University of Victoria **Department of Economics**

ECON 546: Themes in Econometrics
Assignment 1

Due: Monday 1 February, 4:30 p.m.

Question 1

Let y_i (i = 1, 2, 3,.....,n) be independent observations which each follow a distribution whose probability density function is

$$p(y_i \mid \theta) = (1/\theta)e^{-y_i/\theta} \quad ; \quad 0 < y_i < \infty \quad ; \quad \theta > 0$$

(a) Show that the sample average, $\bar{y}$, is a sufficient statistic.

(b) Prove that the characteristic function for this distribution is $(1 - it\theta)^{-1}$.

(c) Prove that $E(y_i) = \theta$ and $Var.(y_i) = \theta^2$.

(d) Derive the expressions for the skewness and ***excess*** kurtosis of this distribution.

(e) Show that the (unique) MLE of θ is $\bar{y}$. (Make sure that you check the second-order condition when you maximize the likelihood function.)

(f) What are the MLE's for the skewness and excess kurtosis coefficients for this distribution?

15 marks

Question 2

Suppose that we have an independent random sample of n observations, y_i, each drawn from a distribution with probability density function given by:

$$p(y_i) = y_i \exp[-y_i^2/(2\theta^2)]/(\theta^2) \quad ; \quad y_i > 0$$

(a) Derive the Maximum Likelihood Estimator (MLE) for θ. (Don't forget to check the second-order condition.)

(b) It can be shown that $E(y_i) = \theta\sqrt{(\pi/2)}$ and $E(y_i^2) = 2\theta^2$. What is the MLE for the variance of the population distribution?

(c) What is the MLE for the mode of the density of y_i?

10 marks

Question 3

The data in the file **S:\Social Sciences\Economics\ECON546\ass1.txt** comprise three columns (*without headings*), each of 25 observations, on the variables capital (K), labour (L) and output (Q) respectively.

Consider a Constant Elasticity of Substitution (CES) aggregate production function:

$$Q_i = \gamma[\delta K_i^{-\rho} + (1-\delta)L_i^{-\rho}]^{-\upsilon/\rho} \exp(\varepsilon_i),$$

where is ε is a well-behaved, Normally distributed error term, and γ, δ, υ, and ρ are unknown parameters, such that $\gamma > 0$, $0 < \delta < 1$, $\upsilon > 0$ and $\rho \geq -1$. The parameter γ is the "efficiency parameter", δ is the "distribution parameter", υ is the "returns to scale parameter", and ρ is the

"substitution parameter". You may assume that the $\varepsilon_i's$ are completely "well-behaved" and normally distributed.

(a) Take (natural) logarithms of both side of the above model and then use EViews to estimate δ, υ, ρ, and log(γ). Edit the coefficient vector (in spreadsheet mode) so that you use initial starting values of 0.4, 0.8, 0.6 and 1.0 respectively.

(b) Try two other sets of starting values to see how sensitive your results are to this choice. (Does the algorithm converge? How different are the estimates?)

(c) Discuss the gradient(s) of the log-likelihood function at the "maximum".

(d) If $\rho \to 0$, the CES model collapses to the Cobb-Douglas production function. Test the validity of the latter model relative to the former, using an asymptotically valid test.

(e) The elasticity of substitution between capital and labour in this model is given by the parameter, $\sigma = 1 / (1+\rho)$. What is the MLE of σ? Test the hypothesis that this elasticity is 0.5 by testing if $1 / (1+\rho) = 0.5$. Now test if $0.5*(1+\rho) = 1$. Do you get the same answer?

12 marks

Question 4

Use the EViews workfile, **S:\Social Sciences\Economics\ECON546\ass1_4.wf1**, and the associated program file, **S:\Social Sciences\Economics\ECON546\ass1_4.prg**, for this question.

The program enables you to conduct a Monte Carlo experiment to determine the bias of the MLE estimator associated with the model in the previous question. **Before running the program, edit the coefficient vector in the Workfile so that you use initial starting values of 2.4, 0.8, 0.6 and 0.4 respectively for c(1) to c(4)**.

(a) Look back at your results for Question 3. Now, what does the comment in the program code, NOTE THAT THE ERROR'S STD. DEVIATION = 0.08, refer to? What is going on in the line after this in the program code?

(b) Use the program, modifying it as necessary, to simulate the percentage biases of the MLE's for each of the four parameters in the model, as well as the MLE for the substitution elasticity. At what point in the parameter space are these biases being determined?

(c) Now, keeping everything else fixed, run the program with a range of different true values for ρ, draw a simple graph to show how the percentage bias of the elasticity of substitution changes as a function of ρ.

13 marks

Total Marks = 50