UNIVERSITY OF VICTORIA

EXAMINATIONS, APRIL 2009

<u>ECONOMICS 546: THEMES IN ECONOMETRICS</u>

TO BE ANSWERED IN BOOKLETS **DURATION: <u>3 HOURS</u>**
INSTRUCTOR: D. Giles

STUDENTS MUST COUNT THE NUMBER OF PAGES IN THIS EXAMINATION PAPER BEFORE BEGINNING TO WRITE, AND REPORT ANY DISCREPANCY IMMEDIATELY TO THE INVIGILATOR.

THIS QUESTION PAPER HAS 6 PAGES.
STATISTICAL TABLES AND A FORMULA SHEET ARE SUPPLIED SEPARATELY.

This is a "closed book/closed notes" examination. Calculators may be used.
Answer <u>ALL FIVE QUESTIONS</u> **(*Total Marks = 90*)**

Question 1:
Suppose that we have "*n*" random observations from a population that is described by the following density function:

$$p(y_i \mid \beta, \rho) = \frac{\beta^{\rho}}{\Gamma(\rho)} y_i^{\rho-1} \exp\{-\beta y_i\} \quad ; \quad y_i > 0 \quad ; \quad \rho > 0 \quad ; \quad \beta > 0$$

where $\Gamma(.)$ is the Gamma function (it is just a constant once we assign a value to ρ).

(a) ***Assuming that ρ is known***, derive the MLE for β. (Don't forget the second-order condition.)

5marks

(b) Derive the Wald test statistic for testing H_0: $\beta = 1$ *vs.* H_A: $\beta \neq 1$. Explain how you would apply this test.

3 marks

(c) Derive the Lagrange Multiplier test statistic for testing H_0: $\beta = 1$ *vs.* H_A: $\beta \neq 1$. Explain how you would apply this test.

3 marks

(d) Suppose that $n = 100$; $\bar{y} = 2/3$; and $\rho = 1$. Apply the Wald and Lagrange Multiplier tests. What do you conclude? Are you results sensitive to the choice of significance level that you choose?

4 marks

Total: 15 Marks

Question 2:

You probably know that J. M. Keynes made many important contributions to probability theory and statistics (as well as to economics, of course). His *Treatise on Probability* is a classic work that makes seminal contributions to the "subjective" theory of probability used by Bayesians. He also provided (Keynes, 1911) the first modern treatment of "Laplace's (1774) first law" - **if we have an odd number of observations, "*n*", then the value of θ that minimizes the expression $\sum_{i=1}^{n} | y_i - \theta |$ is the median of the y_i's.** (An odd number is needed to ensure that the median is unique.)

Now, suppose that we have a random sample of "*n*" (which you can assume to be an odd number of) observations from a Laplace (or "double exponential") distribution. That is, the density function for an individual y_i is:

$$p(y_i \mid \theta, \lambda) = (2\lambda)^{-1} \exp\{-| y_i - \theta | / \lambda\}; \quad -\infty < y_i < \infty \quad ; \quad \lambda > 0 .$$

(a) Prove that the MLE for θ, say $\tilde{\theta}$, is the median of the sample, and that the MLE for λ is

$$\tilde{\lambda} = \frac{1}{n} \sum_{i=1}^{n} | y_i - \tilde{\theta} | .$$

6 marks

(b) It can be shown that $E(y_i) = \theta$ and $E(y_i^2) = (2\lambda^2 + \theta^2)$. Suppose that we want to provide a unitless measure of the variability of the data. One such measure is the "coefficient of variation", $cv = E(y_i) / \sqrt{\text{var.}(y_i)}$. Provide a consistent estimator for *cv*. What else can you say about the asymptotic properties of this estimator?

3 marks

(c) Now, suppose we ***know*** θ, but that we are totally ***ignorant*** about λ. Obtain the posterior density for λ, and show that the Bayes' estimator of this parameter when we have a zero-one loss function is $\hat{\lambda} = \frac{1}{n+1} \sum_{i=1}^{n} | y_i - \tilde{\theta} |$. Do this estimator and the MLE converge in probability to λ at the same rate as each other as $n \to \infty$?

6 marks

Total: 15 Marks

References: Keynes, J. M. (1911), "The principal averages and the laws of error which lead to them", *Journal of the Royal Statistical Society, Series A*, 74, 322-328.

Laplace, P. (1774), "Mémoire sur la probabilité des causes par les èvénemens", *Mémoires de Mathématique et de Physique*, 6, 621-656.

