Department of Economics **University of Victoria**

ECON 546: Themes in Econometrics

Estimation of Simultaneous Equations Models in EViews

In this handout we discuss the estimation of structural simultaneous equations models (SEM's) using various estimation techniques using the EViews econometrics package. In particular, we will use a simple well-known structural model to illustrate the results that are obtained when different "limited information" and "full information" estimators are used.

The idea SEM's for the economy came from Jan Tinbergen, who estimated a 24-eqatuion system for the Dutch economy in 1936. See Tinbergen (1959, pp.37-84) for an English translation. When the first Nobel Prize in Economic Science was awarded in 1969, Tinbergen shared the inaugural honour with Ragnar Frisch (a Norwegian econometrician) for their pioneering work that led to the development of econometrics as a recognized sub-discipline.

Klein's (1950) "Model I" for the U.S. economy was a 6-equation SEM, comprising 3 structural equations and 3 identities. The equations of Klein's model are given below, with the endogenous variables as the dependent variables:

$$C_t = \alpha_0 + \alpha_1 P_t + \alpha_2 P_{t-1} + \alpha_3 (W_t^p + W_t^g) + \varepsilon_{1t} \qquad (\textit{Consumption})$$

$$I_t = \beta_0 + \beta_1 P_t + \beta_2 P_{t-1} + \beta_3 K_{t-1} + \varepsilon_{2t} \qquad \text{(Investment)}$$

$$W_t^p = \gamma_0 + \gamma_1 X_t + \gamma_2 X_{t-1} + \gamma_3 A_t + \varepsilon_{3t} \qquad \text{(Private Wages)}$$

$$X_t \equiv C_t + I_t + G_t \qquad \text{(Equilibrium Demand)}$$

$$P_t \equiv X_t - T_t - W_t^p \qquad \text{(Private Profits)}$$

$$K_t \equiv K_{t-1} + I_t \qquad \text{(Capital Stock)}$$

The predetermined variables in the model are the intercept, G_t (government non-wage spending), T_t (indirect business taxes plus net exports), W_t^g (government wage bill), A_t (time trend, measured as years from 1931), and the lagged dependent variables, P_{t-1}, X_{t-1} and K_{t-1}. Allowing for lags, the sample period for the estimation of the model was 1921 to 1941 inclusive.

(1) Why, do you think, did the sample begin in 1921 and end in 1941, when the model was published in 1950?

(2) The data for this exercise are on the server in the EViews file titled **Klein.wf1**. Note that the variable called "*K*1" is K_{t-1}, and "*W*" is $(W_t^p + W_t^g)$. Estimate each of the three structural equations using OLS. What can you say about the properties of this estimator in the present context? The results that you should obtain are as follows:

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: CONS
Method: Least Squares
Date: 11/20/03 Time: 14:25
Sample(adjusted): 1921 1941
Included observations: 21 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	16.23660	1.302698	12.46382	0.0000
P	0.192934	0.091210	2.115273	0.0495
P(-1)	0.089885	0.090648	0.991582	0.3353
W	0.796219	0.039944	19.93342	0.0000

R-squared	0.981008	Mean dependent var	53.99524
Adjusted R-squared	0.977657	S.D. dependent var	6.860866
S.E. of regression	1.025540	Akaike info criterion	3.057959
Sum squared resid	17.87945	Schwarz criterion	3.256916
Log likelihood	-28.10857	F-statistic	292.7076
Durbin-Watson stat	1.367474	Prob(F-statistic)	0.000000

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: I
Method: Least Squares
Date: 03/14/02 Time: 11:39
Sample(adjusted): 1921 1941
Included observations: 21 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	10.12579	5.465547	1.852658	0.0814
P	0.479636	0.097115	4.938864	0.0001
P(-1)	0.333039	0.100859	3.302015	0.0042
K1	-0.111795	0.026728	-4.182749	0.0006

R-squared	0.931348	Mean dependent var	1.266667
Adjusted R-squared	0.919233	S.D. dependent var	3.551948
S.E. of regression	1.009447	Akaike info criterion	3.026325
Sum squared resid	17.32270	Schwarz criterion	3.225282
Log likelihood	-27.77641	F-statistic	76.87537
Durbin-Watson stat	1.810184	Prob(F-statistic)	0.000000

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: WP
Method: Least Squares
Date: 03/14/02 Time: 11:42
Sample(adjusted): 1921 1941
Included observations: 21 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.497044	1.270032	1.178745	0.2547
X	0.439477	0.032408	13.56093	0.0000
X(-1)	0.146090	0.037423	3.903734	0.0011
A	0.130245	0.031910	4.081604	0.0008

R-squared	0.987414	Mean dependent var	36.36190
Adjusted R-squared	0.985193	S.D. dependent var	6.304401
S.E. of regression	0.767147	Akaike info criterion	2.477367
Sum squared resid	10.00475	Schwarz criterion	2.676324
Log likelihood	-22.01235	F-statistic	444.5682
Durbin-Watson stat	1.958434	Prob(F-statistic)	0.000000

(3) Now estimate by Two Stage Least Squares (2SLS) – this is just I.V. estimation with ***all of the predetermined variables in the model*** used as the instruments. What are we hoping to achieve by using 2SLS? The results that you should obtain are as follows:

'iew | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: CONS
Method: Two-Stage Least Squares
Date: 12/02/03 Time: 09:18
Sample(adjusted): 1921 1941
Included observations: 21 after adjusting endpoints
Instrument list: C P(-1) WG K1 X(-1) A G T

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	16.55476	1.467979	11.27725	0.0000
P	0.017302	0.131205	0.131872	0.8966
P(-1)	0.216234	0.119222	1.813714	0.0874
W	0.810183	0.044735	18.11069	0.0000

