MATH 122 (A02) **Quiz 3**
Date: Mar. 6, 2025 **Time Limit: 25 minutes**

This quiz contains 4 pages (including this one, and a scrap page) and 5 questions.

Before starting your quiz, enter your name and student ID (clearly) on this page.

You may not open the quiz until instructed to do so. In the meantime, please read the following instructions:

- This quiz is closed-book. You are permitted a Sharp EL-510R series calculator (which you potentially may find useful).
- Please organize your work reasonably neatly in the space provided. Work that is scattered and difficult to follow, without clear ordering, will receive little credit.
- Unless otherwise specified, in order to receive full credit for a correct solution you must show your work. Unsupported answers will not receive full credit, even if they are correct.

Question:	1	2	3	4	5	Total
Points:	2	2	3	3	5	15

[2] 1. Check the appropriate circle to indicate whether each statement is true or false. You do not need to provide any reasoning.

	True	**False**	
(a)	◯	◯	If the set S contains 6 elements, then S has $2^6 = 64$ proper subsets.
(b)	◯	◯	Let A and B be sets. If $A \cap B = \varnothing$, then $\lvert A \cup B \rvert = \lvert A \rvert + \lvert B \rvert$.
(c)	◯	◯	In the inductive hypothesis to prove a statement $S(n)$ using weak induction, we assume $S(k)$ holds for all k.
(d)	◯	◯	If a sequence is recursively defined by $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$ where $a_1 = 2$ and $a_2 = -1$, then $a_4 = 0$.

[2] 2. In a group of 130 people surveyed, 47 have a cat, 65 have a dog, and 32 have both a cat and a dog. How many people have neither a cat nor a dog?

[3] 3. Consider the recursive sequence $a_n = a_{n-1} + 4n$ for $n \geq 1$ with $a_0 = 7$.

(a) Find a_1, a_2, a_3, and a_4. You may leave your answers unsimplified.

(b) Use your work in (a) to conjecture (guess) a formula for a_n that depends only on n, and is valid for all $n \geq 0$. Your answer should not involve a sum of many terms. You do not need to prove that your conjecture is correct.

[3] 4. Let A and B be sets. Using any method, prove that $(A \cap B^c) \cup B = A \cup B$.
Hint: There is a short proof using the laws of set theory.

[5] 5. Use induction to prove that $1 + 5 + 5^2 + 5^3 + \cdots + 5^n = \frac{1}{4}(5^{n+1} - 1)$ for all integers $n \geq 0$.

This page is for rough work only, and will not be graded.
If you have work on this page you want marked,
please indicate this next to the original question.