

# Joint vs. Individual Taxation and Intrafamily Distribution

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## Abstract

Apps and Rees (1999 a and b) evaluate a change from individual to joint taxation for couples on efficiency grounds and from an interhousehold distributional perspective, while this paper considers intrafamily distributional effects of such a tax reform. The change in tax schedule either expands or shrinks the utility possibility set (UPS) of a couple. Homothetic and symmetric UPF's are considered and we assume that the husband earns a higher wage rate than the wife. We model how spouses share family resources in two ways. (1) Spouses apply a bargaining rule, namely the Nash Bargaining or the Kalai-Smorodinsky solution, where the disagreement point is the utility of each spouse when single; (2) Spouses divide family full income between them and the allocation of the market good and the household good is competitive. An expansion (reduction) of the UPS does not necessarily imply that both spouses are better off (worse off) with tax reform. (1) Under the bargaining approach, a family tax reform will not affect the disagreement point and both spouses benefit from tax reform if the UPS expands. (2) Under the competitive approach, if spouses receive family resources proportional to their contribution to family full income, the husband benefits from tax reform and the wife is hurt by tax reform. If individual taxation results in a larger UPS than joint taxation, then under both approaches individual taxation leads to lower inequality between spouses than joint taxation.

JEL-Codes: C71, D13, D63, H24, H31.

Keywords: family income taxation, family bargaining models, sharing rules, intrafamily distribution.

I am extremely grateful to Hervé Moulin for his advise and patience, and I would like to thank James Brown, Suchan Chae, Peter Mieszkowski and the participants of the graduate student seminar at Rice University for helpful comments. I am also grateful to my discussant at the SEA meeting in New Orleans 2002, Yang-Ming Chang from Kansas State University, and to Patricia Apps for her comments at the IAFFE meeting in Barbados 2003.

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# 1 Introduction

We analyze the effects of family tax reform on intrafamily distribution when one spouse earns a lower wage rate than the other. Joint taxation taxes both spouses at the same rate, while individual taxation taxes the spouse with lower wage rate at a lower rate. For the rest of the paper, we will refer to the low wage spouse as wife and the high wage spouse as husband. A change from individual to joint taxation of the household does not only change the net-wage ratio between spouses, it also affects the utility possibility set of the household (see Apps and Rees, 1999a and b; Piggott and Whalley, 1996). To fix ideas, we assume that a change from individual to joint taxation of the family results in an outward shift of the utility possibility frontier (UPF). We determine whether both spouses benefit from such an expansion of the utility possibility set.

In order to evaluate the utility of each spouse before and after tax reform, we need a model of how husband and wife share family resources. If families were to maximize some household utility function subject to family resource constraints, the composition of family income would not affect the distribution of utility shares between spouses. If household utility increases both spouses benefit from tax reform. Increasing empirical evidence, however, rejects the income pooling hypothesis suggesting that a change in the composition of family income has an impact on intrafamily distribution.<sup>1</sup> The recent literature on economics of the family often suggests that cooperative conflict between spouses can explain the empirical findings: While both spouses cooperate to maximize the size of the pie (being on the Pareto frontier of the utility possibility set), husband and wife have conflicting ideas of how to share it (selecting a point on the utility possibility frontier).<sup>2</sup>

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<sup>1</sup>See e.g. Bourguignon et al. (1993), Browning et al. (1994), Lundberg et al. (1997), Rubalcava and Thomas (2000), Woolley (2000), and Attanasio and Lechene (2002).

<sup>2</sup>See Manser and Brown (1980), McElroy and Horney (1981), Sen (1996), Lundberg and Pollak (1993) and (1996).

Our work is unique in the sense that it addresses two distinctive ways of modelling intrafamily resource allocation simultaneously. In the bargaining approach, spouses bargain with each other over final utility shares. The disagreement utility of each spouse is given by how much each of them can guarantee him or herself when single. While the Nash Bargaining Solution (NBS) is often proposed to solve this problem, we also focus on a less applied solution concept, the Kalai-Smorodinsky solution (KSS).<sup>3</sup> Alternatively, we model the resource allocation problem using a competitive approach (see Apps and Rees, 1988). Family full income is divided among spouses according to some rule.<sup>4</sup> Each spouse then maximizes his or her utility, given the share of family full income and household prices. We use a simple income sharing rule - we call it Separate Accounts -, in which the income share of a spouse depends only on his or her earning potential when married, i.e. income shares are equal to the stand alone income of each spouse and the husband's share of family full income is therefore larger than the wife's share.<sup>5</sup> We link this rule to Sen's 1996 discussion of perceived contributions.

Spouses consume two goods, a market good and a household good. Both goods are private.<sup>6</sup> Each spouse is endowed with the same amount of time that can be used in market labor or household production. Spouses' utility functions are identical and homogenous of some degree  $r$ ,  $0 < r \leq 1$ . These assumptions imply a simple functional form of the utility possibility frontier (UPF); the UPF is symmetric, and a change in wage rates or tax rates will result in a homothetic shift of the UPF.<sup>7</sup>

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<sup>3</sup>Manser and Brown in their seminal paper in 1980 considered both the NBS and KSS, but in more recent papers in the literature the focus is clearly on NBS.

<sup>4</sup>Family full income is the income a family would receive if both spouses spent all their time in market labor.

<sup>5</sup>This sharing rule has been applied previously on an ad-hoc basis. See e.g. Apps (1982) , Apps and Jones (1986), Apps and Rees (1988 and 1996), and Apps and Savage (1989).

<sup>6</sup>The assumption that the household good is private is not the only possibility. Household public goods are often emphasized in the literature. As public goods play a bigger role in families with children, our model is more relevant for households only consisting of a husband and a wife. Alternative treatments of household production are discussed in section 7.

<sup>7</sup>The class of bargaining problems with the same functional form of the UPF as considered here also

In the bargaining approach our result depends crucially on the direction in which the UPF shifts when we move from individual to joint taxation. Both the Nash Bargaining solution and the Kalai-Smorodinsky solution satisfy the homothetic solidarity property, that is, both spouses benefit from an outward shift of the UPF (Proposition 1). Inequality within the household decreases; gains of cooperation as well as utility shares assigned to each spouse are distributed more equally. This result does not depend on the specifics of the tax reform.<sup>8</sup> Take any two tax schedules for the couple. If tax reform from one tax schedule to the other results in an outward shift of the UPF, both spouses obtain a higher utility and inequality between spouses decreases.

With Separate Accounts the result critically depends on how net-wage rates change with tax reform, but the result does not depend on the direction in which the UPF shifts. The wife's utility decreases while the husband's utility increases after a switch from individual to joint taxation no matter whether tax reform expands or shrinks the utility possibility set of the household (Proposition 2). Inequality between spouses increases, because the gap between net-wage rates of spouses increases under joint taxation leaving the wife with a lower and the husband with a higher stand alone income than before tax reform.

This paper provides a guideline for evaluating a tax reform if policy makers want to reduce inequality between spouses. If a change from individual to joint taxation results in an inward shift of the UPF, our results state that under any resource allocation rule discussed above joint taxation increases inequality between spouses.<sup>9</sup>

The article is organized as follows. We first discuss related literature (section 2). In section 3, we introduce the formal framework, followed by section 4 that stresses the co-

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allows for comparison between different bargaining rules along the UPF. Most recently, Anbarci (2002) shows that KSS assigns a higher utility to the person with lower disagreement utility than NBS, if the disagreement utility of this person is set to be zero. See also Lemma 2.

<sup>8</sup>Provided that tax rates for singles are the same before and after the reform.

<sup>9</sup>See also Nelson (1996) who argues in favor of individual taxation because of its intrafamily distributional effects.

operative features of the model. Section 5 deals with the bargaining approach. Section 6 introduces the competitive approach. Section 7 discusses extensions to the basic model and concludes. All proofs can be found in the appendix, and all figures at the end of the paper.

## 2 Related Literature

### 2.1 Household Production and Family Income Taxation

Household production and tax reform in our model is based on Apps and Rees (1999a). Our model distinguishes itself from Apps and Rees' by specifying how a couple shares resources. Apps and Rees do not model any specific allocation rule since their focus is on whether the utility possibility set expands with tax reform, but not how the welfare of an individual household member changes. They suggest, however, that their model naturally lends itself to the analysis of intrafamily distributional effects.

The argument for individual taxation of couples on efficiency grounds is based on the secondary worker hypothesis. While the primary worker has a very inelastic labor supply, the labor supply of the secondary worker is very high. In the absence of household production, "standard Ramsey rule considerations would argue for taxing primary and secondary workers at different marginal rates (Apps and Rees, 1999a, p. 394)." With household production, however, different tax rates on the income of the wife and the income of the husband distort the shadow price of home production.<sup>10</sup> This distortion would not occur under joint taxation. Piggott and Whalley (1996) suggest that this distortion outweighs the benefits of individual taxation, Apps and Rees (1999a and b) argue the opposite.

Apps and Rees (1999b) focus on different household types in which wives vary in their

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<sup>10</sup>In his seminal work Becker (1981) emphasized the role of household production in a multi-person household.

productivity in household production. They argue that households in which the wife has a lower productivity in household production are the ones most likely to gain from a tax reform from joint towards individual taxation.

This paper puts forth a new argument in favor of individual taxation. It argues that inequality between household members is reduced compared to joint taxation under all of our allocation rules provided that the utility possibility set of the household expands with individual taxation.