R-squared	0.976711	Mean dependent var	53.99524
Adjusted R-squared	0.972601	S.D. dependent var	6.860866
S.E. of regression	1.135659	Sum squared resid	21.92525
F-statistic	225.9334	Durbin-Watson stat	1.485072
Prob(F-statistic)	0.000000	Second-Stage SSR	67.25682

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: I
Method: Two-Stage Least Squares
Date: 12/02/03 Time: 09:19
Sample(adjusted): 1921 1941
Included observations: 21 after adjusting endpoints
Instrument list: C P(-1) WG K1 X(-1) A G T

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	20.27821	8.383249	2.418896	0.0271
P	0.150222	0.192534	0.780237	0.4460
P(-1)	0.615944	0.180926	3.404398	0.0034
K1	-0.157788	0.040152	-3.929751	0.0011

R-squared	0.884884	Mean dependent var	1.266667
Adjusted R-squared	0.864569	S.D. dependent var	3.551948
S.E. of regression	1.307149	Sum squared resid	29.04686
F-statistic	41.20019	Durbin-Watson stat	2.085334
Prob(F-statistic)	0.000000	Second-Stage SSR	41.13794

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

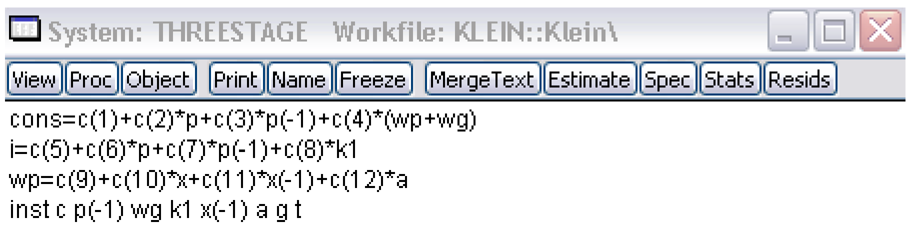
Dependent Variable: WP
Method: Two-Stage Least Squares
Date: 12/02/03 Time: 09:19
Sample(adjusted): 1921 1941
Included observations: 21 after adjusting endpoints
Instrument list: C P(-1) WG K1 X(-1) A G T

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.500297	1.275686	1.176070	0.2558
X	0.438859	0.039603	11.08155	0.0000
X(-1)	0.146674	0.043164	3.398063	0.0034
A	0.130396	0.032388	4.026001	0.0009

R-squared	0.987414	Mean dependent var	36.36190
Adjusted R-squared	0.985193	S.D. dependent var	6.304401
S.E. of regression	0.767155	Sum squared resid	10.00496
F-statistic	424.1940	Durbin-Watson stat	1.963416
Prob(F-statistic)	0.000000	Second-Stage SSR	45.96024

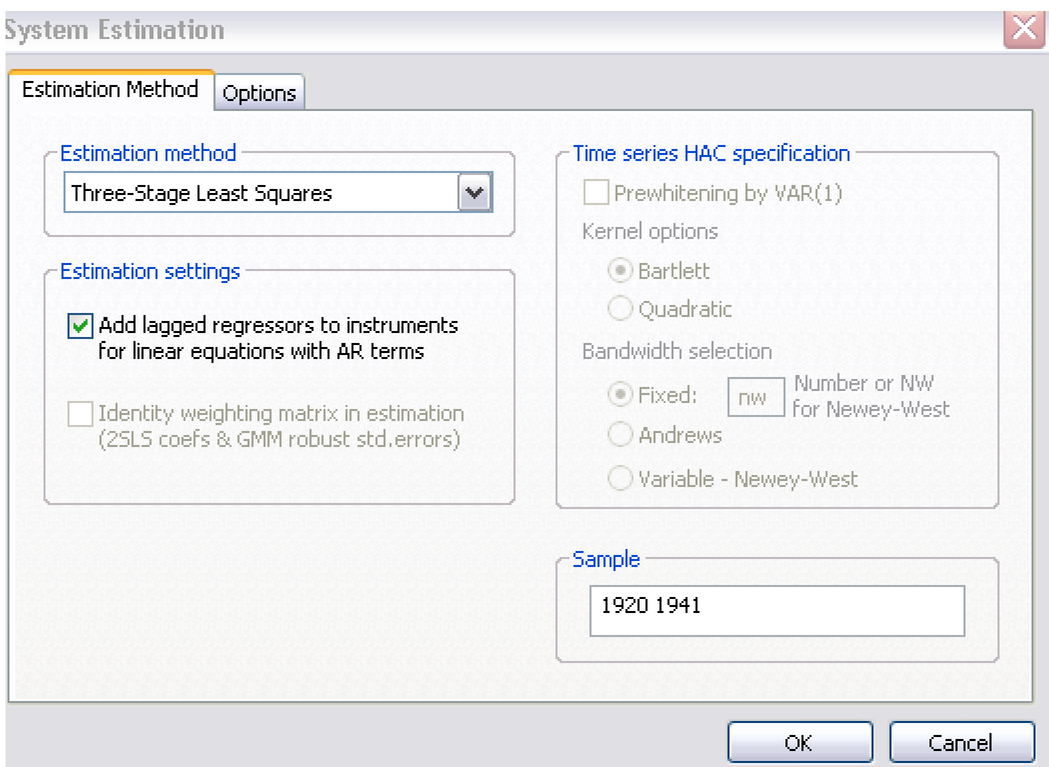
Compare the 2SLS and OLS estimates.