Question 3:
Suppose that we have a simple linear regression model (with no intercept), in which the errors are independent, but the errors' variance may change at some point in the sample:

$$y_i = \beta x_i + \varepsilon_{1i} \; ; \qquad \varepsilon_{1i} \sim N[0, \sigma_1^2] ; \qquad i = 1, 2, 3, \ldots\ldots, n_1$$

$$y_i = \beta x_i + \varepsilon_{2i} \; ; \qquad \varepsilon_{2i} \sim N[0, \sigma_2^2] ; \qquad i = n_1+1, \ldots\ldots, n.$$

(a) Write down the likelihood function for this model, recalling that if $z \sim N[\mu , \sigma^2]$, then its density is $p(z) = (2\pi\sigma^2)^{-1/2} \exp\{-(z-\mu)^2/(2\sigma^2)\}$.

3 marks

(b) Derive the MLE's for β, σ_1^2 and σ_2^2, showing that the first of these estimators is just the OLS estimator based on the "pooled" (combined) sample of data.

7 marks

(c) Consider the hypothesis, H_0: $\sigma_1^2 = \sigma_2^2$. Derive the Likelihood Ratio test statistic for testing H_0 against a two-sided alternative hypothesis.

7 marks

(d) What can you say about the distribution of the LRT statistic? Briefly explain how you would apply this LRT in practice.

3 marks
Total: 20 marks

Question 4:
Consider a Binomial random variable. Let x denote the number of "successes" in n independent Bernoulli trials, and let θ be the probability of a "success". So, the (joint) mass function for the data is

$$p(x \mid \theta) = {}^nC_x \theta^x (1-\theta)^{n-x} \; .$$

Assume that our prior information about θ can be represented by a "Beta" density:

$$p(\theta) \propto \theta^{a-1}(1-\theta)^{b-1} \quad ; a, b > 0; \quad 0 \le \theta \le 1.$$

A Beta distribution (for θ) has the following property:

$$E(\theta^k) = \frac{(a+k-1)}{(a+b+k-1)} E(\theta^{k-1}) \quad ; \qquad k = 1, 2, 3, \ldots\ldots..$$

(a) What is the posterior density for θ?

4 marks

(b) Show that, under a quadratic loss function, the Bayes estimator of θ is

$$\hat{\theta} = (x + a) / (n + a + b) \; .$$

3 marks

(c) Prove that the mode of the ***prior*** density for θ is at $\theta = (a - 1)/(a + b - 2)$. What condition(s) must be satisfied for this mode to be "sensible"?

5 marks

(d) What is the Bayes estimator of θ under an ‘all-or-nothing’ (zero-one) loss function?

2 marks

(e) Calculate the bias of each of these Bayes estimators. Explain why a Bayesian would not be concerned about the magnitudes of these biases.

6 marks

[**<u>Hint:</u>** Recall that for the Binomial distribution, $E[X] = n\theta$.]

Total: 20 marks

Question 5:

The following variables have been used in a study of currency crises in 167 countries:

CRISIS = number of currency crises
DUMINCHI = 1 if the country is a high-income or upper-middle-income country (= 0, otherwise)
DUMJPEG = 1 if the crisis occurred under a “hard peg” (fixed) exchange rate regime (= 0, otherwise)
DUMJFLT = 1 if the crisis occurred under a floating exchange rate regime (= 0, otherwise)
(There are many other types of exchange rate regimes, such as “managed float”, *etc.*)

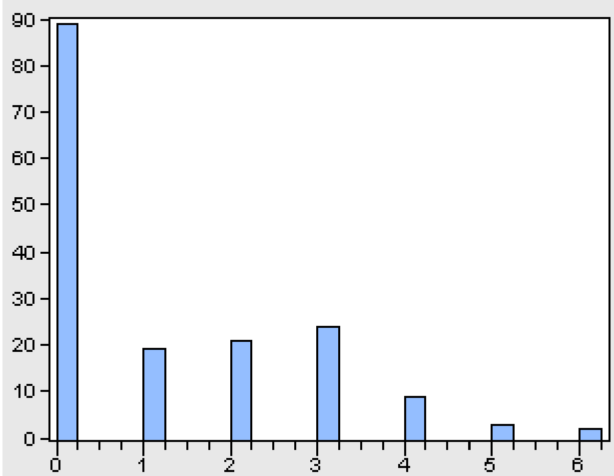
(a) Briefly explain why an OLS regression would not be the ideal way to model the CRISIS data.