## **2.2 The Resource Allocation Problem of the Household**

While the assumption that a multi-person household maximizes a household utility function is still common in the literature, solution concepts from cooperative bargaining theory are increasingly applied to model resource allocation between household members. Family bargaining models stress the importance of bargaining power between spouses and suggest that own income (both from market labor as well as labor-independent wealth) plays a crucial part in determining the fall-back position for each spouse (e.g. Manser and Brown, 1980; McElroy and Horney, 1981; McElroy, 1997; and Lundberg and Pollak, 1993 and 1996). While most authors in the literature of family bargaining apply the Nash Bargaining Solution (NBS) we focus also on the Kalai-Smorodinsky solution (KSS).

The competitive approach to intrafamily distribution is based on Apps and Rees (1988). It is very closely related to the collective approach taken by Chiappori and others (see e.g. Chiappori, 1988 and 1997). Both the competitive and the collective approach require that the family is on the Pareto frontier of the utility possibility set which is achieved by letting spouses separately maximize their utility given a share of family full income. The collective approach leaves the specific rule of allocating income open, but tries to recover it empirically up to a constant. In contrast, we define a specific income sharing rule for the

spouses and weigh its merits against the bargaining rules. Although Apps (1982), Apps and Jones (1986), Apps and Rees (1988 and 1996), and Apps and Savage (1989) have applied this rule previously, the contribution of this paper with regard to this sharing rule is that it gives a motivation why this sharing rule might be used by families.

## 2.3 Cooperative Game Theory and Distributive Justice

An allocation rule that never makes a person worse off when the utility possibility set expands in an arbitrary way, is said to satisfy the solidarity axiom (e.g. Chun and Thomson, 1988; and Keiding and Moulin, 1991). This axiom is the strongest in the class of monotonicity axioms and only the egalitarian solution satisfies it (Chun and Thomson, 1988). Weaker versions of monotonicity require that an agent be no worse off after restricting the way in which the utility possibility set expands (e.g. Kalai and Smorodinsky, 1975; Chun and Thomson, 1988; and Nicolò and Perea, 2002).

Our assumptions about production and consumption guarantee that the UPF can only shift homothetically thus limiting the way in which the utility possibility set can expand. An allocation rule is said to satisfy the *homothetic solidarity property* if no agent is made worse off when the utility possibility set expands homothetically. We show that both NBS and KSS satisfy the homothetic solidarity property (Proposition 1).

Roemer (1986) and others have argued that by focussing on problems of distributive justice one should consider economic environments and distribute resources rather than converting everything into utility. In the competitive approach, spouses distribute family full income and then trade with each other. Sen (1996) points out that the resource allocation within the household does not only depend on outside options, but also on what is perceived as the contribution of each household member to the family well-being right then and there. He states “given other things, if in the accounting of the respective

outcomes, a person is perceived as making a larger contribution to the overall well-being of the group, then the chosen collusive solution will become more favorable to that person (p.68).” Separate Accounts captures the idea that spouses evaluate their time at market prices. Hence the contribution of the wife is perceived as being less than the contribution of the husband.

### 3 The Model

This section is divided into subsections each emphasizing one building block of the model. The reader is referred to section 7 for a discussion of the more controversial assumptions.

#### 3.1 Household Production and Household Income

Wife ( $i = 1$ ) and husband ( $i = 2$ ) divide their time  $T$  between market labor ( $l_i$ ) and work in household production ( $t_i$ ). The couple does not consume leisure. The wife earns a lower wage rate before taxes than the husband, i.e.  $w_1 < w_2$ .

There are two private goods, one market good ( $x$ ) and one household good ( $y$ ). The market good is taken as the numeraire good. The household good is produced with both spouses’ time inputs,  $t_1$  and  $t_2$ . The production function is twice differentiable, and satisfies the following properties

$$CRS \quad : \quad ay = f(at_1, at_2), a > 0, \tag{1}$$

$$\frac{\partial f}{\partial t_i} > 0, \frac{\partial^2 f}{\partial t_1 \partial t_2} \geq 0, \frac{\partial^2 f}{\partial t_i^2} \leq 0. \tag{2}$$

Both spouses are equally productive in the household: Consider the output produced with  $t_1 = p$  and  $t_2 = q$  and the output produced with  $t_1 = q$  and  $t_2 = p$ . For any pair

$$(p, q) \in [0, \infty]^2,$$

$$f(p, q) = f(q, p).$$

Moreover, the household good can be produced by one person alone,

$$f(t_i, 0) > 0 \text{ for } t_i > 0.$$

*Tax Schedule.* Let  $\tau_i$  be the marginal tax rate on the wage rate of a spouse. The hourly net wage after paying  $\tau_i w_i$  in taxes on hourly wages is  $(1 - \tau_i) w_i$ . Under individual taxation  $\tau_i = \beta_i$  with  $\beta_1 < \beta_2$ , and joint taxation taxes wage rates of both spouses at the same rate  $\tau_i = \alpha, i = 1, 2$  where  $\beta_1 < \alpha < \beta_2$ . The tax schedule under individual taxation guarantees people with higher wage rates a higher net-wage rate,  $(1 - \beta_1) w_1 < (1 - \beta_2) w_2$ , but it narrows the gap compared to the before-tax wage rate ratio,  $\frac{w_1}{w_2} < \frac{(1 - \beta_1) w_1}{(1 - \beta_2) w_2}$  while joint taxation maintains the same net-wage ratio as the ratio of before-tax wage rates.<sup>1112</sup>

The couple's feasibility constraints consist of the following. Expenditure on the market good cannot exceed household labor income after taxes, and the couple must consume an amount of the household good that is less or equal to the amount they produced. Denote  $x_i$  the quantity of the market good consumed by spouse  $i$  and denote  $y_i$  the quantity of

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<sup>11</sup>This is the crucial point for Piggott and Whalley's 1996 argument in favor of joint taxation.

<sup>12</sup>Typically, the tax system is also captured by introducing a lump-sum transfer  $\gamma$  from the government to the household, so that the actual income for the household would be  $\gamma + \sum_{i=1,2} (1 - \beta_i) w_i$ . We abstract from this intercept here, because it would not change the results qualitatively. Apps and Rees, for example, assume that the government would only change the marginal tax rates in case of a tax reform, but the lump-sum transfers payed to the household would remain the same. (See Apps and Rees, 1999a, p. 397 and Apps and Rees, 1999b, p.250).

the household good consumed by spouse  $i$ , then

$$x = x_1 + x_2 \leq \sum_{i=1}^2 (1 - \tau_i) w_i l_i, \quad (3)$$

$$y = y_1 + y_2 \leq f(t_1, t_2). \quad (4)$$

The time constraint for each spouse is given by

$$l_i + t_i = T. \quad (5)$$

### 3.2 Utility Functions

Utility functions are identical, quasiconcave, twice differentiable, and homogeneous of some degree  $r$ ,

$$u(ax_i, ay_i) = a^r u(x_i, y_i)$$

where  $0 < r \leq 1$ . In order to compare utility shares, we also need a well-defined lower bound on utility which is implied by the assumption of homogeneity,  $u(0, 0) = 0$ .

We find the utility possibility set for the household in the next section.

## 4 The Utility Possibility Set

We derive the UPF in two steps. We first solve for production efficiency. We then maximize the utility-vector of the spouses subject to the constraint that total consumption of both goods selects a point on the production possibility frontier.

## 4.1 Production Efficiency

Given the technologies for producing the household good and household income, we find that the production possibility set is convex, but it will have a kink.

**Definition** The production possibility set  $Y \in R_+^2$  is the set of all product pairs  $(x, y) \geq 0$  satisfying the household's feasibility constraints (3), (4) and (5).

**Definition** Denote the frontier of the production possibility set  $\partial Y$  where  $\partial Y \subset Y$ .

**Observation 1** (1) *The production possibility frontier  $\partial Y$  has a kink denoted by  $(x', y')$ .*

*For  $0 < y \leq y'$ , the slope of  $\partial Y$  is constant  $\frac{dx}{dy} = -p_y, \frac{d^2x}{dy^2} = 0$ . For  $y > y'$ , if  $\frac{\partial^2 f}{\partial t_1 \partial t_2} \geq 0$ , the slope of  $\partial Y$  becomes steeper  $\frac{d^2x}{dy^2} \leq 0$ . (2) Due to the difference in productivity of household income,  $(1 - \tau_1)w_1 < (1 - \tau_2)w_2$ , there is a unique time allocation of spouses  $((l_1, t_1), (l_2, t_2))$  associated with any  $(x, y) \in \partial Y$ . (3) The production possibility set  $Y$  is convex, compact and comprehensive.*

See Figure 1 a. Figure 1b shows the special case in which time inputs of the spouses are perfect substitutes. Then we have a Ricardian Model of Trade between spouses with  $f(t_1, t_2) = b(t_1 + t_2)$  and  $b > 0$ . Define  $a_i = (1 - \tau_i)w_i$ , the PPF is piecewise linear with opportunity cost of  $\frac{a_1}{b}$  up to the point  $y' = bT$ , beyond this point,  $y > y'$ , the opportunity cost for the household good is  $\frac{a_2}{b}$ .

**Definition** The opportunity cost or the shadow price of household production is equal to the absolute value of the slope of the PPF,  $|\frac{dx}{dy}|$ , evaluated at  $(x, y)$  for all  $(x, y) \in \partial Y$ .

Hence the shadow price for good  $y$  is independent of the particular point on the PPF for  $0 < y \leq y'$ .