(4) We will now estimate the model by Three Stage Least Squares (3SLS) – a “full information” or “systems” estimator that has the same asymptotic efficiency as Full Information Maximum Likelihood (FIML). First, we need to create the system we are going to use. In the EViews workfile, select “Object”, “New Object”, “System”. Name your system THREESTAGE. Then lay out the specification of the structural equations in the model as follows:



(To make things easy for you, the code for these equations is stored in the text-object called “Three_Stage_Spec” in the EViews workfile.)

Then select the “Estimate” tab and choose “Three-Stage Least Squares” as the estimation method:



Select “OK” to obtain the 3SLS estimates:

System: THREESTAGE Workfile: KLEIN::Klein\

View | Proc | Object | Print | Name | Freeze | MergeText | Estimate | Spec | Stats | Resids

System: THREESTAGE
Estimation Method: Three-Stage Least Squares
Date: 10/14/09 Time: 14:48
Sample: 1921 1941
Included observations: 21
Total system (balanced) observations 63
Linear estimation after one-step weighting matrix

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	16.44079	1.304549	12.60266	0.0000
C(2)	0.124890	0.108129	1.155013	0.2535
C(3)	0.163144	0.100438	1.624323	0.1105
C(4)	0.790081	0.037938	20.82563	0.0000
C(5)	28.17785	6.793770	4.147601	0.0001
C(6)	-0.013079	0.161896	-0.080787	0.9359
C(7)	0.755724	0.152933	4.941532	0.0000
C(8)	-0.194848	0.032531	-5.989674	0.0000
C(9)	1.797218	1.115855	1.610619	0.1134
C(10)	0.400492	0.031813	12.58877	0.0000
C(11)	0.181291	0.034159	5.307304	0.0000
C(12)	0.149674	0.027935	5.357897	0.0000

Determinant residual covariance 0.282997

Equation: CONS=C(1)+C(2)*P+C(3)*P(-1)+C(4)*(WP+WG)
Instruments: C P(-1) WG K1 X(-1) A G T
Observations: 21

R-squared	0.980108	Mean dependent var	53.99524
Adjusted R-squared	0.976598	S.D. dependent var	6.860866
S.E. of regression	1.049565	Sum squared resid	18.72696
Durbin-Watson stat	1.424939		

Equation: I=C(5)+C(6)*P+C(7)*P(-1)+C(8)*K1
Instruments: C P(-1) WG K1 X(-1) A G T
Observations: 21

R-squared	0.825805	Mean dependent var	1.266667
Adjusted R-squared	0.795065	S.D. dependent var	3.551948
S.E. of regression	1.607958	Sum squared resid	43.95398
Durbin-Watson stat	1.995884		

Equation: WP=C(9)+C(10)*X+C(11)*X(-1)+C(12)*A
Instruments: C P(-1) WG K1 X(-1) A G T
Observations: 21

R-squared	0.986262	Mean dependent var	36.36190
Adjusted R-squared	0.983838	S.D. dependent var	6.304401
S.E. of regression	0.801490	Sum squared resid	10.92056
Durbin-Watson stat	2.155046		

(5) Let's move ahead to FIML estimation of the model. *Now the identities have to be "solved out" in order for EViews to proceed*. With a larger model, this would be very tedious – some other econometrics packages allow you to include identities explicitly as part of the model specification, but EViews does not, unfortunately. So, in the EViews workfile, select "Object", "New Object", "System". Name your system FIML. Then lay out the specification of the structural equations in the model as follows:

System: FIML Workfile: KLEIN::Klein\

View | Proc | Object | Print | Name | Freeze | MergeText | Estimate | Spec | Stats | Resids

```
cons=c(1)+c(2)*(cons+i+g-t-wp)+c(3)*(cons(-1)+i(-1)+g(-1)-t(-1)-wp(-1))+c(4)*
(wp+wg)
i=c(5)+c(6)*(cons+i+g-t-wp)+c(7)*(cons(-1)+i(-1)+g(-1)-t(-1)-wp(-1))+c(8)*k1
wp=c(9)+c(10)*(cons+i+g)+c(11)*(cons(-1)+i(-1)+g(-1))+c(12)*a
```

(Again, to make things easy for you, these equations are stored in the text-object called "FIML_Spec" in the EViews workfile.)

Now select the "Estimate" tab, choose "Full Information Maximum Likelihood" as the estimation method and then select the "Options" tab. Alter the settings as follows (including 1000 as the maximum number of iterations), and you should obtain the following estimation results and "Gradients Summary" (presented here in reverse order to save space):

System: FIML Workfile: KLEIN::Klein\

View | Proc | Object | Print | Name | Freeze | MergeText | Estimate | Spec | Sta

Gradients of objective function at estimated parameters
System: FIML
Method: Full Information Maximum Likelihood
Computed using analytic derivatives
System specification is linear

Coefficient	Sum	Mean	Newton Dir.
C(1)	0.000428	2.04E-05	0.000113
C(2)	0.007761	0.000370	0.000183
C(3)	0.007698	0.000367	-0.000132
C(4)	0.019870	0.000946	-9.60E-06
C(5)	-0.000226	-1.07E-05	-0.000136
C(6)	-0.004240	-0.000202	0.000353
C(7)	-0.004248	-0.000202	-0.000253
C(8)	-0.045926	-0.002187	-1.72E-06
C(9)	0.000362	1.72E-05	0.000258
C(10)	0.023867	0.001137	2.66E-07
C(11)	0.023839	0.001135	5.43E-08
C(12)	0.001288	6.14E-05	9.21E-06

System: FIML Workfile: KLEIN::Klein\

View | Proc | Object | Print | Name | Freeze | MergeText | Estimate | Spec | Stats | Resids

