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The Econometrics of Temporal Aggregation: 1956-2014

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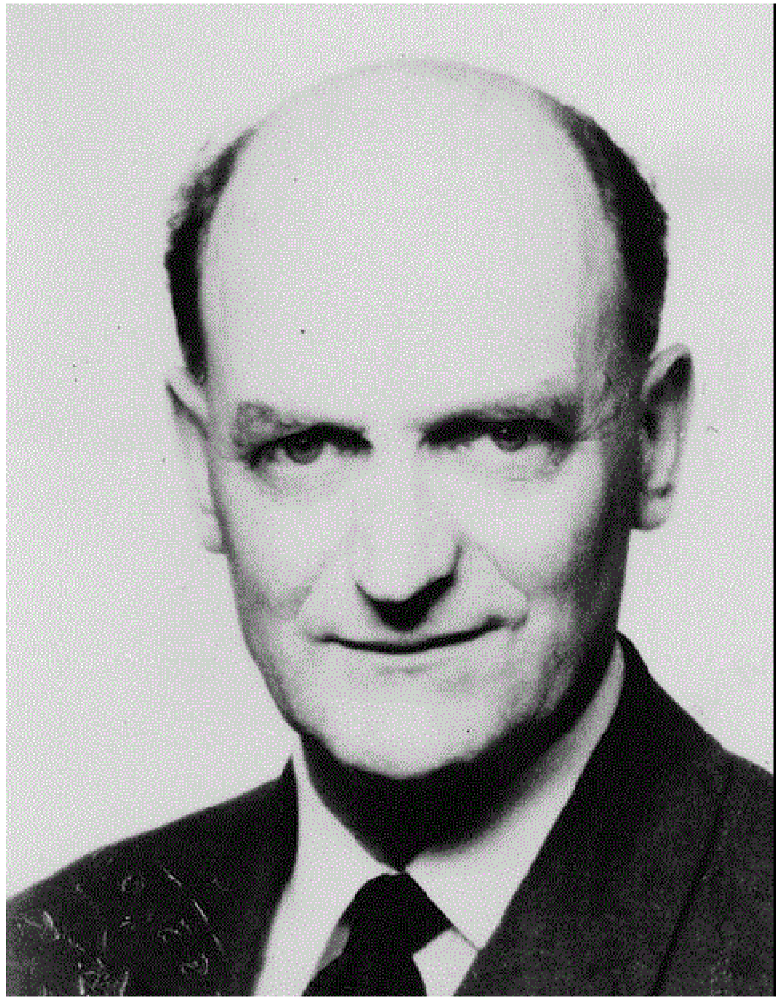
A. W. H. Phillips Memorial Lecture

- Phillips' contributions: stabilization & control, growth, the Phillips Curve, the Lucas critique, & continuous time modelling.
- I'll consider the last of these contributions – summarize its influence on econometric issues surrounding temporal aggregation of data over the past (≈) 60 years.
- This lecture will include some new results on the impact of temporal aggregation on various hypothesis tests used in econometrics.

Road Map

- Continuous time econometrics – a New Zealand contribution
- Temporal aggregation, selective sampling of time-series data:
  1. Unit roots, & Cointegration, Granger causality
  2. Economic dynamics
  3. Model estimation
  4. Hypothesis tests, with some new results
  5. Forecasting performance
- Modelling with mixed data frequencies
- Summary & some open research questions

Bill Phillips & Continuous-Time Modelling

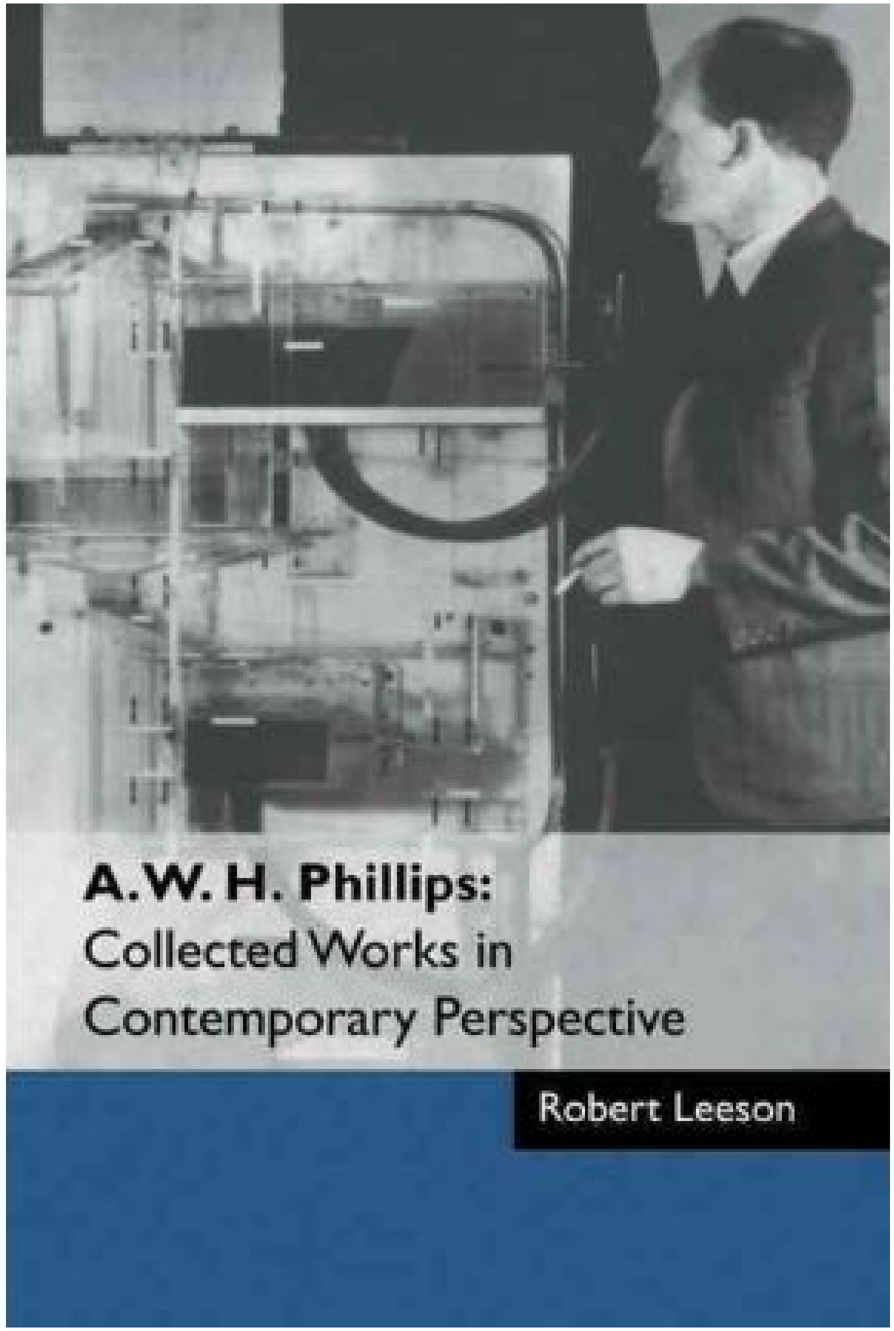


Alban William Housego (Bill) Phillips (1914 – 1975)

- Pre-Phillips: Bachelier (1900), Wiener (1923), Bartlett (1946), Grenander (1950), Koopmans (1950).