## 4.2 Household Efficiency

**Definition** The utility possibility set  $U \in R_+^2$  is the set of all utility pairs  $(u_1, u_2)$  satisfying the household's feasibility constraints (3), (4) and (5).

**Definition** Denote the frontier of the utility possibility set  $\partial U$  where  $\partial U \subset U$ .

**Definition** Denote  $\lambda_i$  the maximal utility a spouse can get if the other spouse does not consume anything,  $\lambda_i = \max \{u_i | u \in U\}$ .

By symmetry the maximal utility for each spouse must be the same,  $\lambda_1 = \lambda_2$ , we refer to it simply as  $\lambda$ . Clearly  $(\lambda, 0) \in \partial U$  and  $(0, \lambda) \in \partial U$ .

We can now describe the utility possibility set for the couple.

**Observation 2** *If  $u(x_i, y_i)$  is homogenous of degree  $0 < r \leq 1$  in  $x_i$  and  $y_i$  and both goods are private, the utility possibility set takes the form  $U = \left\{ u \geq 0 | u_1^{\frac{1}{r}} + u_2^{\frac{1}{r}} \leq \lambda^{\frac{1}{r}} \right\}$ . As a consequence,*

- *a change in the couple's tax rates results in a homothetic shift of  $\partial U$ ;*
- *for any  $u \in \partial U$ , total household consumption  $(x, y)$  is the same;*
- *$U$  is compact, comprehensive and convex.*

Figure 2 captures the idea of deriving the UPF.

By Observation 2, tax reform can shift the UPF homothetically inwards or outwards.

We assume that a change from individual to joint taxation results in an outward shift.<sup>13</sup>

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<sup>13</sup>A tax reform is household efficient if the change in family net wages is positive. (See Apps and Rees, 1999a.)

## 5 Household Behavior 1: The Bargaining Approach

The bargaining problem of the couple is composed of two elements: the utility possibility set  $U \subset R_+^2$  and a disagreement point  $d \in U$ . As stated in Observation 2, our assumptions about utility functions and production guarantee that  $U$  is compact, comprehensive and convex.

We define the disagreement point  $d$  as the utility of each spouse when single. The tax rates for singles are the same as the tax rates they face under individual taxation in marriage. We find  $d_i$  by solving the problem

$$\begin{aligned} \max u(x_i, y_i) \\ \text{s.t.} \quad x_i &= (1 - \beta_i) w_i l_i; \\ y_i &= f(t_i, 0); \\ t_i + l_i &= T. \end{aligned}$$

**Observation 3** *In the present model  $d_1 < d_2$ . Moreover,  $d \in U$ , but  $d \notin \partial U$ .*

Observation 3 guarantees that there are gains from cooperation and that a lower net-wage rate translates into a lower bargaining power for the wife.

**Definition** A bargaining solution is a rule that assigns a solution vector  $\varphi(U, d) \in U$  to every bargaining problem  $(U, d)$  where  $\varphi(U, d) \geq d$  and  $\varphi(U, d) \in \partial U$ .

Before introducing specific bargaining solutions we need more definitions and notation.

**Definition** Spouse  $i$ 's bliss utility: Denote  $\bar{u}_i$   $i$ 's maximal utility if  $j$  receives  $d_j$  :  $\bar{u}_i = \max \{u_i | u_j = d_j \text{ and } (u_i, d_j) \in U\}$ . The bliss gain one spouse can receive is given by  $\bar{u}_i - d_i$ .

**Definition** The point on the UPF where both spouses' utilities are equal is given by  $u^0$  :

$$u^0 = \{(u_1, u_2) | u_1 = u_2 \text{ and } (u_1, u_2) \in \partial U\}.$$

We refer to the vertical and horizontal axes originating in  $d$  as  $d$ -axes. The set of individually rational points on the UPF is given by  $S \subset \partial U$  with  $(d_1, \bar{u}_1)$  and  $(\bar{u}_2, d_2)$  as its endpoints. The area that is contained in the intersection between  $S$  and the  $d$ -axes is denoted  $(S, d)$ ; it is the set of all individually rational utility pairs  $(u_1, u_2)$ . (See Figure 3.)

Suppose  $\varphi(U, d) = (u_1^*, u_2^*)$ . Then  $(u_i^* - d_i)$  denotes  $i$ 's gains from cooperation under bargaining solution  $\varphi(U, d)$ . We will also refer to  $\frac{u_2^*}{u_1^*}$  as the utility ratio and  $\frac{u_2^* - d_2}{u_1^* - d_1}$  as the gains ratio under the bargaining solution  $\varphi(U, d)$ .

We consider four solutions, Egalitarian (EGAL), Equal Split of Cooperative Gains (ES), Nash Bargaining (NBS) and Kalai-Smorodinsky (KSS) Solution.

**Definition** EGAL is the unique  $u^*$  such that 
$$\begin{cases} u^0 & \text{if } d \leq u^0, \\ (\bar{u}_i, d_j) & \text{if } d \not\leq u^0 \text{ and } d_i < d_j. \end{cases}$$

EGAL distributes equal utility to both spouses whenever this is individually rational. Otherwise the wife receives her bliss utility  $\bar{u}_1$ .

**Definition** ES is the unique  $u^*$  that equalizes gains between spouses:  $u_1^* - d_1 = u_2^* - d_2$ .

ES divides gains from cooperation  $(u_i - d_i)$  equally.

EGAL and ES are not independent of affine transformations of the utility function, but the next two bargaining solutions are.

NBS is the unique  $u^*$  that maximizes the Nash Product ( $NP$ ) over  $S$ :

$$NP = (u_1 - d_1)(u_2 - d_2).$$

Figure 4 illustrates the well known construction of NBS. The tangency at  $(u_1^*, u_2^*)$  has its midpoint at  $(u_1^*, u_2^*)$  when the intercepts of this tangency with the  $d$ -axes are taken as its endpoints.

**Definition** KSS is the unique  $u^*$  that equalizes relative gains between spouses

$$\frac{u_1^* - d_1}{\bar{u}_1 - d_1} = \frac{u_2^* - d_2}{\bar{u}_2 - d_2}.$$

Figure 5 illustrates the well known construction of KSS.

All four bargaining solutions allocate at least as big a utility share to the husband as to the wife (Lemma 1), but EGAL and ES are more extreme than NBS and KSS (Lemma 2). EGAL is an extreme solution, because the disagreement point plays only the role of a lower bound. Whenever equal utility shares satisfy individual rationality, EGAL implies that each spouse's utility only depends on family resources, i.e. that the income pooling hypothesis holds. As noted there is substantial empirical work that rejects the income pooling hypothesis, which makes EGAL an unlikely candidate for an allocation rule between spouses.<sup>14</sup> On the other extreme, we have ES. This solution is symmetrically implausible because it preserves entirely the difference between the spouses' utilities in the disagreement outcome. It coincides with NBS and KSS when the UPF is linear (Figure 6), but when  $0 < r < 1$ , NBS and KSS have the normatively appealing property of distributing a higher share of gains to the spouse with lower disagreement utility than ES while still assigning a higher absolute utility to the spouse with higher disagreement point (see Figure 7).<sup>15</sup>

**Lemma 1** *EGAL, ES, NBS and KSS with  $d = (d_2, d_1) : d_2 > d_1$  and  $U$  specified above,*

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<sup>14</sup>See e.g. Bourguignon et al. (1993), Browning et al. (1994), Lundberg et al. (1997), Rubalcava and Thomas (2000), Woolley (2000), and Attanasio and Lechene (2002).

<sup>15</sup>Another argument for focussing on NBS and KSS rather than EGAL and ES is their better non-cooperative foundation.

have solutions that satisfy the following two properties.

1.  $u_1^* \leq u_2^*$
2.  $u_1^* - d_1 \geq u_2^* - d_2$

See Figure 7.

**Lemma 2** *The NB and KS solution vectors,  $u^N$  and  $u^{KS}$ , lie between the egalitarian and equal split of cooperative gains solution vectors,  $u^E$  and  $u^{ES}$ , on the UPF:  $u_1^{ES} \leq u_1^N, u_1^{KS} \leq u_1^E$  with inequalities reversed for the husband.*

See Figure 7. Lemma 2 is closely related to a result very recently obtained by Anbarci (2002), in which he shows that KSS always assigns a larger utility share to the person with lower disagreement utility than NBS if the functional form of the utility possibility frontier is the same as in our model and the disagreement point is given by  $d_1 = 0$  and  $d_2 > 0$ . Lemma 2 does not rank the wife's utility share under KSS and NBS, because with  $d_1 > 0$ , the wife's utility share can be larger or smaller under NBS than under KSS.<sup>16</sup>

## 5.1 Tax Rates within Marriage Change

We are now able to determine the effects of tax reform on intrafamily distribution within the first model of household behavior.

**Question** *Suppose that tax reform from individual to joint taxation shifts the UPF out-*

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<sup>16</sup>For example,  $\partial U : u_1^2 + u_2^2 = 1$ ,

$$\begin{array}{ll}
 d_1 = .2, d_2 = .4 & d_1 = .8, d_2 = .85 \\
 u_1^N = 0.64072 < u_1^{KS} = 0.64747 & u_1^N = 0.67643 > u_1^{KS} = 0.67620 \\
 u_2^N = 0.76777 > u_2^{KS} = 0.76209 & u_2^N = 0.73651 < u_2^{KS} = 0.73672
 \end{array}$$

wards. Let  $(u_1^*, u_2^*)$  be the solution before tax reform, and  $(u_1^{**}, u_2^{**})$  after tax reform.

