University of Victoria **Department of Economics**

ECON 546: Themes in Econometrics
Assignment 2

Due: Thursday 11 February, 4:30 p.m.

Question 1

Consider a sample on n independent observations drawn from a population with probability function given by :

$$p(x_i) = \theta\, x_i^{\theta-1} \quad ; \qquad 0 \le x_i \le 1\,; \quad \theta > 0$$

(a) Obtain the MLE of θ, being careful to check the second-order condition when maximizing the log-likelihood function.

(b) Show that the MLE of exp(-2 / θ) is the square of the geometric mean of the sample data.

(c) Derive Fisher's Information measure for this problem, and its asymptotic counterpart.

(d) Carefully state the asymptotic distribution for the MLE of θ.

(e) How would you obtain a consistent asymptotic standard error for the MLE of θ?

12 marks

Question 2

Suppose that we have a sample of n independent observations from a distribution whose p.d.f. is

$$p(y_i) = (c y_i^{c-1} / b^c)\exp[-(y_i / b)^c] \quad ; 0 < y_i < \infty \ ; b, c > 0\ .$$

(a) Derive the likelihood equations that need to be solved to obtain the MLE's of b and c. (Don't try to solve these equations!)

(b) Suppose that we know that c = 1. Obtain an appropriate expression for the asymptotic distribution of the MLE of b in this case.

(c) For the situation in part (b), obtain the formula for the MLE of the mode of the underlying distribution.

10 marks

Question 3

Let the discrete random variable, Y, follow the Negative Binomial distribution, so that its probability mass function is:

$$\Pr.(Y = y) = \binom{x+y-1}{y} p^x (1-p)^y \quad ; \quad y = 0, 1, 2, 3, \ldots..$$

where x is an observed value, such that $x \geq 1$; and the parameter, p, satisfies $0 < p < 1$.
[This distribution arises, for example, when we have n independent Bernoulli trials, and y is the number of failures before the x^{th} success.]

(a) Prove that the MLE of p is $\tilde{p} = x/(x+\bar{y})$. Don't forget the second-order condition!
(b) Develop a careful statement about the asymptotic distribution for the MLE of p.
(c) Explain how you would construct an asymptotically valid 95% confidence interval for p.
(d) Letting $q = (1 - p)$, it can be shown that the characteristic function for this distribution is:

$$\phi_y(t) = p^x [1 - q\exp(it)]^{-x} \quad .$$

Prove that $E(Y) = (xq/p)$, and that $Var.(Y) = (xq/p^2)$.
(e) What are the MLE's for the mean and variance of Y?

12 marks

Question 4

Consider the following regression model:

$$y_i = x_i'\beta + \varepsilon_i \quad ; \quad \varepsilon_i \sim i.i.d.\ N[0, \sigma_i^2] \qquad (1)$$

$$\sigma_i^2 = (z_i'\alpha)^2 \quad ; \quad i = 1, 2, 3, \ldots\ldots, n \qquad (2)$$

where z_i' is the i^{th} observation on a $(1 \times p)$ vector of known (non-random) variables, the first of which is a "constant" vector of "ones"; and α is a $(p \times 1)$ vector of unknown parameters.

(a) Explain, briefly, why it makes sense that a squared value is being created in equation (2) above; and how restrictions could be placed on certain of the parameters of the model to make the errors homoskedastic.
(b) Formulate the log-likelihood function for the model in (1) and (2).
(c) Derive the "likelihood equations". Can you obtain the MLE's of the parameters analytically?
(d) Use the workfile **S:\Social Sciences\Economics\ECON546\Ass2.wf1** for this part. In the file there is a text-object that defines the variables. *Using just the first 659 observations*, estimate a model of the form (1) and (2) above that explains the quantity of organic apples consumed, using the price of organic apples (relative to the price of regular apples) and family income as the explanatory variables. Assume that the variance of the error term is a function of education and family income. Briefly discuss the statistical "quality" of the results that you obtain, and compare your results with those obtained if you estimated the same basic model by OLS, ignoring equation (2).

16 marks

Total Marks = 50