- Phillips (1956) *Economica* – Model formulation & estimation.
- Phillips*(1959) *Biometrika* – The most general treatment.
- Phillip (1962) Presentation at Nuffield College, Oxford – further estimation issues.
- Phillips (1962) Incomplete paper – VARMA modelling.
- Phillips (1966) Walras-Bowley Lecture, N.A. Meeting of the E.S. – Maximum Likelihood estimation of simultaneous equations models with lagged endogenous variables & MA errors.

- Post-Phillips: Rex Bergstrom, Cliff Wymer, Peter Phillips.


A.W. H. Phillips:
Collected Works in
Contemporary Perspective
Robert Leeson

The Case for Continuous-Time Econometrics

- “The economy does not cease to exist in between observations.” (Bartlett, 1946; Phillips, 1988).
- "In the modern era, news arrives at shorter intervals and economic activities take place in a nonstop fashion." (Bergstrom and Nowman, 2007; Yu, 2014).
- “...the lag functions may be specified in a way which allows the length of the lag to be estimated rather than assumed.”
- “A continuous time model .... can be specified and analysed independently of the observation interval of the sample to be used for estimation, and the forecasting interval is also independent of the observation interval.”   (Wymer, 2009)

"Re-discovered" by Sims (1971 & Geweke (1978).

Some Basic Results

- See Bergstrom (1984).
- Typical discrete-time SEM:

$$\Gamma \boldsymbol{y_t} + B_0 \boldsymbol{x_t} + \sum_{r=1}^{k} B_r \boldsymbol{y_{t-r}} = \boldsymbol{u_t}$$

$$E[\boldsymbol{u_t}] = \boldsymbol{0} \;\; ; E[\boldsymbol{u_s} \boldsymbol{u_t'}] = \boldsymbol{0} \;\; ; E[\boldsymbol{u_t} \boldsymbol{u_t'}] = \Sigma$$

- Lots of (identifying) restrictions on Γ and the B_i matrices.
- Continuous time: $y_1(t), \dots, y_n(t);\; x_1(t), \dots, x_m(t)$.
- Stock variable - $y_{it} = y_i(t)$; *etc*.
- Flow variable - $y_{jt} = \int_{t-1}^{t} y_j(s) ds$; *etc*.

- Continuous-time *system*. For simplicity, if no exogenous variables in the model:

$$d\boldsymbol{y}(t) = [A(\boldsymbol{\theta})\boldsymbol{y}(t) + b(\boldsymbol{\theta})]dt + \zeta(dt)$$

- Then there is an *exact discrete representation* of the continuous-time model:

$$\boldsymbol{y_t} = \sum_{r=1}^{k} F_r\,(\boldsymbol{\theta})\,\boldsymbol{y_{t-r}} + \boldsymbol{\eta_t}$$

$$\boldsymbol{\eta_t} = \sum_{r=0}^{l} C_r(\boldsymbol{\theta})\boldsymbol{\varepsilon_{t-r}}$$

$$E[\boldsymbol{\varepsilon_t}] = \mathbf{0} \;\; ; E[\boldsymbol{\varepsilon_s}\boldsymbol{\varepsilon_t'}] = \mathbf{0} \;\; ; E[\boldsymbol{u_t}\boldsymbol{u_t'}] = K(\boldsymbol{\theta})$$

- Lagged values of *all of the variables* in the model appear in *all of the equations*, and errors follow a vector MA process. It's a VARMA model, with particular restrictions on parameters.
- The form of the VARMA model doesn't depend on the observation period – only on the *form* of the continuous-time system.
- Use FIML estimation to get asymptotically efficient, & super-consistent, estimates of $\boldsymbol{\theta}$. Pretty challenging in 1956!
- Notice that Phillips' work really made the case for (restricted) VARMA(X) modelling. Well ahead of its time.

Temporal Aggregation

- This will be main focus in this lecture.
- Flow variables – monthly to quarterly; quarterly to annual, *etc*.
- *Summing* data over several periods before using them.
- Rather analogous to the shift from Continuous time to Discrete time - *integrating* the data.
- So, expect to encounter some similar modelling & inferential issues.
- These are driven largely by MA effects caused by aggregation.

Selective Sampling

- Stock variables – last quarter of year; middle month of quarter, *etc*.
- As in the case of continuous-time modelling, this tends to be somewhat less problematic than temporal aggregation.
- However, not totally innocuous.
- Relationship to "missing observations" problem, but we're **not** imputing the data.

Some Early Contributions

- Theil (1954), Nerlove (1959), Working (1960), Ironmonger (1959), Mundlak (1961), Telser (1967), Engle (1969, 1970), Moriguchi (1970), Zellner & Montmarquette (1971).
- Temporal aggregation more of a problem for distributed lag, and dynamic, models than for static models.
- The long-run properties of a model are largely unaffected by temporal aggregation, but the short-run properties can be very sensitive.

The **BIG** Picture

- Does the specification of the model suit the form of the data?
- *Analogy* with non-stationary time-series.
- Features of data have implications for modelling, inference.

N-S T-S: Spurious regressions; Unbalanced regressions; ECM, cointegration; implications for estimation, hypothesis testing & forecasting. *Non-standard asymptotics*.