$Q_0$  : *Is the homothetic solidarity property satisfied?*

That is,  $(u_1^{**}, u_2^{**}) \geq (u_1^*, u_2^*)$ .

If so,

$Q_1$  : *Are gains from cooperation shared more equally?*

$Q_2$  : *Do the utilities of the spouses become more equal?*

Note that  $Q_0$  only requires an ordinal utility measure, while  $Q_1$  is only independent of affine transformations of  $u$ . Finally,  $Q_2$  is not independent of a change in the zeros of  $u$ .

The disagreement point does not depend on the new tax rates. Thus, the question whether spouses are better off under the tax reform, depends on the relative position of the new UPF (joint taxation) to the initial UPF (individual taxation), only.

**Proposition 1** *Consider Nash and KS bargaining solutions.*

*The Homothetic Solidarity Property is satisfied*  $(u_1^{**}, u_2^{**}) > (u_1^*, u_2^*)$ .

*The utility ratio becomes more equal*<sup>17</sup>  $\frac{u_1^{**}}{u_2^{**}} > \frac{u_1^*}{u_2^*}$ .

*The cooperative gains ratio becomes more equal*  $\frac{u_1^{**}-d_1}{u_2^{**}-d_2} < \frac{u_1^*-d_1}{u_2^*-d_2}$ .

On a technical note, this result is of some significance, since it is known that for NBS and KSS an arbitrary expansion of the utility possibility set does not necessarily make both persons better off (e.g. Chun and Thomson, 1988; and Keiding and Moulin, 1991). In the class of utility possibility sets considered here, the utility of both spouses increases under NBS and KSS if the utility possibility set expands. By restricting our attention to identical and homogenous utility functions we have found a way to summarize the information of the utility possibility set in two parameters,  $\lambda$  and  $r$ . Although it is not equivalent to the problem of splitting only one good between two individuals it bears some similarity to it. It

is then intuitive that the results for NBS and KSS obtained in the pie-splitting case carry over to the case of homothetic and symmetric utility possibility sets. Instead of restricting the number of goods to be equal to one but allowing for any concave utility functions, we impose strong restrictions on the utility functions but we can allow for any positive number of goods to be shared.

It has to be stressed that the present model is more likely to apply to couples without children for whom household public goods play a minor role. With the presence of children, however, household public goods become more important. In this case we should not ignore the possibility of a disagreement point within marriage determined by the Nash equilibrium of a private-contribution-to-the-public-good game or a disagreement point within marriage in which the division of labor is chosen according to sanctioned gender-roles.<sup>18</sup>

## 6 Household Behavior 2: The Competitive Approach

Instead of settling on a bargaining solution, spouses could base their share of utility on some distribution of family full income (Apps and Rees, 1988, and Chiappori, 1988 and 1997). Family full income is given by

$$I = \Sigma (1 - \tau_i) w_i T.$$

The difference between the bargaining rules proposed in the previous section and the sharing rule here is that while the bargaining rules compare a hypothetical case with the current situation of full cooperation, the sharing rule is based on a measure of current contributions to the output in marriage. Discussions in family law suggest that spouses do not only care about total family income, but it is also essential how much each spouse

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<sup>18</sup>See Lundberg and Pollak (1993).

contributes to it. For example, in Germany lawmakers debated recently whether one spouse should have the right to obtain full information about the income of the other spouse.<sup>19</sup> We therefore focus on a sharing rule of family full income in which one spouse’s own net-wage rate plays a crucial role. As Sen puts it “...notions of who is ‘contributing’ how much have many causal antecedents that call for closer scrutiny. However, the ‘deal’ that women get vis-à-vis men is clearly not independent of these perception problems regarding contribution (p.69).” One might argue that since the wife will work more in household production, her net-wage rate should play a lesser role in evaluating her contribution, but as Sen argues “Division between “gainful” and other activities is quite arbitrary; [...]. But the issue is not whether activities within the household are really less productive, but whether they are perceived as such (p.70).”

We call Separate Accounts the sharing rule that allocates the stand alone income to each spouse. Under Separate Accounts, the ratio between the wife’s and the husband’s income share is equal to the ratio of the wife’s and the husband’s net-wage rate.<sup>20</sup>

$$\textit{Separate Accounts} : I_i = (1 - \tau_i) w_i T.$$

Once family full income is distributed, we find the price vector for the competitive equilibrium allocation. It is given by the price of the market good,  $p_x = 1$ , and by the shadow price of the household good,  $p_y$  (Observation 1). In order to simplify the theoretical analysis in this section, we assume that preferences and technologies are such that the spouses always choose an allocation above the kink of the PPF (see Figure 1a). That

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<sup>19</sup>The issue was referred to the committee of justice in 1999, no final decision has been made.

<sup>20</sup>Even if spouses have a joint checking account, it might be very well possible that the two of them have kept track in their minds of how much each of them has put into the account and is therefore entitled to spend. Woolley (2000) reports that while wives are often involved or even primarily responsible for managing family finances, men are more likely to make cash withdrawals, which “may reflect a partner’s freedom not to account for expenditures.”

is, the optimal amount of the household good requires  $t_1^* < T$  and therefore  $t_2^* < T$  and the opportunity cost of the household good is constant. Each spouse chooses his or her consumption bundle  $(x_i^*, y_i^*)$  by solving the problem

$$\max_{x_i, y_i} u(x_i, y_i)$$

subject to the individual budget constraint

$$x_i + p_y y_i = I_i.$$

**Observation 4** *In Separate Accounts the wife consumes more of the market good than her net wage would afford her to buy, i.e. there is an actual money transfer from the husband to the wife.*

## 6.1 Tax Rates within Marriage Change

We determine now whether the sharing rule satisfies the homothetic solidarity property and whether inequality between spouses' utilities increases with tax reform.

**Proposition 1.2** *Suppose that tax reform from individual to joint taxation shifts the UPF outwards. Let  $(u_1^*, u_2^*)$  be the solution before tax reform, and  $(u_1^{**}, u_2^{**})$  after tax reform. Separate Accounts does not satisfy the homothetic solidarity property. Thus, inequality between spouses increases*

$$u_1^{**} < u_1^*; u_2^{**} > u_2^*.$$

Under Separate Accounts there are losers and winners in the household, even if tax reform expands the utility possibility set. Our result is suggestive of why family tax reform continues to be a subject of vigorous debate in many countries.<sup>21</sup>

## 7 Conclusion

The driving assumption behind our results is that the wife's net-wage rate is lower than the husband's which guarantees the husband a bigger utility share under all allocation rules than the wife. Since the joint tax rate lies between the tax rates under individual taxation, the gap between net-wage rates is wider under joint taxation than it is under individual taxation. This difference plays a role in the competitive approach, because the income sharing rule depends only on the stand alone income of each spouse. In the bargaining approach, the change in the net-wages plays a lesser role in intrafamily distribution, because the disagreement utility of spouses depends on their net-wage rates outside marriage and therefore remains the same before and after tax reform. Yet the change in the net-wages of spouses determines whether both spouses benefit or lose from tax reform by expanding or shrinking the utility possibility set.

Empirical evidence supporting a specific allocation rule is weak.<sup>22</sup> We have given examples of reasonable rules and - in future research - hope to develop empirical tests that will let us distinguish between allocation rules. One immediate testable implication of the present model is the following. The rules under the competitive approach depend only on the net-wage rates within marriage but the bargaining rules depend on both net-wage rates when single and when married. Since the net-wage rates within and outside marriage differ

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<sup>21</sup>Gugl (2003) finds that if spouses bargain over two periods and renegotiation is possible even if the spouses are Nash-bargainers with divorce threat point, it can be the case that one spouse gains and the other loses with tax reform. The reason for this result lies in the assumption that the gross wage of both spouses increases from one period to the next depending on how much they work in the previous period.

<sup>22</sup>Specifically, Bourguignon et al.(1993) find in their paper evidence against NBS.

for couples in countries with joint taxation, we should be able to empirically distinguish between bargaining and income sharing rules.<sup>23</sup>

Several issues arise when characterizing a tax reform. Tax rates in reality depend on earned income and not the wage rate of an individual and the model does not capture the realistic feature of increasing marginal tax rates as we move up in the income tax brackets. Rather it assigns a higher tax rate to every dollar earned of the husband. Using concave functions on the relevant gross income to better capture the actual tax system would not be a problem for both the bargaining and the competitive approach. It would, however, complicate the analysis without adding much insight.

Since we dealt with both the bargaining and the competitive approach, the present model imposes substantial restrictions on the nature and production of the household good, and the utility functions. While the bargaining approach is more sensitive to the utility specifications, the competitive approach imposes more severe restrictions on the production side of the model.

*Increasing Returns to Scale.* The analysis under the bargaining approach can be easily extended to incorporate increasing returns to scale in household production, but it will be difficult to deal with increasing returns under the competitive approach.

*Public Goods.* Housework is often interpreted as a public good. Only if the efficient amount of the public good remains the same along the UPF (e.g. Cobb-Douglas or quasi-linear utility function), will tax reform shift the UPF homothetically. For instance, if preferences are of the Cobb-Douglas form, the UPF is given by

$$\partial U : \lambda^{\frac{1}{\gamma}} = u_1^{\frac{1}{\gamma}} + u_2^{\frac{1}{\gamma}}$$

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<sup>23</sup>With this test, we cannot rule out bargaining models with a disagreement point within marriage, even if the bargaining models with divorce threat point are rejected by the data.

where  $\gamma$  is the taste parameter for the private good. A disagreement point within marriage might then be more plausible than a disagreement point outside marriage and thus the fall-back position of each spouse will depend on tax reform as well. The disagreement point within marriage will be more asymmetric under joint than under individual taxation. It is conceivable that the wife could be worse off with joint taxation even if the UPF shifts outwards, or if both spouses benefit from tax reform, inequality between husband and wife might increase.

