Department of Economics **University of Victoria**

ECON 546: Themes in Econometrics
Term Test, February 2007

Instructor:	David Giles
Instructions:	Answer **ALL QUESTIONS**, & put all answers in the booklet provided.
Time Allowed:	90 minutes (Total marks = 90 – *i.e., one mark per minute*.)
Number of Pages:	**FOUR** (*A separate set of statistical tables is also provided*.)

Question 1:
Write brief notes (and provide diagrams if this helps) to explain what we mean by each of the following:

(a) A mini-max estimator.

(b) An inadmissible estimator.

(c) The Lindeberg-Lévy Central Limit Theorem.

Total: 21 Marks

Question 2:
Consider the distribution whose density function is:

$$p(y_i) = (y_i / \theta^2)\exp[-y_i^2 /(2\theta^2)] \quad ; \quad y_i > 0 .$$

Suppose that we have a sample of n independent observations drawn from this distribution.

(a) Prove that the MLE of θ is $\tilde{\theta} = [\frac{1}{2n}\sum_{i=1}^{n} y_i^2]^{1/2}$. (Don't forget to check the second-order condition.)

10 marks

(b) What is the MLE of the [1/ log(y_m)], where y_m is the mode of this density function?

5 marks
Total: 15 Marks

Question 3:
A Pearson Type III distribution has three parameters. When one of these parameters is set to 0.5, the density function for a random variable, Y, that follows this distribution is:

$$p(y) = \frac{1}{\beta\sqrt{\pi}}\left(\frac{y-\alpha}{\beta}\right)^{-1/2} \exp\{-(y-\alpha)/\beta\} \quad ; \quad y > \alpha$$

and the associated characteristic function can be shown to be:

$$\phi_y(t) = \exp\{i\alpha t\}(1 - i\beta t)^{-1/2}.$$

(a) Suppose we construct a new random variable, $X = Y_1 + Y_2$, where Y_1 and Y_2 are independent. What is the characteristic function for X?

3 marks

(b) Use this characteristic function to prove that $E(X) = 2\alpha + \beta$.

8 marks

(c) Now, suppose that we know that $\alpha = 0$. If we have n independent sample values, show that the MLE of β is $\tilde{\beta} = 2\bar{y} = \frac{2}{n}\sum_{i=1}^{n} y_i$. (Don't forget the second-order condition.)

8 marks

(d) What is the MLE for $E(X)$ in part (b), under the conditions stated in part (c)?

1 mark

Total: 20 Marks

Question 4:

For the Logit model, the likelihood equations may be written as

$$(\partial \log L / \partial\beta) = \sum_{i=1}^{n} (y_i - \Lambda_i)x_i = 0$$

where

$$\Lambda_i = \exp(x_i'\beta)/[1 + \exp(x_i'\beta)]$$

Suppose that our sample comprises $n = n_1 + n_2 + n_3$ observations on y and a single regressor, x. For the first n_1 observations $y = 1$ and $x = 1$; for the second n_2 observations $y = 0$ and $x = 1$; and for the last n_3 observations $y = 1$ and $x = 0$.

Show that the MLE of the scalar, β, is $\tilde{\beta} = \log_e(n_1 / n_2)$.

Total: 10 Marks

Question 5:

A Logit Model has been estimated to explain the incidence of extra-marital affairs, using a cross-section survey of 601 people. The data come from a well-known study by Ray Fair. The definitions of the variables are as follows:

NAFFAIRS	Number of affairs by respondent in past year
MALE	Dummy variable (= 1 if respondent is male; 0 otherwise)
AGE	Age of respondent, in years
CHILDREN	Dummy variable (= 1 if family has children; 0 otherwise)
EDUC	Number of years of education of respondent
RELIGION	Variable taking values 0, 1, 2, 3, 4, 5 to signal how "religious" respondent is ("higher" means "more")
SELFRATING	Variable taking values 1, 2, 3, 4, 5 to signal respondent's perception of quality of their marriage ("higher" means "better")
NYEARS	Number of years married

The results in **Output 1** were obtained with EViews, with Y defined to be unity if NAFFAIRS > 0, and zero if NAFFAIRS = 0:

Output 1

Dependent Variable: Y
Method: ML - Binary Logit (Quadratic hill climbing)
Date: 02/13/07 Time: 13:53
Sample: 1 601
Included observations: 601
Convergence achieved after 5 iterations
QML (Huber/White) standard errors & covariance

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	1.947603	0.649285	2.999610	0.0027
AGE	-0.043925	0.018954	-2.317426	0.0205
SELFRATING	-0.467212	0.090186	-5.180549	0.0000
NYEARS	0.111327	0.029659	3.753629	0.0002
RELIGION	-0.327142	0.092119	-3.551311	0.0004
MALE	0.386122	0.206778	1.867324	0.0619

Mean dependent var	0.249584	S.D. dependent var	0.433133
S.E. of regression	0.410013	Akaike info criterion	1.038035
Sum squared resid	100.0260	Schwarz criterion	1.081948
Log likelihood	-305.9295	Hannan-Quinn criter.	1.055128
Restr. log likelihood	-337.6885	Avg. log likelihood	-0.509034
LR statistic (5 df)	63.51801	McFadden R-squared	0.094048
Probability(LR stat)	2.27E-12		

Obs with Dep=0	451	Total obs	601
Obs with Dep=1	150		

(a) Discuss these results. Pay special attention to the signs and significance of the estimated coefficients, and comment on the overall "quality" of this estimated model.

8 marks

(b) Estimate the probability that a person with the following characteristics will have at least one affair: Female, 30 Years Old, Married for 10 Years, Zero Religion, and the highest possible self-rating of her marriage.

Hint: The distribution function (c.d.f.) for a Logistic random variable, y, is

$$\Lambda(y) = \exp(y)/[1+\exp(y)].$$

8 marks

(c) **Output 2** gives the basic descriptive statistics for the explanatory variables. Use this information to calculate the marginal effect of AGE, ***at the sample means for the data***. Provide an interpretation of what these marginal effects tell us.

> **Hint:** The density function for the Logistic distribution is
>
> $$\lambda(y) = \Lambda'(y) = \exp(y)/[1+\exp(y)]^2 .$$

8 marks

Total: 24 marks

Output 2

	AGE	SELFRATING	NYEARS	RELIGION	MALE
Mean	32.48752	3.931780	8.177696	3.116473	0.475874
Median	32.00000	4.000000	7.000000	3.000000	0.000000
Maximum	57.00000	5.000000	15.00000	5.000000	1.000000
Minimum	17.50000	1.000000	0.125000	1.000000	0.000000
Std. Dev.	9.288762	1.103179	5.571303	1.167509	0.499834
Skewness	0.887000	-0.834126	0.077994	-0.088800	0.096618
Kurtosis	3.220077	2.787923	1.432516	1.990046	1.009335
Jarque-Bera	80.02088	70.81884	62.13686	26.33254	100.1688
Probability	0.000000	0.000000	0.000000	0.000002	0.000000
Sum	19525.00	2363.000	4914.795	1873.000	286.0000
Sum Sq. Dev.	51768.66	730.2030	18623.65	817.8469	149.9002
Observations	601	601	601	601	601

END OF TEST