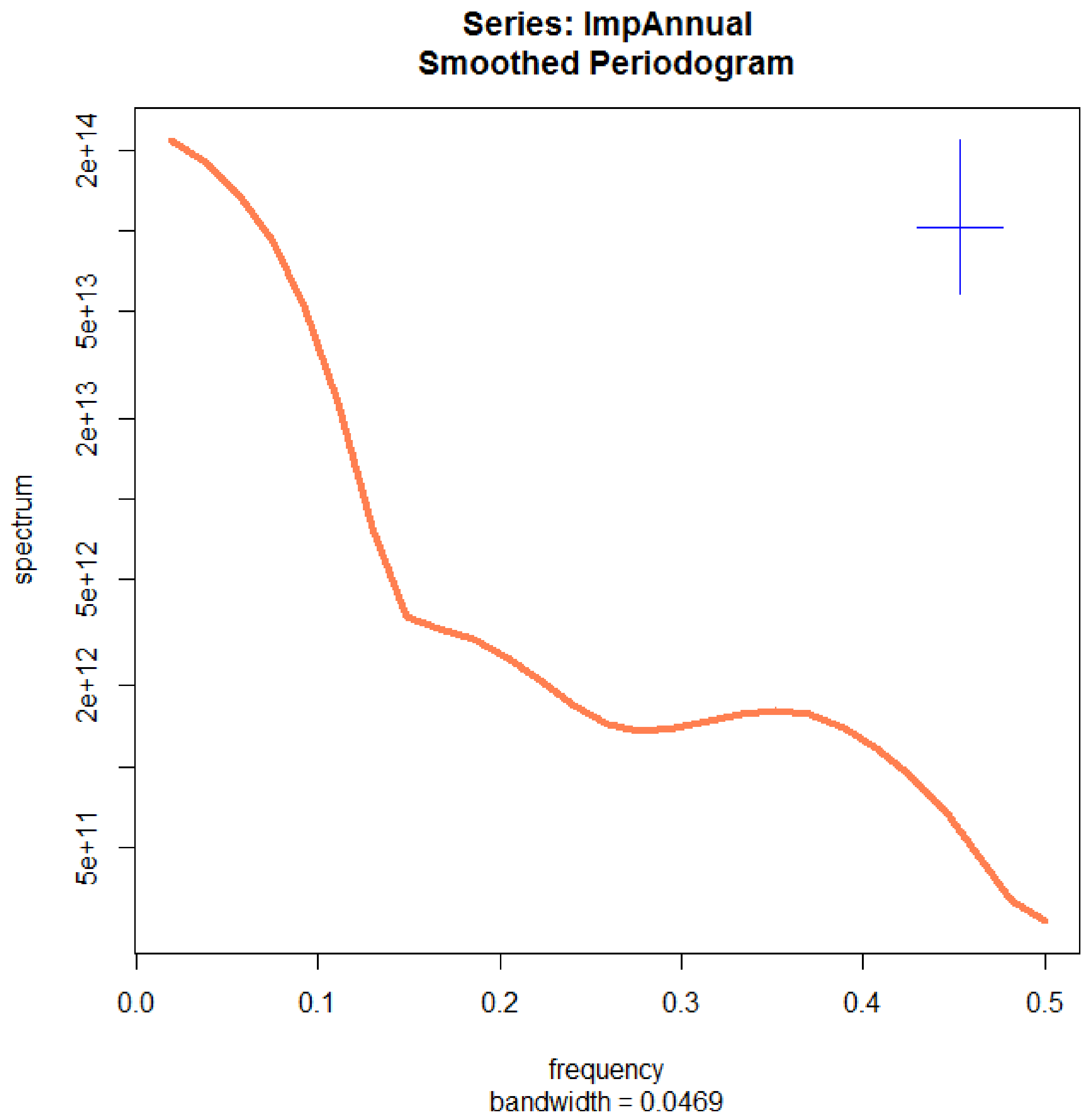
Aggregation: Alters many characteristics of time-series such as model dynamics, lag relationships; can alter causality, non-linearities; implications for estimation, hypothesis testing & forecasting. *Lose asymptotic normality*.

Implications for Time-Series Properties

Spectral shape

Granger (1966)