For the sharing rule approach, we can subtract household expenditure on the public good from family full income and then distribute remaining family full income between spouses. Still, the question remains how much of family income each spouse should receive after the public good is paid for. Do spouses divide the cost for the public good between them (share the mortgage on the house equally) or will the spouse with higher net-wage rate contribute more?

More restrictions on the model are necessary to obtain unambiguous effects of tax reform when the model incorporates household public goods.

*Homothetic Utility Function.* Our results under the competitive approach do not depend on homothetic shifts of the UPF, homothetic preferences would do. For the bargaining approach, we conjecture that still both spouses will be better off under NBS and KSS if the UPF shifts outwards, but inequality between spouses might increase or decrease.

*Other Income Sharing Rules.* Sharing rules like the following two might seem reasonable. (1) Proportional Transfer transfers a fraction of the husband's stand alone income to the wife.

$$\begin{aligned}
 I_1 &= (1 - \tau_1) w_1 T + \phi (1 - \tau_2) w_2 T, \\
 I_2 &= (1 - \phi) (1 - \tau_2) w_2 T,
 \end{aligned}$$

where  $0 < \phi < 1$ . With a switch to joint taxation, the transfer to the wife increases as the husband's stand alone income increases. If the fraction is small enough, it will not compensate the wife for the decrease in utility due to the fall in her own stand alone income. (2) A compromise between Separate Accounts and Equal Split of Income will produce the rule

$$I_i = (1 - \theta) \frac{((1 - \tau_1) w_1 + (1 - \tau_2) w_2) T}{2} + \theta (1 - \tau_i) w_i T,$$

where  $0 < \theta < 1$ .<sup>24</sup> Depending on the size of  $\phi$  and  $\theta$  in the two income sharing rules, it is possible that both spouses gain under joint taxation if the UPF shifts outwards. At the same time, the husband's utility increases by a larger percentage than the wife's utility, increasing inequality between spouses.

One may question why the spouses in Separate Accounts evaluate their contributions to family full income based on their net wage rates. If instead family full income is divided between spouses according to the gross wage rates, tax reform does not change this distribution and both spouses gain with (lose from) tax reform whenever the UPS expands (shrinks).

Analyzing the implications of tax reform and other family policies on intrafamily distribution is important from different points of view. First, neoclassical theory is built on individual choice and welfare analysis should not stop at the family level but consider individual welfare. Second, empirical evidence suggests that families do not pool their income (Hoddinott et al. 1997). Family policies do not necessarily have the same impact on all family members leading to possibly undesired policy effects. As demonstrated in this paper, under Separate Accounts inequality between spouses increases with joint taxation, even if the utility possibility set expands.

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<sup>24</sup>Hervé Moulin suggested this rule.

## 8 Appendix

### 8.1 Proof of Observation 1

Let  $(1 - \tau_i) w_i = a_i$ . Then production efficiency requires

$$\begin{aligned} \max_{x,y} & (x, y) \\ \text{s.t.} & \quad x = a_1 l_1 + a_2 l_2 \\ & \quad y = f((T - l_1), (T - l_2)) \cdot \\ & \quad l_i + t_i = T \end{aligned}$$

and the condition for an interior solution is

$$\frac{\partial f / \partial t_1}{\partial f / \partial t_2} = \frac{a_1}{a_2}. \tag{6}$$

Since  $a_1 < a_2$ , the wife spends more time in household production than the husband. Condition (6) is satisfied as long as household production requires less than the full amount of the wife's time, i.e.  $t_1 < T$ . By CRS, condition (6) also implies that spouses sacrifice the same amount of household income in order to produce an additional unit of  $y$  as they did in order to produce the previous unit. Thus, the PPF is linear up to the point  $(x', y')$  where

$$\frac{f_1(T, t_2)}{f_2(T, t_2)} = \frac{a_1}{a_2}$$

holds. For  $y > y'$  opportunity cost of increasing the household good further will increase, since only the husbands' time (which has a higher productivity in producing income) can now be used. For each additional unit of the husband's time that we use in household production, we lose  $a_2$  units of income. Since  $\frac{\partial^2 f}{\partial t_1 \partial t_2} \geq 0$ , each additional unit of household

good will require at least as much time of the husband as the previous. Opportunity cost of the household good increases. Figure 1a gives the idea.

Given this shape of the production possibility frontier and free disposal, the production possibility set is compact, comprehensive and convex.

## 8.2 Proof of Observation 2

Figure 2 captures the idea of deriving the UPF. Take any pair  $(x, y) \in \partial Y$  and divide it efficiently between spouses. Since the utility function is identical for both spouses, homogenous and quasiconcave, provided  $(x, y) > 0$

$$\frac{\partial u_1 / \partial x_1}{\partial u_1 / \partial y_1} = \frac{\partial u_2 / \partial x_2}{\partial u_2 / \partial y_2}.$$

With homothetic and identical preferences, efficient division of  $(x, y)$  requires that spouses consume the two goods in the same ratio. We can write

$$\begin{aligned} u_1 &= u(x_1, y_1) = u(ax, ay) \\ u_2 &= u(x_2, y_2) = u((1-a)x, (1-a)y) \\ 1 &> a > 0. \end{aligned}$$

By homogeneity of the utility function

$$u_1 = a^r u(x, y) \tag{7}$$

$$u_2 = (1-a)^r u(x, y). \tag{8}$$

Solving for  $a$ ,

$$u(x, y)^{\frac{1}{r}} = u_1^{\frac{1}{r}} + u_2^{\frac{1}{r}}.$$

Choosing optimal  $(x, y)$  for  $u(x, y)$  is therefore all we need to determine  $\partial U$ . Let  $u(x^*, y^*) = \lambda$ . Then

$$\partial U : u_1^{\frac{1}{r}} + u_2^{\frac{1}{r}} = \lambda^{\frac{1}{r}}, \quad (9)$$

where  $(x^*, y^*)$  is to be obtained by

$$\max_{l_1, l_2} u((1 - \tau_1) w_1 l_1 + (1 - \tau_2) w_2 l_2, f((T - l_1), (T - l_2))).$$

A change in wage rates or tax rates will have the same proportional effect on  $u_1, u_2$ , and  $\lambda$ .

We have just determined the Pareto frontier of the utility possibility set. Allowing for free disposal, the utility possibility set is given by

$$U = \left\{ u \geq 0 \mid u_1^{\frac{1}{r}} + u_2^{\frac{1}{r}} \leq \lambda^{\frac{1}{r}} \right\};$$

it is compact, comprehensive and convex.

### 8.3 Proof of Observation 3

We find  $d_i$  by solving the problem

$$\max_{l_i} u((1 - \beta_i) w_i l_i, f(0, T - l_i)).$$

Suppose the single female chooses the bundle  $(x_1^d, y_1^d)$  which she produces with  $(l_1^d, t_1^d)$ . With the same division of his time the single male is able to consume more of good  $x$ , since  $(1 - \beta_1) w_1 l_1^d < (1 - \beta_2) w_2 l_1^d$ . Whichever allocation of time the single male chooses he must be better off than the female.

Let  $U(\alpha)$  denote the utility possibility set under joint taxation, and  $U(\beta)$  denote the

utility possibility set under individual taxation. By assumption,  $U(\beta) \subset U(\alpha)$ . We first show that the disagreement point lies strictly below the Pareto frontier when we have individual taxation, i.e.  $d \notin \partial U(\beta)$ . Clearly,  $d_i > 0$ , and two people when married can choose to do exactly what they have done when they are single, thus  $d \in U(\beta)$ . If  $\frac{\partial^2 f}{\partial t_1 \partial t_2} > 0$ ,

$$f(t_1^d, t_2^d) > f(t_1^d, 0) + f(0, t_2^d).$$

With the same time inputs as singles, spouses produce more of the household good,  $d \notin \partial U(\beta)$ .<sup>25</sup> It follows  $d \in U(\alpha)$  and  $d \notin \partial U(\alpha)$ .

## 8.4 Proof of Observation 4

Under Separate Accounts and with identical and homothetic preferences, spouses consume the market good and the household good in proportion to their net wage rate. That is,

$$\frac{x_1}{x_2} = \frac{(1 - \tau_1) w_1}{(1 - \tau_2) w_2}.$$

There will be a transfer of actual income from the husband to the wife if

$$x_1 > (1 - \tau_1) w_1 l_1 \wedge x_2 < (1 - \tau_2) w_2 l_2$$

or equivalently if

$$\frac{x_1}{x_2} > \frac{(1 - \tau_1) w_1 l_1}{(1 - \tau_2) w_2 l_2}.$$

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<sup>25</sup>In the special case where  $\frac{\partial^2 f}{\partial t_1 \partial t_2} = 0$ , we can apply Ricardo's argument of comparative advantage in international trade. Being single is analogous to autarky, and marriage to international trade.

Since the wife spends more time in household production than the husband,  $l_1 < l_2$  and above inequality holds.

