ECON 546: Themes in Econometrics
Supplementary Problems, II

The following questions are designed to give you additional practice with the material that is being covered in class. You should note that many of them are more challenging than the questions that you should expect in the term test or final exam – so don't be put off by this!

A solution sheet is available on request. (This gives you the opportunity to attempt these questions without seeing the answers.)

Question 1

Suppose that we have a sample of n independent observations from a **Laplace distribution**, so that the marginal probability density function (p.d.f.) for the i^{th} observation is

$$p(y_i \mid \theta, \mu) = [1/(2\theta)]\exp[-\frac{|y_i - \mu|}{\theta}] \quad ; \quad -\infty < y_i < \infty \, ; \quad \theta > 0$$

(a) Assuming that the location parameter, μ, is known, prove that the MLE of the scale parameter, θ, is just the mean absolute deviation (about μ) in the sample.

(b) The characteristic function for this distribution is

$$\phi_y(t) = e^{\mu i t} / [1 + \theta^2 t^2]$$

Prove that the mean and variance of this distribution are μ and $2\theta^2$ respectively, and that the excess kurtosis is 3. What is the MLE for the variance?

Question 2

Suppose that y_i (i = 1, 2, 3, ..., n) are independent drawings from a **Pareto distribution** (with parameter α). That is, the marginal probability density function (p.d.f.) for y_i is

$$p(y_i \mid \alpha) = (\alpha b^{\alpha}) / (y_i^{\alpha+1}) \quad ; \quad y_i > b \quad ; \quad \alpha > 0$$

This distribution is useful for modelling income distributions, for example. Obtain the Maximum Likelihood Estimator for α, assuming that b is known. (Check that you have obtained a maximum, and not a minimum.)

Question 3

Use the Newton-Raphson algorithm to obtain a local minimum of the function:

$$f(\theta) = \theta^5 - 5\theta^3 .$$

(a) Use an initial value of θ_0 = -2.0.

(b) What happens if you use an initial value of θ_0 = 2.0 ? What happens if you use an initial value of θ_0 = 0.5 ?

Question 4
Consider a sample on n independent observations drawn from a population with probability density function given by

$$p(y_i \mid \theta) = \theta\, y_i^{\theta-1} \quad ; 0 < y_i < 1 \quad ; \quad \theta > 0$$

(a) Obtain the MLE of θ, being careful to check the second-order condition when maximizing the log-likelihood function.
(b) Show that the MLE of $exp(-2\theta)$ is the square of the geometric mean of the sample data.
(c) Derive Fisher's Information Measure for this problem, and its asymptotic counterpart.
(d) Carefully state the asymptotic distribution for the MLE of θ.
(e) How would you obtain a consistent asymptotic standard error for the MLE of θ ?

Question 5
Let y_i (i = 1, 2, 3,........,n) follow a distribution with the following p.d.f.:

$$p(y_i \mid \theta) = (\theta/(2\pi))^{1/2} e^{\theta} y_i^{-3/2} \exp[-(\theta/2)(y_i + y_i^{-1})] \quad ; \quad \theta > 0.$$

(a) Assuming independent sampling, write down the Likelihood Function, and derive the MLE for θ. (Check that you have actually *maximized* the Likelihood Function.)
(b) In what sense is the estimator you have obtained a "good" estimator of θ ?

Question 6
Let y_i (i = 1, 2, 3,........,n) follow a **Gumbel distribution**:

$$p(y_i \mid \alpha, \beta) = (1/\beta)\exp[(y_i - \alpha)/\beta]\exp\{-\exp[(y_i - \alpha)/\beta]\} \quad ; \ -\infty < y_i < \infty$$

(a) Discuss how you would obtain the MLE's of the two parameters of this distribution, assuming an independent sample of 'n' observations.
(b) Derive the MLE for the mode of the above density function.
(c) It can be shown that the variance of this distribution is $(\pi^2\beta^2/6)$. Obtain a consistent and asymptotically efficient estimator of the standard deviation of this population.

Question 7
Suppose that we have a sample of n independent observations from a **Lévy distribution**, whose p.d.f. is

$$p(y_i \mid \theta) = \sqrt{(\theta/(2\pi)}[e^{-\theta/(2y_i)} y_i^{-3/2}] \quad ; \qquad y_i > 0 .$$

The single parameter, θ, is the (positive) "scale parameter".

(a) Prove the MLE of the scale parameter is just the harmonic mean of the sample.
(b) Obtain the MLE for the mode of this distribution.
(c) Derive Fisher's Information Measure for this problem, and its asymptotic counterpart.
(d) Carefully state the asymptotic distribution for the MLE of θ.