- Granger & Mortenstern (1963), Hatanaka (1963), Medel (2014).
- Implications for modelling – *e.g.*, DSGE models – Sala (2014).
- Periodogram relatively invariant to temporal aggregation or selective sampling, and to length of sample.
- N.Z. merchandise imports (c.i.f.), 1984 – 2014: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0.4
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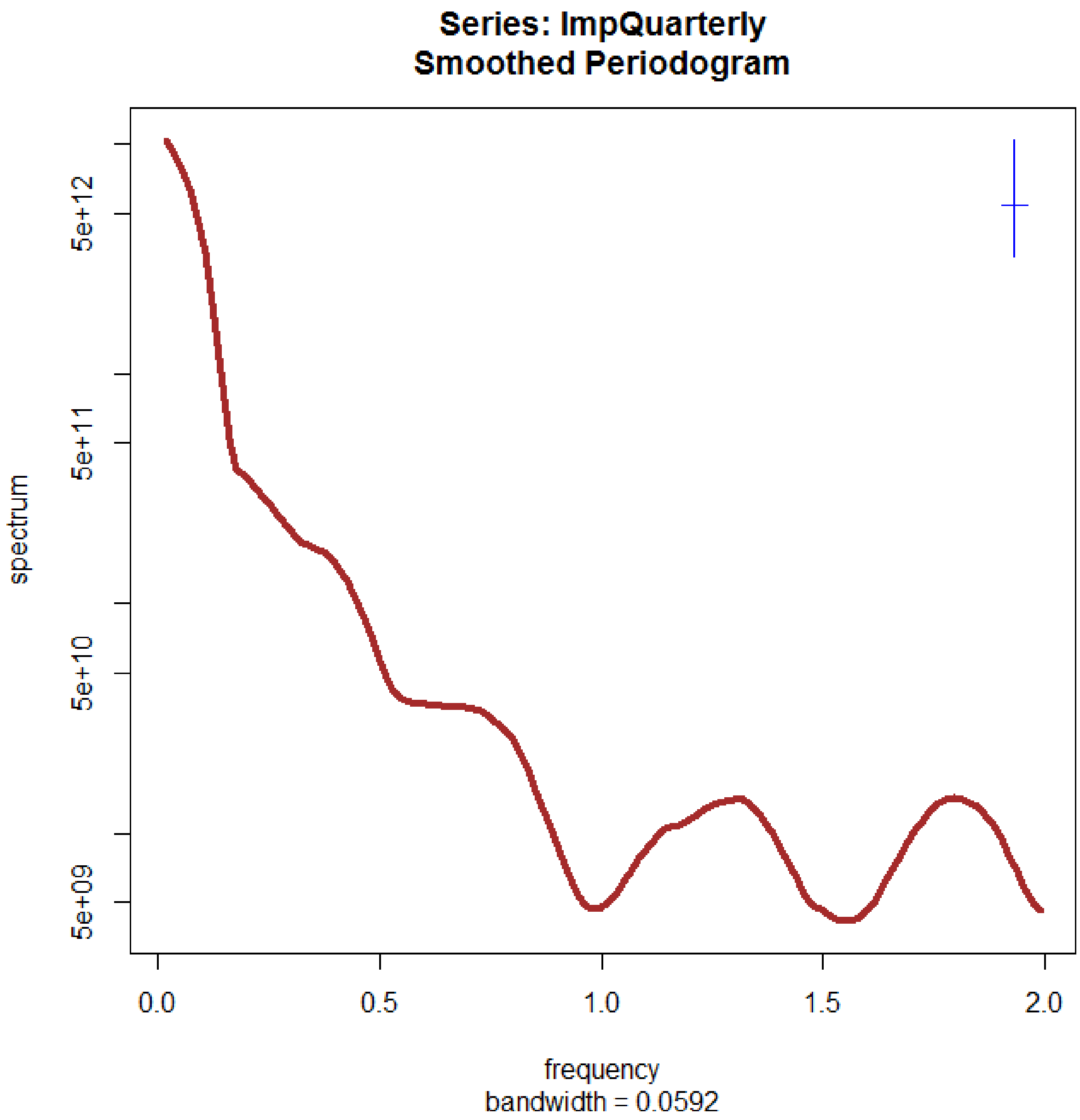
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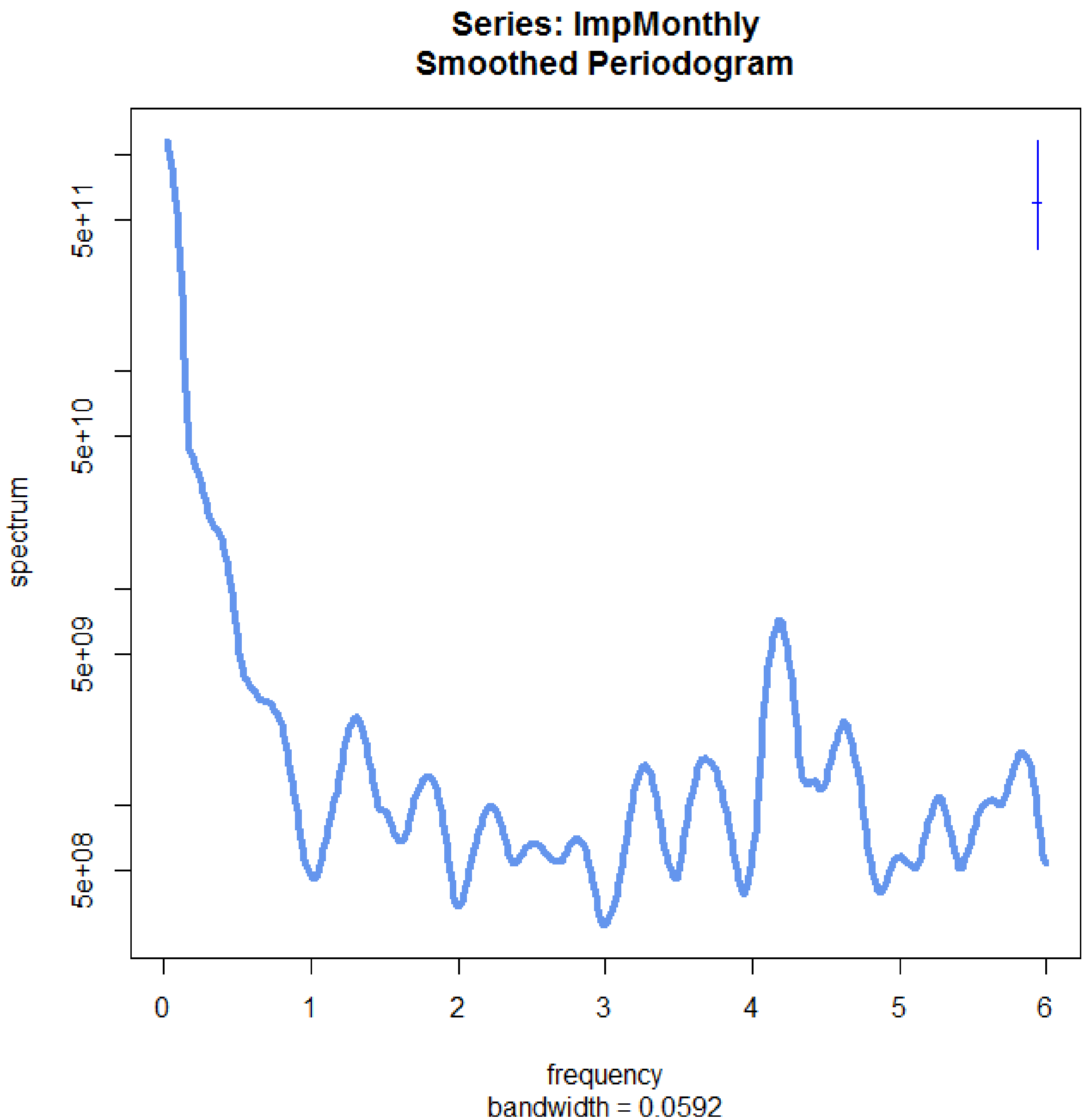
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1
2
3
4
5
6
frequency
bandwidth = 0.0592

White noise data

- *Aggregation* using m observations introduces MA(m-1) effect
- $Y_t = y_t + y_{t-1} + y_{t-2}$ (m = 3 ; monthly to quarterly)
- $y_t \sim i.i.d.\ [0, \sigma^2]$
- $Y_t = y_t + \theta_1 y_{t-1} + \theta_2 y_{t-2}$; $\theta_1 = \theta_2 = 1$
- Y follows a ***non-invertible*** MA(2) process.
- Fails conditions: $\theta_1 + \theta_2 < 1$; $\theta_2 - \theta_1 < 1$; $|\theta_2| < 1$
- *Implications for MLE* & testing,

White noise data

- *Systematic Sampling* every m^{th} observation implies White Noise
- $y_t \sim i.i.d.\ [0\,,\sigma^2]$
- $Y_t = y_t$ (m = 3 ; monthly, end of quarter)
- $Y_t = y_{t-1}$ (m = 3 ; monthly, middle of quarter)
- $V(Y_t) = \sigma^2 I_{T^*}$; $T^* = (T/m)$

However:

- $Y_t = (y_{t-2} + y_{t-1} + y_t)/3$ (m = 3 ; average over quarter)
- Non-invertible MA(2) process, again.
- In general, $Y_t = (y_{t-m+1} + \dots \dots + y_{t-1} + y_t)/m$ ~ MA(m-1) process

ARIMA processes

- $y_t \sim \text{ARIMA}(p, d, q)$.
- Y_t is the temporally aggregated or selectively sampled series.
- *Aggregation* using *m* observations.
- $Y_t \sim \text{ARIMA}(p, d, r)$ where $r = \lfloor (p + d) + (q - p - d)/m \rfloor$.
- If y_t is AR(1), then Y_t is ARMA(1,1) .
- If y_t is a random walk, then Y_t is IMA(1,1) .
- *Systematic Sampling* every *m*th observation.
- $Y_t \sim \text{ARIMA}(p, d, r)$ where $r = \lfloor (p + d + 1) + (q - p - d - 1)/m \rfloor$.
- If y_t is a random walk, then Y_t is also a random walk.

