SOLUTION

[2] 1. Check the appropriate circle to indicate whether each statement is true or false. You do not need to provide any reasoning.

	True	False	
(a)	○	⌀	The negation of $\exists\, x, p(x)$ is $\forall\, x, p(x)$.
(b)	⌀	○	If $\forall\, x, p(x)$ and $\exists\, x, q(x)$ are both true, then $\exists\, x, p(x) \wedge q(x)$ is true.
(c)	⌀	○	For the universe of integers, $\exists\, x, (x > 5) \rightarrow (x < 2)$.
(d)	○	⌀	If $\forall\ x, p(x) \vee q(x)$ is true, then $[\forall x, p(x)] \vee [\forall x, q(x)]$ is true.

[3] 2. Let the universe be $\mathbb{Z}$, and let $p(x)$ be the statement "x is divisible by 5". Consider the statement $\forall\, m, \exists\, n, p(m+n) \wedge \neg p(mn)$. Write out this statement in words, and determine its truth value.

For every integer m, there exists an integer n such that m+n is divisible by 5 and mn is not divisible by 5.

The statement is <u>false</u>. Let m=5, then there is no integer n for which mn is not div. by 5.

[3] 3. Let n be an integer. Prove that if n is odd, then $n^2 + 5$ is even.

Suppose n is odd. Then there exists $k \in \mathbb{Z}$ such that $n = 2k+1$. Then

$$\begin{aligned} n^2+5 &= (2k+1)^2 + 5 \\ &= (4k^2+4k+1)+5 \\ &= 4k^2+4k+6 \\ &= 2(2k^2+2k+3) \end{aligned}$$

Since $2k^2+2k+3 \in \mathbb{Z}$, then n^2+5 is even

[3] 4. Let A, B, and C be sets. Using the laws of logic, known logical equivalences, and the definition of set operations, prove that $x \in (A \cap B) \cap C$ is logically equivalent to $x \in A \cap (B \cap C)$

$x \in (A \cap B) \cap C \iff (x \in A \cap B) \wedge (x \in C)$ defn of $\cap$

$\iff [(x \in A) \wedge (x \in B)] \wedge (x \in C)$ defn of $\cap$

$\iff (x \in A) \wedge [(x \in B) \wedge (x \in C)]$ associativity

$\iff (x \in A) \wedge (x \in B \cap C)$ defn of $\cap$

$\iff x \in A \cap (B \cap C)$ defn of $\cap$

[2] 5. Let A and B be sets. Prove that $A \setminus B \subseteq (A \cap B)^c$ using an argument that starts with "Let $x \in A \setminus B$...".

Let $x \in A \setminus B$. Then $x \in A$ and $x \notin B$. Since $x \notin B$, then $x \notin A \cap B$. Therefore $x \in (A \cap B)^c$, and so $A \setminus B \subseteq (A \cap B)^c$.

Another way...

Let $x \in A \setminus B$. Then $x \in A$ and $x \notin B$, so $x \in B^c$. Then by the defn of union $x \in A^c \cup B^c = (A \cap B)^c$. Therefore, $A \setminus B \subseteq (A \cap B)^c$.

[2] 6. Check the appropriate circle to indicate whether each statement is true or false. You do not need to provide any reasoning. Let A and B be sets.

	True	False	
(a)	⊘	○	If $x \in B$, then $\{x\} \in \mathcal{P}(B)$.
(b)	○	⊘	The sets $A \setminus B$ and $B \setminus A$ are never equal.
(c)	○	⊘	If $x \in A$ and $x \in B$, then $A \subseteq B$.
(d)	○	⊘	For any sets A and B, $B \subsetneq A \cup B$.