## 8.5 Proof of Lemmas 1 and 2

1. EGAL. Whenever the UPF intersects with the  $d_2$ -axis below or at the  $45^\circ$ -line,  $u_1^* = u_2^*$ . If the UPF intersects with  $d_2$ -axis above the  $45^\circ$ -line and  $u_1^* = \bar{u}_1$ ,  $u_2^* = d_2$ .
2. ES. Obvious.
3. NBS. Let  $u_i^{ES} - d_i = A_i$ , that is  $A_1 = A_2 = A$ , and  $u_i^E - d_i = B_i$ .<sup>26</sup> Then  $B_2 < A < B_1$ . Let  $\sigma(X)$  denote the absolute value of the slope at point  $X$  on the UPF. Then  $\sigma(A) < 1$ , and  $\sigma(B) = 1$ . First, we show that the Nash Product increases by a small enough shift to the left of  $B$ ,

$$dNP = B_2 dB_1 + B_1 dB_2 > 0$$

since we have  $dB_1 + dB_2 = 0$  by  $\sigma(B) = 1$ . Second, we show that the Nash Product increases by a small enough shift to the right of  $A$ ,

$$dNP = A_2 dA_1 + A_1 dA_2 > 0$$

since  $dA_2 = -\lambda dA_1$  by  $\sigma(A) = \lambda < 1$ . Thus the NBS lies between  $A$  and  $B$  and therefore  $u_1^* < u_2^*$ ,  $u_1^* - d_1 \geq u_2^* - d_2$ .

4. KSS. First we show that the bliss-utility point  $(\bar{u}_1, \bar{u}_2)$  is above the  $45^\circ$  - line. For given  $\lambda$  and  $d$ ,  $\bar{u}_i = \left(\lambda^{\frac{1}{r}} - d_j^{\frac{1}{r}}\right)^r$ . Clearly,  $\bar{u}_i$  decreases in  $d_j$  and hence  $\bar{u}_1 < \bar{u}_2$ . Both disagreement point and bliss utility point  $\bar{u}$  lie above the  $45^\circ$  - line, therefore  $u^*$  lies

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<sup>26</sup>It is obvious that  $u = (\bar{u}_1, d_2)$  cannot maximize the Nash Product.

above the  $45^\circ$  - *line*. Next we show that  $\bar{u}_1 - d_1 > \bar{u}_2 - d_2$  which is equivalent to  $u_1^* - d_1 > u_2^* - d_2$ . Note that  $\bar{u}_1 - d_1 > \bar{u}_2 - d_2 \Leftrightarrow \bar{u}_1 + d_2 > \bar{u}_2 + d_1$ . We can write

$$\bar{u}_i + d_j = f(d_j) + d_j$$

We need

$$f(d_2) + d_2 > f(d_1) + d_1.$$

We know that  $d_2 < f(d_1)$  by Observation 3. We have to distinguish between two cases. Recall that the point on the UPF where  $u_1 = u_2$  is given by  $u^0$ . By symmetry of  $\partial U$ ,  $f(u^0) = u^0$ . The slope of  $f(u^0)$  is  $-1$ . Notice that  $f(z) + z$  is increasing in  $z$  for  $0 \leq z \leq u^0$ , because the slope at  $f(z)$  is greater than  $-1$ . But for  $u^0 \leq z \leq \lambda$ ,  $f(z) + z$  is decreasing in  $z$  because the slope at  $f(z)$  is less than  $-1$ . Case 1:  $u^0 < d_2 < f(d_1)$ . For this range of  $d_2$ ,  $f(d_2) + d_2$  reaches its lower bound at  $d_2 = f(d_1)$ . In this case

$$f(f(d_1)) + f(d_1) \geq f(d_1) + d_1.$$

By symmetry  $f(f(z)) = z$  and hence

$$d_1 + f(d_1) \geq f(d_1) + d_1.$$

Case 2:  $d_1 < d_2 < u^0$ . For this range of  $d_2$ ,  $f(d_2) + d_2$  reaches its lower bound at  $d_2 = d_1$ . In this case

$$f(d_1) + d_1 \geq f(d_1) + d_1.$$

This completes the proof.

## 8.6 Proof of Proposition 1

### 8.6.1 Nash Bargaining Solution

See Figure 8.

If the utility ratio remains the same, we are at  $\alpha u^*$  on UPF 2. If the gains ratio remains the same, we are at  $u'$  on UPF 2 where

$$\frac{u_2^* - d_2}{u_1^* - d_1} = \frac{u_2' - d_2}{u_1' - d_1}.$$

At  $u'$  a parallel line to the tangency  $\overrightarrow{AB}$  must have  $u'$  as its midpoint. But by homothetic expansion we know that the tangency at  $\alpha u^*$  has the same slope as  $\overrightarrow{AB}$  and therefore the slope of the tangency at  $u'$  – being to the southeast of  $\alpha u^*$  – must be steeper than the slope of  $\overrightarrow{AB}$ . Call the points where the tangency at  $u'$  intercepts with the  $d$ -axes  $C$  and  $D$  respectively. Then  $|Cu'| > |u'D|$ ; the NBS must lie to the northwest of  $u'$ . In point  $\alpha u^*$  the slope of the tangency is the same as in  $u^*$ . But since this point lies to the northwest of  $u'$ ,  $\alpha u^*$  cannot be the midpoint of  $\overrightarrow{EF}$ . Then  $|E\alpha u^*| < |\alpha u^*F|$ . Therefore NBS must lie between  $\alpha u^*$  and  $u'$ .

### 8.6.2 Kalai-Smorodinsky Solution

Given  $\lambda$  before tax reform, we want to know how the gains ratio changes with tax reform, i.e. as  $\lambda$  increases. We use the following equality. By KSS

$$\frac{\left(\lambda^{\frac{1}{r}} - d_1^{\frac{1}{r}}\right)^r - d_2}{\left(\lambda^{\frac{1}{r}} - d_2^{\frac{1}{r}}\right)^r - d_1} = \frac{u_2 - d_2}{u_1 - d_1}.$$

Taking elasticity instead of the derivative will not change the sign and will give us a less complicated expression.

$$\frac{\partial \ln \left( \frac{\left( \lambda^{\frac{1}{r}} - d_1^{\frac{1}{r}} \right)^r - d_2}{\left( \lambda^{\frac{1}{r}} - d_2^{\frac{1}{r}} \right)^r - d_1} \right)}{\partial \ln \lambda} = \frac{\left( \lambda^{\frac{1}{r}} - d_1^{\frac{1}{r}} \right)^{r-1} \lambda^{\frac{1}{r}}}{\left( \lambda^{\frac{1}{r}} - d_1^{\frac{1}{r}} \right)^r - d_2} - \frac{\left( \lambda^{\frac{1}{r}} - d_2^{\frac{1}{r}} \right)^{r-1} \lambda^{\frac{1}{r}}}{\left( \lambda^{\frac{1}{r}} - d_2^{\frac{1}{r}} \right)^r - d_1} \quad (10)$$

Below we show that the sign of (10) is positive. Since both denominators are positive we just need to know whether

$$\left( \left( \lambda^{\frac{1}{r}} - d_2^{\frac{1}{r}} \right)^r - d_1 \right) \left( \lambda^{\frac{1}{r}} - d_2^{\frac{1}{r}} \right)^{1-r} - \left( \left( \lambda^{\frac{1}{r}} - d_1^{\frac{1}{r}} \right)^r - d_2 \right) \left( \lambda^{\frac{1}{r}} - d_1^{\frac{1}{r}} \right)^{1-r} > 0. \quad (11)$$

First, note that for  $r \rightarrow 1$  the left hand side of (11) is zero, and for  $r \rightarrow 0$  inequality (11) holds. Inequality (11) is also satisfied for  $r = \frac{1}{2}$ . We now check if for any  $0 < r < 1$  we can sign (11). Dividing both sides by  $\lambda^{\frac{1}{r}}$  and after a change in variables, such that  $\delta_i = \left( \frac{d_i}{\lambda} \right)^{\frac{1}{r}}$ ,

$$\delta_1 - \delta_2 - \delta_1^r (1 - \delta_2)^{1-r} + \delta_2^r (1 - \delta_1)^{1-r} > 0. \quad (12)$$

In Observation 3, we have established that  $\bar{u}_i - d_i > 0$  which is equivalent to

$$(1 - \delta_2)^r - \delta_1^r > 0.$$

Let

$$\varphi = \delta_2^r (1 - \delta_1)^{1-r} - \delta_1^r (1 - \delta_2)^{1-r} - \delta_2.$$

Inequality (11) holds if and only if for  $\delta_1 < \delta_2 < 1 - \delta_1$  and  $\delta_1 < \frac{1}{2}$ ,

$$\varphi > -\delta_1.$$

Observe that at  $\delta_2 = \delta_1$  and  $\delta_2 = 1 - \delta_1$

$$\varphi = -\delta_1. \tag{13}$$

We make the following two arguments that will prove  $\varphi > -\delta_1$ .

1.  $\varphi$  is concave in  $\delta_2$  on  $[\delta_1, \frac{1}{2}]$  and
2.  $\varphi$  decreases in  $\delta_2$  on  $[\frac{1}{2}, 1 - \delta_1]$ .

