ECON 546: Themes in Econometrics
Supplementary Problems, IV

The following questions are designed to give you additional practice with the material that is being covered in class. You should note that many of them are more challenging than the questions that you should expect in the final exam – so don't be put off by this!

A solution sheet is available on request. (This gives you the opportunity to attempt these questions without seeing the answers.)

Question 1

Let x be a (continuous) random variable that follows a (standard) Gamma Distribution. Then, its p.d.f. is:

$$p(x) = x^{\gamma-1}e^{-x} / \Gamma(\gamma) \quad ; \qquad x > 0 \; ; \quad \gamma > 0$$

where $\Gamma(.)$ is the usual Gamma function:

$$\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}\,dt \, ,$$

which satisfies the recursion relationship:

$$\Gamma(z+1) = z\Gamma(z) \, .$$

The parameter γ is the shape parameter for this distribution. It can be shown that $E(x) = \gamma$. Some examples of this density function appear below.

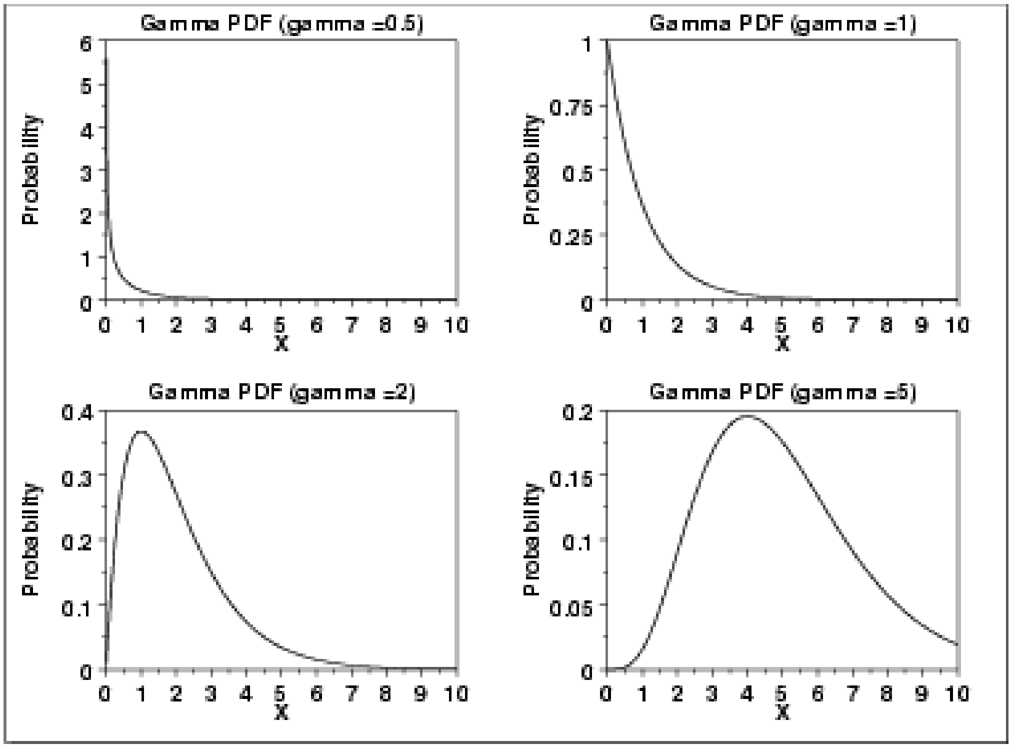
Let y be a (discrete) random variable that follows a Poisson Distribution. Then, its p.m.f. is:

$$p(y) = e^{-\lambda}\lambda^x / x! \qquad ; \qquad x = 0, 1, 2, \ldots\ldots ; \qquad \lambda > 0.$$

(The parameter λ is both the mean and the variance of this distribution.)

(a) Now, suppose we have a random sample of n independent observations from a Poisson Distribution, and that we wish to estimate λ. Taking a Bayesian approach, we need to specify a prior p.d.f. for this parameter. Prove that the Gamma prior is the natural conjugate prior in this case.

(b) What is the Bayes estimator for λ, first under a quadratic loss function; and second under a 'zero-one' loss function? Are either of these Bayes estimators unbiased?



Question 2

Suppose that we have a random sample of n non-negative observations from an exponential distribution with mean ($1 / \theta$). So, the p.d.f. for the i^{th} observation is:

$$p(y_i) = \theta \exp\{-\theta y_i\} \quad .$$

Suppose also that the prior for θ is a gamma density, with parameters α and β, and mean equal to ($\alpha\beta$), and mode at $[(\alpha - 1)\beta]$, if $\alpha > 1$:

$$p(\theta) = \theta^{\alpha-1} e^{-(\theta/\beta)} / [\beta^{\alpha}\Gamma(\alpha)] \quad .$$

(a) Prove that the posterior for θ is a gamma density, with parameters ($\alpha + n$) and $(\beta^{-1} + \Sigma y_i)^{-1}$.

(b) What is the Bayes estimator of θ under a quadratic loss function?

(c) What is the Bayes estimator of θ under a "zero – one" loss function?

Question 3

Consider the Natural Conjugate Bayes estimator of β in the standard Normal multiple linear regression model, under a quadratic loss function.

(a) Show that this estimator is biased.

(b) Why does this really not matter to a pure Bayesian econometrician?

(c) If the conditional prior covariance matrix for β were chosen to be equal to the covariance matrix for the Maximum Likelihood estimator of β in this model, show that the expected value of the Bayes estimator of β is a simple average of the conditional prior mean for β, and β itself.

(d) Why would the choice of prior covariance matrix in (c) really be a "non-Bayesian" choice?

Question 4

Suppose that we have a sample of 'n' random observations from a normal population whose mean is ***known***, but whose variance is unknown. We wish to estimate the latter parameter using Bayesian inference.

(a) Show that the natural-conjugate prior for the "precision", $\tau = \sigma^{-2}$, is a Gamma density.

(b) Give the formulae for the Bayes estimator of this precision parameter under both quadratic and zero-one loss functions.

(c) What would be a consistent estimator for the variance itself under each of these loss functions?