ECON 546: Themes in Econometrics

Lab Exercises #3
(27 January, 2010)

(a) The 'Box-Cox' model allows for a flexible non-linear functional form by transforming the '*y*' and '*x*' variables as follows:

$$y^{(\lambda)} = (y^{\lambda} - 1)/\lambda \quad ; \quad x^{(\lambda)} = (x^{\lambda} - 1)/\lambda$$

Show that this function converges to a linear transformation as $\lambda \to 1$, and a log-log transformation as $\lambda \to 0$.

Note, for example, that a CES production function with CRTS is just a Box-Cox model:

$$Q = [\alpha K^{\lambda} + (1-\alpha)L^{\lambda}]^{1/\lambda}$$

So, when $\lambda = 1$ we get the CRTS linear production function:

$$Q = \alpha K + (1-\alpha)L$$

and when $\lambda \to 0$ we get the Cobb-Douglas production function with CRTS:

$$Q = K^{\alpha} L^{(1-\alpha)}$$

or,

$$\log(Q) = \alpha \log(K) + (1-\alpha)\log(L) \quad .$$

(b) Consider the model: $y_t^{(\lambda)} = \alpha + \beta x_t^{(\lambda)} + \varepsilon_t$. Show that the Jacobian of the transformation from ε_t to y_t is $y_t^{\lambda-1}$.

(c) Now look at the Excel file, **S:\Social Sciences\Economics\ECON546\LAB3.XLS** on the server. Create an EViews file and estimate a Box-Cox regression that explains expenditure on Research & Development ('*y*') in terms of Profit ('*x*'), and an intercept. Unlike some other econometrics packages, EViews does not have the Box-Cox model as a standard option, but we can estimate the model by MLE by using the following commands within a **LOGL Object**, assuming independent and normally distributed errors:

```
@logl ll1
yt = (y^c(1)-1)/c(1)
xt = (x^c(1)-1)/c(1)
res = yt-c(2)-c(3)*xt
ll1 =  -0.5*log(c(4)) + (c(1)-1)*log(y)-(res^2)/(2*c(4))
```

(d) Edit the 'c' (coefficient) vector in the workfile so that the first 4 elements are 0.1, 10, 0, and 10 respectively, and then experiment with the starting values until you obtain satisfactory results. How would you choose between different, apparently satisfactory, estimated models?

(e) Use z-tests to check if the specification should be linear, log-log, or neither.

(f) Show, algebraically, that the elasticity for y_t with respect to x_t in the Box-Cox model is $\beta(x_t / y_t)^{\lambda}$.

(g) Now estimate this elasticity for your model, at each sample point and at the sample mean of the data. Compare your answer with the estimated elasticity from either the linear model, or the log-log model, depending on what you found in part (e) above.