Taking first and second order derivatives with respect to  $\delta_2$  yields

$$\varphi_2 = r\delta_2^{r-1}(1-\delta_1)^{1-r} + (1-r)\delta_1^r(1-\delta_2)^{-r} - 1, \tag{14}$$

$$\varphi_{22} = r(1-r)(\delta_1^r(1-\delta_2)^{-r-1} - \delta_2^{r-2}(1-\delta_1)^{1-r}). \tag{15}$$

Observe that  $\varphi_{22}$  is increasing in  $\delta_1$ . Therefore we just have to check whether for fixed  $\delta_2$ ,  $\varphi_{22} \leq 0$  when  $\delta_1$  is largest. This is the case for  $\delta_1 = \delta_2 \leq \frac{1}{2}$ .

For argument 2, we show first that  $\varphi_2$  is increasing in  $\delta_1$  for  $\frac{1}{2} < \delta_2 \leq 1 - \delta_1$ . For  $\delta_2 \in [\frac{1}{2}, 1 - \delta_1]$  it follows that

$$\delta_1 < 1 - \delta_2 < \delta_2. \tag{16}$$

Taking the derivative of  $\varphi_2$  with respect to  $\delta_1$ ,

$$\frac{\partial \varphi_2}{\partial \delta_1} = \varphi_{21} = r(1-r) \left[ \frac{1}{1-\delta_2} \left( \frac{1-\delta_2}{\delta_1} \right)^{1-r} - \frac{1}{\delta_2} \left( \frac{\delta_2}{1-\delta_1} \right)^r \right]. \tag{17}$$

The first expression in the square bracket of (17) is larger than the second term by constraint (16). Hence  $\varphi_{21} > 0$ . To complete the argument we need  $\varphi_2 \leq 0$  at the largest possible value of  $\delta_1$ . At  $\delta_1 = 1 - \delta_2$ , in (14) we get  $\varphi_2 = 0$ .

We have thus shown that the husband receives proportionally more gains from cooperation as the UPF expands. Since  $d$  does not change, this implies  $u_2^* < u_2^{**}$ . Next we show that the wife's utility increases with an expansion of the UPF as well. In order to evaluate the change in  $u_1$ , we solve a system of two equations

$$\begin{aligned} \frac{\left(\lambda^{\frac{1}{r}} - d_2^{\frac{1}{r}}\right)^r - d_1}{\left(\lambda^{\frac{1}{r}} - d_1^{\frac{1}{r}}\right)^r - d_2} &= \frac{u_1 - d_1}{u_2 - d_2} \\ u_1^{\frac{1}{r}} + u_2^{\frac{1}{r}} - \lambda^{\frac{1}{r}} &= 0. \end{aligned}$$

We change the variables to  $\lambda^{\frac{1}{r}} = a$ ,  $d_i^{\frac{1}{r}} = \varepsilon_i$ ,  $u_i^{\frac{1}{r}} = v_i$ .<sup>27</sup> Then we can write above system of equations as

$$\begin{aligned} ((a - \varepsilon_2)^r - \varepsilon_1^r)(v_2^r - \varepsilon_2^r) - ((a - \varepsilon_1)^r - \varepsilon_2^r)(v_1^r - \varepsilon_1^r) &= 0 \\ v_1 + v_2 - a &= 0 \end{aligned}$$

Totally differentiating yields

$$\begin{aligned} V_1 \frac{dv_1}{da} + V_2 \frac{dv_2}{da} &= -A \\ \frac{dv_1}{da} + \frac{dv_2}{da} &= 1 \end{aligned}$$

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<sup>27</sup>Since  $a$  and  $v_i$  are positive monotonic transformation of  $\lambda$  and  $u_i$ , the sign of the change in  $u_1$  when  $\lambda$  increases will be the same as the change in  $v_1$  as  $a$  increases.

where

$$\begin{aligned}
V_1 &= -rv_1^{r-1}((a - \varepsilon_1)^r - \varepsilon_2^r) < 0, \\
V_2 &= rv_2^{r-1}((a - \varepsilon_2)^r - \varepsilon_1^r) > 0, \\
-A &= -r[(a - \varepsilon_2)^{r-1}(v_2^r - \varepsilon_2^r) - (a - \varepsilon_1)^{r-1}(v_1^r - \varepsilon_1^r)].
\end{aligned}$$

By Cramer's rule we find

$$\widehat{v}_1 = \frac{-A - V_2}{V_1 - V_2} > 0 \text{ if } -A < V_2. \quad (18)$$

We divide  $V_1, V_2$  and  $-A$  by  $a^{2r-1}$  and get

$$\begin{aligned}
\frac{V_1}{a^{2r-1}} &= \Psi_1 = -rz_1^{r-1}((1 - \delta_1)^r - \delta_2^r) < 0, \\
\frac{V_2}{a^{2r-1}} &= \Psi_2 = rz_2^{r-1}((1 - \delta_2)^r - \delta_1^r) > 0, \\
-\frac{A}{a^{2r-1}} &= \Lambda = -r[(1 - \delta_2)^{r-1}(z_2^r - \delta_2^r) - (1 - \delta_1)^{r-1}(z_1^r - \delta_1^r)].
\end{aligned}$$

where  $\delta_i = \left(\frac{d_i}{\lambda}\right)^{\frac{1}{r}}$  and  $z_i = \left(\frac{u_i}{\lambda}\right)^{\frac{1}{r}}$ . The sign of  $\Lambda$  is the same as the sign of the change in the gains ratio and thus  $\Lambda > 0$ . Inequality (18) holds if and only if

$$\frac{z_1^r - \delta_1^r}{(1 - \delta_1)^{1-r}} - \frac{z_2^r - \delta_2^r}{(1 - \delta_2)^{1-r}} < \left(\frac{(1 - \delta_2)^r - \delta_1^r}{z_2^{1-r}}\right).$$

Note that  $\left(\frac{(1 - \delta_2)^r - \delta_1^r}{z_2^{1-r}}\right) > \frac{z_1^r - \delta_1^r}{(1 - \delta_1)^{1-r}}$  because  $(1 - \delta_2) > z_1$  and  $(1 - \delta_1) > z_2$ . Thus we have shown that  $\Lambda < \Psi_2$  and therefore  $\widehat{v}_1 > 0$ . As the UPF shifts outwards, the utility assigned to the wife by KSS increases. The homothetic solidarity property is satisfied.

## 8.7 Proof of Proposition 2

From Observation 1 we know that each unit of the household good is produced with the same amount of each spouse's time inputs  $(\tilde{t}_1, \tilde{t}_2)$  as long as  $y \leq y'$ . Then the shadow price of the household good  $p_y$  must satisfy

$$p_y = \sum_{i=1}^2 (1 - \tau_i) w_i \tilde{t}_i.$$

We can now rewrite the household budget constraint in terms of family full income ( $I$ )

$$x + p_y y = \sum (1 - \tau_i) w_i T = I,$$

where the left hand side is virtual household expenditures and the right hand side denotes family full income. By Shepard's lemma taking the derivative of the cost function (given by  $p_y y$ ) with respect to the price of an input factor yields the demand for the input factor and hence

$$\frac{\partial p_y}{\partial (1 - \tau_i) w_i} = \frac{t_i}{y} = \tilde{t}_i.^{28}$$

Indirect utility for each spouse is given by  $v(p_y, I_i)$ . We can write the individual income share under Separate Accounts as

$$I_i = (1 - \beta_i) w_i T.$$

Taking the derivative of the indirect utilities with respect to each spouse's net-wage rate yields

$$\begin{aligned}\frac{\partial v_i}{\partial (1 - \tau_i) w_i} &= -\lambda_i y_i \frac{\partial p_y}{\partial (1 - \tau_i) w_i} + \lambda_i T \\ \frac{\partial v_i}{\partial (1 - \tau_j) w_j} &= -\lambda_i y_i \frac{\partial p_y}{\partial (1 - \tau_j) w_j},\end{aligned}$$

where  $\lambda_i$  here denotes the Lagrangian Multiplier, i.e. the marginal utility of the income share.<sup>29</sup> We evaluate an infinitesimal change in the tax rates such that  $d(1 - \beta_1) < 0 < d(1 - \beta_2)$ .<sup>30</sup>

$$\begin{aligned}dv_1 &= \lambda_1 \left( w_1 (y - y_1) \frac{t_1}{y} d(1 - \beta_1) - w_2 y_1 \frac{t_2}{y} d(1 - \beta_2) + w_1 l_1 d(1 - \beta_1) \right) < 0 \\ dv_2 &= \lambda_2 \left( -w_1 y_2 \frac{t_1}{y} d(1 - \beta_1) + w_2 (y - y_2) \frac{t_2}{y} d(1 - \beta_2) + w_2 l_2 d(1 - \beta_2) \right) > 0\end{aligned}$$

A further increase in the wife's tax rate and a further decrease in the husband's would again have the same implications as above. Thus we find that the wife is worse off and the husband better with tax reform from individual to joint taxation. Separate Accounts does not satisfy the homothetic solidarity property.

Since our sharing rule always guarantees the husband a bigger share of resources than the wife, utility shares between wife and husband are more equal under individual taxation.

## 9 References

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<sup>29</sup>By Roy's Identity  $\frac{\partial v_i}{\partial p_y} = -\lambda_i y_i$ ,  $\frac{\partial v_i}{\partial I_i} = \lambda_i$ .

<sup>30</sup>We follow Apps and Rees (1999a and b) who use the same method in analyzing the efficiency of tax reform.

