**UNIVERSITY OF VICTORIA**

**EXAMINATIONS, APRIL 2008**

**<u>ECONOMICS 546: THEMES IN ECONOMETRICS</u>**

**TO BE ANSWERED IN BOOKLETS** | **DURATION: <u>3 HOURS</u>**
**INSTRUCTOR: D. Giles**

**STUDENTS MUST COUNT THE NUMBER OF PAGES IN THIS EXAMINATION PAPER BEFORE BEGINNING TO WRITE, AND REPORT ANY DISCREPANCY IMMEDIATELY TO THE INVIGILATOR.**

**THIS QUESTION PAPER HAS 7 PAGES.**
**STATISTICAL TABLES ARE SUPPLIED SEPARATELY.**

This is a "closed book/closed notes" examination.
Calculators may be used.
**Answer <u>ALL QUESTIONS</u>** (***Total Marks = 90***)

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**Question 1:**
Suppose that we have a sample of "$n$" independent observations from a continuous distribution whose mean is $\theta$ and whose p.d.f. is:

$$p(y_i) = (1 / \theta) \exp[-y_i / \theta] ; \quad \theta > 0 ; 0 < y_i < \infty ; i = 1, 2, ......., n.$$

(a) Show that the MLE of $\theta$ is the arithmetic mean of the $y_i$ values. (Check the second-order condition, because you will need this information below.)

**(4 marks)**

(b) Suppose that we want to test the hypothesis that $\theta = 1$, against a two-sided alternative hypothesis. Construct the LRT and Wald statistics for this test, and explain how the tests would be conducted.

**(8 marks)**

(c) Construct the LM test for this same null hypothesis, and apply it for the case where $n = 10$ and the sample mean is 1.5.

**(6 marks)**
**Total: 18 Marks**

**Question 2:**

Consider the standard linear multiple regression model, $y = X\beta + \varepsilon$, with all of the usual "ideal" assumptions about the regressors and the error term, *except* suppose that the error vector, $\varepsilon$, has the following ***joint*** density function :

$$p(\varepsilon \mid \nu) = c[\nu + \varepsilon' \varepsilon]^{-(n+\nu)/2}$$

where $v$ ( > 0) is a ***known*** parameter; and $c$ is a positive "normalizing" constant that ensures that the density integrates to unity.

(a) Show that the MLE for $\beta$ is just the usual OLS estimator.

**(4 marks)**

(b) Show that the Hessian matrix, ***when evaluated at the MLE for*** $\boldsymbol{\beta}$, is the matrix $-[(n + v)(X'X)/(v + ns^2)]$, where $ns^2$ is the sum of the squared OLS residuals.

**(5 marks)**

(c) Using this Hessian matrix, construct a Wald-type statistic for testing a standard set of exact independent linear restrictions, $R\beta = q$. What can you say about the distribution of this test statistic?

**(4 marks)**

(d) Suppose that we are totally ignorant, a priori, about $\beta$. What is the Bayes estimator for $\beta$ under a zero-one loss function?

**(5 marks)**
**Total: 18 Marks**

**Question 3:**

let $X$ be a (continuous) random variable that follows a Gamma Distribution. Then, its p.d.f. is:

$$p(x) = (x/n)^{c-1} e^{-(x/b)} / [b\Gamma(c)] \qquad ; \qquad x > 0\ ; \quad b, c > 0$$

where $\Gamma(.)$ is the usual Gamma function:

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt\ ,$$

which satisfies the recursion relationship:

$$\Gamma(z+1) = z\Gamma(z)\ .$$

The parameter $c$ is the shape parameter, and $b$ is the scale parameter for this distribution. It can be shown that the moments (about the origin) of this distribution are:

$$E(X^r) = b^r \prod_{i=0}^{r-1} (c+i)\ ;\ r = 1, 2, 3, \ldots\ldots\ldots$$

(a) Derive Method of Moments estimators for $b$ and $c$.

**(9 Marks)**

(b) What desirable property (ies) will these estimators have?

**(1 mark)**
**Total: 10 Marks**

**Question 4:**

Consider the Natural Conjugate Bayes estimator of $\beta$ in the standard Normal multiple linear regression model, under a quadratic loss function. That is, $\breve{\beta} = [A + X'X]^{-1}[A\overline{\beta} + X'y]$, where $\overline{\beta}$ is the mean of the (conditional) prior density for $\beta$, and $\sigma^2 A^{-1}$ is the covariance matrix of the (conditional) prior for $\beta$.

(a) Show that this estimator is biased. Why does this really not matter to a Bayesian econometrician?

**(5 Marks)**

(b) If the conditional prior covariance matrix for $\beta$ were chosen to be equal to the covariance matrix for the Maximum Likelihood estimator of $\beta$ in this model, show that the expected value of the Bayes estimator of $\beta$ is a simple average of the conditional prior mean for $\beta$, and $\beta$ itself.

