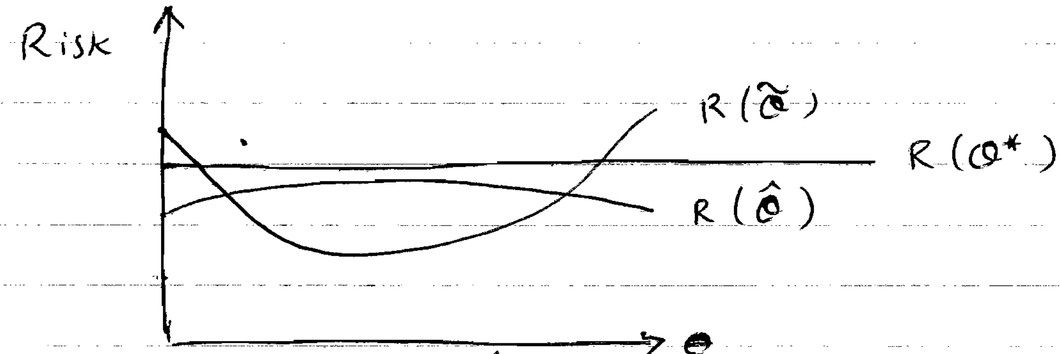
ECON 546 - 2007

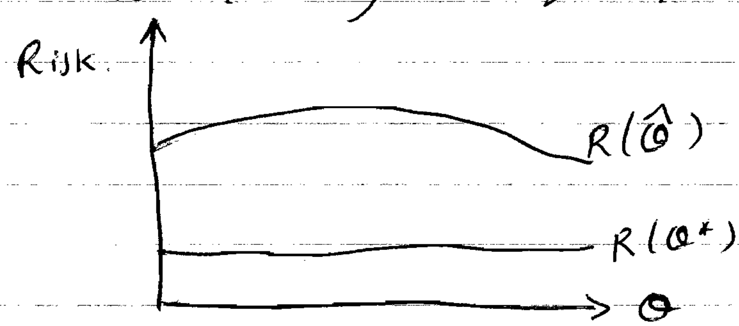
Midterm Test - Solution

Q.1. (a) An estimator is "mini-max" within some family of estimators if its maximum risk is the minimum of the maximum risks of the other estimators. For example:



In this family, $\hat{\theta}$ is mini-max.

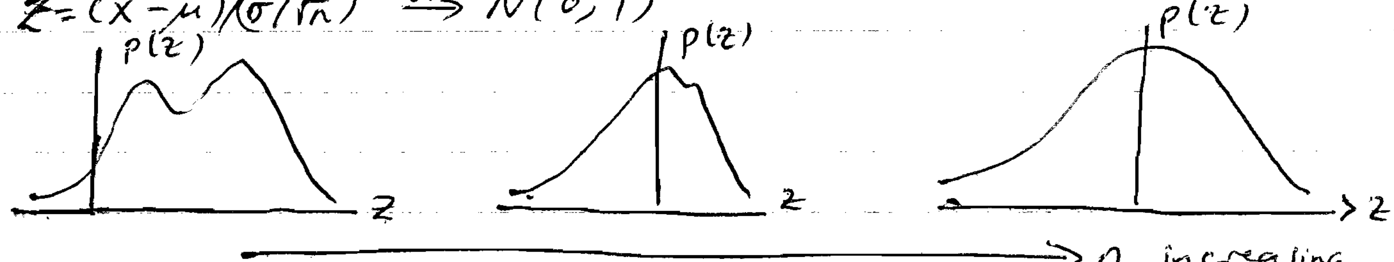
(b) An estimator is inadmissible if, for the same loss fctn. there exists any other estimator whose risk is less than or equal to this estimator's risk for all θ, and strictly less for some θ. So:



The existence of θ^* $\Rightarrow$ $\hat{\theta}$ is inaddmissible

(c) If x_i's are iid then once we average them, they follow a Normal distn. if there enough obs. Specifically,

$z = (\bar{x} - \mu)/(\sigma/\sqrt{n}) \xrightarrow{d} N(0,1)$



Q.2. (a) Independence $\Rightarrow$ $L(\theta|\underset{\sim}{y}) = \prod_i p(y_i|\theta)$

So, $L(\theta|y) = \dfrac{\prod_i y_i}{\theta^{2n}} \exp\left[-\dfrac{1}{2\theta^2}\sum_i y_i^2\right]$

& $\log L = \sum_i \log y_i - 2n\log\theta - \dfrac{1}{2\theta^2}\sum_i y_i^2$

$(\partial \log L/\partial\theta) = -(2n/\theta) + \dfrac{1}{\theta^3}\sum_i y_i^2 = 0$

$\Rightarrow \tilde{\theta} = \left[\dfrac{1}{2n}\sum_i y_i^2\right]^{1/2}$

Check the s.o.c. :
$(\partial^2 \log L/\partial\theta^2) = 2n/\theta^2 - 3\sum_i y_i^2/\theta^4$.

When $\theta = \tilde{\theta}$,
$(\partial^2\log L/\partial\theta^2)|_{\tilde{\theta}} = \dfrac{1}{\tilde{\theta}^4}\left[2n\tilde{\theta}^2 - 3\sum_i y_i^2\right]$

$= 1/\tilde{\theta}^4\left[\sum_i y_i^2 - 3\sum_i y_i^2\right]$

$= -(2\sum_i y_i^2)/\tilde{\theta}^4 < 0.$ (Max.)