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# 10 Figures

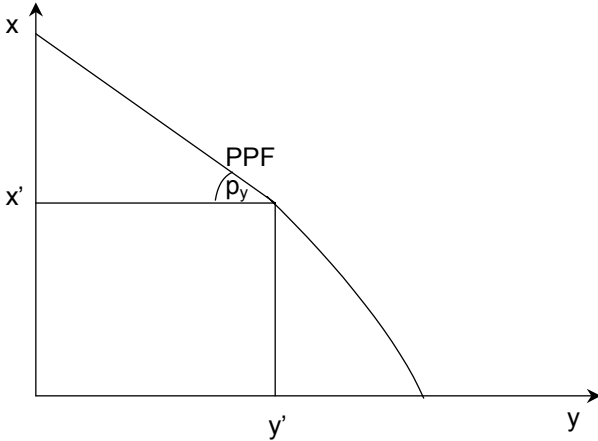


Figure 1a: PPF when inputs are imperfect substitutes.

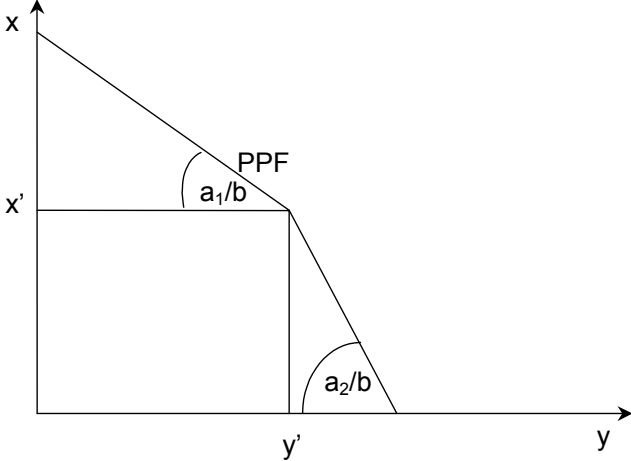


Figure 1b: PPF when inputs are perfect substitutes.

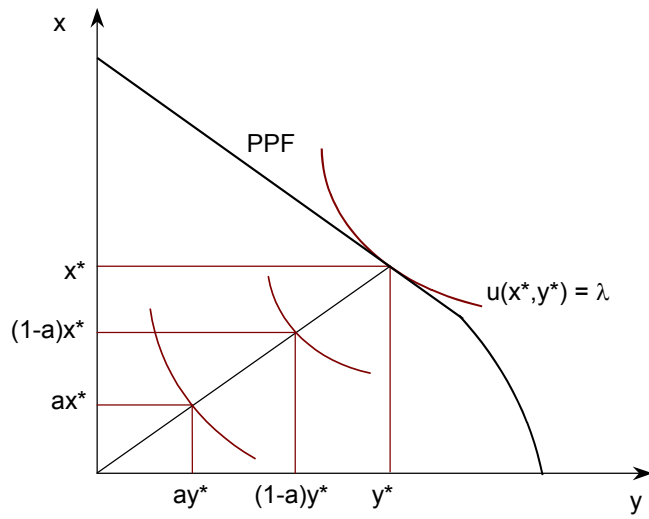


Figure 2: Deriving UPF.

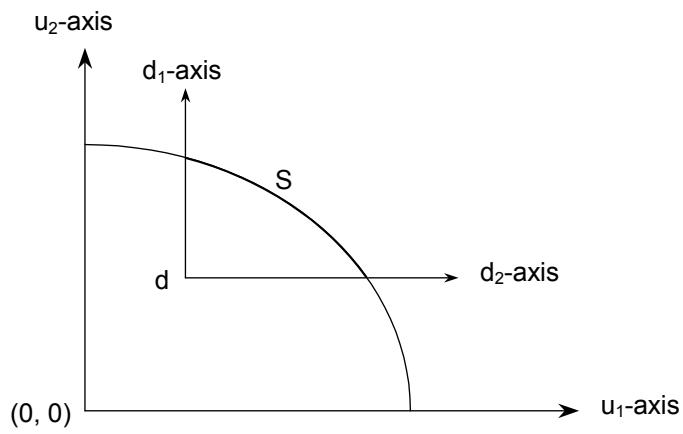


Figure 3: Notation.

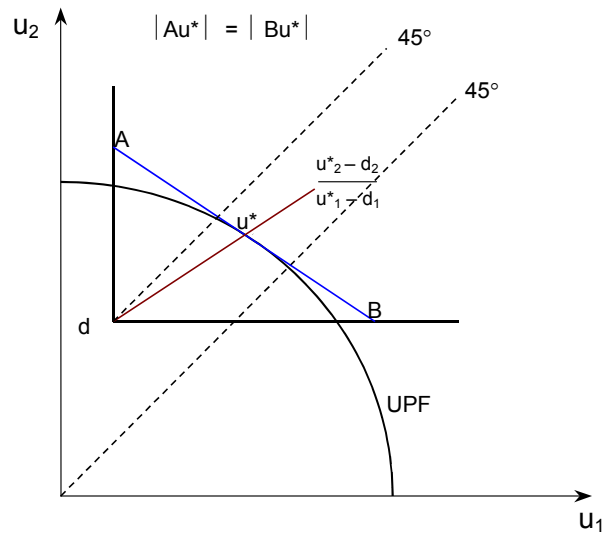


Figure 4: NBS.

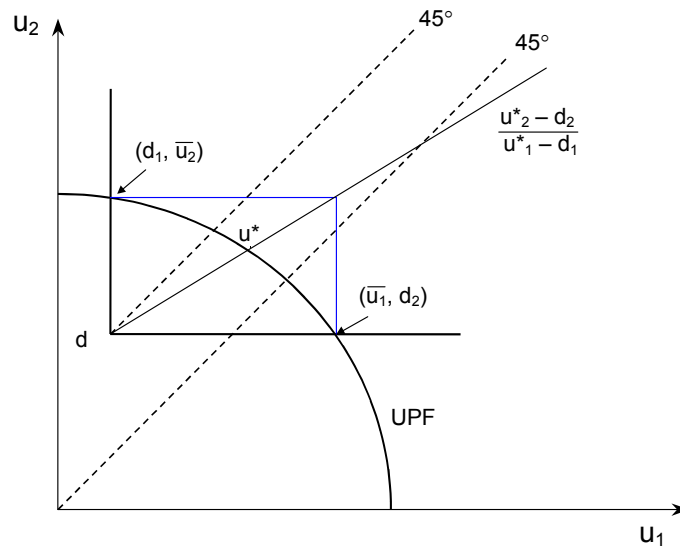


Figure 5: KSS.

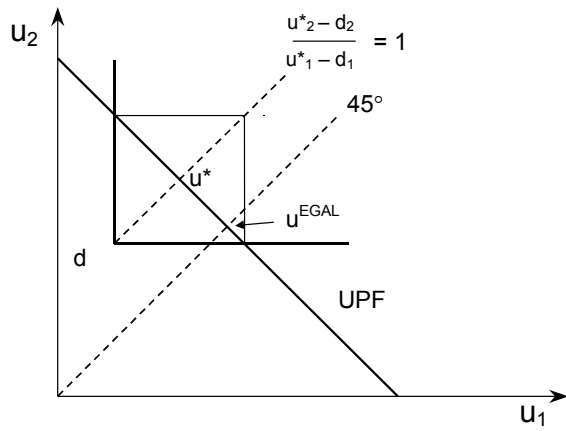


Figure 6:  $r = 1$ .

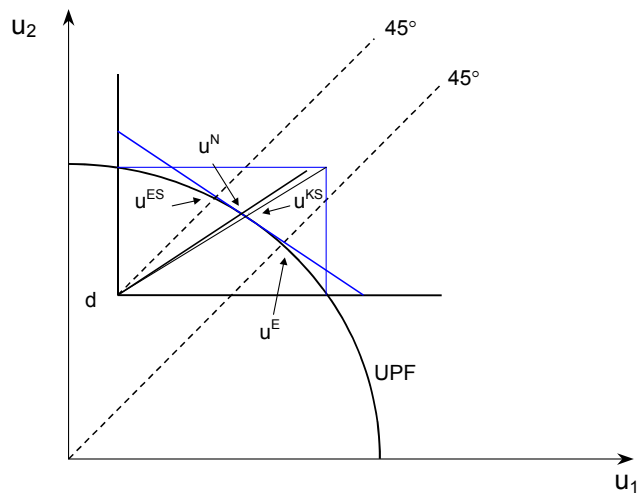


Figure 7: NBS and KSS lie between ES and EGAL.

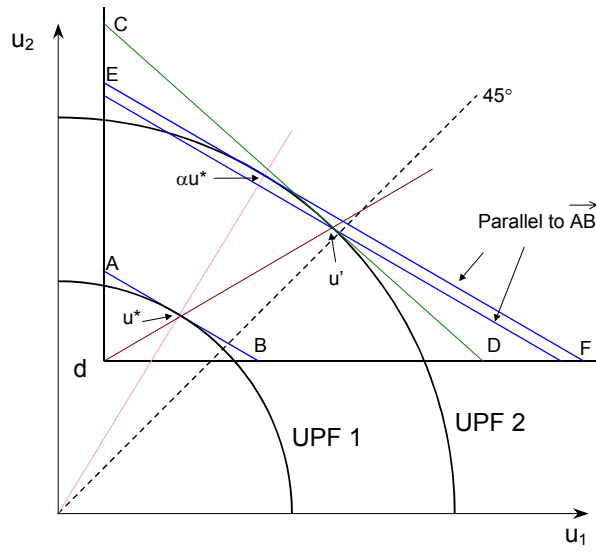


Figure 8: Both spouses gain under NBS.