SOLUTION

[2] 1. Check the appropriate circle to indicate whether each statement is true or false. You do not need to provide any reasoning.

	True	**False**	
(a)	⌀	○	If $(p \wedge q) \to r$ is false, then q is true.
(b)	⌀	○	If $\neg(p \to q)$ is true, then $q \to p$ is true.
(c)	○	⌀	It is possible for a statement and its negation to both be true.
(d)	⌀	○	The sentence "UVic is on an island." is a statement.

[3] 2. Use the laws of logic and known logical equivalences to show that $(p \vee q) \wedge \neg(p \wedge \neg q)$ is logically equivalent to q.

$$(p \vee q) \wedge \neg(p \wedge \neg q) \Leftrightarrow (p \vee q) \wedge (\neg p \vee q) \quad \text{(De Morgan, double neg)}$$
$$\Leftrightarrow (p \wedge \neg p) \vee q \quad \text{(distributivity)}$$
$$\Leftrightarrow 0 \vee q \quad \text{(known LE)}$$
$$\Leftrightarrow q \quad \text{(identity)}$$

$p \to (q \wedge r)$

[3] 3. Consider the statement "If it is snowing, then it is cold outside and it is cloudy." Write the contrapositive, converse, and negation of this statement as sentences in straightforward English. You do not need to provide any reasoning.

(a) Contrapositive: $\neg(q \wedge r) \to \neg p \Leftrightarrow (\neg q \vee \neg r) \to \neg p$

If it is not cold or not cloudy, then it is not snowing.

(b) Converse: $(q \vee r) \to p$

If it is cold and cloudy, then it is snowing.

(c) Negation: $\neg[p \to (q \wedge r)] \Leftrightarrow \neg[\neg p \vee (q \wedge r)] \Leftrightarrow p \wedge (\neg q \vee \neg r)$

stated in a nice way:

It is not cold or not cloudy, but it is snowing.

[2] 4. Give a counterexample to show that the following argument is invalid. Briefly explain your reasoning.

$$\begin{array}{r} \neg p \to q \\ q \vee r \\ \hline \therefore r \to \neg q \end{array}$$

both $p=0$, $q=1$, $r=1$

and $p=1$, $q=1$, $r=1$

make the premises true and the conclusion false, making the argument invalid.

(you only needed to give one counterexample)

[3] 5. Use rules of inference and known logical equivalences to show the argument below is valid.

$$\begin{array}{r} \neg q \to r \\ p \to \neg q \\ \neg r \\ \hline \therefore\ \neg p \end{array}$$

Lots of possible paths! Here is one:

$$\begin{array}{r} \left.\begin{array}{r} \neg q \to r \\ p \to \neg q \end{array}\right\} \\ \neg r \\ \hline \therefore \neg p \end{array} \xRightarrow{\text{chain rule}} \begin{array}{r} \left.\begin{array}{r} p \to r \\ \neg r \end{array}\right\} \\ \hline \therefore \neg p \end{array} \xRightarrow{\text{modus tollens}} \begin{array}{r} \neg p \\ \hline \therefore \neg p \end{array}$$

so, valid!

[2] 6. Check the appropriate circle to indicate whether each statement is true or false. You do not need to provide any reasoning.

	True	False	
(a)	○	⌀	The law of distributivity is one of the rules of inference.
(b)	○	⌀	If $s_1 \Rightarrow s_2$ then the truth table columns for s_1 and s_2 are the same.
(c)	○	⌀	If a premise in a logical argument is false, then the argument is invalid.
(d)	⌀	○	$(p \vee q) \wedge r$ is logically equivalent to $r \wedge (q \vee p)$.