System: FIML
Estimation Method: Full Information Maximum Likelihood (BHHH)
Date: 10/14/09 Time: 14:59
Sample: 1921 1941
Included observations: 21
Total system (balanced) observations 63
Convergence achieved after 873 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	18.34295	12.87763	1.424404	0.1543
C(2)	-0.232347	1.930369	-0.120364	0.9042
C(3)	0.385657	1.083031	0.356090	0.7218
C(4)	0.801841	0.084178	9.525593	0.0000
C(5)	27.26372	21.47140	1.269769	0.2042
C(6)	-0.800945	2.334309	-0.343118	0.7315
C(7)	1.051836	1.403898	0.749225	0.4537
C(8)	-0.148102	0.099173	-1.493370	0.1353
C(9)	5.794027	4.645271	1.247296	0.2123
C(10)	0.234127	0.095283	2.457170	0.0140
C(11)	0.284672	0.061742	4.610703	0.0000
C(12)	0.234830	0.077642	3.024526	0.0025

Log likelihood	-83.32381	Schwarz criterion	9.675328
Avg. log likelihood	-1.322600	Hannan-Quinn criter.	9.207994
Akaike info criterion	9.078458		
Determinant residual covariance	1.442705		

Equation: CONS=C(1)+C(2)*(CONS+I+G-T-WP)+C(3)*(CONS(-1)+I(-1)+G(-1)-T(-1)-WP(-1))+C(4)*(WP+WG)
Observations: 21

R-squared	0.953069	Mean dependent var	53.99524
Adjusted R-squared	0.944787	S.D. dependent var	6.860866
S.E. of regression	1.612123	Sum squared resid	44.18200
Durbin-Watson stat	1.367116		

Equation: I=C(5)+C(6)*(CONS+I+G-T-WP)+C(7)*(CONS(-1)+I(-1)+G(-1)-T(-1)-WP(-1))+C(8)*K1
Observations: 21

R-squared	-0.062797	Mean dependent var	1.266667
Adjusted R-squared	-0.250349	S.D. dependent var	3.551948
S.E. of regression	3.971753	Sum squared resid	268.1720
Durbin-Watson stat	1.235548		

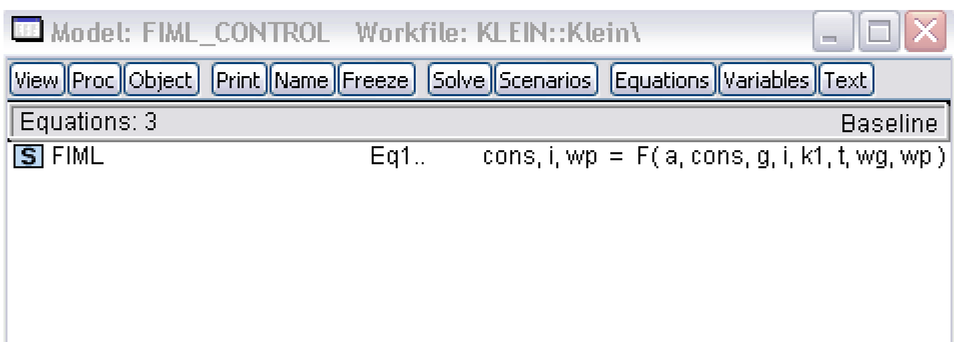
Equation: WP=C(9)+C(10)*(CONS+I+G)+C(11)*(CONS(-1)+I(-1)+G(-1))+C(12)*A
Observations: 21

R-squared	0.952421	Mean dependent var	36.36190
Adjusted R-squared	0.944025	S.D. dependent var	6.304401
S.E. of regression	1.491562	Sum squared resid	37.82089
Durbin-Watson stat	1.493348		

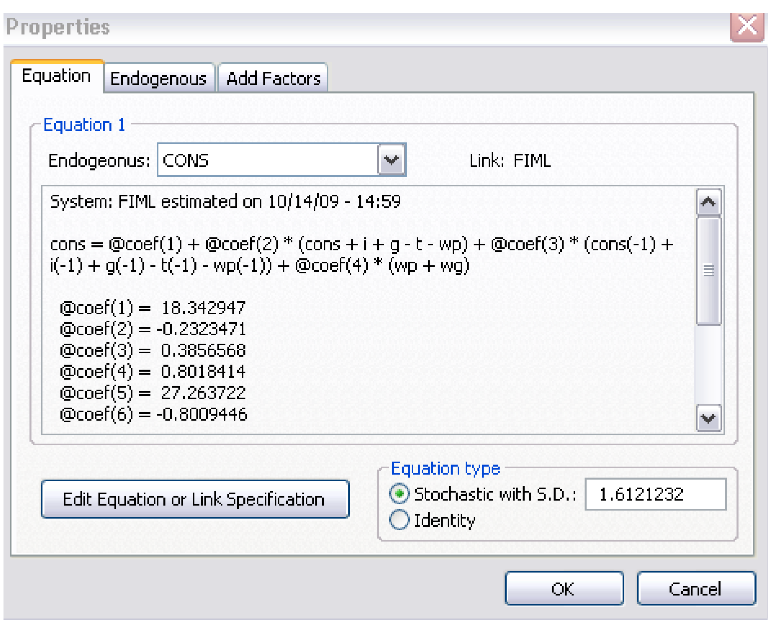
If you look on page 385 of Greene (2008) you will see a summary of the OLS, 2SLS, 3SLS and FIML results, together with some other estimates. His results agree closely with ours.

(6) The next thing is to see how we can "solve" the estimated structural form of the model for the restricted reduced form. In our case, the model is linear in both the endogenous variables and the parameters, so this is achieved by straightforward matrix manipulations. However, if the model were non-linear in the endogenous variables, this solution would have to be achieved iteratively as we would then have a system of non-linear equations to be solved. In that case techniques such as the Gauss-Seidel method or Newton's method would be used. Note that this "solution" process has nothing to do with estimation – that has been done already – what we are now doing is converting the structural form equations into the corresponding restricted reduced form equations so that we can either forecast, or else perform policy simulations.