For *either temporal aggregation, or systematic sampling*:

- If data are ***generated*** over a time interval that is small relative to the ***observation*** interval, then *m* will be large.
- In this case the AR component of the process becomes irrelevant; the unit root components are unaltered; & the MA component simplifies.
- For large enough *m*, $Y_t \sim \text{IMA}(d, d)$. See Tiao (1972).
- In the case of a *seasonal* time-series, if $m \geq s$, then process becomes regular *non-seasonal* ARIMA. See Wei (1979, 2006).

Unit Roots and Cointegration

Unit roots

- Integrated time-series remain integrated under temporal aggregation. See Lütkepohl (1987), Marcellino (1999).

$$y_t \sim I(1) \Rightarrow w_t \equiv (y_t - y_{t-1}) \sim I(0)$$

$$\Rightarrow W_\tau \equiv (Y_\tau - Y_{\tau-1}) = [\sum_{j=0}^{m-1} y_{\tau-j} - \sum_{j=0}^{m-1} y_{\tau-1-j}]$$

$$= (y_\tau - y_{\tau-1}) + (y_{\tau-1} - y_{\tau-2}) + \ldots\ldots + (y_{\tau-m+1} - y_{\tau-m})$$

$$= (w_\tau + w_{\tau-1} + \cdots \ldots + w_{\tau-m+1}) \sim I(0)$$

$$\Rightarrow Y_\tau \sim I(1) \,.$$

- *However*, what about ***tests*** for stationarity/non-stationarity?

- Pierce and Snell (1995):

$$y_t = \rho y_{t-1} + u_t \quad ; \quad u_t \sim \text{ARMA}(p, q) \text{ ; } p \text{ and } q \text{ finite}$$

$$H_0 \colon \rho = 1 \quad \textit{vs.} \quad H_A \colon \rho = e^{-c/T} \qquad ; \ c > 0$$

(*Sequence of local alternatives*.)

Temporal aggregation or selective sampling; interval = m.

"Any test that is asymptotically independent of nuisance parameters under both H_0 and H_A has a limiting distribution under both H_0 and H_A that is independent of m."

- **A**DF, PP, Hall-IV, tests, *etc*. (Similarly for KPSS, *etc*.)

- What matters is the temporal *span*, not the *number* of obs.
- Intuition – loss of power due to less observations is made up by increased "separation" of H_0 and H_A.
- This is also essentially true in finite samples – see the Monte Carlo evidence of Pierce and Snell (1995), and others.

N.Z. Imports data

	T	ADF	lag	p
1960M1 – 2014M3	651	-2.05	2	0.57
1960Q1 – 2014Q1	217	-2.03	0	0.58
1960 – 2013	54	-1.409	4	0.85

Cointegration

- If y_t & x_t are *cointgrated*, so are Y_t & X_t (Granger, 1988).

  $x \sim I(1)$ and $y_t \sim I(1)$

  There exists a unique α such that $z_t \equiv (y_t - \alpha x_t) \sim I(0)$.

$$\Rightarrow Z_\tau \equiv (Y_\tau - \alpha X_\tau) = [\,\sum_{j=0}^{m-1} y_{\tau-j} - \alpha \sum_{j=0}^{m-1} X_{\tau-j}]$$

$$= (y_\tau - \alpha x_\tau) + (y_{\tau-1} - \alpha x_{\tau-1}) + \ldots\ldots + (\,y_{\tau-m+1} - \alpha x_{\tau-m+1})$$

$$= (z_\tau + z_{\tau-1} + \ldots\ldots + z_{\tau-m+1}) \sim I(0)$$

$\Rightarrow Y_\tau$ and X_τ are also cointegrated.

- Cointegration implies existence of an ECM for y_t & x_t of form:

$$\Delta x_t = -\rho_1 z_{t-1} + lagged\{\Delta x_t \,;\, \Delta y_t\} + residual$$

$$\Delta y_t = -\rho_2 z_{t-1} + lagged\{\Delta x_t \,;\, \Delta y_t\} + residual$$

  and at least one of ρ_1 and ρ_2 is non-zero.

- $\Delta X_\tau \equiv (X_\tau - X_{\tau-1}) = \left[\sum_{j=0}^{m-1} x_{\tau-j} - \sum_{j=0}^{m-1} x_{\tau-1-j}\right]$

$$= \Delta x_\tau + \Delta x_{\tau-1} + \cdots + \Delta x_{\tau-m}$$

$$= -\rho_1 z_{\tau-1} + lagged\{\Delta x_\tau \,;\, \Delta y_\tau\} - \cdots$$

$$-\rho_1 z_{\tau-m-1} + lagged\{\Delta x_{\tau-m} \,;\, \Delta y_{\tau-m}\} + residual$$

$$= -\rho_1 Z_{\tau-1} + lagged\{\Delta X_\tau \,;\, \Delta Y_\tau\} + residual$$

- So, there must also be an ECM for Y_t and X_t , but its lag structure may be different from that for y_t and x_t
- Recall "Early Contributions".
- The results of Pierce and Snell also apply to *tests* of cointegration – $e.g.$, Engle-Granger, Johansen.
- What matters is the *span* of the sample, not the sample *size*.
- Marcellino (1996): If $\vec{x} \sim C(d, d - c)$ then
  1. The number & composition of the cointegrating vectors are invariant to temporal aggregation.
  2. Loadings of aggregated & diasggregated ECT's are same.

