Department of Economics **University of Victoria**

ECON 546: Themes in Econometrics
Term Test, February 2010

Instructor: David Giles
Instructions: Answer **ALL QUESTIONS**, & put all answers in the booklet provided.
Time Allowed: 90 minutes (Total marks = 90 – *i.e., one mark per minute*.)
Number of Pages: **THREE** (*A separate set of statistical tables is also provided*.)

Question 1:
Write brief notes (and provide diagrams if this helps) to explain what we mean by each of the following:

(a) The "moments" of a probability distribution.
(b) The asymptotic "Cramér-Rao lower bound".
(c) The "invariance" property of maximum likelihood estimators.

Total: 21 Marks

Question 2:
Let y_i (i = 1, 2, 3,....., n) follow a Wald distribution, whose density function is

$$p(y_i \mid \theta) = [\theta/(2\pi)]^{1/2} e^{\theta} y_i^{-3/2} \exp[-(\theta/2)(y_i + y_i^{-1})] \quad ; \quad \theta > 0.$$

(a) Assuming independent sampling, write down the likelihood function, and derive the MLE for θ. (Check that you have actually *maximized* the Likelihood Function.)

8 marks

(b) Obtain Fisher's information measure for this problem.

3 marks

(c) Derive the likelihood ratio test statistic for testing the null hypothesis that $\theta = 1$, against the alternative hypothesis that $\theta \neq 1$.

5 marks

(d) Explain, with the aid of a diagram, how you would apply this test in practice.

4 marks

Total: 20 Marks

Question 3:
Suppose that we have a sample of n independent observations from a Lévy distribution, whose p.d.f. is

$$p(y_i \mid \theta) = \sqrt{\theta/(2\pi)} \exp[-\theta/(2y_i)] y_i^{-3/2} \; ; \quad y_i > 0 \; ; \; \theta > 0 \, .$$

(a) Show that the MLE for θ is the Harmonic Mean of the y_i's. [**Note:** The Harmonic Mean is the reciprocal of the simple (arithmetic) mean of the reciprocals of the data.]

6 marks

(b) Derive Fisher's information measure, and the asymptotic information measure. Suggest a consistent estimator for the asymptotic information measure.

5 marks

(c) Carefully state the asymptotic distribution for the MLE of θ.

3 marks

(d) Show that the mode of this density function occurs at $y_i = (\theta/3)$. Provide an estimator for this mode that is weakly consistent and asymptotically efficient.

6 marks

(e) Explain how you would construct an asymptotically valid 95% confidence interval for θ.

5 marks

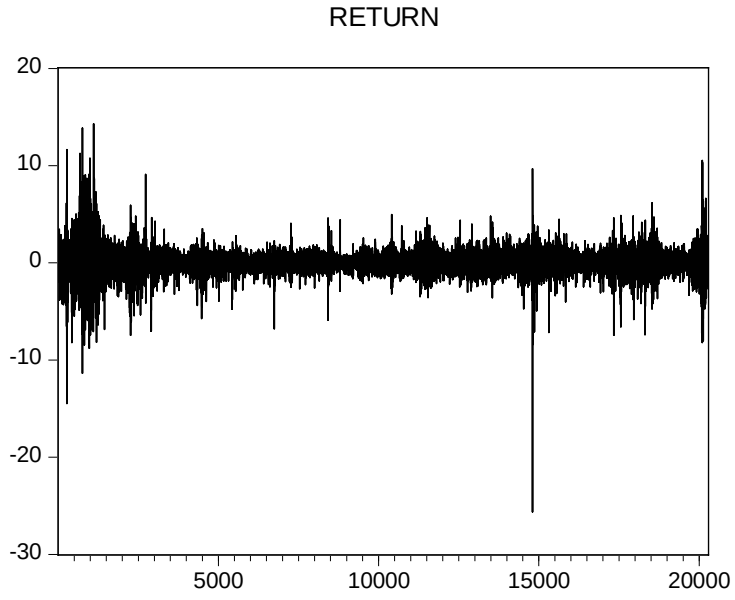
(f) The characteristic function for the Lévy distribution is $\phi_y(t) = \exp[-\sqrt{-2i\theta t}]$, where $i^2 = -1$. Prove that this distribution has an infinite mean.