First, select "Object", "New Object", "Model", and name your new model FIML_CONTROL. There are various ways to get the estimated equations into this model. The easiest way at this stage is to copy and paste your FIML ***system*** into the blank window for the FIML_CONTROL ***model***. You should then see this:

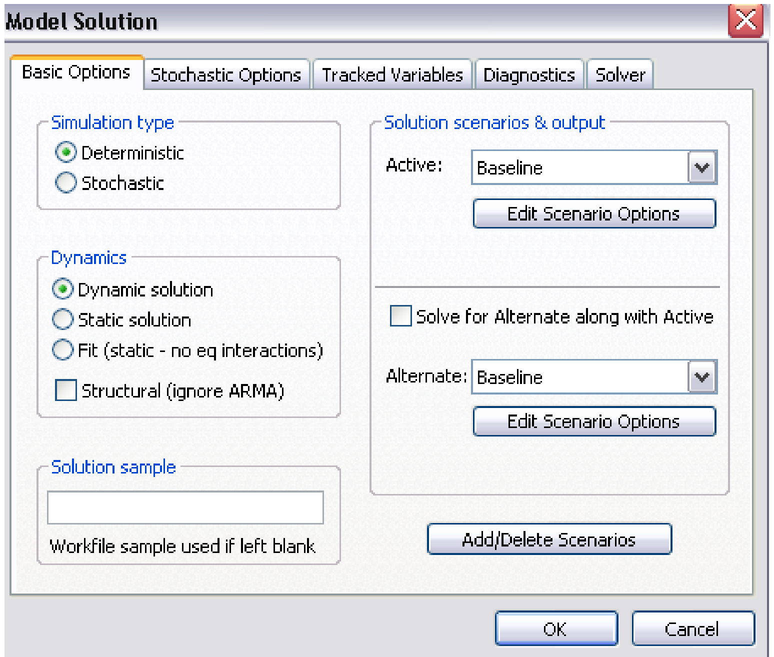


Click on the blue "S" logo and you will see:



You can then scroll the endogenous variables to see the specifications of the other two equations in the model.

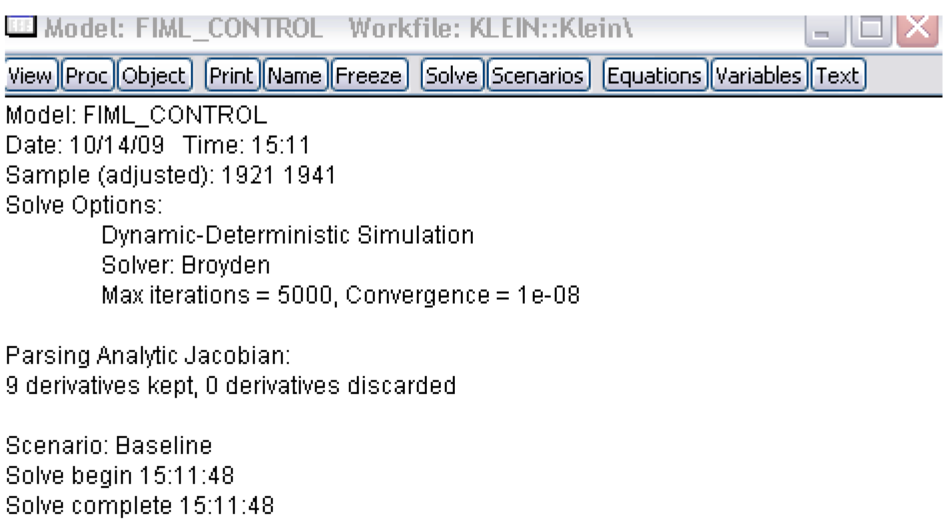
To solve the model, select “Solve:



What is the difference between a “Deterministic” and “Stochastic” simulation?

What are the differences between “Dynamic Solution”, “Static Solution” and “Fit” in the “Dynamics box?

Select “OK” and you will see:



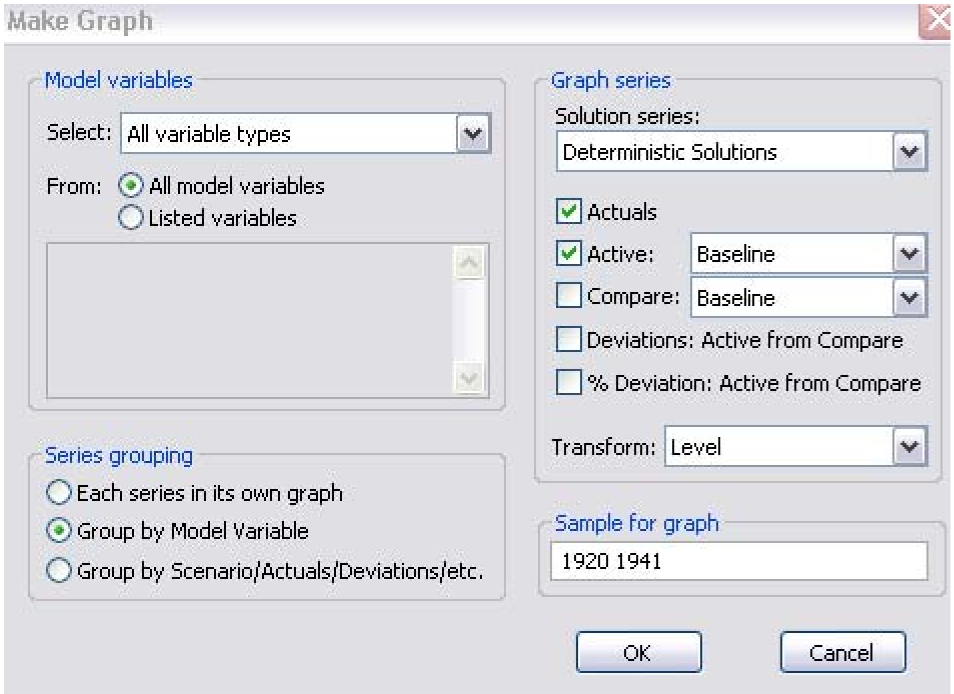
Model: FIML_CONTROL
Date: 10/14/09 Time: 15:11
Sample (adjusted): 1921 1941
Solve Options:
Dynamic-Deterministic Simulation
Solver: Broyden
Max iterations = 5000, Convergence = 1e-08