Granger Causality

- **(One) Definition:**

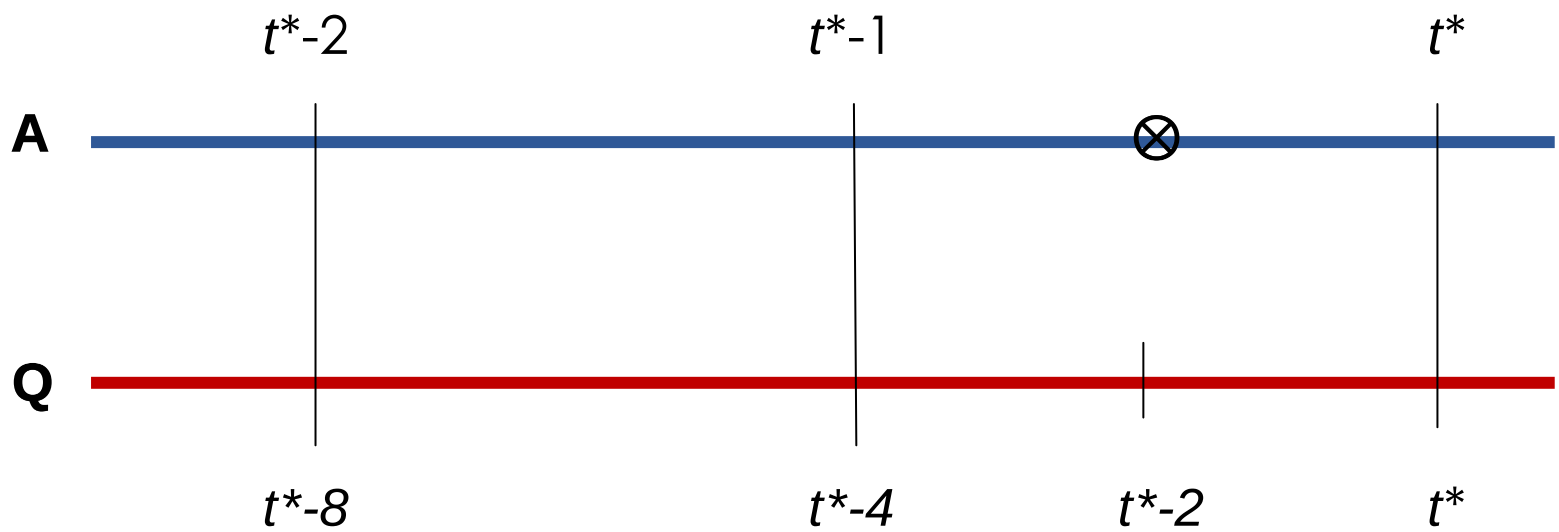
  Let $\Omega_t = \{x_{t-j}, \boldsymbol{z_{t-j}}\,; j \geq 0\}$ and $\Omega'_t = \{x_{t-j}, y_{t-j,}\boldsymbol{z_{t-j}}\,; j \geq 0\}$.

  If there exists a $k > 0$ such that $E(x_{t+k}\,|\,\Omega'_t\,) \neq E(x_{t+k}\,|\,\Omega_t)$,

  then $y_t \overset{G}{\Rightarrow} x_{t+k}$ with respect to Ω'_t .

- Usually test with $k = 1$, using least squares optimal forecast .
- Test of linear restrictions – special case – more later.

- Temporal aggregation can distort information sets. Past and future values of the data get “mixed up”.



- A legitimate high-frequency VAR model will have a VARMA representation when data are temporally aggregated. See McCrorie & Chambers (2004).

- In the case of temporal aggregation:

1. If $y_t \nRightarrow x_t$, then $Y_t \nRightarrow X_t$.

2. If $y_t \Longrightarrow x_t$ or $x_t \Longrightarrow y_t$, then we *can find* $Y_t \Leftrightarrow X_t$;

   or $Y_t \nRightarrow X_t$; and/or $X_t \nRightarrow Y_t$.

3. If $y_t \Leftrightarrow x_t$, then we *can find* only $Y_t \Longrightarrow X_t$; or *vice versa*.

   See Sims (1971), Wei (1982), Christiano & Eichenbaum (1987), Marcellino (1999), Gulasekaran & Abeysinghe (2002), Breitung and Swanson (2002).

- Same issues arise if data are non-stationary, and/or seasonal. See Gulasekaran & Abeysinghe (2002).

Example

Giles (2014)

Crude Oil and Wholesale Gasoline Prices (2009 - 2013)

	Daily	***Weekly***	***Monthly***
$\boldsymbol{P_C \nRightarrow P_W}$			
v	28	8	2
$\chi^2_{(v)}$	49.693	4.909	0.192
p	0.007	0.767	0.908
$\boldsymbol{P_W \nRightarrow P_C}$			
v	28	8	2
$\chi^2_{(v)}$	32.714	6.584	3.659
p	0.246	0.582	0.161

- Granger-causality testing has been extended to the continuous-time case. See Harvey & Stock (1989), Hansen & Sargent (1991), and McCrorie & Chambers (2004).
- Empirical example given by McCrorie & Chambers –

Money $\Longrightarrow$ Income ? Monthly U.S. data, 1960M1 – 2001M12.

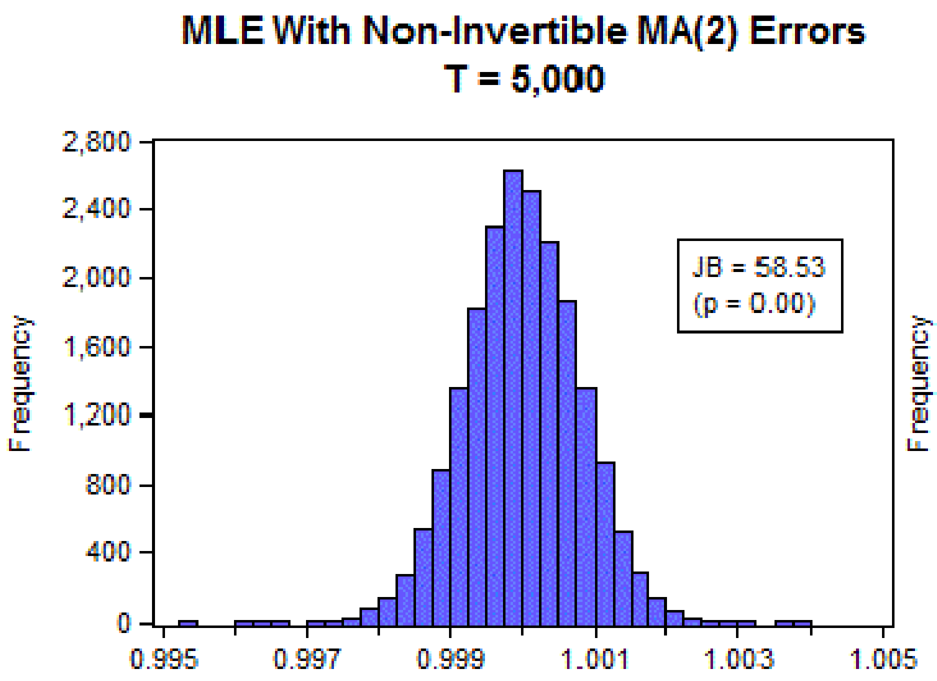
	Continuous-time; MA(3)	**Discrete Monthly**
$\boldsymbol{H_0: M \nRightarrow Y}$		
LRT	34.701	10.761
v	2	12
p	0.000	0.549