5 marks

Total: 30 Marks

Question 4:

The following graph shows the daily "returns" (*i.e.*, the percentage change from the end of one day to the end of the next day) for the Dow Jones share index in the U.S.A.. One feature of such data is that they exhibit "clusters" of volatility over time – when there is a large daily fluctuation, this tends to be followed by further large fluctuations, for a while. One way of modelling such data is by using the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model.



The EViews **Output 1** on the next page gives the results from estimating a particular form of GARCH model for these data, assuming that the errors follow a Student-t distribution. *I know that you have probably not have seen output of this type before.*

(a) What general method of estimation has been used here? How large is the sample?

3 marks

(b) Construct a 95% confidence interval for the coefficient of RETURN(-1).

5 marks

(c) Test the hypothesis that the degrees of freedom for the Student-t error distribution are 7, against a 2-sided alternative hypothesis.

5 marks

(d) Note the highlighted lines in **Output 1** and **Output 2**. What restrictions have been imposed on the parameters of the model in **Output 2**? Use the Likelihood Ratio Test to test if these restrictions are valid.

6 marks
Total: 19 marks

Output 1

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: RETURN
Method: ML - ARCH (Marquardt) - Student's t distribution
Date: 02/23/10 Time: 14:21
Sample (adjusted): 5 20280
Included observations: 20276 after adjustments
Convergence achieved after 11 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(6) + C(7)*RESID(-1)^2 + C(8)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.047043	0.005008	9.393950	0.0000
RETURN(-1)	0.084296	0.007123	11.83410	0.0000
RETURN(-2)	-0.036987	0.007115	-5.198763	0.0000
RETURN(-3)	-0.003864	0.007058	-0.547466	0.5841
RETURN(-4)	0.005657	0.006961	0.812602	0.4164
Variance Equation				
C	0.006482	0.000802	8.083419	0.0000
RESID(-1)^2	0.066897	0.003405	19.64450	0.0000
GARCH(-1)	0.928171	0.003380	274.6022	0.0000
T-DIST. DOF	6.503847	0.250915	25.92054	0.0000

R-squared	-0.004398	Mean dependent var	0.017385
Adjusted R-squared	-0.004596	S.D. dependent var	1.166289
S.E. of regression	1.168966	Akaike info criterion	2.534482
Sum squared resid	27699.94	Schwarz criterion	2.537996
Log likelihood	-25685.58	Hannan-Quinn criter.	2.535631
Durbin-Watson stat	2.133124		

Output 2

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: RETURN
Method: ML - ARCH (Marquardt) - Student's t distribution
Date: 02/23/10 Time: 14:15
Sample (adjusted): 3 20280
Included observations: 20278 after adjustments
Convergence achieved after 10 iterations
Presample variance: backcast (parameter = 0.7)
t-distribution degree of freedom parameter fixed at 7
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.046951	0.005024	9.345851	0.0000
RETURN(-1)	0.085118	0.007151	11.90284	0.0000
RETURN(-2)	-0.036904	0.007101	-5.196687	0.0000
Variance Equation				
C	0.006448	0.000783	8.239812	0.0000
RESID(-1)^2	0.066458	0.003301	20.13154	0.0000
GARCH(-1)	0.927816	0.003318	279.5992	0.0000

R-squared	-0.004427	Mean dependent var	0.017439
Adjusted R-squared	-0.004526	S.D. dependent var	1.166250
S.E. of regression	1.168886	Akaike info criterion	2.534394
Sum squared resid	27701.62	Schwarz criterion	2.536737
Log likelihood	-25690.22	Hannan-Quinn criter.	2.535160
Durbin-Watson stat	2.134602		

END OF TEST