Parsing Analytic Jacobian:
9 derivatives kept, 0 derivatives discarded

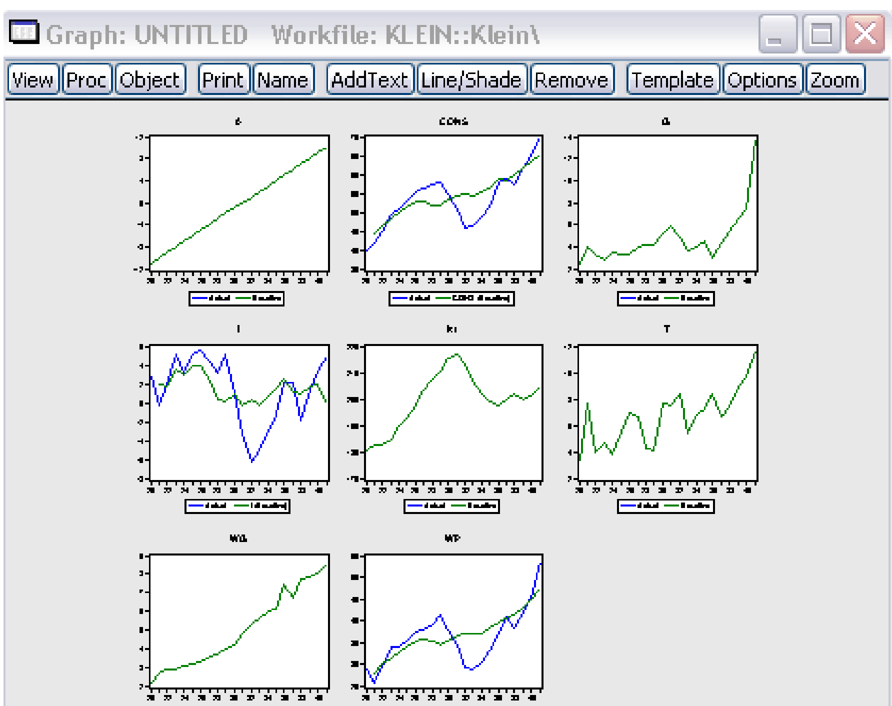
Scenario: Baseline
Solve begin 15:11:48
Solve complete 15:11:48

If you look at the main EViews workspace, you should also see that three new variables have been created. What are they?

Now select “Proc”, “Make Graph”, and edit the window so that it looks like this:



Select “OK”:

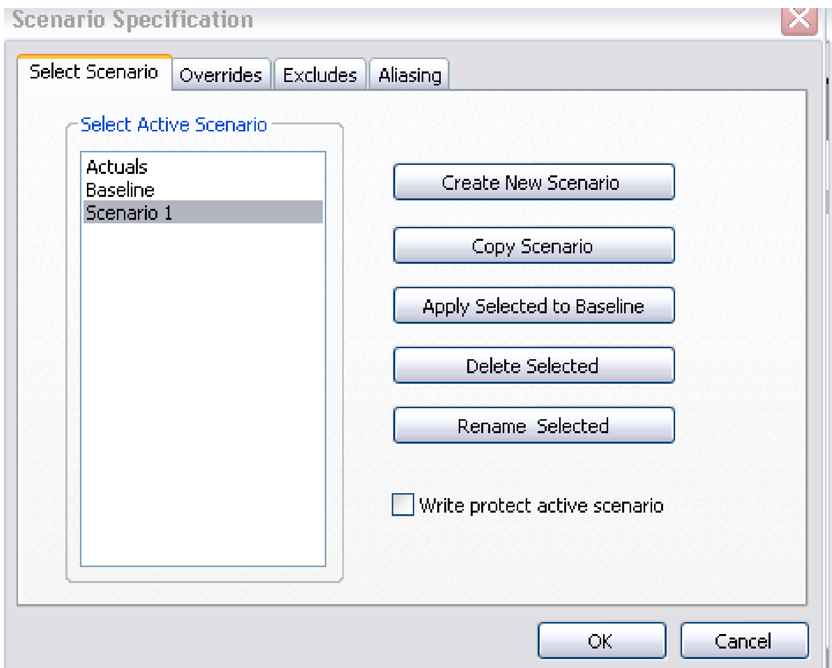


Why are there two lines on some of the graphs and only one on others?

What does "Baseline" refer to in the legends?

