University of Victoria **Department of Economics**

ECON 546: Themes in Econometrics

Assignment 4

(Due: 29 March, 2010, 4:30p.m.)

Question 1

Suppose that we have a 2-equation SURE model, with regressor matrices X_1 and X_2. There are different numbers of regressors in each equation, and the errors of the two equations are correlated.

Prove that the OLS and GLS estimators of all of the coefficients will be identical if

$$X_1(X_1'X_1)^{-1}X_1' = X_2(X_2'X_2)^{-1}X_2'$$

[Actually, this is both a necessary and sufficient condition.]

Total: 10 marks

Question 2

Consider the p.d.f.

$$p(y|\theta) = \exp\{-(y-\theta)\} \quad ; \quad y > 0$$

Let the prior p.d.f. for θ be Cauchy:

$$p(\theta) = [\pi(1+\theta^2)]^{-1}$$

Given just one observation, y, find the Bayes estimator for the parameter, θ, under a "zero-one" loss function.

Total: 6 marks

Question 3

(a) Write down a careful definition of a Bayes decision rule (or estimator, if you wish). Prove that, provided this rule is defined, it will be admissible.

[**Hint:** Assume that the converse is true, and prove that this leads to a contradiction.]

(**4 marks)**

(b) We know that if we have a diffuse prior, the MEL estimator of the coefficient vector in the usual linear regression model with normal errors is just the OLS estimator. However, we also know from Stein's early work that this estimator is *inadmissible* when the model contains three or more regressors. Can you reconcile Stein's result with your answer to part (a) above?

(2 marks)

Total: 6 marks

Question 4

Consider a Binomial random variable. Let x denote the number of "successes" in n independent trials, and let θ be the probability of a "success". Assume that out prior information about θ can be represented by a "Beta" density:

$$p(\theta) \propto \theta^{a-1} (1 - \theta)^{b-1} \;\; ; \;\; a, b > 0 \;\; ; \;\; 0 \le \theta \le 1.$$

(a) Prove that, under a quadratic loss function, the Bayes estimator of θ is

$$\theta^* = (x + a) / (n + a + b).$$

(6 marks)

(b) Prove that the mode of $p(\theta)$ is at $\theta = (a - 1)/(a + b - 2)$.

(3 marks)

(c) What is the Bayes estimator of θ under an 'all-or-nothing' (zero-one) loss function?

(2 marks)

(d) Compare the biases of these two Bayes estimators with that of the MLE of θ.

(4 marks)

Total: 15 marks

Question 5

Suppose that we have a Normal data-generating process with an unknown mean of μ and a known variance of 300. A sample of size n = 6 is taken from this distribution, the sample values being 45, 72, 68, 54, 80, 65.

(a) If your prior judgments about μ can be represented by a Normal distribution with mean 60 and variance 80, what is your posterior distribution for μ?

(3 marks)

(b) Now suppose that some more (independent) data come available from the same underlying population. You can treat the above posterior information, as now being *prior* information for this second stage of the analysis. The 4 new data points take the values 80, 95, 105, 90. What is your *new* posterior distribution for μ?

(3 marks)

(c) Calculate Pr.($50 < \mu < 80$ | Both sets of data).

(5 marks)

(d) If you have a zero-one loss function in mind, what is your *final* point estimator of μ?

(2 marks)

Total: 13 marks

(Total: 50 Marks)