER	57.41200	32.19129	1.783464	0.0745
EXPERSQ	-0.781939	0.809279	-0.966216	0.3339
AGE	-37.14717	12.04116	-3.085017	0.0020
KIDSLT6	-534.8157	150.6138	-3.550907	0.0004
KIDSGE6	1.396108	42.76065	0.032649	0.9740
Error Distribution				
SCALE:C(10)	1112.469	103.8358	10.71373	0.0000

Mean dependent var	740.5764	S.D. dependent var	871.3142
S.E. of regression	577.0212	Akaike info criterion	9.899979
Sum squared resid	2.47E+08	Schwarz criterion	9.961388
Log likelihood	-3717.342	Hannan-Quinn criter.	9.923637
Avg. log likelihood	-4.936709		

Left censored obs	325	Right censored obs	0
Uncensored obs	428	Total obs	753

OUTPUT 2

Wald Test:
Equation: TOBIT

Test Statistic	Value	df	Probability
F-statistic	0.729663	(3, 743)	
Chi-square	2.188988	3	

Null Hypothesis: C(4)=C(6)=C(9)=0

OUTPUT 3

Dependent Variable: HOURS
Method: ML - Censored Extreme Value (Newton-Raphson / Marquardt steps)
Date: 02/17/16 Time: 13:36
Sample: 1 753
Included observations: 753
Left censoring (value) at zero
Convergence achieved after 9 iterations
Coefficient covariance computed using the Huber-White method

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	1636.636	531.8561	3.077216	0.0021
FAMINC	0.205555	0.021262	9.667904	0.0000
NWIFEINC	-213.6543	22.38386	-9.545016	0.0000
EXPER	35.44226	13.79912	2.568443	0.0102
AGE	-39.27781	13.94171	-2.817287	0.0048
KIDSLT6	-562.0874	156.2687	-3.596928	0.0003
Error Distribution				
SCALE:C(7)	1113.790	104.0929	10.69996	0.0000

Mean dependent var	740.5764	S.D. dependent var	871.3142
S.E. of regression	585.3175	Akaike info criterion	9.896655
Sum squared resid	2.56E+08	Schwarz criterion	9.939641
Log likelihood	-3719.091	Hannan-Quinn criter.	9.913216
Avg. log likelihood	-4.939032		
Left censored obs	325	Right censored obs	0
Uncensored obs	428	Total obs	753

OUTPUT 4

Gradients of the Objective Function
Gradients evaluated at estimated parameters
Equation: TOBIT
Method: ML - Censored Extreme Value
Specification: HOURS C FAMINC NWIFEINC EXPER AGE KIDSLT6

Variable	Sum	Mean	Weighted Grad.
C	1.47E-16	1.95E-19	4.55E-10
FAMINC	4.74E-12	6.29E-15	-1.74E-17
NWIFEINC	8.61E-15	1.14E-17	1.00E-12
EXPER	1.15E-15	1.53E-18	-1.06E-12
AGE	1.03E-14	1.37E-17	2.73E-13
KIDSLT6	5.24E-17	6.95E-20	-4.50E-10
SCALE:C(7)	3.13E-16	4.16E-19	1.05E-10

END OF TEST