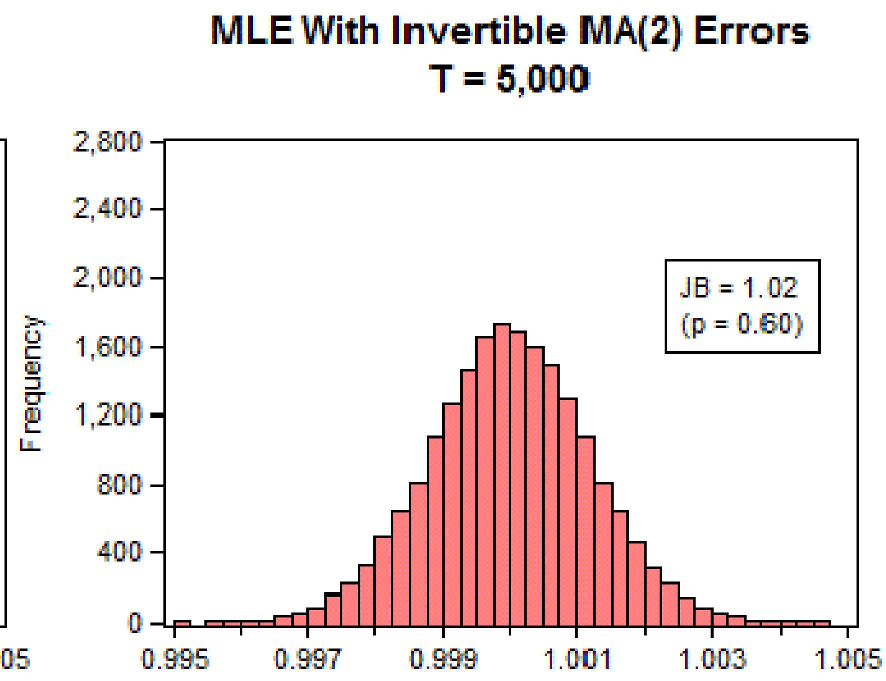
Temporal Aggregation & Economic Dynamics

- Not surprisingly, the characteristics of a dynamic model can be altered by temporal aggregation of the data.
- Early contributions related to Distributed Lag models – *e.g.*, Mundlak (1961), Moriguchi (1970), Wei (1978).
- Aggregation introduces a *specification bias* in such models.
- In Partial Adjustment models this can lead to estimator inconsistency.
- Important implications for evaluation of multipliers & economic policy analysis – very much what Phillips was concerned with.

Temporal Aggregation & Model Estimation

- One issue - loss of estimator efficiency due to MA effect.
- More complex than originally thought.
- Plosser & Schwert (1977) – consider *non-invertible* MA error processes due to over-differencing.
- Results have implications for effects of temporal aggregation.
- Estimation and testing when parameters take values on boundary of parameter space. *e.g.*, Moran (1971).
- MLE's & test statistics don't have usual desirable asymptotic properties. *e.g.*, Sargan & Bhargava (1983).





- Monte Carlo experiment
- Replications = 20,000
- DGP – linear model with MA(2) errors
- MLE – allowing for MA(2) errors
- True parameter value = 1.0

Impact of Aggregation on Hypothesis Tests

General observations

- Recall, temporal aggregation introduces *special* MA effects.
- These are likely to show up in errors of regression models.
- Expect this to distort sizes and impact on powers of tests, at least in finite-sample case.
- Look at some standard model specification tests.
- Discussion is only illustrative – not comprehensive.

Tests of linear restrictions

- The main issue is impact of MA process on the tests (*e.g.*, t, F).
- Regression model t-statistics: Plosser & Schwert – leptokurtic.
- Effect on tests depends on form of the data & restrictions, and also on parameters of MA process – "nuisance" parameters.
- "Bounds tests" of Watson (1955), Watson & Hannan (1956), Vinod (1976), Kiviet (1979, 1980), Giles & Lieberman (1993).
- Bounds diverge as we approach non-invertible case.
- Also Rothenberg (1984), and "exact tests" – Dufour (1990).
- Krämer (1989) – AR errors; Giles & Godwin (2014) - **MA errors**.

Example

Giles & Godwin (2014)

$$y_t = \beta_0 + \beta_1 x_t + u_t \quad ; \quad u_t \sim i.i.d.N[0\,,1]$$

$$x_t = 0.1t + N[0,1]$$

$$Y_t = \beta_0 + \beta_1 X_t + v_t \quad ; \; m = 3$$

$$H_o: \beta_1 = 0 \qquad vs. \qquad H_A: \beta_1 > 0$$

t-test is UMP

20,000 replications in MC experiment

Actual sizes

α^* \ T:	12	50	100	500	5000	5000
1%	3.1	8.7	8.8	8.7	9.9	24.9
	(11.6)	*(4.6)*	*(3.1)*	*(1.9)*	*(1.2)*	*(3.4)*
5%	10.5	16.6	17.2	16.9	17.1	31.8
	(21.4)	*(11.2)*	*(9.1)*	*(6.9)*	*(5.6)*	*(9.8)*
10%	17.2	22.6	23.3	22.9	22.9	35.9
	(27.4)	*(17.0)*	*(15.1)*	*(12.4)*	*(10.8)*	*(15.9)*

(Tests based on Newey-West std. errors)

Results for $m = 12$

Tests of linearity

- Effects of both temporal aggregation & systematic sampling tend to simplify non-linearities and reduce the power of associated tests.
- Brännäs & Ohlsson (1999), Granger & Lee (1999), Teles & Wei (2000).
- Models - Bilinear, Threshold, Sign, Rational Nonlinear AR: TAR, SGN, NAR.
- Tests – White's Neural Network, Tsay, White's Dynamic Information, Ramsey's RESET, Hinich's Bispectral.
- Illustrative Monte Carlo results from Granger & Lee. H_0: Linear; k corresponds to "m"; T = 200; Replications = 1,000; $\alpha = 5\%$.

