University of Victoria **Department of Economics**

ECON 546: Themes in Econometrics

Lab Exercises #5
(10 February, 2010)

In this Lab. class we will use Monte Carlo simulation to investigate the finite-sample performance of the Likelihood Ratio Test (LRT) for a simple hypothesis testing problem that arise in the context of a regression model with possibly autocorrelated errors.

Specifically we will:

1. See how much the null distribution of the test statistic differs from its asymptotic chi-square counterpart.
2. Determine how much the true size (significance level) of the test is distorted away from the size that we *think* we are adopting when we use the critical value associated with the asymptotically valid chi-square distribution for the test statistic.
3. Determine how this size distortion compares with that of the Wald test, which is probably the test you would have naturally used for the problem under study.
4. Evaluate the power curves of the LRT and Wald tests once we have "corrected" for the size distortion.

The EViews workfile and program are at **S:\Social Sciences\Economics\ECON546\LAB5.WF1** and **S:\Social Sciences\Economics\ECON546\LAB5.PRG**. The initial version of the program file is attached to this sheet, and you will need to amend it as you proceed through the exercises.

Here is the problem that we are going to study:
Consider the regression model

$$y_t = \beta_1 + \beta_2 x_t + u_t \quad ; \quad u_t = \rho u_{t-1} + \varepsilon_t \quad ; \quad \varepsilon_t \sim N[0, \sigma^2] \quad ; |\rho| < 1$$

We want to test $H_0 : \rho = 0$ *vs.* $H_A : \rho \neq 0$. By way of interest, note that it can be shown (Anderson, 1948) that ***there is no UMP test for this problem***.

I am sure that what most of you would do is to estimate the model by (non-linear) MLE, allowing for AR(1) errors, and then use the t-statistic associated with ρ to test H_0. Asymptotically, this statistic is standard normally distributed. As you know, this is just an asymptotically valid Wald test. (Squaring the "t-statistic" would give a statistic that is asymptotically chi-square distributed, with one degree of freedom, under the null hypothesis.)

Alternatively, we could construct a LRT of H_0, by constructing the statistic

$$\lambda = -2[\log \tilde{L}_R - \log \tilde{L}_U]$$

where $\log \tilde{L}_R$ and $\log \tilde{L}_U$ are the maximized values of the log-likelihood function associated with OLS (MLE) and NLLS (asymptotically equivalent to MLE). If H_0 is true, $\lambda \to \chi^2_{(1)}$.

(a) Work your way through the program and check that you understand what is being done at each stage.

  (i) What significance level are we considering in this experiment – 10%, 5% or 1%?
  (ii) In which line of the code do we assign the null hypothesis to be true?
  (iii) What sample size is being considered?
  (iv) Which line(s) would we alter if we wanted to consider a sample size of (say) $n = 200$?

(b) Run the program.

  (i) What are the values of POWER_LRT and POWER_WALD?
  (ii) What do these values actually represent?
  (iii) What are the values of CRIT_LRT and CRIT_WALD?
  (iv) What do these values actually represent?
  KEEP A NOTE OF THESE TWO VALUES FOR LATER.

(c) Look at the histograms of the series LRT and WALD. Do these distributions look like those for a chi-square distribution with one degree of freedom?

(d) Increase the sample size.

  (i) Does the "size distortion" of each test get smaller as n increases?
  (ii) Which test seems to be better in this respect?

(e) Now, restore the sample size to $n = 20$. Set the critical values to their true finite-sample values (see part (b) above). Either delete or "comment out" the last 2 lines of the program code. Run the program several times, with different positive values of ρ, and compare the "size-adjusted" powers of the LRT and Wald tests.

(f) Plot the two power curves.

  (i) Why was the "size-adjustment" needed?
  (ii) Which of these two tests would you prefer to use **FOR THIS PARTICULAR PROBLEM, THIS PARTICULAR SAMPLE OF X DATA, AND FOR THESE PARTICULAR TRUE VALUES OF THE INTERCEPT AND SLOPE PARAMETERS?**

```
rndseed 123456
!nrep=2000
!n=20
smpl 1  !n
scalar c1=1
scalar c2=4
scalar rho=0.0
series(20000) y
vector(!nrep) lrtv
vector(!nrep) waldv
scalar power_lrt=0.0
scalar power_wald=0.0
scalar asymp_crit=3.84
'=============================
for !i=1 to !nrep
genr e=0
smpl 2 !n
genr e=rho*e(-1)+@rnorm
y=c1+c2*x+e
equation eq1.ls y c x
scalar loglr=@logl
equation eq2.ls  y c x ar(1)
scalar loglu=@logl
lrtv(!i)=-2*(loglr-loglu)
if lrtv(!i)>asymp_crit  then
power_lrt=power_lrt+1
endif
waldv(!i)=@tstats(3)^2
if waldv(!i)>asymp_crit then
power_wald=power_wald+1
endif
next
'===============================
smpl 1 !nrep
mtos(lrtv,lrt)
mtos(waldv,wald)
power_lrt=power_lrt/!nrep*100
power_wald=power_wald/!nrep*100
scalar crit_lrt=@quantile(lrt,0.95)
scalar crit_wald=@quantile(wald,0.95)
```