3 marks

(b) A summary of the CRISIS data appears in Figure 1. What does this summary suggest about the way in which the data might be modelled.

2 marks

FIGURE 1



Series: CRISIS	
Sample 1 167	
Observations 167	
Mean	1.173653
Median	0.000000
Maximum	6.000000
Minimum	0.000000
Std. Dev.	1.512924
Skewness	1.064639
Kurtosis	3.178994
Jarque-Bera	31.77082
Probability	0.000000

(c) OUTPUT 1 (on the next page) gives the results for a model of CRISIS. Discuss these results.

4 marks

(d) What is the “LR statistic” in OUTPUT 1? What hypothesis is being tested, and what do you conclude?

3 marks

OUTPUT 1

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: CRISIS
Method: ML/QML - Poisson Count (Quadratic hill climbing)
Date: 04/03/09 Time: 16:00
Sample: 1 167
Included observations: 167
Convergence achieved after 5 iterations
QML (Huber/White) standard errors & covariance

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-1.052824	0.168459	-6.249726	0.0000
DUMINCHI	0.349450	0.143521	2.434830	0.0149
DUMJPEG	1.393652	0.234612	5.940245	0.0000
DUMJFLT	0.924900	0.184706	5.007403	0.0000

R-squared	0.486538	Mean dependent var	1.173653
Adjusted R-squared	0.477087	S.D. dependent var	1.512924
S.E. of regression	1.094037	Akaike info criterion	2.301486
Sum squared resid	195.0973	Schwarz criterion	2.376168
Log likelihood	-188.1741	Hannan-Quinn criter.	2.331798
Restr. log likelihood	-278.2981	LR statistic	180.2480
Avg. log likelihood	-1.126791	Prob(LR statistic)	0.000000

(e) OUTPUT 2 shows the results of estimating an alternative model. What model has been used, and why?

3 marks

OUTPUT 2

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: CRISIS
Method: ML - Negative Binomial Count (Quadratic hill climbing)
Date: 04/03/09 Time: 16:03
Sample: 1 167
Included observations: 167
Convergence achieved after 15 iterations
QML (Huber/White) standard errors & covariance

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-1.052830	0.168457	-6.249857	0.0000
DUMINCHI	0.349453	0.143523	2.434829	0.0149
DUMJPEG	1.393655	0.234610	5.940297	0.0000
DUMJFLT	0.924908	0.184710	5.007359	0.0000
Mixture Parameter				
SHAPE:C(5)	-14.72929	48.07171	-0.306402	0.7593

R-squared	0.486534	Mean dependent var	1.173653
Adjusted R-squared	0.473856	S.D. dependent var	1.512924
S.E. of regression	1.097412	Akaike info criterion	2.313462
Sum squared resid	195.0986	Schwarz criterion	2.406815
Log likelihood	-188.1741	Hannan-Quinn criter.	2.351352
Restr. log likelihood	-278.2981	LR statistic	180.2480
Avg. log likelihood	-1.126791	Prob(LR statistic)	0.000000

(f) Discuss OUTPUT 3. What do you conclude?

OUTPUT 3

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Wald Test:
Equation: EQNEGBIN

Test Statistic	Value	df	Probability
F-statistic	0.000433	(1, 162)	0.9834
Chi-square	0.000433	1	0.9834

Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
EXP(C(5))	4.01E-07	1.93E-05

Delta method computed using analytic derivatives.

2 marks

(g) OUTPUT 4 gives the summary statistics for the predictions of the conditional mean of the dependent variable, based on the results in OUTPUT 1. LAM0 is calculated with DUMINCHI = 0, and LAM1 is calculated with DUMINCHI = 1. Use this information to calculate a marginal effect for the income dummy variable. Interpret this marginal effect.

OUTPUT 4

	LAM0	LAM1
Mean	1.007648	1.429133
Median	0.348951	0.494912
Maximum	3.545670	5.028776
Minimum	0.348951	0.494912
Std. Dev.	1.071735	1.520028
Skewness	1.633210	1.633210
Kurtosis	4.284233	4.284233
Jarque-Bera	85.71800	85.71800
Probability	0.000000	0.000000
Sum	168.2772	238.6652
Sum Sq. Dev.	190.6703	383.5404
Observations	167	167

3 marks
Total: 20 marks

END OF EXAMINATION