**(4 Marks)**

(c) What estimator will $\breve{\beta}$ converge to if (i) the sample size becomes very large; (ii) the elements of $A^{-1}$ become very large?

**(4 Marks)**
**Total: 13 Marks**

**Question 5:**

Suppose that we use a Poisson regression model to model some "count" data, $y_1$, $y_2$, $y_3$, ……..
So, we specify:

$$\Pr.[Y = y_i] = \exp(-\lambda_i)\lambda_i^{y_i} / y_i!$$

and then set

$$\lambda_i = \exp[x_i'\beta] \qquad ; \quad i = 1, 2, 3, \ldots, n$$

(a) Write down the log-likelihood function for this model, based on $n$ independent observations, and show that the first-order condition for maximizing it is:

$$\sum_{i=1}^{n}[y_i - \exp(x_i'\beta)]x_i = 0 .$$

**(4 Marks)**

(b) Show that the marginal effect associated with the $j^{th}$ (continuous) covariate at observation $i$ is $ME_{ij} = [\beta_j \exp(x_i'\beta)]$.

**(4 Marks)**

(c) Explain how you would calculate the marginal effect if the the $j^{th}$ covariate is a zero-one dummy variable.

**(3 Marks)**
**Total: 11 Marks**

**Question 6:**

The following variables have been used to estimate a model to explain the number of bad credit reports that individuals receive:

NBR = Number of bad credit reports
AGE = Age in years+ 12ths of a year,
INCOME = Annual income, in dollars, divided by 10,000
AVGEXP = Average monthly credit card expenditure
OWNRENT = Dummy variable: 1 if individual owns home; 0 if home is rented
SELFEMPL = Dummy variable: 1 if individual is self-employed; 0 if not

**OUTPUT 1**

Dependent Variable: NBR
Method: ML/QML - Poisson Count (Quadratic hill climbing)
Date: 04/15/08 Time: 12:17
Sample: 1 100
Included observations: 100
Convergence achieved after 22 iterations
QML (Huber/White) standard errors & covariance

| Variable | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| C | -1.321828 | 0.949746 | -1.391770 | 0.1640 |
| AVGEXP | -0.006787 | 0.003049 | -2.225847 | 0.0260 |
| INCOME | 0.223979 | 0.099589 | 2.249026 | 0.0245 |
| AGE | 0.005081 | 0.027190 | 0.186856 | 0.8518 |
| SELFEMPL | -46.56396 | 0.582881 | -79.88587 | 0.0000 |
| OWNRENT | 0.172965 | 0.694080 | 0.249200 | 0.8032 |

| | | | |
|---|---|---|---|
| R-squared | 0.121087 | Mean dependent var | 0.360000 |
| Adjusted R-squared | 0.074336 | S.D. dependent var | 1.010250 |
| S.E. of regression | 0.971976 | Akaike info criterion | 1.673128 |
| Sum squared resid | 88.80535 | Schwarz criterion | 1.829438 |
| Log likelihood | -77.65640 | Hannan-Quinn criter. | 1.736390 |
| Restr. log likelihood | -91.93738 | Avg. log likelihood | -0.776564 |
| LR statistic (5 df) | 28.56196 | LR index (Pseudo-R2) | 0.155334 |
| Probability(LR stat) | 2.83E-05 | | |

(a) What type of model has been estimated in OUTPUT 1? Why? What method has been used to estimate the parameters of this model?

**(4 marks)**

(b) Interpret the “LR statistic” that is highlighted in OUTPUT 1. What are the null and alternative hypotheses, and what do you conclude?

**(3 marks)**

**OUTPUT 2**

Dependent Variable: NBR
Method: ML/QML - Poisson Count (Quadratic hill climbing)
Date: 04/15/08 Time: 12:25
Sample: 1 100
Included observations: 100
Convergence achieved after 22 iterations
QML (Huber/White) standard errors & covariance

| Variable | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| C | -1.122311 | 0.584739 | -1.919337 | 0.0549 |
| AVGEXP | -0.006751 | 0.002902 | -2.326263 | 0.0200 |
| INCOME | 0.232016 | 0.095207 | 2.436948 | 0.0148 |
| SELFEMPL | -32.90722 | 0.592579 | -55.53223 | 0.0000 |

| | | | |
|---|---|---|---|
| R-squared | 0.117150 | Mean dependent var | 0.360000 |
| Adjusted R-squared | 0.089561 | S.D. dependent var | 1.010250 |
| S.E. of regression | 0.963950 | Akaike info criterion | 1.636651 |
| Sum squared resid | 89.20315 | Schwarz criterion | 1.740858 |
| Log likelihood | -77.83257 | Hannan-Quinn criter. | 1.678826 |
| Restr. log likelihood | -91.93738 | Avg. log likelihood | -0.778326 |
| LR statistic (3 df) | 28.20962 | LR index (Pseudo-R2) | 0.153418 |
| Probability(LR stat) | 3.28E-06 | | |

