Department of Economics **University of Victoria**

ECON 546: Themes in Econometrics
Term Test, February 2008

Instructor:	David Giles
Instructions:	Answer **ALL QUESTIONS**, & put all answers in the booklet provided.
Time Allowed:	80 minutes (Total marks = 80 – *i.e., one mark per minute*.)
Number of Pages:	**FOUR** (*A separate set of statistical tables is also provided*.)

Question 1:

Write brief notes (and provide diagrams if this helps) to explain what we mean by each of the following:

(a) The "risk" of an estimator.

(b) An asymptotically efficient estimator.

(c) The Neyman-Pearson Lemma.

Total: 24 Marks

Question 2:

Suppose that we have a sample of n independent observations from a distribution whose p.d.f. is

$$p(y_i \mid \theta) = \sqrt{(\theta/(2\pi)}[e^{-\theta/(2y_i)} y_i^{-3/2}] \quad ; \qquad y_i > 0 \ ; \theta > 0.$$

(a) Show that the MLE of θ is $\tilde{\theta} = [(1/n)\sum_{i=1}^{n}(1/y_i)]^{-1}$.

7 marks

(b) Derive Fisher's Information Measure for this problem, and its asymptotic counterpart. Suggest a consistent estimator for the latter.

5 marks

(c) Carefully state the asymptotic distribution for the MLE of θ.

2 marks

(d) The characteristic function for this distribution is $\phi_y(t) = \exp\{-\sqrt{-2i\theta t}\}$. Prove that the mean of the distribution is infinite. Would you expect the variance to be finite?

6 marks

Total: 20 Marks

Question 3:

The density for a random variable, *Y*, that follows a Rayleigh distribution is:

$$p(y \mid \theta) = (y/\theta^2)\exp\{-y^2/(2\theta^2)\} \quad ; \quad y > 0\ ;\ \theta > 0.$$

and the k^{th} moment about the origin is $\mu_k' = \theta^k 2^{k/2}\Gamma[1+(k/2)]$. Here, the Gamma function satisfies the recurrence relationship, $\Gamma(x+1) = x\Gamma(x)$; $\Gamma(1) = 1$ and $\Gamma(1/2) = \sqrt{\pi}$.

(a) If we have *n* independent sample values, show that the MLE of θ is $\tilde{\theta} = +\sqrt{(1/2n)\sum_i y_i^2}$.

7 marks

(b) Show that the mean of *Y* is $\theta\sqrt{\pi/2}$ and the variance of *Y* is $\theta^2(4-\pi)/2$ .What are the MLE's for the mean, variance *and standard deviation* of *Y*, and what desirable properties will these estimator have?

7 marks

(c) Derive the Likelihood Ratio Test statistic for testing H_0: $\theta = 1$ against H_1: $\theta \neq 1$.

5 marks

(d) Suppose that $n = 100$ and $\sum_{i=1}^{n} y_i^2 = 180$. Apply the LRT. What assumptions have you made? Is your conclusion sensitive to your choice of significance level?

5 marks

Total: 24 Marks

Question 4:

We can estimate a Box-Cox model by MLE by using the following commands within a LOGL Object in EViews:

```
@logl ll1
yt = (y^c(1)-1)/c(1)
xt = (x^c(1)-1)/c(1)
res = yt-c(2)-c(3)*xt
ll1 =  -0.5*log(c(4)) + (c(1)-1)*log(y)-(res^2)/(2*c(4))
```

We can also estimate the model by MLE when the errors are heteroskedastic, with $\text{var}(\varepsilon_t) = \exp(\delta_0 + \delta_1\sqrt{Z_t})$, by modifying our LOGL Object to:

```
@logl ll2
yt = (y^c(1)-1)/c(1)
xt = (x^c(1)-1)/c(1)
res = yt-c(2)-c(3)*xt
var=exp(c(4)+c(5)*z^0.5)
ll2 =  -0.5*log(var) + (c(1)-1)*log(y)-(res^2)/(2*var)
```

The following results have been obtained:

LogL: LOGL01
Method: Maximum Likelihood (Marquardt)
Date: 02/14/07 Time: 08:47
Sample: 1 18
Included observations: 18
Evaluation order: By observation
Convergence achieved after 28 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.132921	0.194739	0.682560	0.4949
C(2)	0.307780	2.405941	0.127925	0.8982
C(3)	0.764673	0.218204	3.504400	0.0005
C(4)	5.549571	15.12861	0.366826	0.7137

Log likelihood	-135.3922	Akaike info criterion	15.48803
Avg. log likelihood	-7.521791	Schwarz criterion	15.68589
Number of Coefs.	4	Hannan-Quinn criter.	15.51531

LogL: LOGL02
Method: Maximum Likelihood (Marquardt)
Date: 02/14/07 Time: 08:49
Sample: 1 18
Included observations: 18
Evaluation order: By observation
Convergence achieved after 40 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.294795	0.637398	0.462497	0.6437
C(2)	0.427104	4.608686	0.092674	0.9262
C(3)	0.659183	0.450980	1.461670	0.1438
C(4)	3.023551	5.238874	0.577137	0.5638
C(5)	0.003591	0.015383	0.233442	0.8154

Log likelihood	-135.2106	Akaike info criterion	15.57896
Avg. log likelihood	-7.511701	Schwarz criterion	15.82628
Number of Coefs.	5	Hannan-Quinn criter.	15.61306

(a) Use a ***Likelihood Ratio Test*** to test the hypothesis that the errors are homoskedastic.

4 marks

(b) Use the information in the second output above to apply an alternative test that the errors are homoskedastic. Do you come to the same conclusion as in part (a)?

2 marks

(c) Construct two asymptotically valid 95% confidence intervals for C(3), using the information in each of the above outputs separately. Use these intervals to address the statement: "The value of C(3) is the same in both versions of the model."

5 marks

(d) Do you think these tests are likely to be "reliable" in the present context?

1 mark

Total: 12 Marks

END OF TEST

"Examinations are formidable even to the best prepared, for the greatest fool may ask more than the wisest man can answer."

- Charles Caleb Colton (English clergyman and writer, 1780-1832)