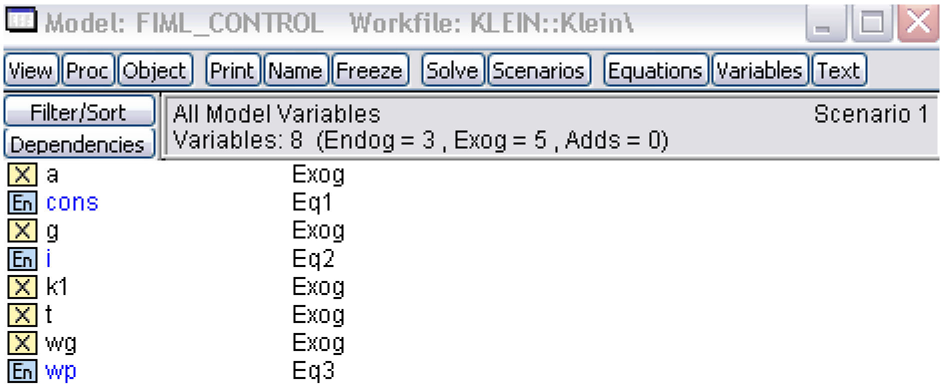
(7) Finally, let's simulate the effect of a simple policy change. Specifically, we are going to see what the model predicts *would have happened* if Government Non-Wage Spending (G) had been 5 units larger in each of the years 1937 to 1941 inclusive. What follows shows how to conduct a dynamic/deterministic simulation and compare the "policy-on" results with both the "policy off" "(control", or "baseline") results and the actual data. You can experiment with other types of simulations.

In the Model window, select the "Scenarios" tab, and you will see:

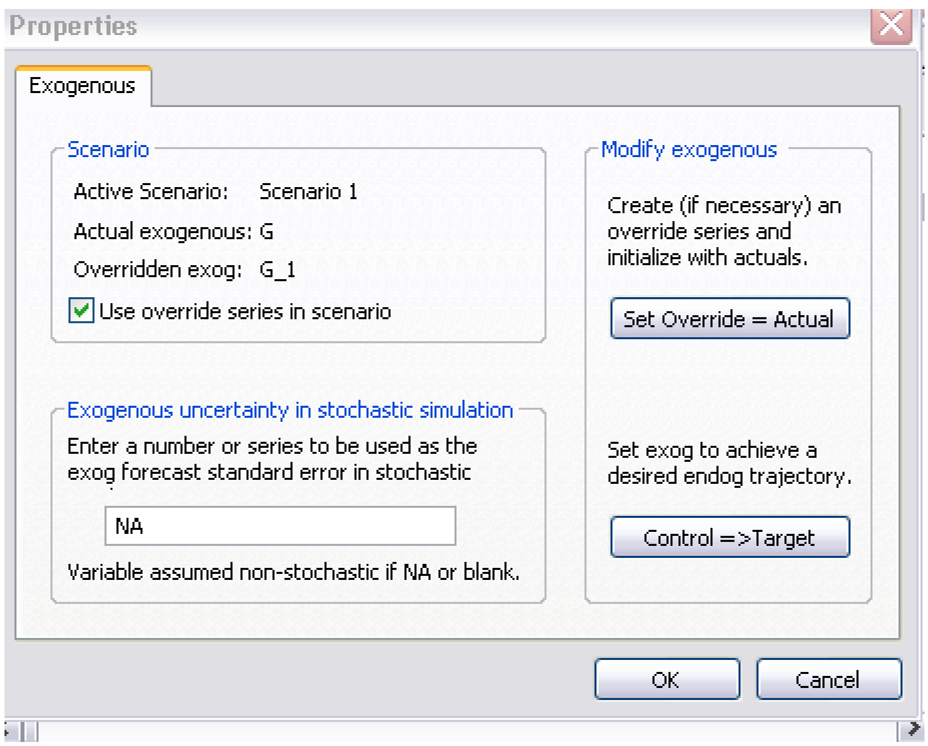


Highlight Scenario 1, as shown, and select "OK".

Then, in the Model window and select the "Variables" tab:

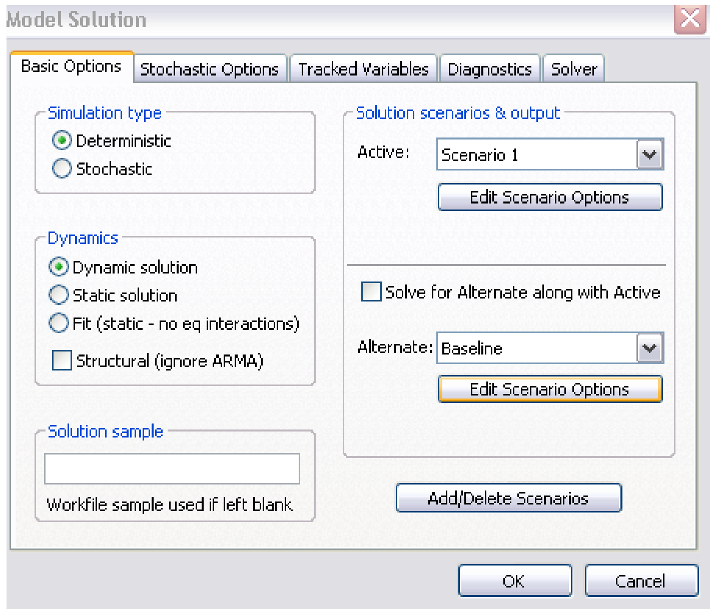


Next, right mouse-click on the variable "g", and select "Properties". Check the "override" box as shown, and press "Select Override = Actual":