**OUTPUT 3**

| | AVEXP | INCOME | SELFEMPL | OWNRENT |
|---|---|---|---|---|
| Mean | 189.0231 | 3.369300 | 0.360000 | 0.050000 |
| Median | 81.29500 | 3.000000 | 0.000000 | 0.000000 |
| Maximum | 1898.030 | 10.00000 | 1.000000 | 1.000000 |
| Minimum | 0.000000 | 1.500000 | 0.000000 | 0.000000 |
| Std. Dev. | 294.2446 | 1.629013 | 0.482418 | 0.219043 |
| Skewness | 3.241249 | 1.853352 | 0.583333 | 4.129483 |
| Kurtosis | 16.75127 | 7.293857 | 1.340278 | 18.05263 |

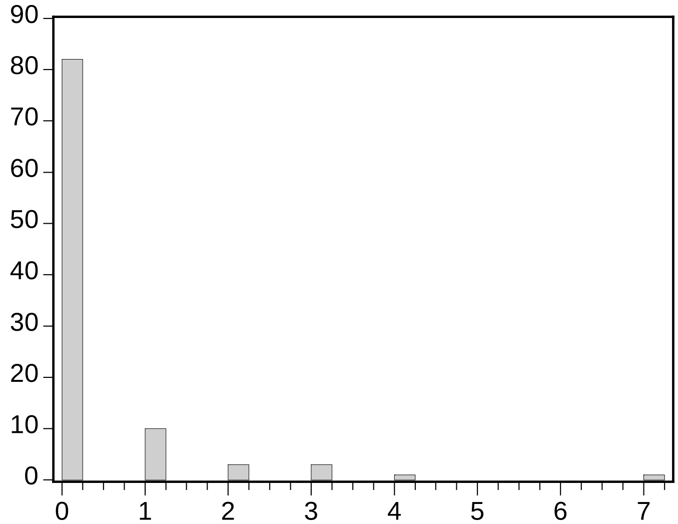
(c) Use the information in OUTPUT 2 and OUTPUT 3 to compute the marginal effect for INCOME, at the medians of the data, and interpret its value. [**Hint:** see Question 5.]

**( 5 marks)**

(d) The following histogram and table (OUTPUT 4) shows the characteristics of the dependent variable in our model. What does this information suggest about the suitability of the model we have used above?

**( 2 marks)**

**OUTPUT 4**




| | |
|---|---|
| Series: NBR | |
| Sample 1 100 | |
| Observations 100 | |
| Mean | 0.360000 |
| Median | 0.000000 |
| Maximum | 7.000000 |
| Minimum | 0.000000 |
| Std. Dev. | 1.010250 |
| Skewness | 4.019249 |
| Kurtosis | 22.43030 |
| Jarque-Bera | 1842.308 |
| Probability | 0.000000 |

**OUTPUT 5**

Sample: 1 100
Included observations: 100
Convergence achieved after 22 iterations
QML (Huber/White) standard errors & covariance

| | Coefficient | Std. Error | z-Statistic | Prob. |
|---|---|---|---|---|
| C | -1.398950 | 0.631889 | -2.213917 | 0.0268 |
| AVGEXP | -0.005800 | 0.002073 | -2.797558 | 0.0051 |
| INCOME | 0.293698 | 0.157700 | 1.862389 | 0.0625 |
| SELFEMPL | -31.91880 | 0.559869 | -57.01121 | 0.0000 |
| Mixture Parameter | | | | |
| SHAPE:C(5) | 1.127512 | 0.439562 | 2.565082 | 0.0103 |

| | | | |
|---|---|---|---|
| R-squared | 0.100654 | Mean dependent var | 0.360000 |
| Adjusted R-squared | 0.062787 | S.D. dependent var | 1.010250 |
| S.E. of regression | 0.978021 | Akaike info criterion | 1.421938 |
| Sum squared resid | 90.86990 | Schwarz criterion | 1.552196 |
| Log likelihood | -66.09688 | Hannan-Quinn criter. | 1.474656 |
| Restr. log likelihood | -91.93738 | Avg. log likelihood | -0.660969 |
| LR statistic (4 df) | 51.68099 | LR index (Pseudo-R2) | 0.281066 |
| Probability(LR stat) | 1.61E-10 | | |

(e) What type of model has been estimated in OUTPUT 5? Why?

**(3 marks)**

**OUTPUT 6**

Wald Test:
Equation: Untitled

| Test Statistic | Value | df | Probability |
|---|---|---|---|
| F-statistic | 5.175592 | (1, 95) | 0.0252 |
| Chi-square | 5.175592 | 1 | 0.0229 |

Null Hypothesis Summary:

| Normalized Restriction (= 0) | Value | Std. Err. |
|---|---|---|
| EXP(C(5)) | 3.087964 | 1.357351 |

Delta method computed using analytic derivatives.

(f) Explain what test is being conducted in OUTPUT 6. What are the null and alternative hypotheses? What do you conclude from these results?

**(3 marks)**
**Total: 20 Marks**

**END OF EXAMINATION**