ECON 546: Themes in Econometrics
Supplementary Problems, III

The following questions are designed to give you additional practice with the material that is being covered in class. You should note that many of them are more challenging than the questions that you should expect in the term test or final exam – so don't be put off by this!

A solution sheet is available on request. (This gives you the opportunity to attempt these questions without seeing the answers.) *You may wish to refer back to Supplementary Problems II, where certain of the distributions were met in the context of MLE.*

Question 1

Suppose that we have a random sample of n observations, x_i, each drawn from a distribution with a probability density function given by:

$$p(x_i) = \theta^{x_i}(1-\theta)^{1-x_i} \quad ; \quad 0 \le \theta \le 0.5$$

(a) Derive and compare the LR, Wald and LM test statistics for testing $H_0 : \theta = 0.25$ against $H_A : \theta \neq 0.25$.

(b) Explain how you would apply each of these tests, at the 10% significance level. What assumptions are you using?

Question 2

Consider a sample of n independent observations drawn from a population with probability function given by

$$p(y_i \mid \theta) = \theta\, y_i^{\theta-1} \quad ; 0 < y_i < 1 \quad ; \quad \theta > 0$$

(a) Derive and compare the LR, Wald and LM test statistics for testing $H_0 : \theta = 1$ against $H_A : \theta \neq 1$.

(b) Explain how you would apply each of these tests, at the 5% significance level. What assumptions are you using?

Question 3

Let y_i (i = 1, 2, 3,........,n) follow a Wald distribution:

$$p(y_i \mid \theta) = (\theta/(2\pi))^{1/2} e^{\theta} y_i^{-3/2} \exp[-(\theta/2)(y_i + y_i^{-1})] \quad ; \quad \theta > 0.$$

Derive and compare the LR, Wald and LM test statistics for testing $H_0 : \theta = 1$ against $H_A : \theta \neq 1$.

Question 4

The Poisson distribution arises when we model "count data": 0, 1, 2, 3, 4, ………

The probability mass function for a Poisson-distributed random variable is:

$$p(y \mid \lambda) = [\exp(-\lambda)\lambda^y]/y! \quad ; \quad y = 0, 1, 2, \ldots..$$

The sole parameter of this distribution is both its mean and its variance (which can be quite a limitation.) Assume that we have an independent sample of 'n' observations.

(a) Derive the test statistics for the LR, Wald and LM tests of $H_0 : \lambda = 5$ against $H_A : \lambda \neq 5$.

(b) Suppose that n = 500 and $\sum_{i=1}^{n} y_i$ = 2,300. Apply each of the above three tests, and compare their outcomes in this case.

Question 5

Suppose that two random variables, 'x' and 'y' have a joint distribution such that:

$$p(x, y) = \theta \exp[-(\beta + \theta)y][(\beta y)^x] / [x!] \; ; \; \beta, \theta > 0 \; ; \; y > 0 \; ; x = 0, 1, 2, \ldots\ldots$$

[Note that 'y' is a continuous random variable, and 'x' is a discrete random variable.]

(a) Obtain the MLE's for β, θ. Don't forget to check the second-order condition.

(b) What is the MLE of $[\theta / (\beta + \theta)]$?

(c) Show that the marginal p.m.f. for 'x' is of the form, $p(x) = \lambda(1 - \gamma)^x$, and find the MLE of γ.

(d) Construct the LRT test of the hypothesis, H_0: β = 1 against a two-sided alternative hypothesis.