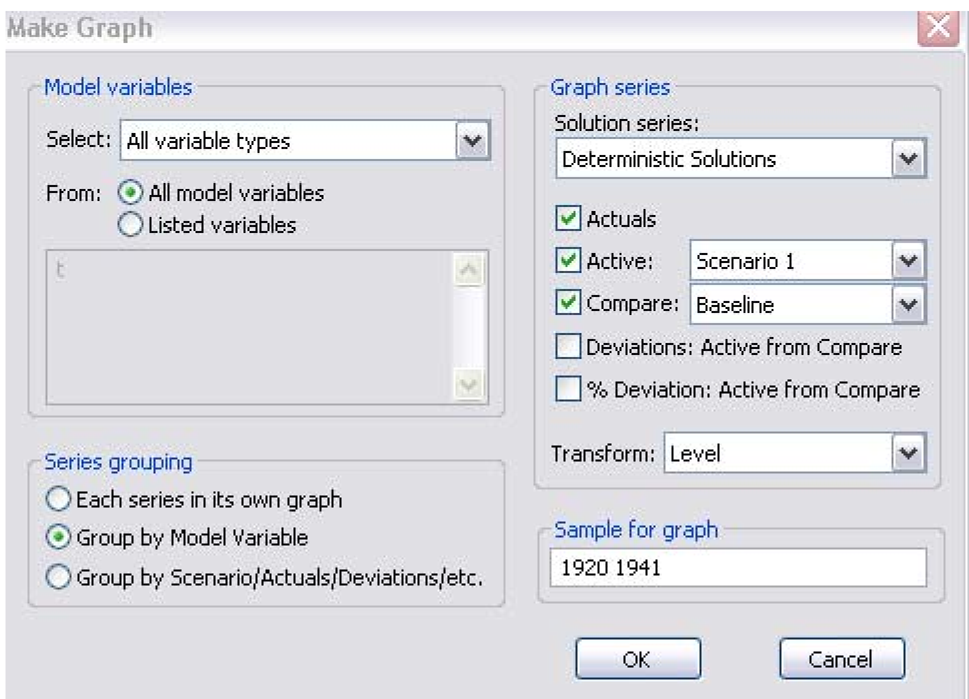


What new variable has been created in the Workfile?
Note that "g" has changed colour to red in the Model box.

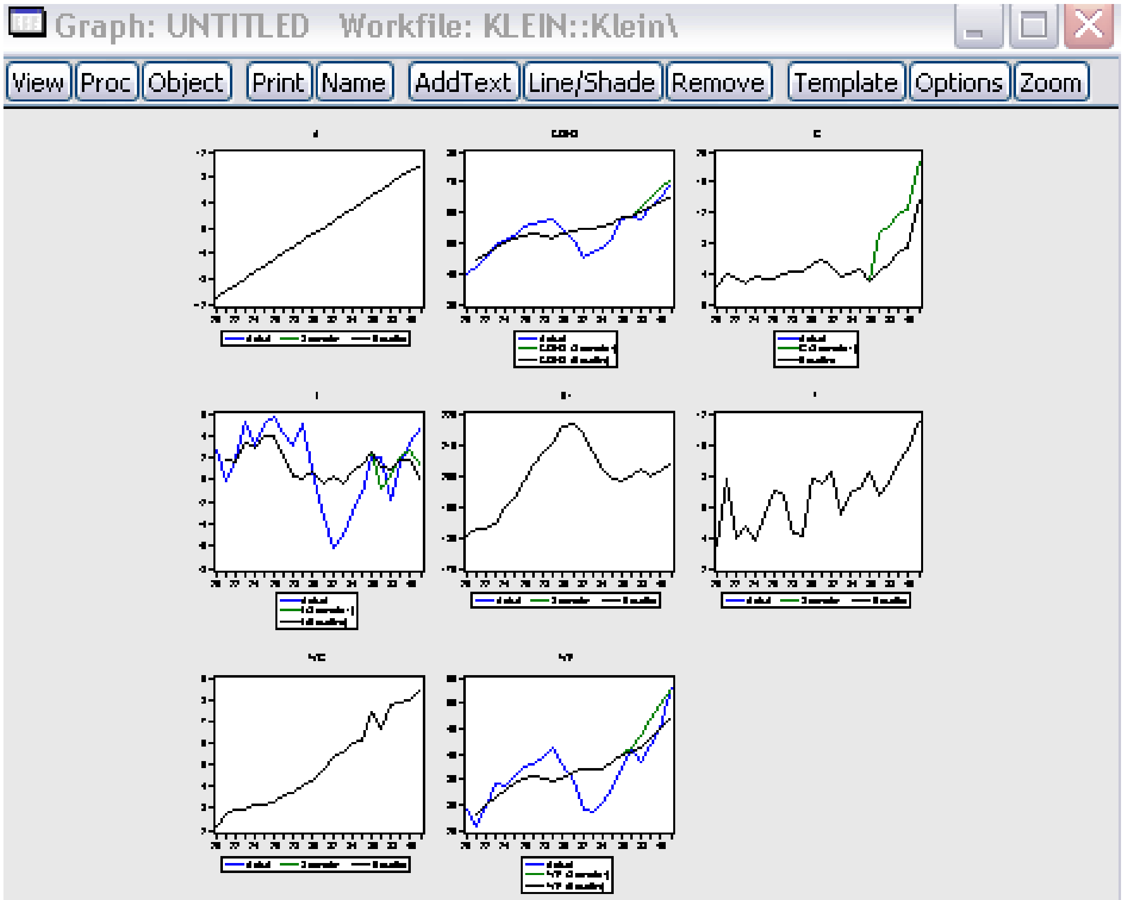
Now edit the series "G_1"by increasing each of the last five values by 5 units. (*e.g.*, the 1941 value will now be 18.8.) Select the "Solve" tab in the Model window and you will now see:



Select "OK". Then select "Proc", "Make Graph" and edit the window as follows:



Select "OK", and interpret the graphs:



References:

Greene, W. H. (2008), *Econometric Analysis*, 6th ed., Pearson Prentice Hall, Upper Saddle River, NJ.

Klein, L. R. (1950), *Economic Fluctuations in the United States, 1921-1941*, Wiley, New York.

Tingbergen, J. (1959), *Selected Papers*, edited by L. H. Klaassen, L. M. Koyck and J. H. Witteveen, North-Holland, Amsterdam.