Effects of Temporal Aggregation
Dynamic Information Matrix Test

Model	No aggregation	Systematic $k=4$	sampling $k=10$	Temporal $k=4$	aggregation $k=10$
BILINEAR	995	303	91	133	50
	(995)	(290)	(86)	(128)	(45)
TAR	46	55	43	63	50
	(41)	(54)	(37)	(56)	(44)
SGN	876	123	53	65	53
	(868)	(110)	(51)	(57)	(50)
NAR	114	51	41	49	46
	(104)	(47)	(38)	(47)	(40)

Rejection frequencies (i.e., powers)using simulated (asymptotic) critical values.

Tests of normality

Giles & Godwin (2014)

$$y_t = \beta_0 + \beta_1 x_t + u_t \quad ; \quad u_t \sim i.i.d.N[0\,,1]$$

$$x_t = 0.1t + N[0,1]$$

$$Y_t = \beta_0 + \beta_1 X_t + v_t \quad ; \qquad m = 3; m = 12$$

$$H_o:\ u_t \sim Normal \quad \text{vs.} \quad H_A: u_t \sim Non - Normal$$

Jarque-Bera (Bowman-Shenton, 1975) "Omnibus Test"

20,000 replications in MC experiment

Actual sizes of J-B Test

α^* \ T:	12	50	100	500	5000
1%	0.2	1.7	1.8	1.6	1.0
5%	0.8	3.6	4.0	4.5	4.9
10%	1.5	5.2	6.3	8.1	10.0

Actual sizes of J-B Test

$(m = 3)$

$\alpha^* \backslash T$:	12	50	100	500	5000
1%	0.2	1.7	1.8	1.6	1.0
	0.2	2.1	3.1	4.4	5.0
5%	0.8	3.6	4.0	4.5	4.9
	0.7	4.3	6.6	11.3	13.9
10%	1.5	5.2	6.3	8.1	10.0
	1.3	6.6	10.2	17.9	21.6

(Mean & Variance of sampling distribution $\longrightarrow 3$ & 10 as $T \rightarrow \infty$)

Powers of J-B Test

(t_3 **errors**; $\boldsymbol{m = 3}$)

α^* \ T:	12	25	50	100	250	500
1%	3.5	27.3	55.7	84.2	99.4	100
	0.6	6.8	22.2	45.6	78.8	95.9
5%	7.3	33.9	63.1	88.8	99.8	100
	1.4	11.1	29.0	53.5	88.4	97.6
10%	10.0	38.1	67.4	91.2	99.9	100
	2.1	14.3	33.6	58.4	87.1	98.2

Test can be “biased” (*even without aggregation*)

Temporal Aggregation & Forecast Performance

- Consider linear (possibly seasonal) time-series models.
- Temporal aggregation usually reduces forecast performance.
- This is because the full information set is no longer available.
- Formalize this (Wei, 1979).
- $\hat{Y}_{T+h}$: h – period ahead optimal forecast based on $\{y_t\}_{t=1}^{T}$.
- $\hat{\hat{Y}}_{T+h}$: h – period ahead optimal forecast based on $\{Y_t\}_{t=1}^{T}$.
- $\zeta(m,h) = Var.[Y_{T+h} - \hat{Y}_{T+h}]$ / $Var.[Y_{T+h} - \hat{\hat{Y}}_{T+h}]$.
- Then:
  - $0 \leq \zeta(m,h) \leq 1$; for all m and h .
  - $\zeta(h) \equiv \lim_{m\to\infty}[\zeta(m,h)] = 1$; for all h, if $d = 0$.
  - $\zeta(h) \ll 1$ and $\zeta(h) \to 1$ if $h \to \infty$, only if $d > 0$.

Example (Anathanasopoulos *et al.*, 2011)

Tourism Forecasting With 366 Monthly Time-Series

Effects of Temporal Aggregation on Forecast Performance (MAPE, %)

	ETS		ARIMA		ForePro	
	$h = 1$	$h = 2$	$h = 1$	$h = 2$	$h = 1$	$h = 2$
Yearly	11.79	16.49	10.99	14.59	11.44	15.36
***m* = 4**	10.32	14.32	9.94	13.98	9.95	14.48
***m* = 12**	10.29	14.29	9.93	13.96	9.92	14.46

- Similar results found for SVAR models by Georgoutsus *et al.* (1998). Preferable to forecast with disaggregated data & then aggregate, rather than forecast with aggregate data.

Quarterly Ex-post Forecasting Results (h = 16)

Model		σ	$\hat{\sigma}$	U_{66}	U_B	U_S	U_C
	M	0.015	0.012	0.268	0.020	0.002	0.982
Qrtly	Y	0.087	0.304	0.891	0.040	0.049	0.907
	r	0.003	0.014	0.174	0.002	0.013	0.984
	M	0.015	0.087	0.652	0.096	0.696	0.206
Mthly	Y	0.087	0.312	1.276	0.094	0.668	0.236
	r	0.003	0.106	0.623	0.088	0.723	0.188

MIDAS Modelling

- **MI**xed **DA**ta **S**ampling regression models.
- Eric Ghysels & co-authors.
- Making the most of multi-frequency time-series data, without resorting to imputation.
- Avoids unnecessary temporal aggregation.
- Easy to implement in R, or in MATLAB.
- Lots of recent developments – 2013, 2014.

Summary

- Bill Phillips pioneered continuous-time modelling in economics.
- Many of the issues that his work revealed also arise when we use discrete data that have been temporally aggregated.
- Aggregation affects the time-series properties of our data due to Moving-Average effects isolated by Phillips.
- These, in turn, impact on virtually all of our estimators, tests, forecasts. *Asymptotics can become non-standard.*
- Important implications for policy analysis.
- Don't aggregate if you don't have to!

Some Open Research Questions

- "The topic of mixed frequency data, temporal aggregation and linear interpolation is being researched again more intensely in recent years ....." Ghysels & Miller (2014)
- Can we use tests of MA process non-invertibility to help assess the magnitude of temporal aggregation "problems"?

  Tests: Tanaka & Satchell (1989), Tanaka (1990), Larsson (2014).
- What are the effects of temporal aggregation on various tests?
- How far can we go with MIDAS modelling?
- What are the gains of modelling in the frequency domain?

Slides and Bibliography

davidegiles.com (“Downloads” ; “NZAE 2014”)

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