(b) First, find the mode of the density – this is when $(\partial p(y_i)/\partial y_i) = 0$:

$(\partial p(y_i)/\partial y_i) = \dfrac{1}{\theta^2} e^{-y_i^2/2\theta^2} + y_i/\theta^2\, e^{-y_i^2/2\theta^2}(-y_i/\theta^2)$

$= 1/\theta^2 e^{-y_i^2/2\theta^2}\left[1 - y_i^2/\theta^2\right]$

$= 0$

$\Rightarrow y_m = \theta.$

So, the MLE of $[1/\log(y_m)] = [1/\log(\tilde{\theta})]$.

Q. 3.

(a) If we add 2 r.v.'s, the c.f. of the sum is the product of the individual c.f.'s (if they are independent). So –

$$\phi_X(t) = \exp\{2i\alpha t\}(1-i\beta t)^{-1}.$$

(b) $$\phi_X'(t) = \exp\{2i\alpha t\}(-1)(-i\beta)(1-i\beta t)^{-2}$$

$$+ \exp\{2i\alpha t\}(2i\alpha)(1-i\beta t)^{-1}$$

$$= i\exp\{2i\alpha t\}(1-i\beta t)^{-2}[\beta + 2\alpha(1-i\beta t)]$$

$$(\phi_X'(0)/i) = \exp(0)(1)[\beta + 2\alpha(1-0)]$$

$$= (2\alpha + \beta.)$$

(c) Let $\alpha = 0$. Then $p(y_i|\beta) = \frac{1}{\beta\sqrt{\pi}}\left(\frac{y_i}{\beta}\right)^{-1/2} e^{-y_i/\beta}$

& $$L = \left(\frac{1}{\beta^n}\right)\left(\frac{1}{\sqrt{\pi}}\right)^n\left(\frac{\prod y_i}{\beta^n}\right)^{-1/2} e^{-\frac{1}{\beta}\sum y_i}$$

$$\log L = -n\log\beta - n\log\sqrt{\pi} + \frac{n}{2}\log\beta - \frac{1}{2}\sum_i \log y_i - \frac{1}{\beta}\sum_i y_i$$

$$(\partial \log L/\partial\beta) = -n/\beta + \frac{n}{2\beta} + (\textstyle\sum_i y_i)/\beta^2 = 0$$

$$= -\frac{n}{2\beta} + (\textstyle\sum_i y_i)/\beta^2 = 0$$

$$\Rightarrow \tilde{\beta} = \frac{2}{n}\sum_i y_i = 2\bar{y}.$$

$$(\partial^2 \log L / \partial\beta^2) = \frac{n}{2\beta^2} - 2\sum_i y_i/\beta^3$$
$$= n/\beta^2 \left[\frac{1}{2} - 2\bar{y}/\beta\right]$$

& when $\beta = \tilde{\beta}$,

$$(\partial^2 \log L/\partial\beta^2)_{\tilde{\beta}} = \frac{n}{\tilde{\beta}^2}\left[\frac{1}{2} - 1\right] = -n/2\tilde{\beta}^2 < 0. \text{ (Max.)}$$

(d) $E(X) = (2\alpha + \beta) = \beta$, if $\alpha = 0$.

So, MLE for $E(X)$ is just $\tilde{\beta} = \frac{2}{n}\sum_i y_i$.

Q.4. The likehood equation is

$$\sum_{i=1}^{n} (y_i - \Lambda_i) x_i = 0,$$

or, $\sum_{i=1}^{n} x_i y_i = \sum_{i=1}^{n} x_i \Lambda_i$

So, $\sum_{i=1}^{n_1} (1.1) + 0 + 0 = \sum_{i=1}^{n_1} 1.\Lambda_i + \sum_{i=1}^{n_2} 1.\Lambda_i$

i.e. $n_1 = (n_1 + n_2)\Lambda_i$

& $\Lambda_i = [\exp(1.\beta)] / [1 + \exp(1.\beta)]$

$= e^{\beta}/(1 + e^{\beta})$

So, $n_1(1 + e^{\beta}) = (n_1 + n_2)e^{\beta}$

or, $n_1 = n_2 e^{\beta}$

or, $\tilde{\beta} = \log_e(n_1/n_2)$.

Q.5

(a) All of the covariates are significant at the 10% if we have no prior idea of their signs, and at the 5% if we do know the signs.
The older and "more religious" the respondent, the less likely are they to have an affair. Similarly if they rate their marriage as being "good". On the other hand, males, & respondents who have been married for a longer time are more likely to have one or more affairs.
The McFadden R^2 is only 0.094, which is very low – this model doesn't explain very much.

(b) MALE = 0 ; AGE = 30 ; NYEARS = 10 ; RELIGION = 0
SELFRATING = 5.

$$\text{So, } x_i'\hat{\beta} = [1.947603 + (30)(-0.043925) + (5)(-0.467212) + 10(0.111327)]$$
$$= -0.592937$$

$$\& \quad Pr.(y=1) = P_i = \Lambda(x_i'\beta)$$

$$= \exp(-0.5929)/[1 + \exp(-0.5929)]$$

$$= 0.35595$$

(c) At the sample means:
$$x_i'\hat{\beta} = [1.947603 + (-0.043925)(32.48752) + (-0.467212)(3.93178)$$
$$+ (0.111327)(8.177696) + (-0.327142)(3.116473) + (0.386122)(0.4759$$
$$= -1.24178$$

So, marginal effect $= \hat{\beta}_K \Lambda'(x_i'\hat{\beta})$

For AGE:

$$\text{Marginal Effect} = \frac{-0.043925 \exp(-1.24176)}{[1 + \exp(-1.24176)]^2}$$

$$= -0.0076$$

An increase of 1 year in age $\Rightarrow$ Reduction in probability of having one or more affairs of 0.0076