Department of Economics **University of Victoria**

ECON 546: Themes in Econometrics
Term Test, February 2009

Instructor: David Giles
Instructions: Answer **ALL QUESTIONS**, & put all answers in the booklet provided.
Time Allowed: 80 minutes (Total marks = 80 – *i.e., one mark per minute*.)
Number of Pages: **THREE** (*A separate set of statistical tables is also provided*.)

Question 1:

Write brief notes (and provide diagrams if this helps) to explain what we mean by each of the following:

(a) An "admissible" estimator.

(b) The "Cramér-Rao lower bound" for an unbiased estimator.

(c) A "sufficient" statistic.

Total: 21 Marks

Question 2:

Suppose that we have a sample of n independent observations from an Erlang distribution, whose p.d.f. is

$$p(y_i \mid \lambda) = \frac{\lambda^k y_i^{k-1} \exp(-\lambda y_i)}{(k-1)!} \quad ; \quad y_i > 0 \ ; \lambda > 0.$$

Here, λ is called the "rate parameter", and for simplicity we are going to treat the "shape" (k) of the distribution as a ***known constant***.

(a) Write down the joint data density, and show that $\bar{y}$ (the sample average of the data) is a sufficient statistic.

4 marks

(b) Show that the MLE of λ is $\tilde{\lambda} = (k / \bar{y})$.

7 marks

(c) Derive Fisher's information measure, and the asymptotic information measure. Suggest a consistent estimator for the asymptotic information measure.

5 marks

(d) Carefully state the asymptotic distribution for the MLE of λ.

2 marks

(e) Explain how you would construct an asymptotically valid 95% confidence interval for λ.

4 marks

(f) The characteristic function for the Erlang distribution is $\phi_y(t) = (1 - it/\lambda)^{-k}$, where $i^2 = -1$. Prove that the mean of this distribution is (k / λ), and that the variance is (k / λ^2).

7 marks

(g) What are the MLE's for the mean and variance of this distribution?

2 marks

(h) Derive the Likelihood Ratio Test statistic for testing H_0: $\lambda = 1$ against H_A: $\lambda \neq 1$.

7 marks

(i) Suppose that $k = 2$, $n = 1{,}000$ and $\sum_{i=1}^{n} y_i = 1{,}800$. Apply the LRT. What assumptions have you made? Is your conclusion sensitive to your choice of significance level?

6 marks

Total: 44 Marks

Question 3:

Sometimes, in empirical economics, we encounter data that have been "censored". A censored data-point is one whose value is incomplete due to random factors. There are various types of censoring. As you might guess, censoring has implications for the form of the underlying probability distribution, and hence for the formulation of the likelihood function.

The EViews **Output 1** on the next page gives the results from estimating a model in which a large part of the sample is "left censored" – there are a lot of zeros for the dependent variable. *I know that you have probablyl not have seen output of this type before.*

(a) What indications are there that this model has been estimated by the method of Maximum Likelihood?

3 marks

(b) Something special been done when computing the standard errors. What is this (in words), and why?

2 marks

(c) Construct a 90% confidence interval for the coefficient of NWIFEINC. Under what circumstances is this interval valid?

5 marks

(d) What restriction has been imposed on the coefficients of the model in **Output 2** below? Use the Likelihood Ratio Test to test if this restriction is valid.

5 marks

Total: 15 marks

Output 1

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: HOURS
Method: ML - Censored Normal (TOBIT) (Newton-Raphson)
Date: 02/19/09 Time: 11:32
Sample: 1 753
Included observations: 753
Left censoring (value) at zero
Convergence achieved after 7 iterations
QML (Huber/White) standard errors & covariance

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	1287.811	394.1732	3.267119	0.0011
NWIFEINC	-8.465080	4.469402	-1.894007	0.0582
EDUC	86.14619	21.72331	3.965611	0.0001
EXPER	77.41802	6.350441	12.19097	0.0000
AGE	-58.25464	6.773509	-8.600363	0.0000
KIDSLT6	-919.9445	114.9780	-8.001048	0.0000
Error Distribution				
SCALE:C(7)	1131.124	43.11603	26.23443	0.0000

Mean dependent var	740.5764	S.D. dependent var	871.3142
S.E. of regression	753.6533	Akaike info criterion	10.17880
Sum squared resid	4.24E+08	Schwarz criterion	10.22179
Log likelihood	-3825.319	Hannan-Quinn criter.	10.19536
Avg. log likelihood	-5.080105		
Left censored obs	325	Right censored obs	0
Uncensored obs	428	Total obs	753

Output 2

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: HOURS
Method: ML - Censored Normal (TOBIT) (Newton-Raphson)
Date: 02/19/09 Time: 12:21
Sample: 1 753
Included observations: 753
Left censoring (value) at zero
Convergence achieved after 7 iterations
QML (Huber/White) standard errors & covariance

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-1537.121	364.2047	-4.220487	0.0000
NWIFEINC	-24.44749	5.103039	-4.790770	0.0000
EDUC	142.3052	22.48846	6.327919	0.0000
EXPER+AGE	14.01973	4.028089	3.480491	0.0005
KIDSLT6	-696.7109	123.3360	-5.648883	0.0000
Error Distribution				
SCALE:C(6)	1270.338	45.44937	27.95061	0.0000

Mean dependent var	740.5764	S.D. dependent var	871.3142
S.E. of regression	822.7842	Akaike info criterion	10.37228
Sum squared resid	5.06E+08	Schwarz criterion	10.40913
Log likelihood	-3899.164	Hannan-Quinn criter.	10.38647
Avg. log likelihood	-5.178172		
Left censored obs	325	Right censored obs	0
Uncensored obs	428	Total obs	753

END OF TEST